Lecture 06

Design of Reinforced Concrete Beam for Shear

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Lecture Contents

- Shear Stresses in Rectangular Beams
- Diagonal Tension in RC Beams subjected to Flexure and Shear Loading
- Types of Cracks in Reinforced Concrete Beam
- Shear Strength of Concrete
- Web Reinforcement Requirement in RC Beams
- ACI Code Provisions for Shear Design
- Design of RC beam for Flexure and Shear (Example)
- References

Learning Outcomes

- **At the end of this lecture, students will be able to;**
  - ***Explain*** mechanics of shear in reinforced concrete beams
  - ***Classify*** cracks in reinforced concrete beam
  - ***Analyze*** reinforced concrete beam for shear
  - ***Design*** reinforced concrete beam for shear as per ACI code provisions

Shear Stresses in Rectangular Beam

- **Shear stresses in homogeneous Elastic Rectangular Beam**
  - The shear stress (v) at any point in the cross section is given by

$$v = \frac{VQ}{Ib}$$

Where;

V = Total shear at section

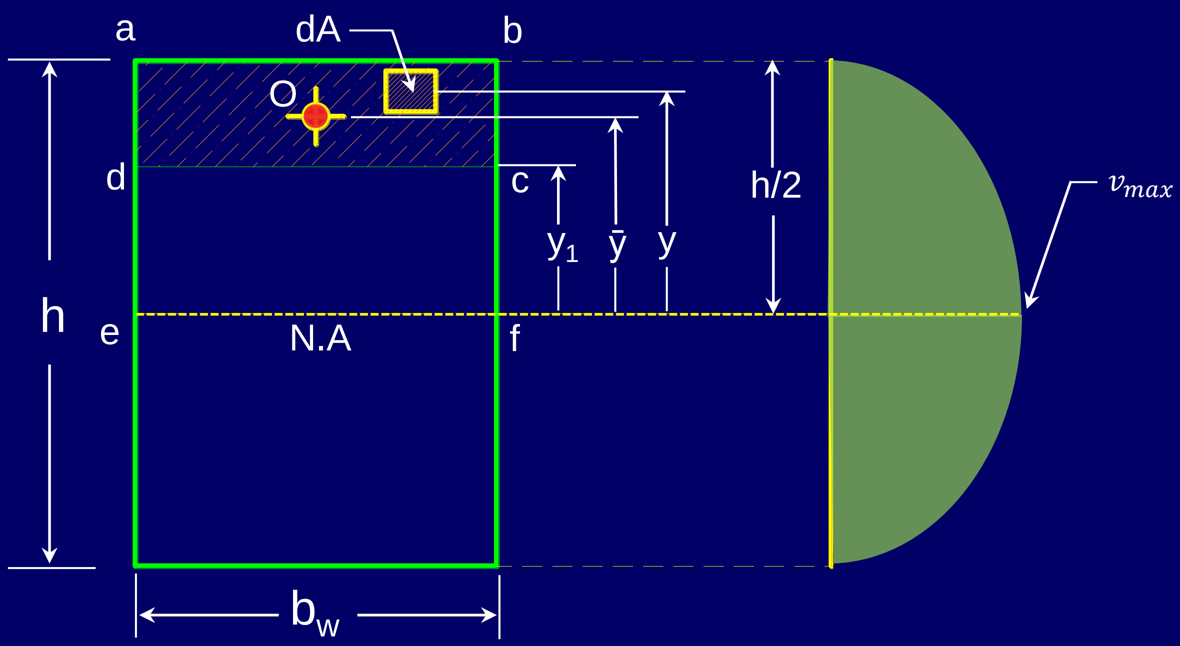
I = Moment of inertia of cross section about neutral axis

b = Width of beam at a given point

Q = Statical moment about neutral axis of that portion of cross section lying between a line through point in question parallel to neutral axis and nearest face (upper or lower) of beam

Shear Stresses in Rectangular Beam

- **Shear stresses in homogeneous Elastic Rectangular Beam**
  - For the calculation of shear stress at level d-c in the given figure, Q will be equal to $A_{abcd}\ \bar{y}$, where A_{abcd} is area abcd and $\bar{y}$ is the centroidal distance of area abcd from N.A

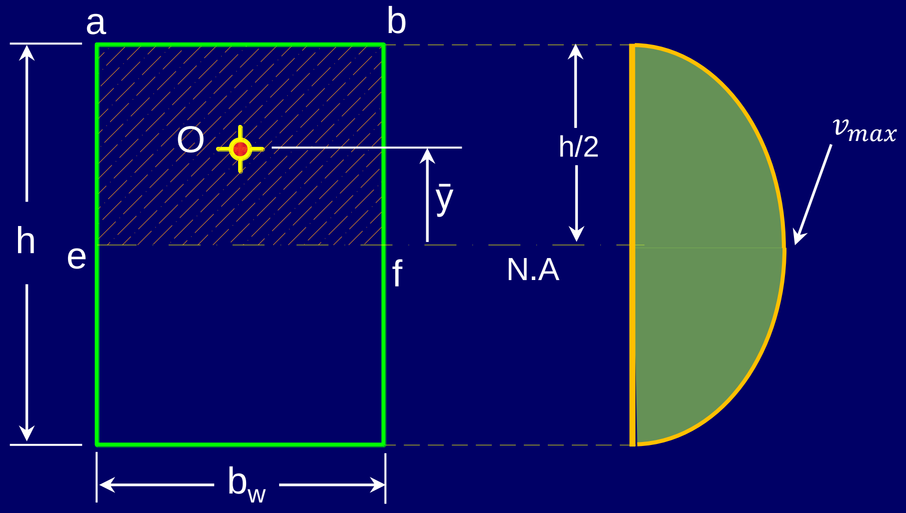


Shear Stresses in Rectangular Beam

- **Shear stresses in homogeneous Elastic Rectangular Beam**
  - For shear at neutral axis, we have

$$Q = A_{abef}\bar{y} = \frac{b_w h}{2} \times \frac{h}{4}$$

$$I = \frac{b_w h^3}{12}$$



Therefore,

$$v = \frac{VQ}{Ib} = \frac{V(b_w h^2/8)}{(b_w h^3/12) \times b_w}$$

which on simplification gives,

$$v_{max} = \frac{1.5V}{b_w h}$$

Shear Stresses in Rectangular Beam

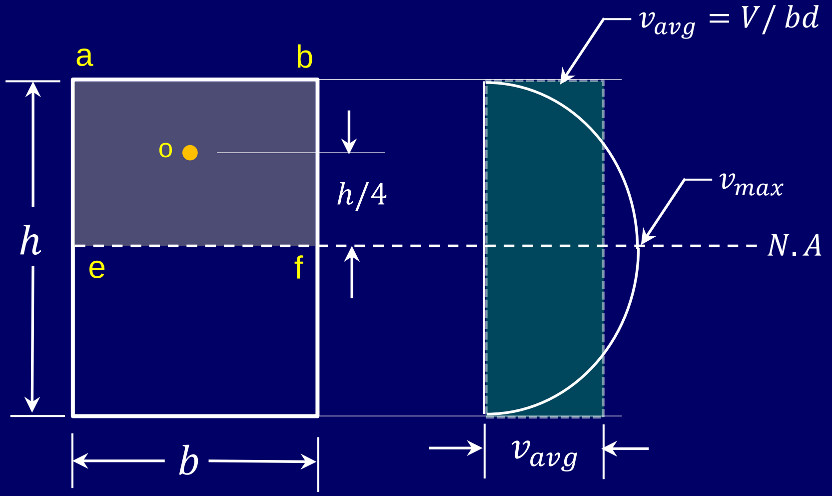
- **Shear stresses in Reinforced concrete Beam**
  - The shear stresses in reinforced concrete member cannot be computed from $v = VQ/Ib$, because concrete is not an elastic homogeneous material.

Shear Stresses in Rectangular Beam

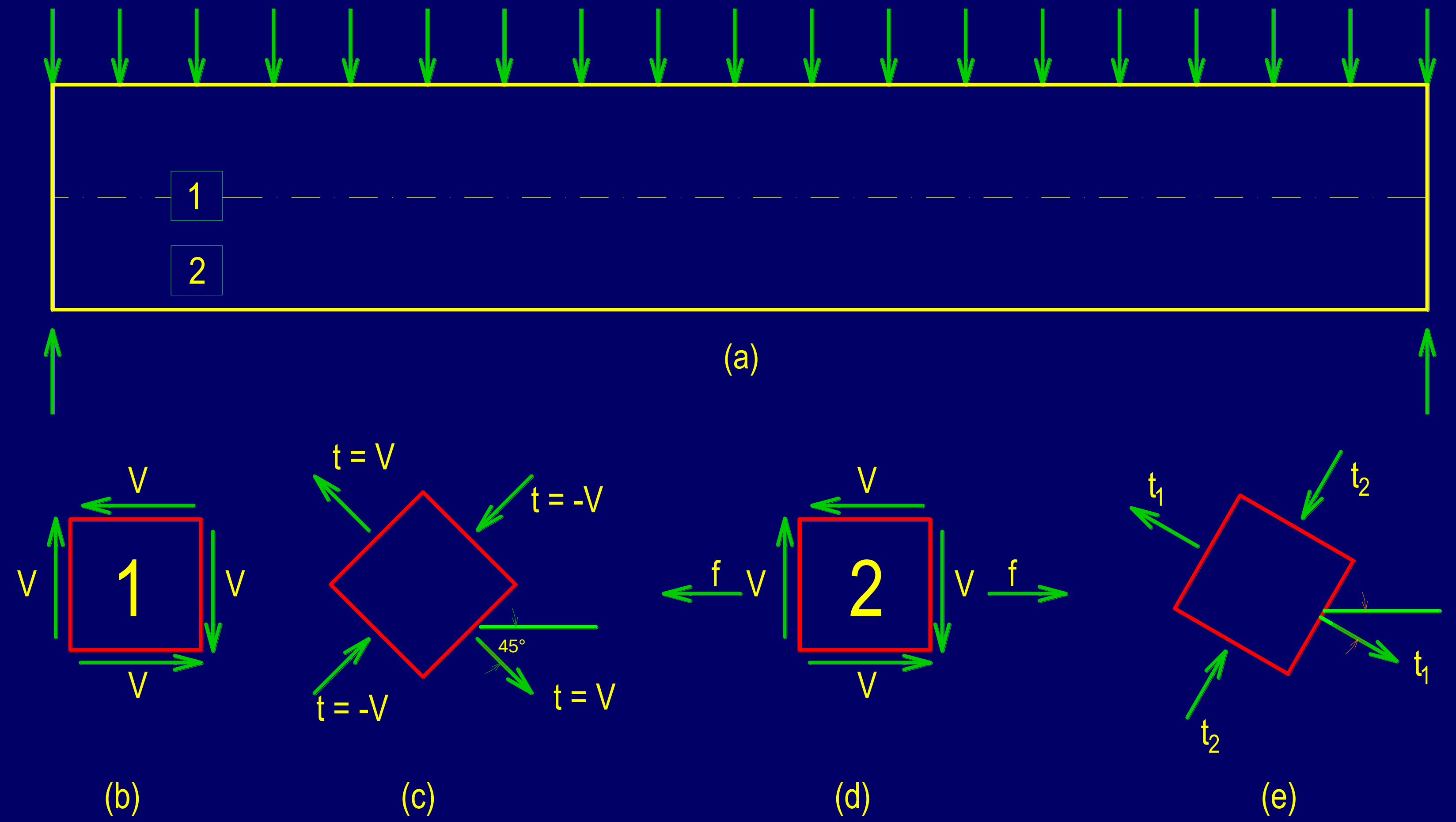
- **Shear stresses in Reinforced concrete Beam**
  - Tests have shown that the average shear stress in a RC beam can be expressed by :

$$v_{avg} = V / bd$$

  - The maximum value, which occurs at the neutral axis, will exceed this average by an unknown but moderate amount.



Diagonal Tension in RC Beams subjected to Flexure and Shear Loading



Diagonal Tension in RC Beams subjected to Flexure and Shear Loading

- **Types of Cracks in Reinforced Concrete Beam**
  - There are basically two types of cracks

1. Flexural Cracks
2. Diagonal Tension Cracks

   I. Web-shear cracks:

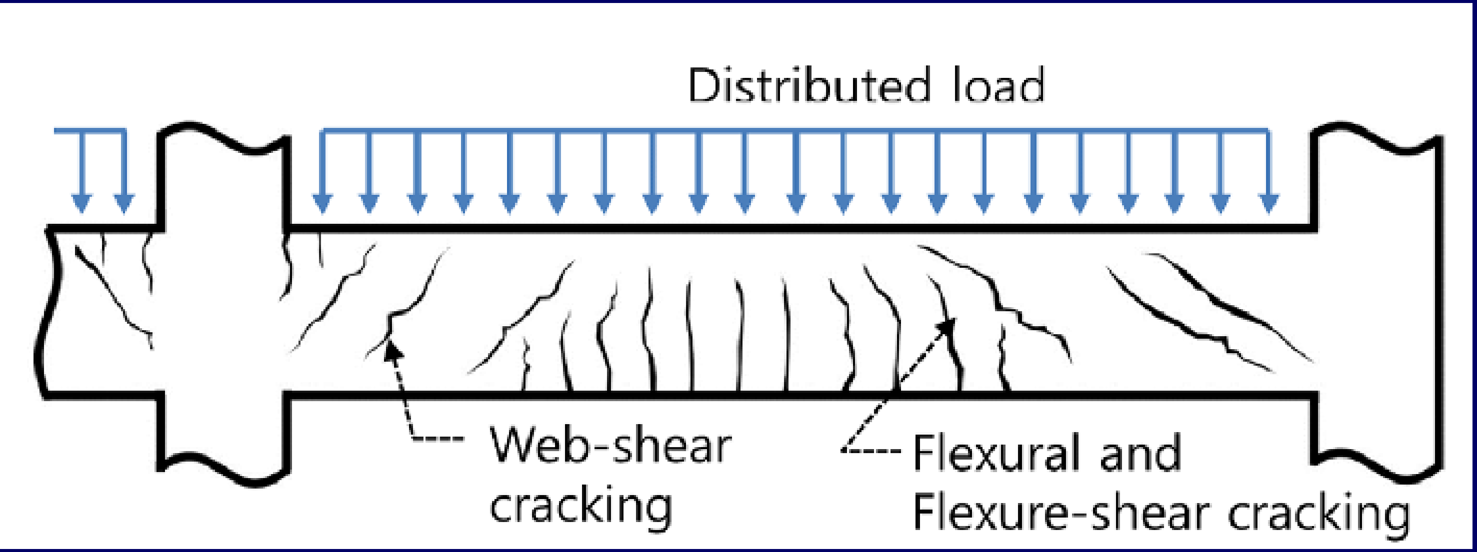
   These cracks are formed at locations where flexural stresses are negligibly small.

   II. Flexure shear cracks:

   These cracks are formed where shear force and bending moment have large values.

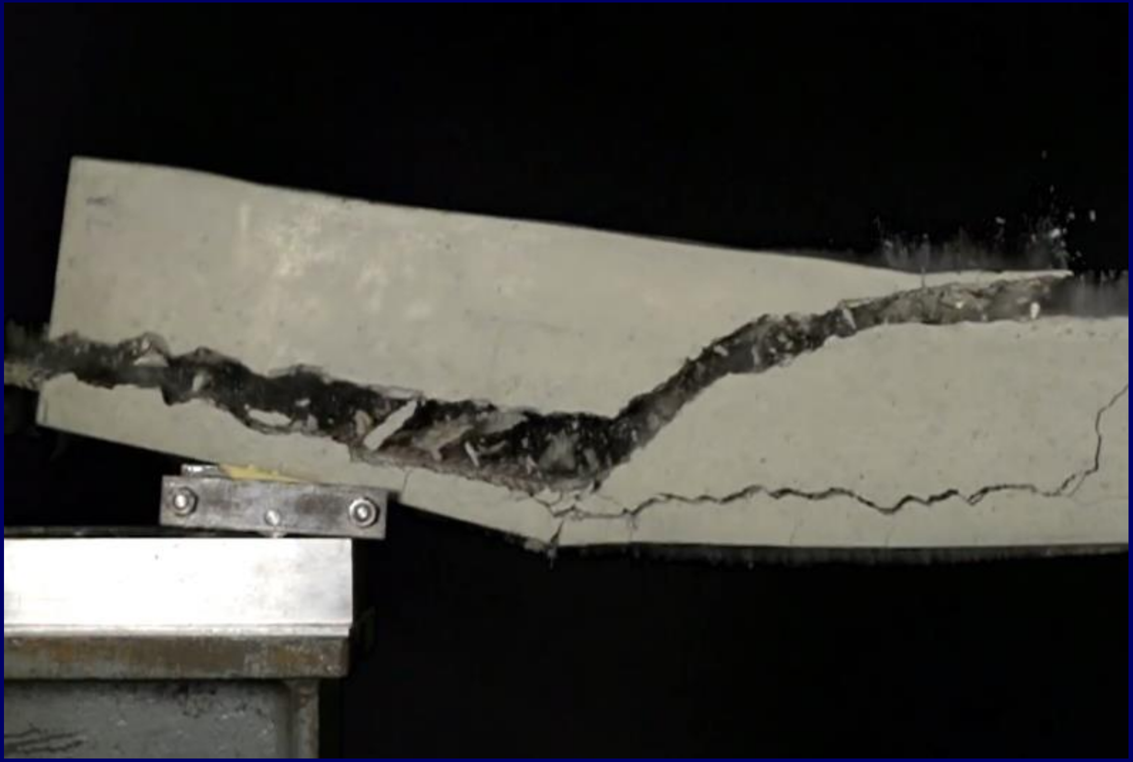
Diagonal Tension in RC Beams subjected to Flexure and Shear Loading

- **Types of Cracks in Reinforced Concrete Beam**



Diagonal Tension in RC Beams subjected to Flexure and Shear Loading

- **Shear Failure of beam**



Web Reinforcement Requirement in RC Beams

- **Nominal Shear Capacity V_n**
  - As per ACI 318-19, section 22.5.1.1, the general expression for shear capacity of reinforced concrete beam is given as:

    $V_n = V_c + V_s$ ------ (6.1)

    Where;

    V_c =Nominal shear capacity of concrete,

    V_s = Nominal shear capacity of shear reinforcement.

  - Note that in case of flexural capacity, $M_n = M_c + M_s$ where $M_c = 0$, at ultimate load. However, in case of shear capacity, the term $V_c \neq 0$.

Web Reinforcement Requirement in RC Beams

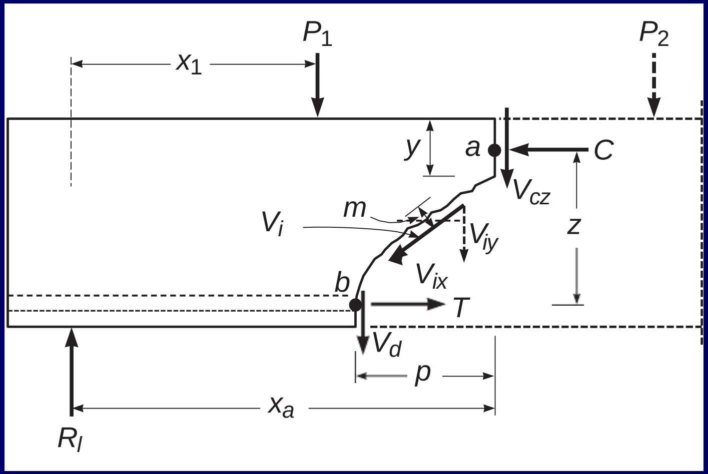
- **Nominal Shear Capacity V_n**
  - **Calculation of V_c**
    - Test evidence have led to the conservative assumption that just prior to failure of a web-reinforced beam, three internal shear components contributing to the total shear, the sum of which is referred as shear capacity of concrete V_c.

$$V_c = V_{cz} + V_d + V_{iy}$$

V_{cz} = Internal vertical forces in the uncracked portion of the concrete.

V_d = Internal vertical forces across the longitudinal steel, acting as a dowel.

V_{iy} = Vertical component of sizable interlock forces.



Web Reinforcement Requirement in RC Beams

- **Nominal Shear Capacity V_n**
  - **Calculation of V_c**
    - ACI 318 -19 Table 22.5.5.1 provides three equations for calculating nominal shear capacity of concrete.The equation which is applicable in most of the cases is given as follows;

$$V_c = \left[2\lambda\sqrt{f_c'} + \frac{N_u}{6A_g} \right] b_w d$$

    - In case of beams, Axial force N_u is very small and can be ignored. Taking $N_u = 0$ and $\lambda = 1(\text{normalweight concrete})$, the above equation reduces to;

$$V_c = 2\sqrt{f_c'} b_w d \quad \text{------ (6.2)}$$

Web Reinforcement Requirement in RC Beams

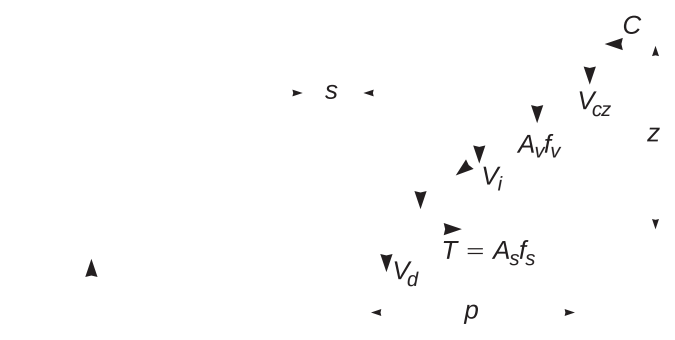
- **Nominal Shear Capacity V_n**
  - **Calculation of V_s**
    - As per ACI 318-19, section 22.5.8.5.3, V_s can be determined as

$$V_s = nA_v f_y$$

n = number of stirrups

A_v = Shear reinforcement

f_y = Yield tensile strength of steel



Here, "n" can be conservatively assumed as d/s_d, therefore

$V_s = A_v f_y d / s_d$ ------ (6.3)

Web Reinforcement Requirement in RC Beams

- **Design Shear Capacity $\varnothing V_n$**

From equation (6.1) we have

$$V_n = V_c + V_s$$

From which design shear capacity is calculated as;

$$\varnothing V_n = \varnothing V_c + \varnothing V_s$$

Substituting values of V_c and V_s, we obtain

$$\varnothing V_n = \varnothing V_c + \frac{\varnothing A_v f_y d}{s_d}$$

Web Reinforcement Requirement in RC Beams

- **Design Shear Capacity $\emptyset V_n$**

For no failure, $\emptyset V_n \geq V_u$

Setting $\emptyset V_n = V_u$, we get

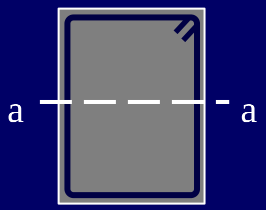
$$\emptyset V_c + \frac{\emptyset A_v f_y d}{s_d} = V_u$$

Which on solving for s gives

$$s_d = \frac{\emptyset A_v f_y d}{V_u - \emptyset V_c} \quad \text{------ (6.4)}$$

A_v is the cross-sectional area of web reinforcement within a distance "s", for single loop stirrups (2 legged), $A_v = 2A_s$

A_s = cross sectional area of the stirrup bar



At section a-a, if #3 bar is used $A_s = 0.11$ in^2, $A_v = 2 \times 0.11 = 0.22$ in^2

ACI 318 Code Provisions for Shear Design

- **Location of Critical Section for ultimate shear V_u**
  - In most of the cases, for the design of shear, critical shear is taken at a distance “d” from the support instead of maximum shear at the face of the support.

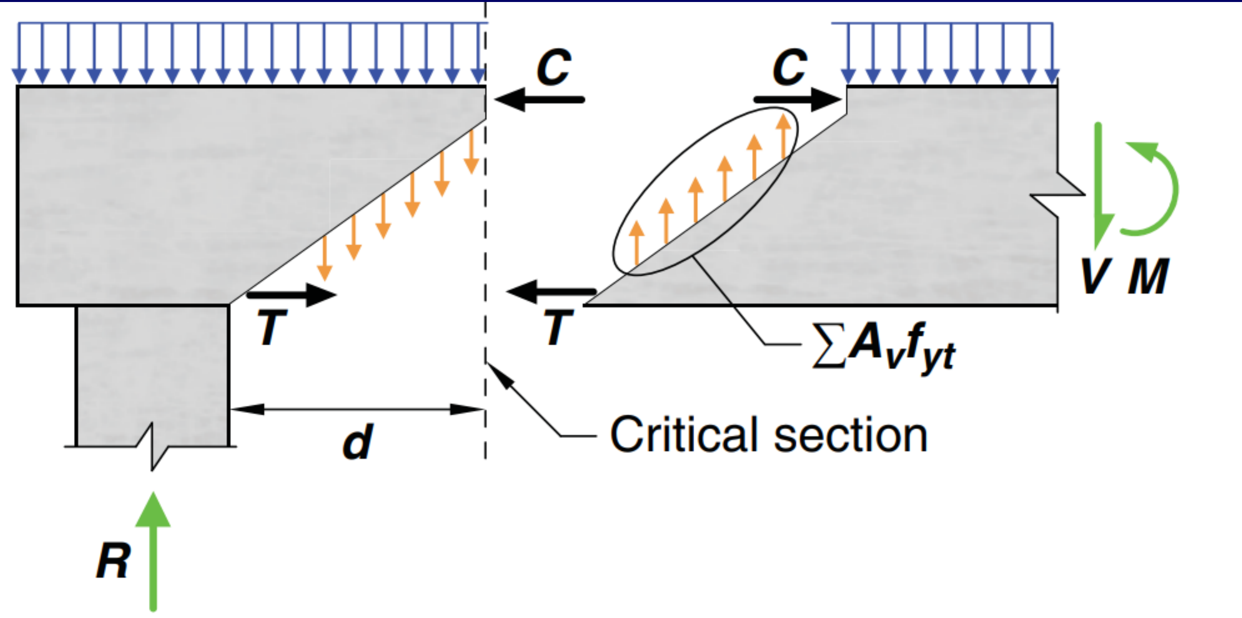
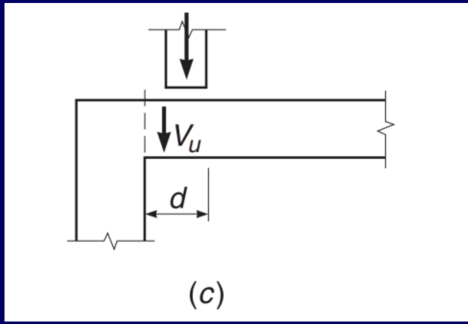


Fig. R9.4.3.2a—Free body diagrams of the end of a beam.

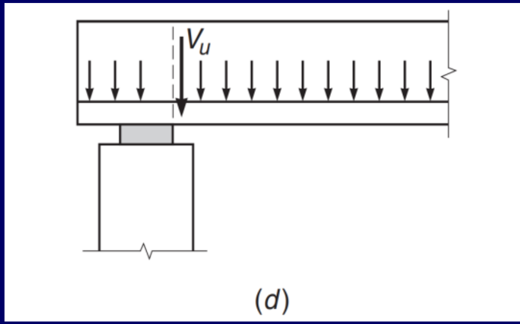
ACI 318 Code Provisions for Shear Design

- **Location of Critical Section for ultimate shear V_u**

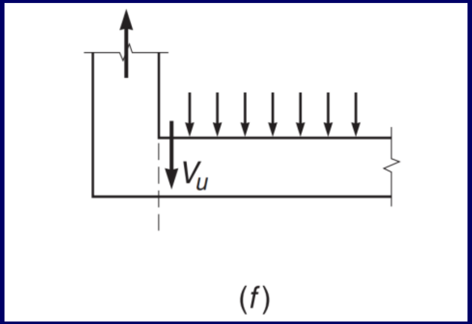
The critical section for shear is taken at different positions in different situations, as shown in the diagrams below.



Concentrated load within distance “d”



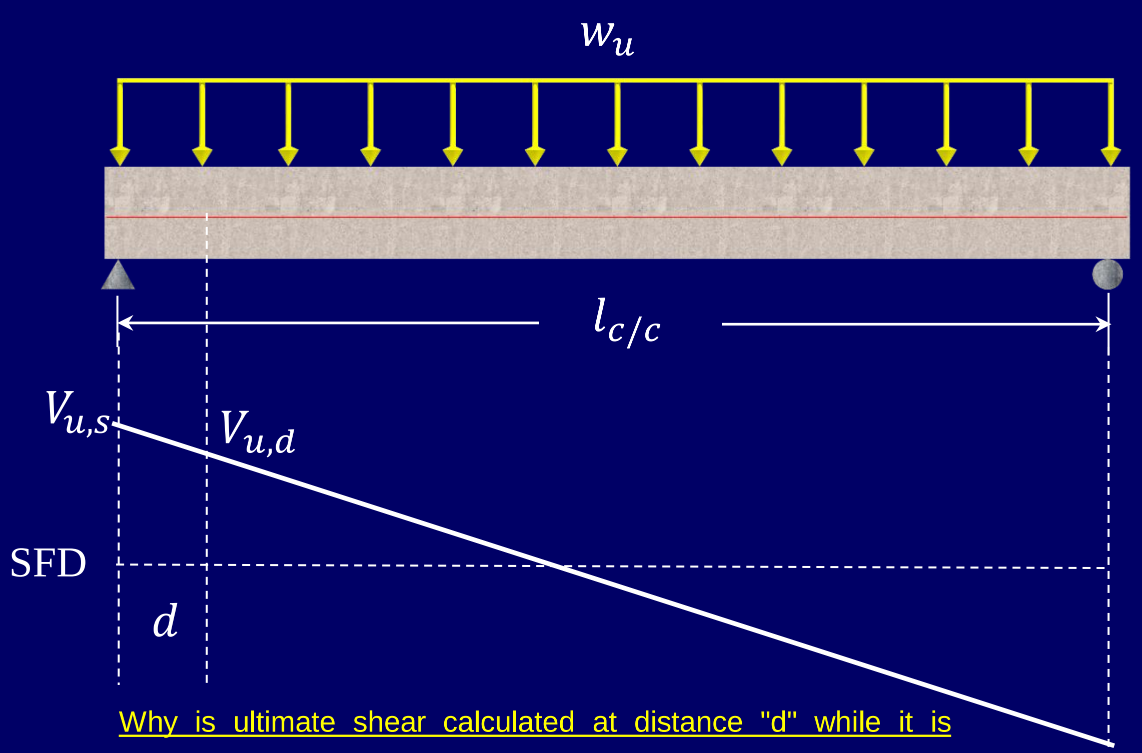
Inverted T beam



Suspended slab held by tie column

ACI 318 Code Provisions for Shear Design

- **Calculation of ultimate shear V_u**
  - Critical shear is taken at a distance "d" from the face of support.



$V_{u,s}$ = Factored shear forced at the center of support.

$V_{u,d}$ = Factored shear forced at a distance "d" from the face of support.

Why is ultimate shear calculated at distance "d" while it is greatest at the support?

ACI 318 Code Provisions for Shear Design

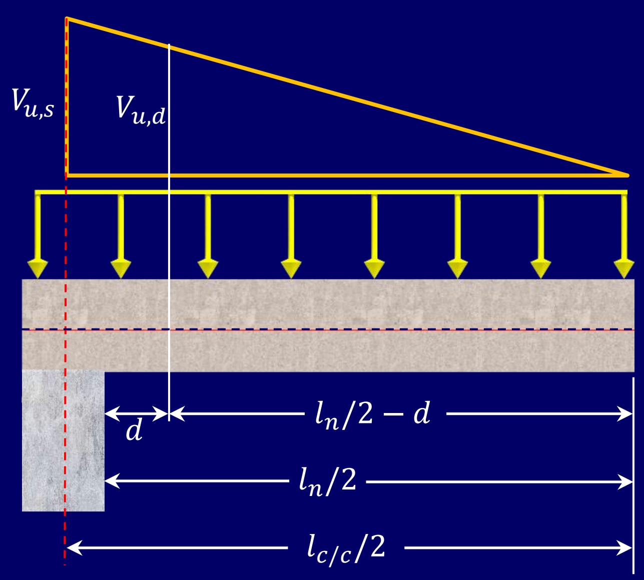
- **Calculation of ultimate shear $V_{u,d}$**
  - The ultimate shear at distance d can be calculated by similarity of triangles as follows;

$$\frac{V_{u,d}}{l_n/2 - d} = \frac{V_{u,s}}{l_{c/c}/2}$$

$$\Rightarrow V_{u,d} = \frac{V_{u,s}}{l_{c/c}/2}\left(\frac{l_n}{2} - d\right)$$

Since $\frac{V_{u,s}}{l_{c/c}/2} = w_u$

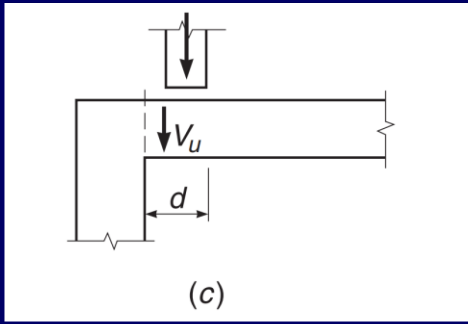
$V_{u,d} = w_u(l_n/2 - d)$------ (6.6)



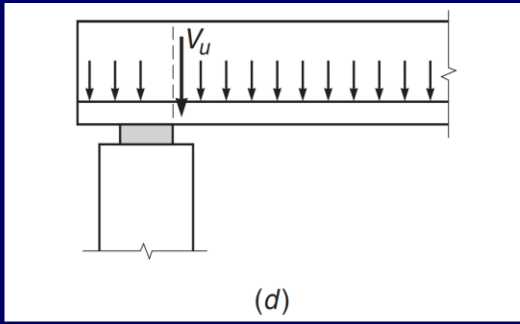
ACI 318 Code Provisions for Shear Design

- **Location of Critical Section for ultimate shear $V_{u,cr}$**

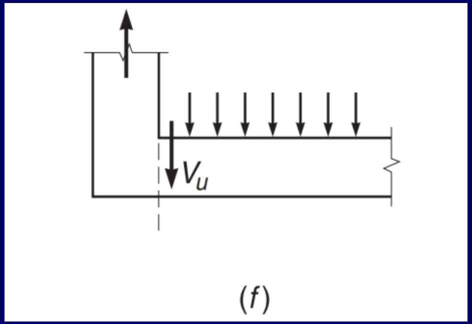
  The critical section for shear is taken at different positions in different situations, as shown in the diagrams below.



Concentrated load within distance “d”



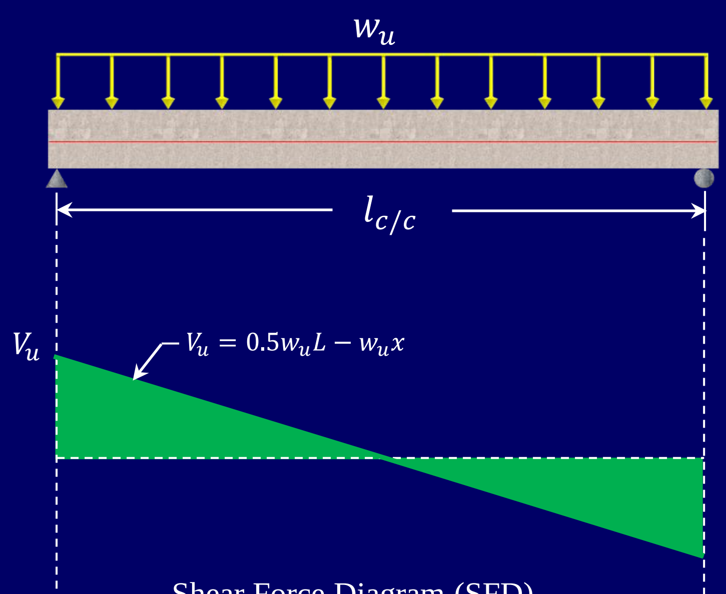
Inverted T beam



Suspended slab held by tie column

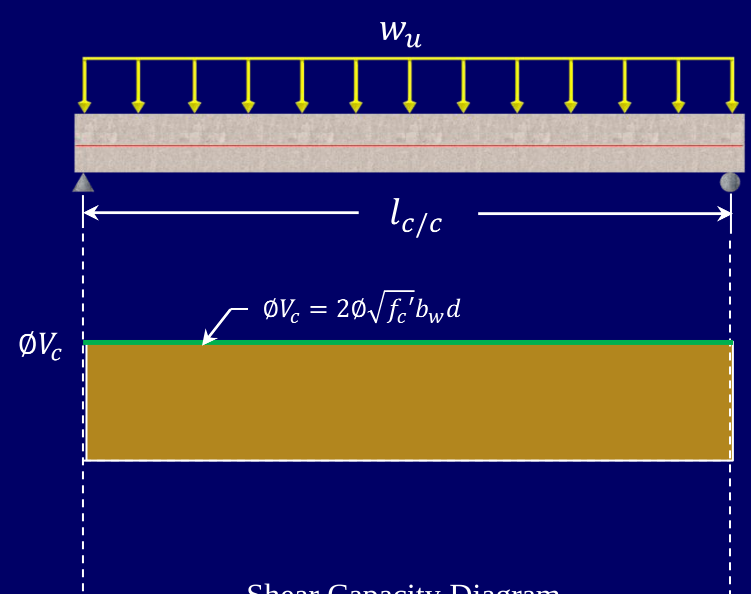
ACI 318 Code Provisions for Shear Design

- **Design for critical shear demand V_u**
  - Consider the following typical Shear force diagram, and shear capacity diagram of beam.



Shear Force Diagram (SFD)

(Varies along the length of beam)



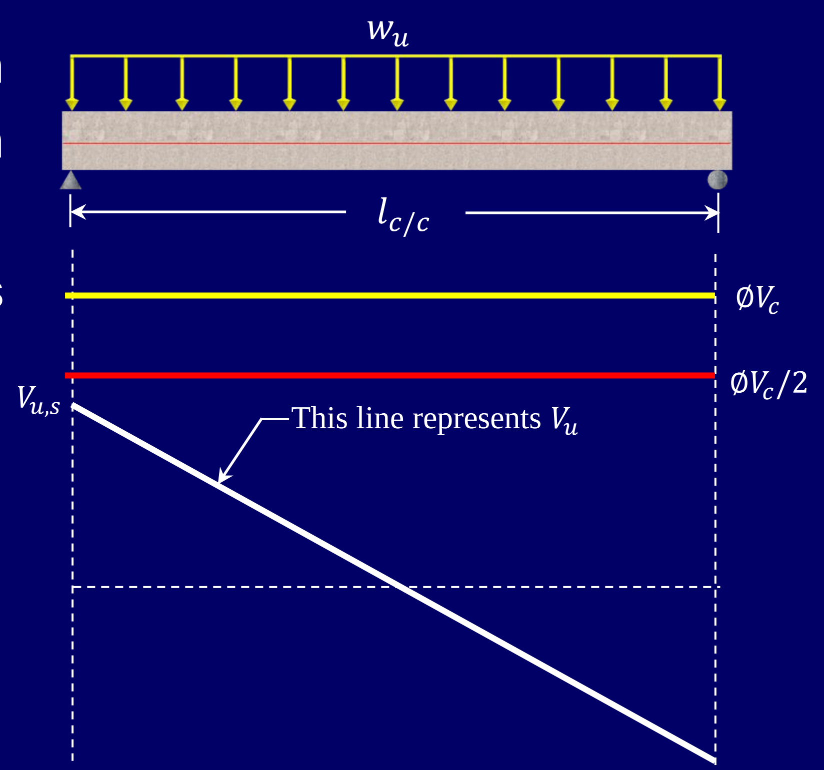
Shear Capacity Diagram

(Remains constant throughout the length of beam)

ACI 318 Code Provisions for Shear Design

- **Design for critical shear demand V_u**
  - If we plot shear capacity on shear force diagram, then depending on the value of V_u, the following three conditions are possible.

    1. $\emptyset V_c/2 \geq V_u$

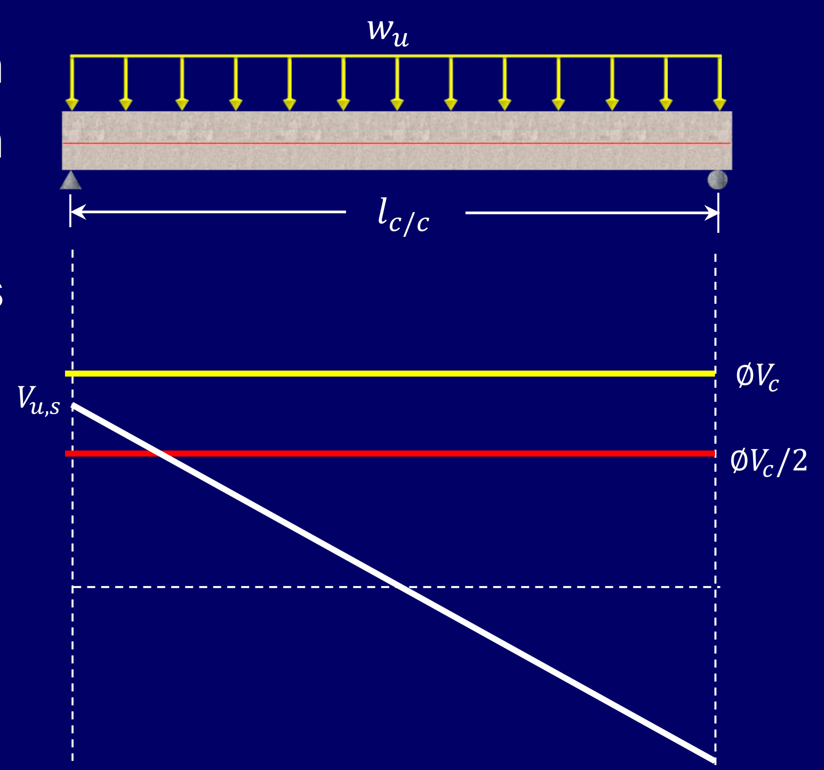


Shear Force Diagram (SFD)

ACI 318 Code Provisions for Shear Design

- **Design for critical shear demand V_u**
  - If we plot shear capacity on shear force diagram, then depending on the value of V_u, the following three conditions are possible.

    1. $\emptyset V_c/2 \geq V_u$
    2. $\emptyset V_c/2 < V_u$ but $\emptyset V_c \geq V_u$

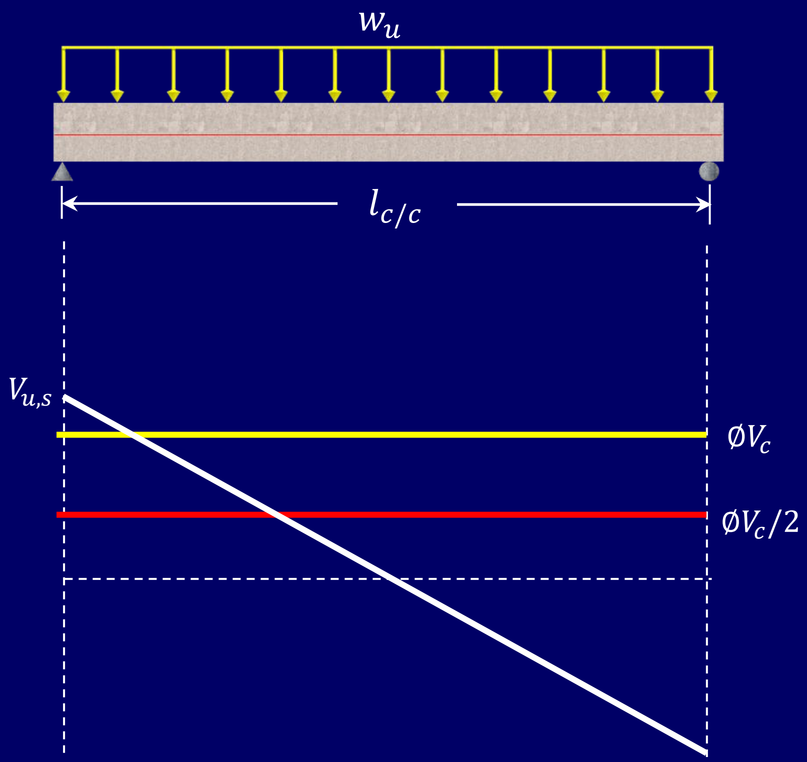


Shear Force Diagram (SFD)

ACI 318 Code Provisions for Shear Design

- **Design for critical shear demand V_u**
  - If we plot shear capacity on shear force diagram, then depending on the value of V_u, the following three conditions are possible.

1. $\emptyset V_c/2 \geq V_u$
2. $\emptyset V_c/2 < V_u$ but $\emptyset V_c \geq V_u$
3. $\emptyset V_c < V_u$



Shear Force Diagram (SFD)

ACI 318 Code Provisions for Shear Design

- **Design for ultimate shear V_u**

1. When $\emptyset V_c /2 > V_u$, no web reinforcement is required.

2. When $\emptyset V_c \geq V_u$ but $\emptyset V_c /2 < V_u$, theoretically no web reinforcement is required. However, ACI 9.7.6.2.2 and 10.6.2.2 recommend that minimum web reinforcement in the form of maximum spacing S_{max} shall be provided:

$$s_{max} = minimum\ of \left\{ \frac{A_v f_y}{50 b_w}, \frac{A_v f_y}{0.75\sqrt{f_c'} b_w}, \frac{d}{2}, 24" \right\} \quad \text{------ (6.6)}$$

Minimum Spacing s_{min} is 3 inches.

ACI 318 Code Provisions for Shear Design

- **Design for ultimate shear V_u**

  3. When $\varnothing V_c < V_u$, web reinforcement is required.

  - In this case, the required spacing s_d can be calculated using eq. (6.4)

$$s_d = \frac{\varnothing A_v f_y d}{V_u - \varnothing V_c}$$

  - If s_d is greater than s_{max}, use s_{max}

ACI 318 Code Provisions for Shear Design

- **Necessary Checks**

  1. **Check for Depth of Beam:**

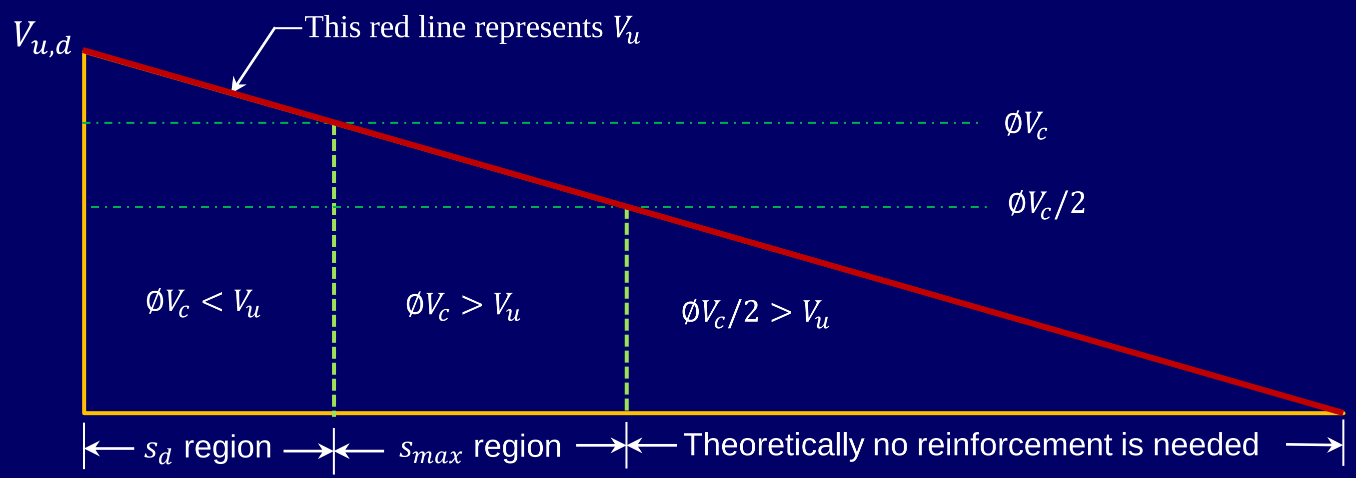
     $\phi V_s \leq \phi 8\sqrt{f_c'} b_w d$ → Depth of beam is OK!, otherwise increase depth

  2. **Check for maximum Spacing of stirrups:**

     $\phi V_s \leq \phi 4\sqrt{f_c'} b_w d$ → S_{max} is OK!, otherwise divide S_{max} by 2

ACI 318 Code Provisions for Shear Design

- **Placement of Shear reinforcement**

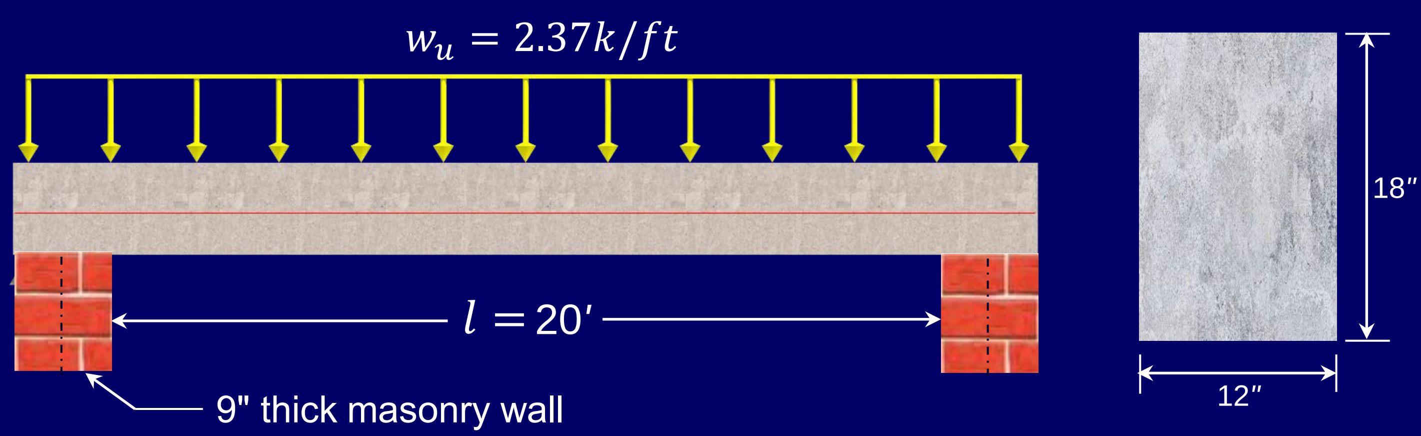


- For $\emptyset V_c < V_u \rightarrow$ use s_d
- For $\emptyset V_c > V_u$, use s_{max}
- For $\emptyset V_c/2 > V_u$, no reinforcement is required

Design of RC Beam for Flexure and Shear

- **Example 6.1**
  - A simply supported beam with given cross sectional dimensions is subjected to a uniformly distributed factored load of 2.37 kip/ft as shown in the figure below.

Analyze and ***Design*** the beam for flexure and shear in accordance with ACI 318-19. Take $f_c{}' = 3ksi$ and $f_y = 40ksi$



Design of RC Beam for Flexure and Shear

- **Solution**
  - **Step No.1: Selection of sizes**

    Cross sectional dimensions are already given

    $b_w = 12"$

    $h = 18"$

    Assuming $\bar{y} = 2.5"$

    $d = 18 - 2.5 = 15.5"$

  - **Step No.2: Calculation of loads**

    $w_u = 2.37k/ft$

Design of RC Beam for Flexure and Shear

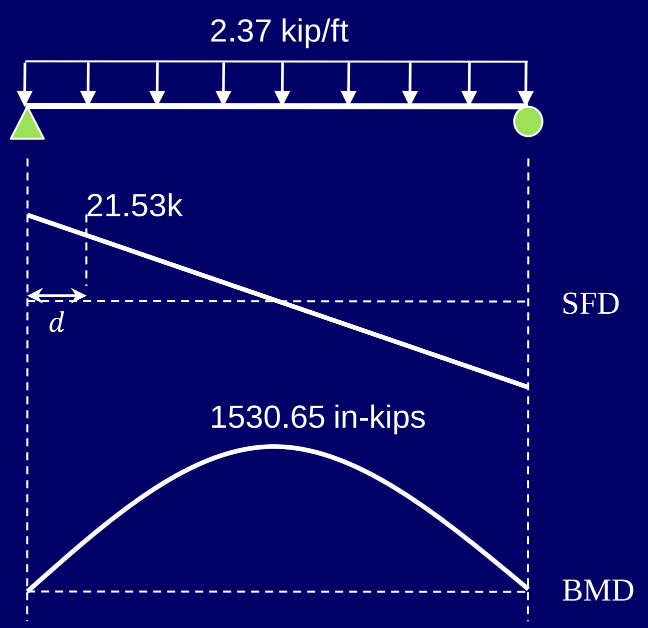
- **Solution**

  - **Step No.3: Analysis**

1. Analysis for Flexure

$$M_u = \frac{w_u l^2}{8} = \frac{2.37(20.75)^2}{8} \times 12 = 1530.65\ in.kip$$

2. Analysis for Shear



From eq.(6.5), we have

$$V_u = w_u(l/2 - d)$$

$$V_u = 2.37(20.75/2 - 15.5/12)$$

$$V_u = 21.53 kips$$

Design of RC Beam for Flexure and Shear

- **Solution**
  - **Step No.4: Determination of flexural steel area**

Using direct method, we have

$$a = d - \sqrt{d^2 - \frac{2.614 M_u}{f_c{}' b_w}} = 15.5 - \sqrt{15.5^2 - \frac{2.614 \times 1530.65}{3 \times 12}} = 4.14"$$

Putting $a = 3.795$ and $\emptyset = 0.90$, we get

$$A_s = \frac{M_u}{\emptyset f_y (d - \frac{a}{2})} = \frac{1530.65}{0.9 \times 40 \left(15.5 - \frac{4.14}{2}\right)}$$

$$A_s = 3.17 in^2$$

Design of RC Beam for Flexure and Shear

- **Solution**

  - **Step No.5: Check for flexural steel area**

    - Minimum reinforcement limit

$$A_{s,min} = \frac{200}{f_y} b_w d = \frac{200}{40000} \times 12 \times 15.5 = 0.93 in^2$$

    - Maximum reinforcement limit

$$A_{s,max} = \frac{f_c' b_w d}{136} = \frac{3 \times 12 \times 15.5}{136} = 4.1 in^2$$

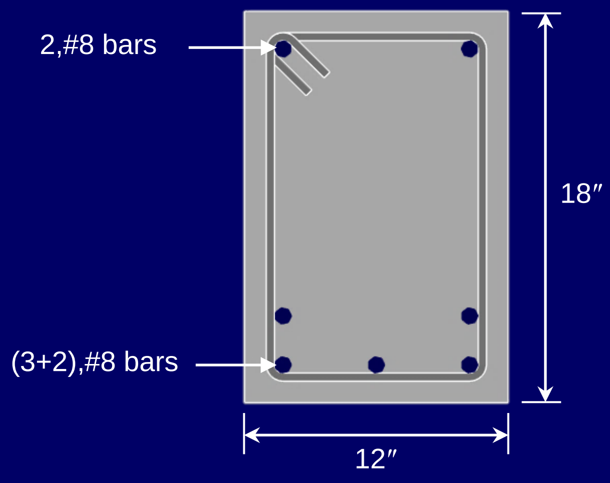
$$A_{s,min} < A_s < A_{s,max} \Rightarrow OK!$$

Design of RC Beam for Flexure and Shear

- **Solution**
  - **Step No.6: Detailing of flexural reinforcement**
    - Using #8 bar with $A_b = 0.79 in^2$

$$No. of\ bars = \frac{A_s}{A_b} = \frac{3.17}{0.79} = 4.01 \approx 5$$

- Provide 5,#8 bars in two layers
  - 3 in first layer and
  - 2 in second layer



Design of RC Beam for Flexure and Shear

- **Solution**
  - **Step No.7: Check for requirement of Shear reinforcement**

    From eq.(6.2), Design shear capacity of concrete can be calculated as;

$$\emptyset V_c = 2\emptyset\sqrt{f_c'}b_w d$$

$$= 2 \times 0.75\sqrt{3000}\ 12 \times 15 = 15281.5lb$$

$$\emptyset V_c = 15.28k$$

$\emptyset V_c < V_u$ $\rightarrow$ Shear reinforcement is required

Design of RC Beam for Flexure and Shear

- **Solution**
  - **Step No.8: Determination of stirrup spacing**

Calculate design spacing s_d using eq. (6.4)

$$s_d = \frac{\emptyset A_v f_y d}{V_u - \emptyset V_c}$$

Using 2 legged #3 stirrups, $A_v = 2A_b = 2(0.11) = 0.22in^2$

$$s_d = \frac{0.75 \times 0.22 \times 40 \times 15.5}{21.53 - 15.28}$$

$$s_d = 16.4"$$

Design of RC Beam for Flexure and Shear

- **Solution**
  - **Step No.8: Determination of stirrup spacing**

Calculate maximum spacing s_{max} using eq. (6.6)

$$s_{max} = Least\ of \begin{cases} \dfrac{A_v f_y}{50 b_w} = \dfrac{0.22 \times 40{,}000}{50 \times 12} = 14.67'' \\ \dfrac{A_v f_y}{0.75\sqrt{f_c{}'} b_w} = \dfrac{0.22 \times 40{,}000}{0.75\sqrt{3000} \times 12} = 17.85'' \\ \dfrac{d}{2} = \dfrac{15.5}{2} = 7.75'' \\ 24'' \end{cases}$$

$$s_{max} = 7.75'' < s_d$$

Design of RC Beam for Flexure and Shear

- **Solution**
  - **Step No.9: Apply necessary checks**

1. Check for Depth of Beam:

$$\emptyset V_s \leq \emptyset 8\sqrt{f_c{}'}b_w d$$

$$\emptyset V_s = V_u - \emptyset V_c = 21.53 - 15.28 = 6.25kip$$

$$\emptyset 8\sqrt{f_c{}'}b_w d = 4\emptyset V_c = 4 \times 15.28 = 61.12kip > \emptyset V_s \rightarrow OK!$$

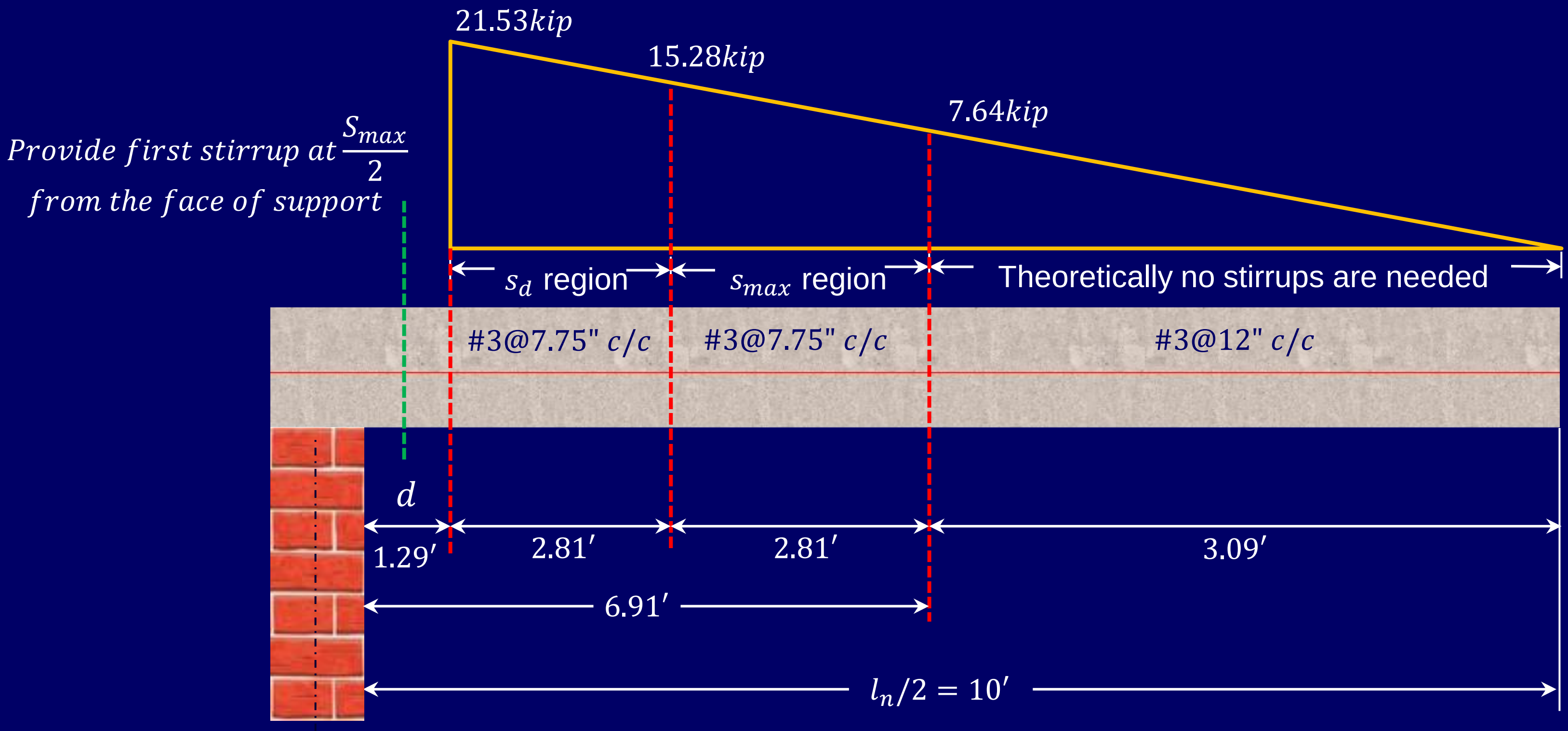
2. Check for Maximum spacing of stirrups:

$$\emptyset V_s \leq \emptyset 4\sqrt{f_c{}'}b_w d \quad , \quad \emptyset V_s = 6.25kip$$

$$\emptyset 4\sqrt{f_c{}'}b_w d = 2\emptyset V_c = 2 \times 15.28 = 30.56kip > \emptyset V_s \rightarrow OK!$$

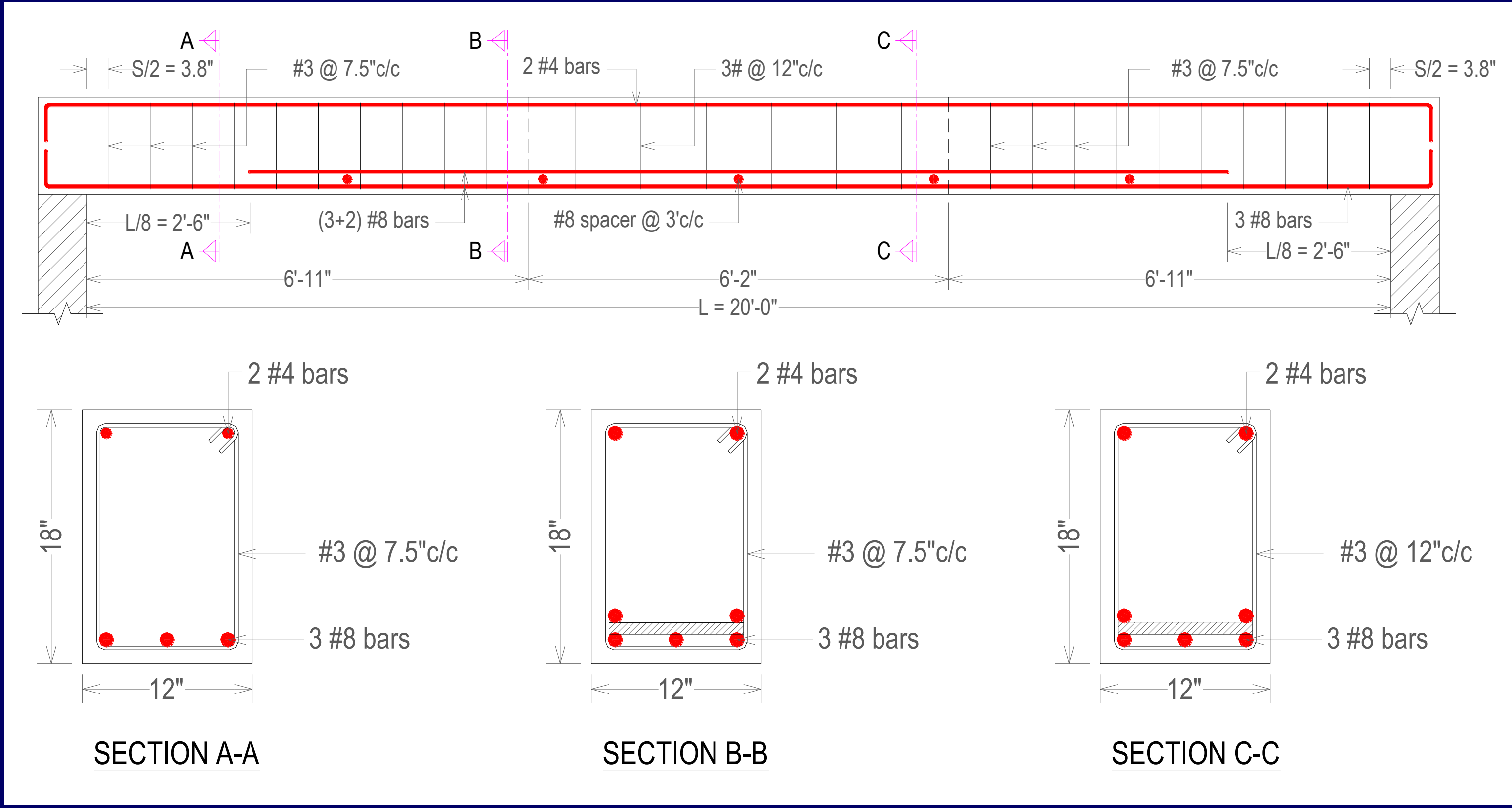
Design of RC Beam for Flexure and Shear

- **Solution**
  - **Step No.10: Detailing of shear reinforcement**



Design of RC Beam for Flexure and Shear

- **Solution**
  - **Step No.11: Drafting**



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- **Example 6.2 (class activity)**
  - Using the data provided in the following table
    - ***Design*** 12″ x 18″, 20 feet long beam for shear
    - ***Design*** 12″ x 24″, 20 feet long beam for shear

S.No.	Concrete Compressive Strength f_c' (ksi)	Rebar Tensile yield Strength f_y (ksi)	Shear force V_u (kips)
1	3	60	35
2	4	60	45
3	4	40	30

References

- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-19)