



# Reinforced Concrete Design II

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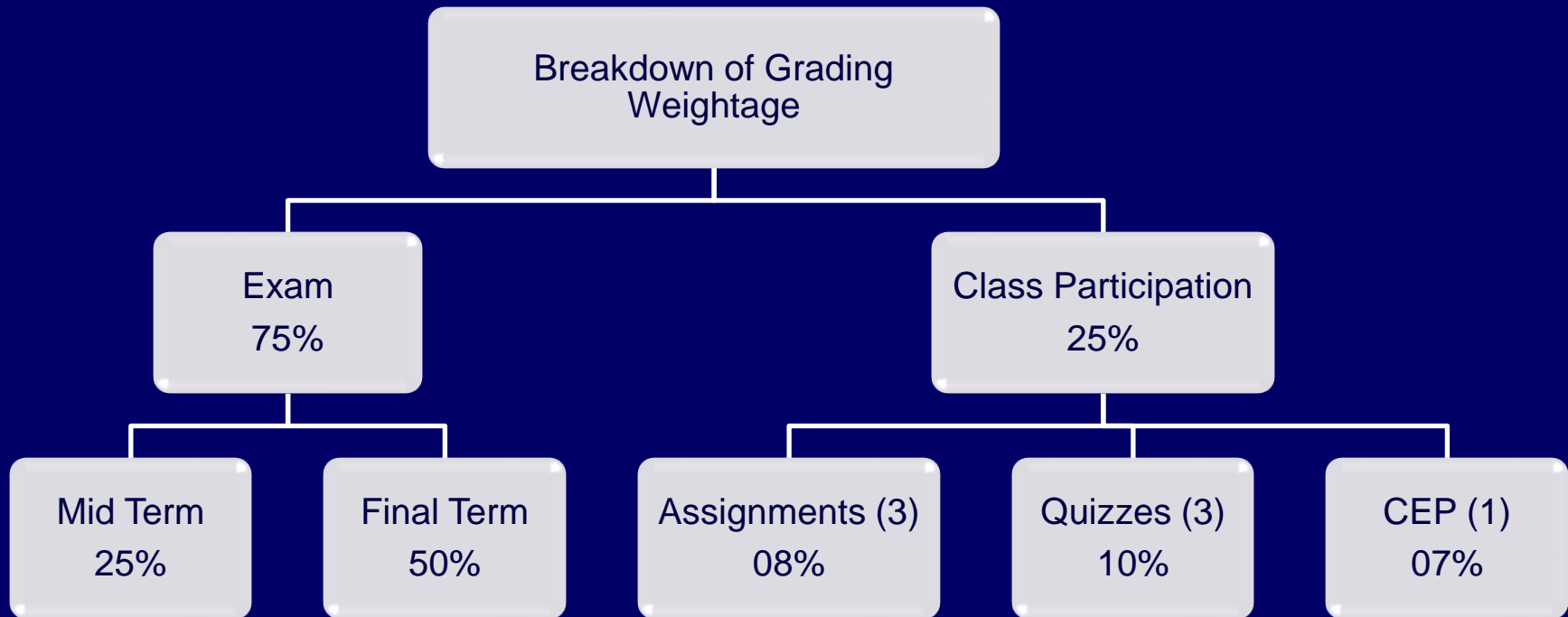


# Course Contents

## OBE Course Contents Fall 2022



# Grading Policy



## Assignments & Quizzes Schedule



# Lecture Availability

- Previous version of lectures are available on the website.

[www.drqaisarali.com/lectures](http://www.drqaisarali.com/lectures)

- Updated lectures upon completion will be uploaded on website as well as on Google Classroom.



# Lecture 01

## Introduction

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# Learning Outcomes

- **At the end of this lecture, students will be able to;**
  - *Explain* Difference between Demand and Capacity
  - *Compare* Working stress method with Strength Design method
  - *Analyze* and Design Beams for flexure and shear using ACI Recommendations



# Lecture Contents

- Concept of Demand and Capacity
- A Glimpse of RCD – I
  - Load Combinations and Strength Reduction Factors
  - ACI Code Provisions related to Flexural Design of beams
  - ACI Code Provisions related to Shear Design of beams
- Flexural and Shear Design of Beam (Example)
- References
- Appendix



# Concept of Demand and Capacity

- **Demand**

- Demand on a structure refers to all external actions.
- Gravity, wind, earthquake, snow are external actions.
- These actions when act on the structure will induce internal disturbance(s) in the structure in the form of stresses (such as compression, tension, bending, shear and torsion).
- The internal stresses are also called **Load effects**.





# Concept of Demand and Capacity

- **Capacity**

- Capacity refers to the **overall ability** of a structure to carry an imposed demand.

Beam resists the applied load up to its capacity.



Beam fails when demand exceeds the capacity.





# Concept of Demand and Capacity

- **Failure**

- Failure occurs when Capacity is less than Demand.
- To avoid failure, capacity to demand ratio should be kept greater than one, or at least equal to one.
- It is, however, intuitive to have some margin of safety i.e., to have capacity to demand ratio more than one. How much?



# Concept of Demand and Capacity

- **Failure**
  - An Experimental Test on Beam's Capacity under Point load

*Depth = 12 in*  
*Width = 8 in*  
*Length = 7 ft*  
*Bars = 4, #4*  
*Stirrup Spacing = 5 in c/c*  
*Calculated Load Capacity = 90 KN*



# Concept of Demand and Capacity

- Failure



Failure (Capacity < Demand)



# Concept of Demand and Capacity

- **About Ton**

- **Ton** is the name which basically describes the unit of weight. Different types of Ton are tabulated below.

| S.No. | Name                       | Quantity                   |
|-------|----------------------------|----------------------------|
| 1     | <b>Long Ton (US)</b>       | 2240 pounds                |
| 2     | <b>Short Ton (US)</b>      | 2000 pounds                |
| 3     | <b>Tonne or Metric Ton</b> | 2204.6226 pounds Or 1000kg |

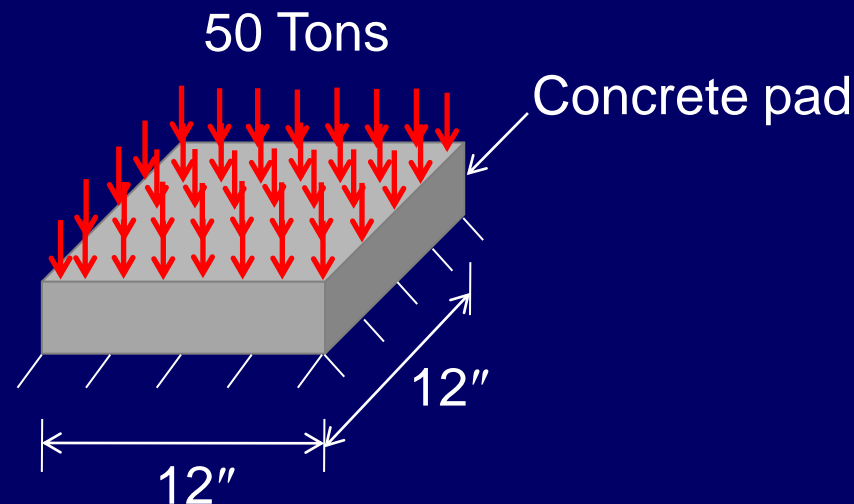
- In Pakistan, the use of metric ton is very common; therefore; we will refer to Metric Ton in our discussion.



# Concept of Demand and Capacity

- **Example 1.1**

- Calculate demand in the form of stresses or load effects on the given concrete pad of size 12" × 12".





# Concept of Demand and Capacity

- **Solution**

- **Given Data**

- Load,  $P = 50 \text{ Tones} = 50 \times 2204 = 110200 \text{ lb.}$
- Area of concrete Pad,  $A = 12 \times 12 = 144 \text{ in}^2$

- **Required Data**

- Calculate the Capacity of Concrete Pad for the given Demand



# Concept of Demand and Capacity

- **Solution**

- Based on convenience either the loads or the load effects as demand are compared to the load carrying capacity of the structure in the relevant units.

As we know that

$$\text{Capacity} = \text{Strength} \times \text{Area} \Rightarrow C = \sigma_{max} \times A$$

For concrete pad;  $\sigma_{max} = f_s$

$$C = f_s \times 144$$





# Concept of Demand and Capacity

- **Solution**

- For safety, Capacity should be equal to or greater than demand.

For *Capacity = Demand*;

$$f_s \times 144 = 110200$$

$$f_s = \frac{110200}{144} = 765.28psi$$

- This means Capacity of the pad in the form of resistance should be able to carry a stress of 765.28 psi.
- In other words, the compressive strength of concrete pad (capacity) should be more than 765.28 psi (demand).

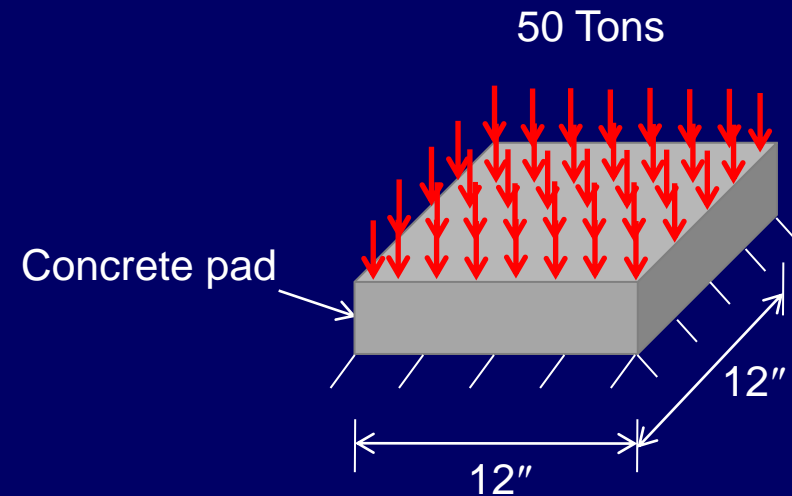


# Concept of Demand and Capacity

- **Example 1.2**

- Determine capacity to demand ratio for the pad of example 1.1 for the following capacities given in the form of compressive strength of concrete. Comment on the results

- 500 psi
- 765.28 psi
- 1000 psi
- 2000 psi.





# Concept of Demand and Capacity

- Solution**

| Part | Capacity/Demand       | Remarks                                |
|------|-----------------------|--|
| i    | $500/765.28 = 0.7$    | Capacity is less than Demand           |
| ii   | $765.28/765.28 = 1.0$ | Capacity is just equal to Demand       |
| iii  | $1000/765.28 = 1.3$   | Capacity is 1.3 times less than Demand |
| iv   | $1200/765.28 = 1.6$   | Capacity is 1.6 times less than Demand |

- In (iii) and (iv), there is some margin of safety normally called as factor of safety.



# Concept of Demand and Capacity

- **Factor of Safety**

- The factor by which the capacity exceeds the demand is known as **Factor of Safety**.
- It is always better to have a factor of safety in our designs.
- It can be achieved easily if we fix the ratio of capacity to demand greater than 1.0, say 1.5, 2.0 or so, as shown in example 1.2.



# Concept of Demand and Capacity

- **Factor of Safety**

- For certain reasons, however, let say we insist on a factor of safety such that capacity to demand ratio remains 1.0. Then there are three ways of doing this:
  1. Take an increased demand instead of actual demand (load), e.g., 70 ton instead of 50 ton in the previous example,
  2. Take a reduced capacity instead of actual capacity such as 1500 psi for concrete whose actual strength is 3000 psi
  3. Doing both.
- How are these three situations achieved?



# Concept of Demand and Capacity

- **Factor of Safety**

- Following are the two methods of achieving appropriate factor of safety in Design.

1. **Working Stress Method**

- In the Working Stress or Allowable Stress Design method, the material strength is knowingly taken less than the actual one.

2. **Strength Design Method**

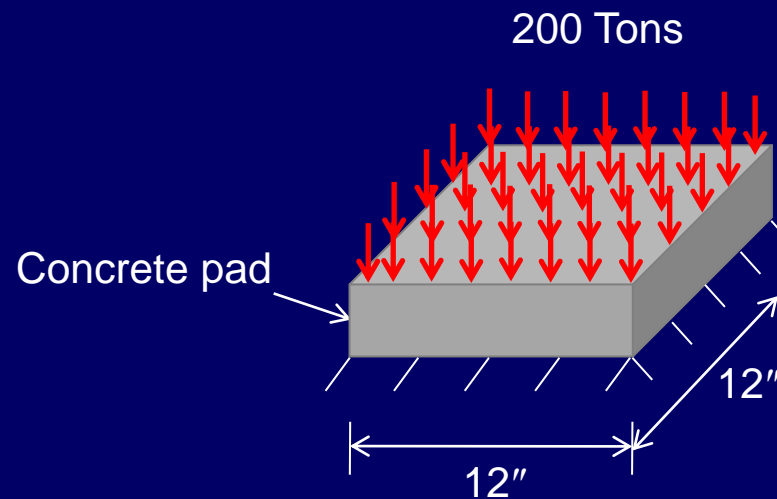
- In the Strength Design method, the increased loads and the reduced strength of the material are considered, but both based on scientific rationale.



# Concept of Demand and Capacity

- **Example 1.3**

- Design the 12" × 12" pad to carry a load of 200 tons. The area of the pad cannot be increased for some reasons. Take Concrete compressive strength,  $f_c' = 3$  ksi.





# Concept of Demand and Capacity

- **Solution**

- **Given Data**

- Load,  $P = 200$  Tones
- Area of concrete Pad,  $A = 12 \times 12 = 144 \text{ in}^2$
- Concrete Strength,  $f'_c = 3000 \text{ psi}$

- **Required Data**

- Design Concrete Pad for the given demand





# Concept of Demand and Capacity

- **Solution**

- First calculate Demand in the form of load effect

$$\sigma = \frac{P}{A} = \frac{200 \times 2204}{144} = 3061.11 \text{psi}$$

- Now, determine Factor of Safety

$$FOS = \frac{C}{D} = \frac{3000}{3061.11} = 0.98 < 1$$

**Food for Thought**

What are some possible solutions to this problem?



# Concept of Demand and Capacity

- **Solution**

- There are three possible options to resolve this problem:
  1. Increase area of the pad (geometry)
  2. Increase the strength (by using high strength concrete, steel or other material)
  3. Using combination of steel with concrete.
- The first option is not possible as the size of pad is restricted in the given example.
- Now Let us assume that we want to use steel bar reinforcement of yield strength  $f_y = 40$  ksi. Now again calculate the capacity using Working Stress Method.



# Concept of Demand and Capacity

- **Solution**

- For Demand = Capacity; and using Working Stress Method

$$P = R_c + R_s$$

$$P = \left(\frac{f'_c}{2}\right)(A_g - A_s) + \left(\frac{f_y}{2}\right)A_s$$

By Substituting the relevant values, we get

$$200 \times 2.204 = \left(\frac{3}{2}\right) \times (144 - A_s) + \left(\frac{40}{2}\right)A_s$$

Which on solving for  $A_s$  gives

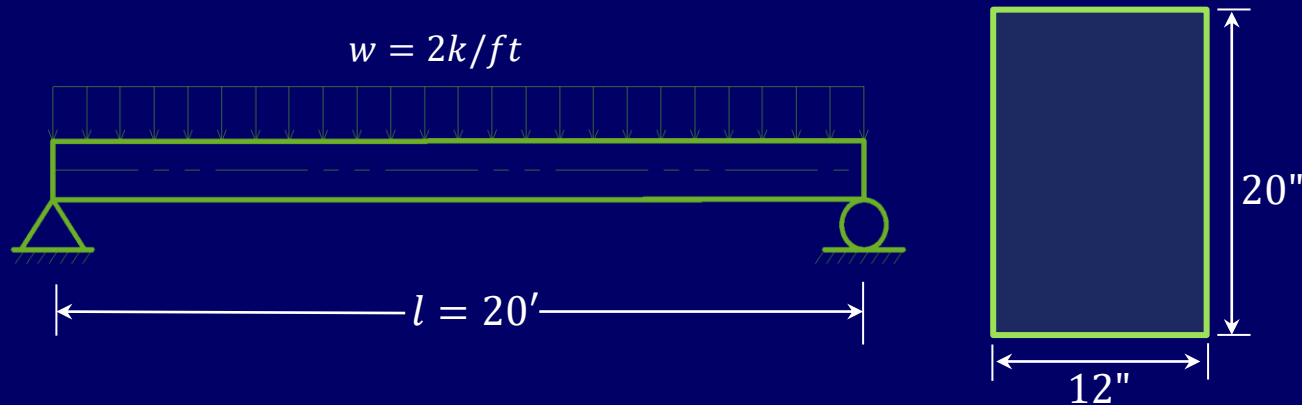
$$A_s = 12.15 \text{ in}^2$$



# Concept of Demand and Capacity

- **Example 1.4**

- Check the capacity of the plain concrete beam given in figure below against flexural stresses within the linear elastic range. Concrete compressive strength ( $f_c'$ ) = 3 ksi. Consider Self weight of the beam.





# Concept of Demand and Capacity

- **Solution**

- **Given Data**

- Load on beam (excluding self weight) ,  $W = 2 \text{ kip/ft}$
- Width,  $b = 12''$
- Depth,  $h = 20''$
- Concrete Strength,  $f'_c = 3000 \text{ psi}$

- **Required Data**

- Calculate the Capacity of the beam



# Concept of Demand and Capacity

- **Solution**

- **Step No.1: Calculate Demand Moment on Beam**

- Self weight of beam,  $SW = \frac{12 \times 20}{144} \times 0.145 = 0.242 \text{kip/ft}$

- Maximum Bending Moment at the midspan of the beam is given by;

$$M_u = \frac{W_u l^2}{8}$$

$$M_u = \frac{(2 + 0.242)20^2}{8} \times 12$$

$$M_u = 1345.2 \text{in. kip}$$



# Concept of Demand and Capacity

- **Solution**

- **Step No.2: Calculate Flexural Capacity of beam**

- In the linear elastic range, flexural stress in concrete beam can be calculated as:

$$f = \frac{Mc}{I}$$

Where;

$$I = \frac{bh^3}{12} = \frac{12 \times 20^3}{12} = 8000in^4$$

and

$$c = \frac{h}{2} = \frac{20}{2} = 10in$$



# Concept of Demand and Capacity

- **Solution**

- **Step No.2: Calculate Flexural Capacity of beam**

- As we know that for a plain concrete beam Flexural strength is equal to modulus of rupture which is given by;

$$f = f_r = 7.5\sqrt{f_c'} = 7.5\sqrt{3000} = 410.79\text{psi}$$

Now,

$$M = \frac{fI}{c} = \frac{410.79 \times 8000}{10} = 328632 \text{ lb.in or } 328.63 \text{ in.kip}$$

- Hence, Demand = 1345.2 in-kips and Capacity = 328.63 in-kips





# A Glimpse of RCD – I

- In RCD – I, you have already studied the detailed procedure of designing Beams, Slabs, Columns and Footings.
- However, for the sake of revision, we will briefly go over some of those concepts here.



# A Glimpse of RCD – I

- **Load Combinations**

| Table 5.3.1 - Load combinations   |   |
|---|---|
| Load combination  | Notations   |
| $U = 1.4D$  | D = Dead Load<br>L = Live Load<br>L <sub>r</sub> = Roof Live Load<br>R = Rain Load<br>S = Snow Load<br>W = Wind Load<br>E = Earthquake Load |
| $U = 1.2D + 1.6L + 0.5 (L_r \text{ or } S \text{ or } R)$                   |   |
| $U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$ |   |
| $U = 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$             |   |
| $U = 1.2D + 1.0E + 1.0L + 0.2S$   |   |
| $U = 0.9D + 1.0W$   |   |
| $U = 0.9D + 1.0E$   |   |



# A Glimpse of RCD – I

- **Strength Reduction Factors**

| <b>Table 21.2.1—Strength reduction factors</b> |   |                          |
|--|---|--------------------------|
| <b>S.No.</b>                                   | <b>Action or structural element</b>           | <b><math>\phi</math></b> |
| 1  | Tension controlled regions (Moment)           | 0.90                     |
| 2  | Compression controlled regions (Axial force ) | 0.65                     |
| 3  | Shear   | 0.75                     |
| 4  | Torsion                                       | 0.75                     |



# A Glimpse of RCD – I

- **Flexural Design of Beam**

- The **Design Flexural Capacity** of an RC beam is given by

$$\phi M_n = \phi A_s f_y \left( d - \frac{a}{2} \right)$$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

Where  $A_s$  which is tensile reinforcement, is subjected to the following restrictions.

$$A_{s,min} < A_s < A_{s,max}$$



# A Glimpse of RCD – I

- **Flexural Design of Beam**
  - **Minimum Reinforcement Limit**

$$A_{s,min} = \frac{3\sqrt{f'_c}}{f_y} bd \quad \text{or} \quad \frac{200}{f_y} bd \quad \text{which ever is greater}$$

- **Maximum Reinforcement Limit**

$$A_{s,max,40} = \frac{f'_c}{136} bd$$

$$A_{s,max,60} = \frac{f'_c}{223} bd$$

## Food for Thought

Why have these restrictions been imposed on the steel area ?



# A Glimpse of RCD – I

- **Flexural Design of Beam**

- **Reinforcement ratio**

- The table below provides Minimum and maximum reinforcement ratios for various values of  $f_c'$  and  $f_y$ .

| <b>Minimum and Maximum reinforcement ratios</b> |        |        |        |        |
|---|--------|--------|--------|--------|
| $f_c'$ (psi)                                    | 3000   |        | 4000   |        |
| $f_y$ (psi)                                     | 40,000 | 60,000 | 40,000 | 60,000 |
| $\rho_{min}$                                    | 0.005  | 0.0033 | 0.005  | 0.0033 |
| $\rho_{max}$                                    | 0.0221 | 0.0135 | 0.0294 | 0.0181 |



# A Glimpse of RCD – I

- **Shear Design of Beam**

Design Shear Capacity of beam is given by

$$\phi V_n = \phi V_c + \phi V_s$$

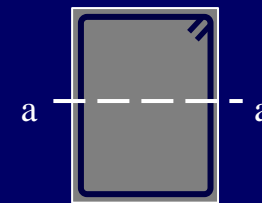
Where;

$$\phi V_c = 2\phi\sqrt{f_c'}bd$$

and

$$\phi V_s = \frac{\phi A_v f_y d}{s_d}$$

$A_v$  is the cross-sectional area of web reinforcement within a distance “s”, for single loop stirrups (2 legged),  $A_v = 2A_s$   
 $A_s$  = cross sectional area of the stirrup bar



At section a-a, if #3 bar is used  $A_s = 0.11 \text{ in}^2$ ,  
 $A_v = 2 \times 0.11 = 0.22 \text{ in}^2$



# A Glimpse of RCD – I

## • Shear Design of Beam

- The beam is designed for the ultimate shear force  $V_u$  at critical location (which in most cases is at a distance “d” from the face of support.)
1. When  $\phi V_c / 2 > V_u$ , no web reinforcement is required.
  2. When  $\phi V_c \geq V_u$  but  $\phi V_c / 2 < V_u$ , theoretically no web reinforcement is required. However, minimum web reinforcement in the form of maximum spacing  $S_{max}$  shall be provided:

$$S_{max} = \text{minimum of } \left\{ \frac{A_v f_y}{50 b_w}, \frac{A_v f_y}{0.75 \sqrt{f_c'} b_w}, \frac{d}{2}, 24'' \right\}$$





# A Glimpse of RCD – I

- **Shear Design of Beam**

3. When  $\phi V_c < V_u$ , web reinforcement is required.

- In this case, the required spacing  $s_d$  can be calculated using

$$s_d = \frac{\phi A_v f_y d}{V_u - \phi V_c}$$

- If  $s_d$  is greater than  $s_{max}$ , use  $s_{max}$



# A Glimpse of RCD – I

- **Shear Design of Beam**

- **Necessary Checks**

1. **Check for Depth of Beam:**

$\phi V_s \leq \phi 8 \sqrt{f'_c} b_w d \rightarrow$  Depth of beam is OK!, otherwise increase depth

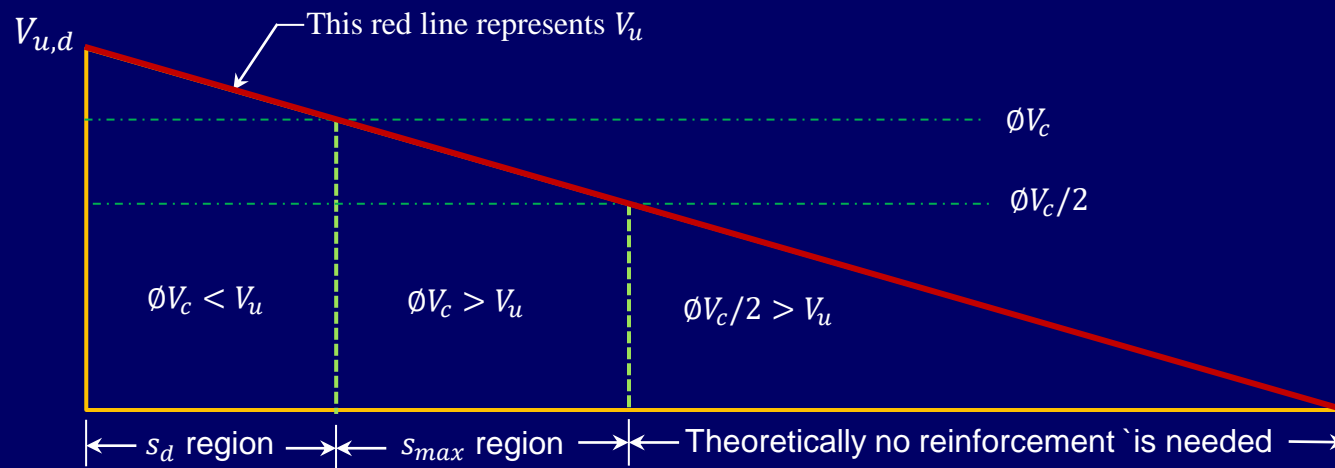
2. **Check for maximum Spacing of stirrups:**

$\phi V_s \leq \phi 4 \sqrt{f'_c} b_w d \rightarrow S_{max}$  is OK!, otherwise divide  $S_{max}$  by 2



# A Glimpse of RCD – I

## • Shear Design of Beam



- For  $\phi V_c < V_u \rightarrow$  use  $s_d$
- For  $\phi V_c > V_u$ , use  $s_{max}$
- For  $\phi V_c/2 > V_u$ , no reinforcement is required

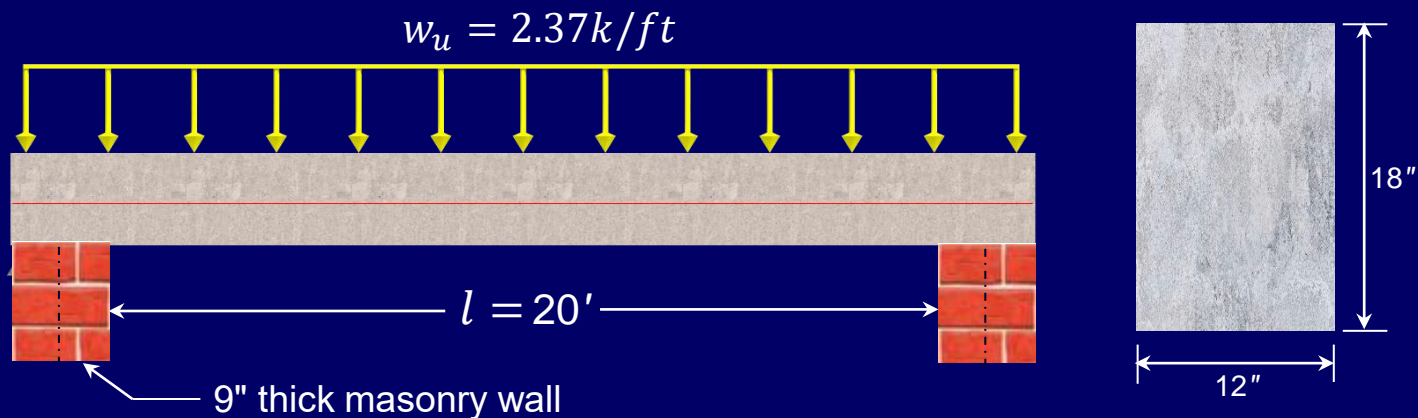


# Flexural and Shear Design of RC Beam

## • Example 1.5

- A simply supported beam with given cross sectional dimensions is subjected to a uniformly distributed factored load of 2.37 kip/ft as shown in the figure below.

*Analyze* and *Design* the beam for **flexure and shear** in accordance with ACI 318-19. Take  $f'_c = 3\text{ksi}$  and  $f_y = 40\text{ksi}$





# Flexural and Shear Design of RC Beam

- **Solution**

- **Step No.1: Selection of sizes**

Cross sectional dimensions are already given

$$b_w = 12''$$

$$h = 18''$$

Assuming  $\bar{y} = 2.5''$

$$d = 18 - 2.5 = 15.5''$$

- **Step No.2: Calculation of loads**

$$w_u = 2.37k/ft$$



# Flexural and Shear Design of RC Beam

## • Solution

### ➤ Step No.3: Analysis

#### 1. Analysis for Flexure

$$M_u = \frac{w_u l^2}{8} = \frac{2.37(20.75)^2}{8} \times 12 = 1530.65 \text{ in. kip}$$

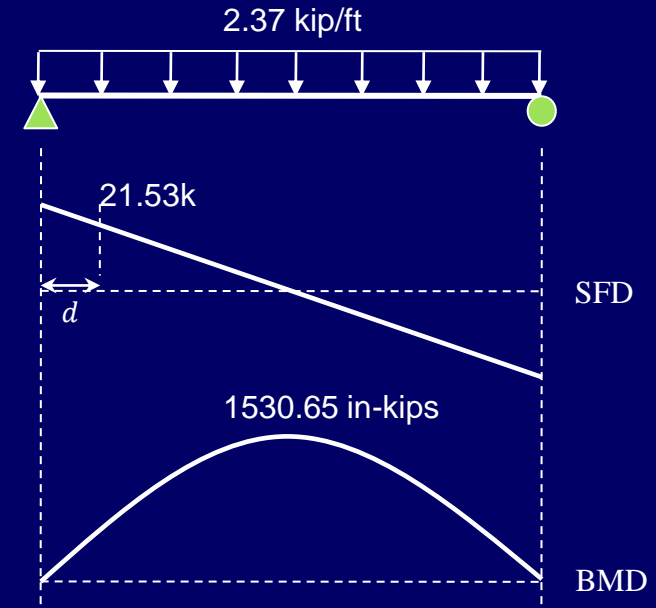
#### 2. Analysis for Shear

From eq.(6.5), we have

$$V_u = w_u(l/2 - d)$$

$$V_u = 2.37(20.75/2 - 15.5/12)$$

$$V_u = 21.53 \text{ kips}$$





# Flexural and Shear Design of RC Beam

- **Solution**

- **Step No.4: Determination of flexural steel area**

Using direct method, we have

$$a = d - \sqrt{d^2 - \frac{2.614M_u}{f_c' b_w}} = 15.5 - \sqrt{15.5^2 - \frac{2.614 \times 1530.65}{3 \times 12}} = 4.14''$$

Putting  $a = 4.14''$  and  $\phi = 0.90$ , we get

$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2}\right)} = \frac{1530.65}{0.9 \times 40 \left(15.5 - \frac{4.14}{2}\right)}$$

$$A_s = 3.17 \text{ in}^2$$



# Flexural and Shear Design of RC Beam

- **Solution**

- **Step No.5: Check for flexural steel area**

- **Minimum reinforcement limit**

$$A_{s,min} = \frac{200}{f_y} b_w d = \frac{200}{40000} \times 12 \times 15.5 = 0.93 in^2$$

- **Maximum reinforcement limit**

$$A_{s,max} = \frac{f'_c b_w d}{136} = \frac{3 \times 12 \times 15.5}{136} = 4.1 in^2$$

$$A_{s,min} < A_s < A_{s,max} \Rightarrow OK!$$





# Flexural and Shear Design of RC Beam

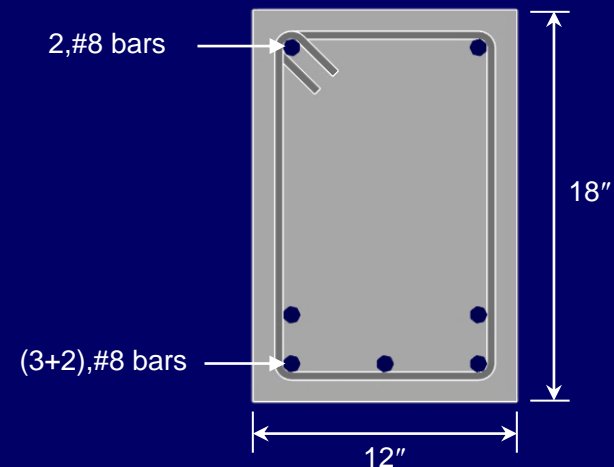
## • Solution

### ➤ Step No.6: Detailing of flexural reinforcement

- Using #8 bar with  $A_b = 0.79in^2$

$$No. of bars = \frac{A_s}{A_b} = \frac{3.17}{0.79} = 4.01 \approx 5$$

- Provide 5,#8 bars in two layers
  - 3 in first layer and
  - 2 in second layer





# Flexural and Shear Design of RC Beam

- **Solution**

- **Step No.7: Check for requirement of Shear reinforcement**

From eq.(6.2), Design shear capacity of concrete can be calculated as;

$$\begin{aligned}\phi V_c &= 2\phi\sqrt{f'_c}b_wd \\ &= 2 \times 0.75\sqrt{3000} 12 \times 15 = 15281.5lb\end{aligned}$$

$$\phi V_c = 15.28k$$

$\phi V_c < V_u \rightarrow$  Shear reinforcement is required



# Flexural and Shear Design of RC Beam

- **Solution**

- **Step No.8: Determination of stirrup spacing**

Calculate design spacing  $s_d$  using eq. (6.4)

$$s_d = \frac{\phi A_v f_y d}{V_u - \phi V_c}$$

Using 2 legged #3 stirrups,  $A_v = 2A_b = 2(0.11) = 0.22in^2$

$$s_d = \frac{0.75 \times 0.22 \times 40 \times 15.5}{21.53 - 15.28}$$

$$s_d = 16.4''$$



# Flexural and Shear Design of RC Beam

- **Solution**

- **Step No.8: Determination of stirrup spacing**

Calculate maximum spacing  $s_{max}$  using eq. (6.6)

$$s_{max} = \text{Least of } \left\{ \begin{array}{l} \frac{A_v f_y}{50 b_w} = \frac{0.22 \times 40,000}{50 \times 12} = 14.67'' \\ \frac{A_v f_y}{0.75 \sqrt{f_c'} b_w} = \frac{0.22 \times 40,000}{0.75 \sqrt{3000} \times 12} = 17.85'' \\ \frac{d}{2} = \frac{15.5}{2} = 7.75'' \\ 24'' \end{array} \right.$$

$$s_{max} = 7.75'' < s_d$$



# Flexural and Shear Design of RC Beam

- **Solution**

- **Step No.9: Apply necessary checks**

1. **Check for Depth of Beam:**

$$\phi V_s \leq \phi 8 \sqrt{f_c'} b_w d$$

$$\phi V_s = V_u - \phi V_c = 21.53 - 15.28 = 6.25 \text{ kip}$$

$$\phi 8 \sqrt{f_c'} b_w d = 4 \phi V_c = 4 \times 15.28 = 61.12 \text{ kip} > \phi V_s \rightarrow \text{OK!}$$

2. **Check for Maximum spacing of stirrups:**

$$\phi V_s \leq \phi 4 \sqrt{f_c'} b_w d \quad , \quad \phi V_s = 6.25 \text{ kip}$$

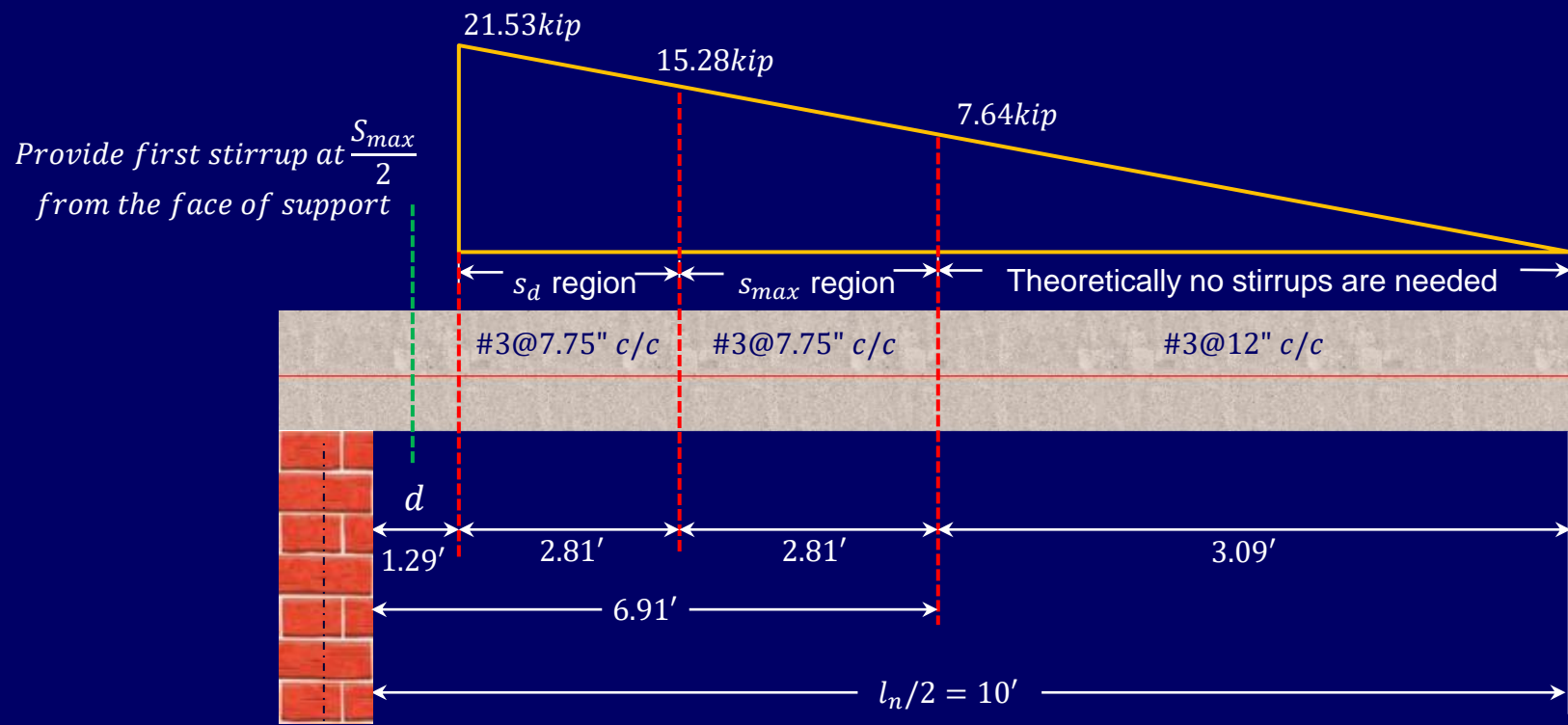
$$\phi 4 \sqrt{f_c'} b_w d = 2 \phi V_c = 2 \times 15.28 = 30.56 \text{ kip} > \phi V_s \rightarrow \text{OK!}$$



# Flexural and Shear Design of RC Beam

- Solution**

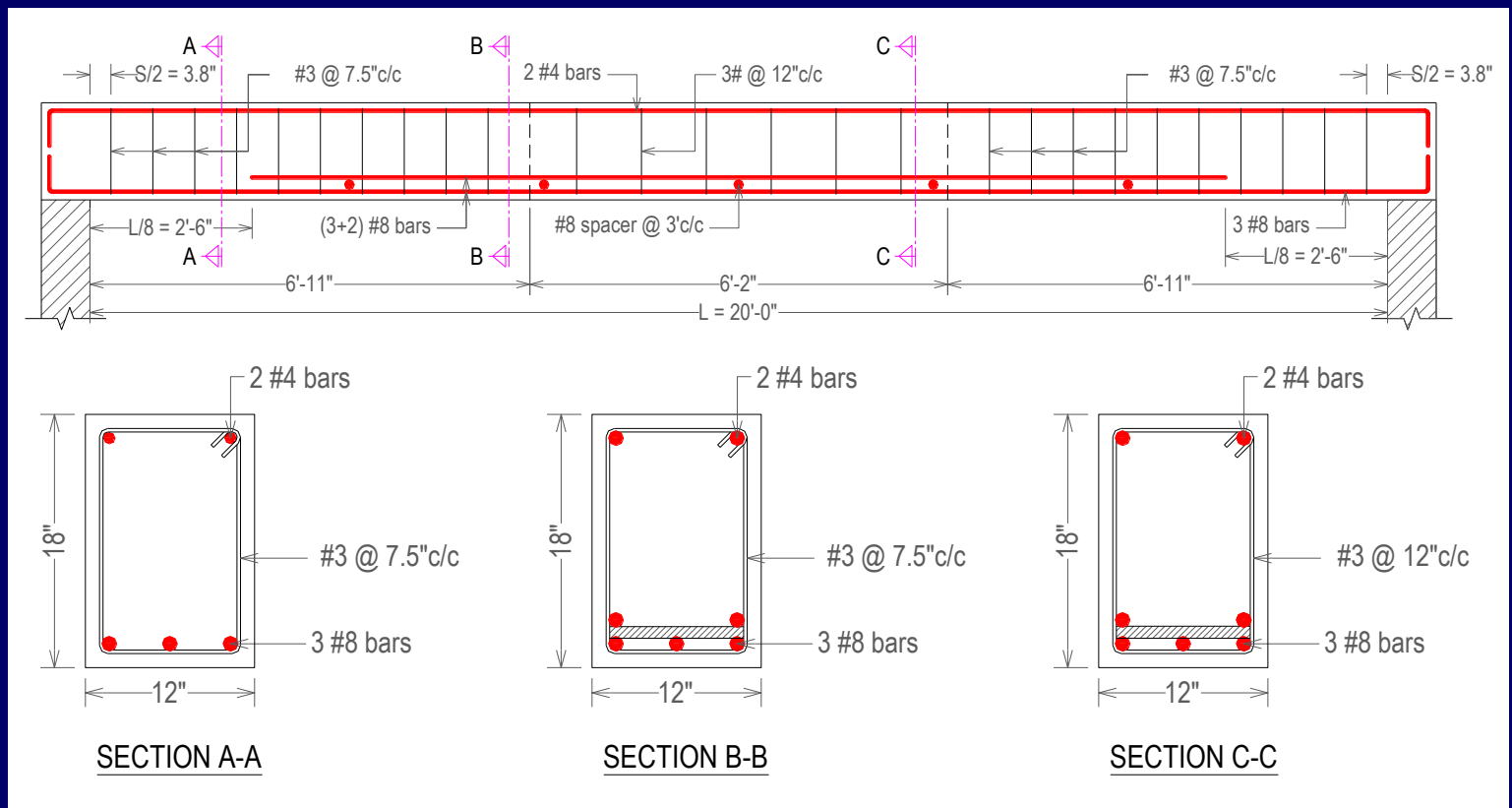
- Step No.10: Detailing of shear reinforcement**





# Flexural and Shear Design of RC Beam

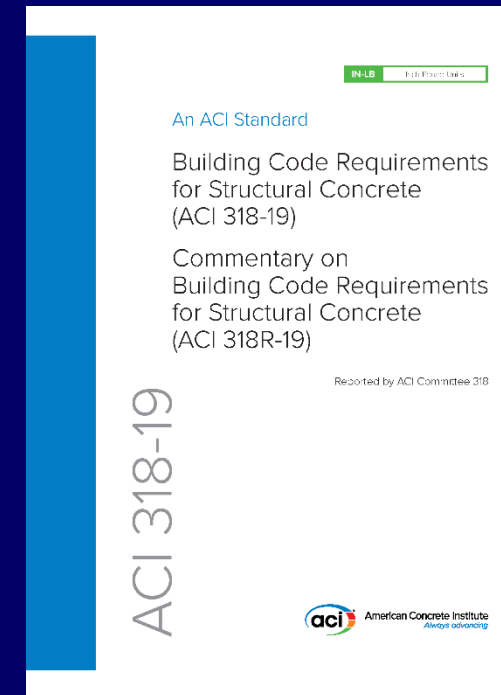
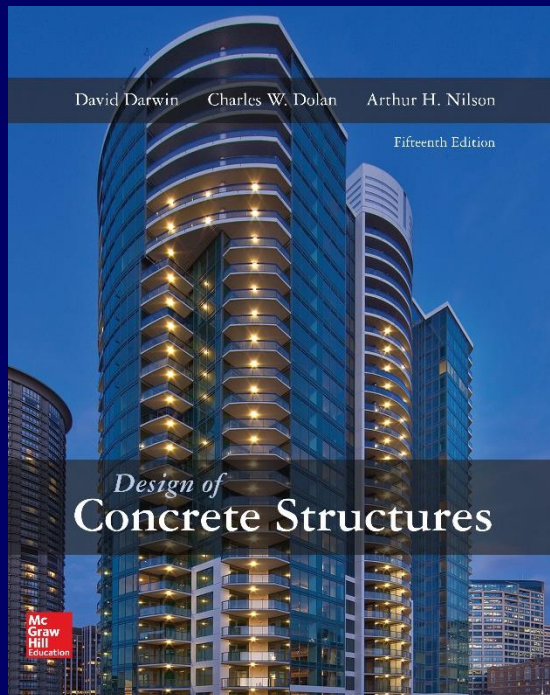
- **Solution**
  - **Step No.11: Drafting**





# References

- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-19)



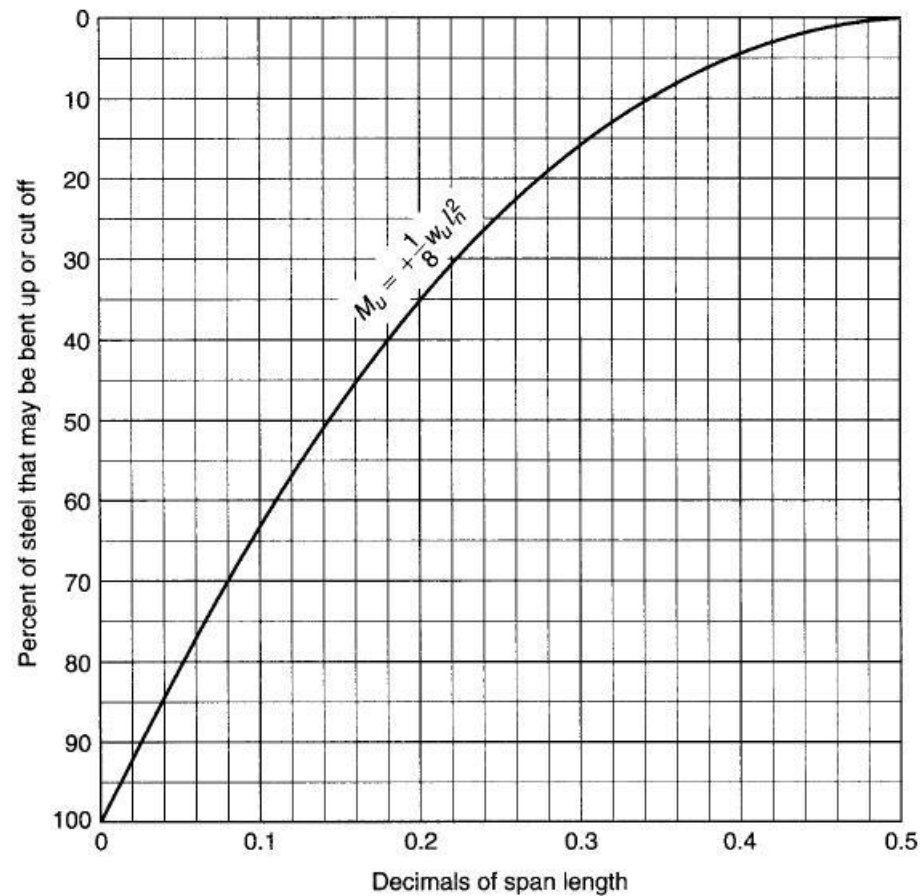




# Appendix

**GRAPH A.2**

Location of points where bars can be bent up or cut off for simply supported beams uniformly loaded.



Exact curtailments lengths for simply supported positive moments (to be measured from face of the support)



# Appendix

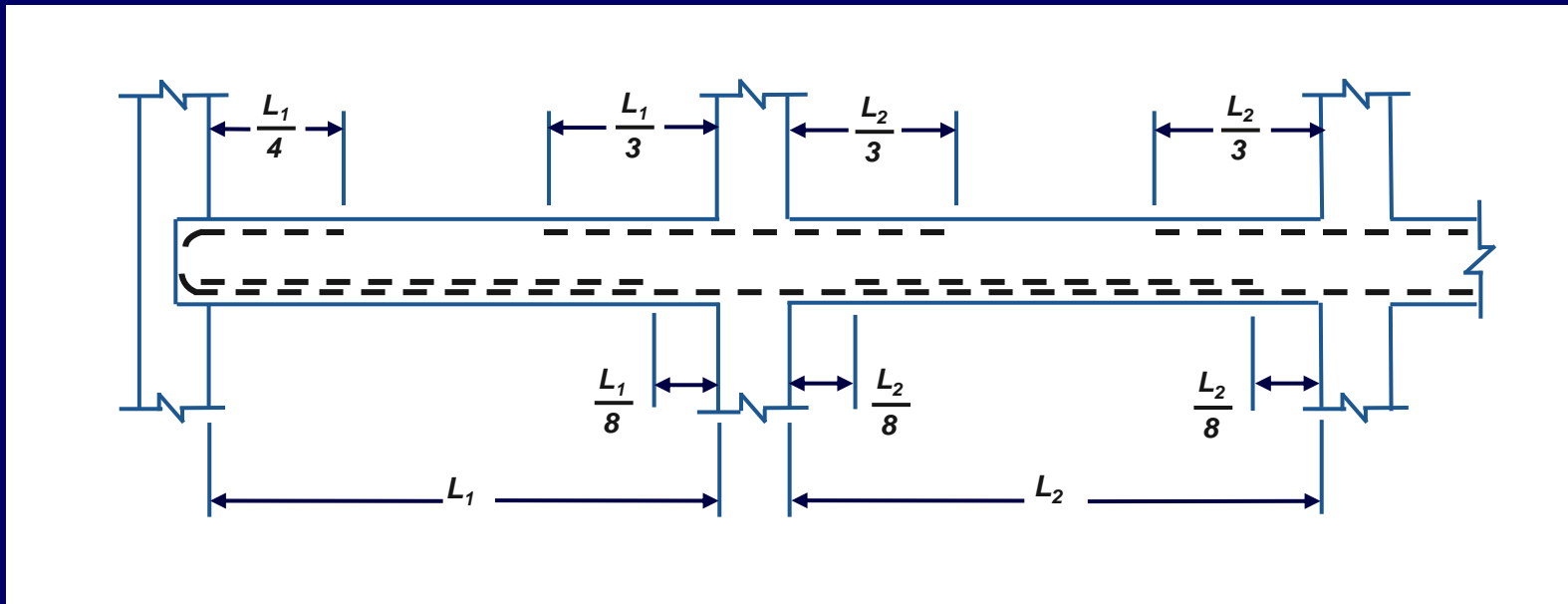


Figure: Cutoff for bars in approximately equal spans with uniformly distributed loads for 50% curtailment