# Lecture 10

# Deflection and Control of Cracking in Reinforced Concrete Members

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# Learning Outcomes

- **At the end of this lecture, students will be able to;**
  - ***Define*** short-term and long-term deflection
  - ***Explain*** crack formation
  - ***Identify*** relevant ACI codes to control deflection and crack width

# Part – I

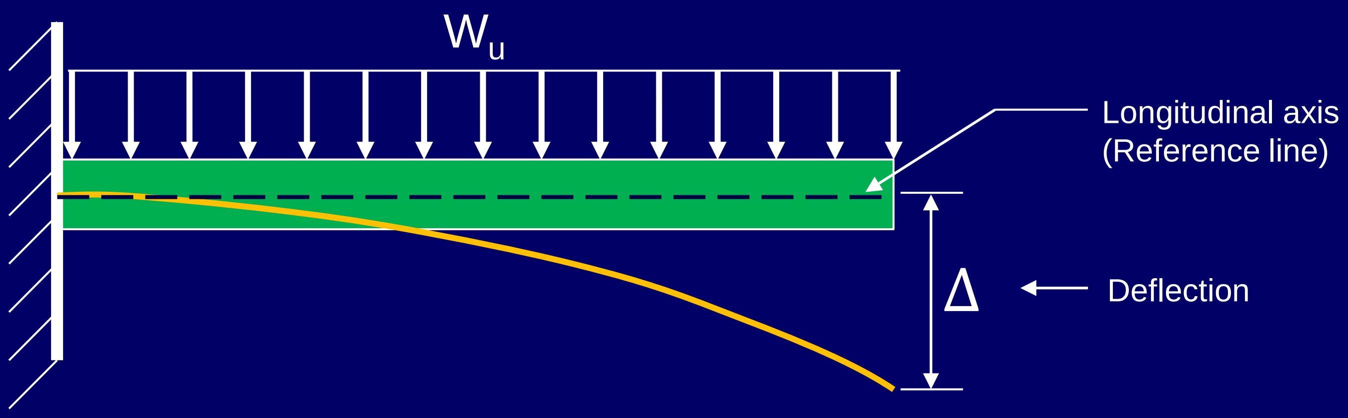
# Deflections in RC Frames

# Introduction to Deflections

- **Background**
  - In previous lectures, we have dealt primarily with the strength design of reinforced concrete beams.
  - Methods have been developed to ensure that beams will have a proper safety margin against failure in flexure or shear, or due to inadequate bond and anchorage of the reinforcement.
  - However, in addition to safety, serviceability requirements must also be ensured so that the structure performs well under service load conditions.
  - Deflection control is an important serviceability consideration in the structural design of concrete buildings.

# Introduction to Deflections

- **Deflection**
  - Deflection as defined by ACI concrete Terminology is the movement of a point on a structure or structural element, usually measured as a linear displacement or as succession displacements transverse to a reference line or axis.



# Introduction to Deflections

- **Deflection effects**
  - It is important to maintain control of deflections so that members designed mainly for strength at prescribed overloads will also perform well in normal service.
  - Excessive deflections can lead to cracking of supported walls and partitions, ill-fitting doors and windows, poor roof drainage, misalignment of sensitive machinery and equipment, and visually offensive sag etc.

# Introduction to Deflections

- **Types of deflections**

  1. **Short-term deflection**
     - The immediate deflection after casting and application of partial or full service loads.
  2. **Long-term deflection**
     - Deflection that occurs over time as a result of shrinkage and creep of concrete.

# Deflection in RC One-way Slabs and Beams

- **Immediate (short-term) Deflection**
  - Immediate deflection at a given load in a structure is calculated using equations of elastic deflection.

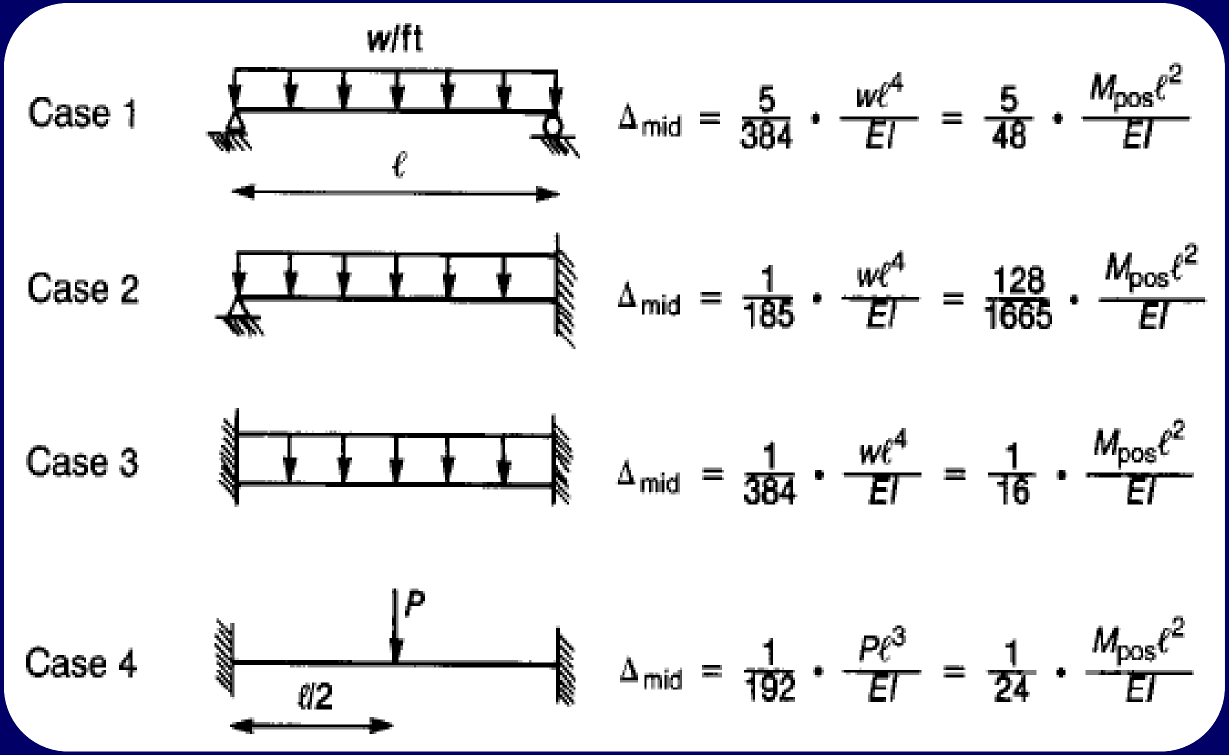
$$\Delta = \frac{f(loads, spans, supports)}{EI}$$

Where;

- $f(loads, spans, supports)$ is a function of load, span and support arrangement and
- $EI$ is the flexural rigidity.

# Deflection in RC One-way Slabs and Beams

- **Immediate (short-term) Deflection**
  - Deflections can be directly computed for different conditions of loading and end conditions as below:



# Deflection in RC One-way Slabs and Beams

- **Immediate (short-term) Deflection**
  - Determination of Modulus of Elasticity of concrete
    - As per ACI 318-19, Section 19.2.2.1, Modulus of elasticity $E_c$ for concrete shall be accordance with (a) or (b):
      - For values of $w_c$ between 90 and 160 lb/ft$^3$

$$E_c = w_c^{1.5} 33\sqrt{f_c'} \quad (psi) \qquad (19.2.2.1.a)$$

      - For Normalweight concrete

$$E_c = 57000\sqrt{f_c'} \quad (psi) \qquad (19.2.2.1.b)$$

    - $w_c$ in equation (a) is the equilibrium density of concrete mixture.

# Deflection in RC One-way Slabs and Beams

- **Immediate (short-term) Deflection**
  - Determination of Effective moment of Inertia ($I_e$)
    - In elastic deflection equation, the effective moment of inertia $I_e$ is calculated in accordance with ACI 318-19, Section 24.2.3.9a

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g - \left[\left(1 - \frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g$$

Where;

$M_a$ = Maximum service load moment for which deflections are being considered.

$M_{cr}$ = Cracking Moment = $f_r I_g / y_t$

$I_g$ = Gross moment of inertia

$I_{cr}$ = Moment of inertia of cracked section

# Deflection in RC One-way Slabs and Beams

- **Long-term Deflections**
  - Shrinkage and creep due to sustained loads cause additional long-term deflections over and above those which occur when loads are first placed on the structure.
  - Such deflections are influenced by:
    - Temperature,
    - Humidity,
    - Curing conditions,
    - Age at the time of loading,
    - Quantity of compression reinforcement, and
    - Magnitude of the sustained load.

# Deflection in RC One-way Slabs and Beams

- **Long-term Deflections**
  - Additional Long-term deflection resulting from the combined effect of creep and shrinkage is determined by multiplying the immediate deflection caused by the sustained load with the factor lD as given in ACI table 24.2.4.1.3.
  - $\Delta_{(cp+sh)} = \lambda_{\Delta} (\Delta_i)_{sus}$

$$\lambda_{\Delta} = \frac{\xi}{1 + 50\rho'}$$

$\rho' = A_s'/bd$

| Table 24.2.4.1.3 | |
|---|---|
| **Sustained load duration, months** | **Time dependent factor, $\xi$** |
| 3 months | 1.0 |
| 6 months | 1.2 |
| 12 months | 1.4 |
| 60 or more | 2.0 |

# Deflection in RC One-way Slabs and Beams

- **Long-term Deflections**
  - It is important to note here that long term deflections are function of immediate deflections due to sustained load only i.e.

    $\Delta_{(cp+sh)} = \lambda_{\Delta}(\Delta_i)_{sus}$

  - Sustained loads are loads that are permanently applied on the structure e.g., dead loads, superimposed dead loads and live loads kept on the structure for long period.

# Deflection in RC One-way Slabs and Beams

- **Deflection Control according to ACI code:**

  1. **Direct Approach of deflection control**

     - In direct approach, the deflections are controlled by restricting their magnitude to the permitted limits recommended by ACI 318 Code.

     - Deflections are said to be within limits if the combined effect of immediate and long-term deflections does not exceed the limits specified in ACI table 24.2.2.(shown on next slide)

# Deflection in RC One-way Slabs and Beams

- **Deflection Control according to ACI code:**

1. **Direct Approach of deflection control**

<table>
<tr><th>Member</th><th colspan="2">Condition</th><th>Deflection to be considered</th><th>Deflection limitation</th></tr>
<tr><td>Flat roofs</td><td colspan="2" rowspan="2">Not supporting or attached to nonstructural elements likely to be damaged by large deflections Immediate deflection due to L</td><td>Immediate deflection due to maximum of Lr, S, and R</td><td>$l/180$[1]</td></tr>
<tr><td>Floors</td><td>$l/360$</td><td></td></tr>
<tr><td rowspan="2">Roofs or floors</td><td rowspan="2">Supporting or attached to non-structural elements</td><td>Likely to be damaged by large deflections</td><td rowspan="2">That part of the total deflection occurring after attachment of nonstructural elements, which is the sum of the time-dependent deflection due to all sustained loads and the immediate deflection due to any additional live load[2]</td><td>$l/480$[3]</td></tr>
<tr><td>Not likely to be damaged by large deflections</td><td>$l/240$[4]</td></tr>
</table>

[1]Limit not intended to safeguard against ponding.

[2]Time-dependent deflection shall be calculated in accordance with 24.2.4 but shall be permitted to be reduced by amount of deflection calculated to occur before attachment of nonstructural elements.

[3] Limit shall be permitted to be exceeded if measures are taken to prevent damage to supported or attached elements.

[4]Limit shall not exceed tolerance provided for nonstructural elements.

# Deflection in RC One-way Slabs and Beams

- **Deflection Control according to ACI code:**

  2. **Indirect Approach of deflection control**

     - Deflections can also be controlled indirectly by limiting the thickness (depth) of structural members.
     - Deflections are said to be within limits if the thickness of beams and one-way slabs are greater than the minimum requirements given in ACI table 7.3.1.1 and 9.3.1.1 (as shown on next slide).
     - However, this method is applicable only to the cases of loadings and spans commonly experienced in buildings and cannot be used for unusually large values of loading and span.

# Deflection in RC One-way Slabs and Beams

- **Deflection Control according to ACI code:**

2. **Indirect Approach of deflection control**

**Table 7.3.1.1— Minimum thickness of solid nonprestressed one-way slabs**

| Support condition | Minimum h |
|---|---|
| Simply supported | $l/20$ |
| One end continuous | $l/24$ |
| Both ends continuous | $l/28$ |
| Cantilever | $l/10$ |

**Table 9.3.1.1— Minimum depth of nonprestressed beam**

| Support condition | Minimum h |
|---|---|
| Simply supported | $l/16$ |
| One end continuous | $l/18.5$ |
| Both ends continuous | $l/21$ |
| Cantilever | $l/8$ |

- $l$ = Span length (already defined)
- For $f_y$ other than 60,000 psi, the expressions in Table 7.3.1.1 and 9.3.1.1 shall be multiplied by (0.4 + $f_y$ /100,000)

# Part – I

# Cracking in RC Frames

# Cracking in RC Members

- **Crack Formation**
  - All RC beams crack, generally starting at loads well below service level, and possibly even prior to loading due to restrained shrinkage.
  - In a well-designed beam, flexural cracks are fine, so-called hairline cracks, almost invisible to a casual observer, and they permit little if any corrosion to the reinforcement.

# Cracking in RC Members

- **Crack Formation**
  - As loads are gradually increased above the cracking load, both the number and width of cracks increase, and at service load level a maximum width of crack of about 0.016 inch (0.40 mm) is typical.
  - If loads are further increased, crack widths increase further, although the number of cracks do not increase substantially.
  - The limiting value of crack width both for interior and exterior exposures is taken as 0.016 inch.

**(ACI 318-19, Section 24.3.2.1)**

# Cracking in RC Members

- **ACI Code Provisions for crack control**
  - There are two approaches of controlling crack in reinforced concrete members.

1. **Direct approach:**
   - Expected crack width is calculated and is compared with the maximum permissible limit provided by the code.
2. **Indirect approach:**
   - Cracking in RC member can be controlled by imposing limit on the maximum bar spacing.

# Cracking in RC Members

- **ACI Code Provisions for crack control**
  - The maximum center-to-center spacing between the adjacent bars shall not exceed the limits in Table 24.3.2.
  - For deformed bars or wires, maximum spacing $s_{max}$ is given by;

$$s_{max} = Least\ of\ \ 15\left(\frac{40{,}000}{f_s}\right) - 2.5C_c \quad and\ \ 12\left(\frac{40{,}000}{f_s}\right)$$

Where;

$f_s$ = The bar stress in $ksi$ under service condition.

$C_c$ = The clear cover in inches from the nearest surface in tension to the surface of the flexural tension reinforcement.

# Cracking in RC Members

- **ACI Code Provisions for crack control**
  - As per ACI 318-19, Section 24.3.2.1, stress $f_s$ in deformed reinforcement closest to the tension face at service loads shall be calculated based on the unfactored moment, or it shall be permitted to take $f_s$ as $(2/3)_y$.
  - Hence by putting the value of $f_s$ in previous equations, we get

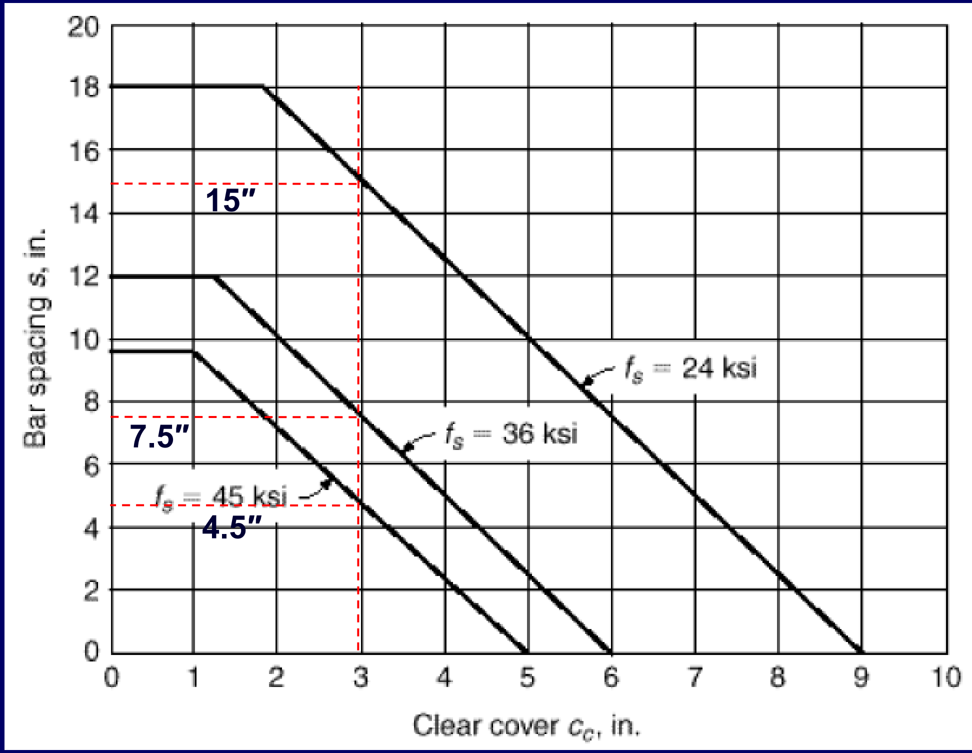
$$s_{max} = Least\ of\ \ \frac{900}{f_y} - 2.5C_c\ \ \ and\ \ \frac{720}{f_y}$$

(where $f_s$ $is\ in\ ksi$ )

  - Note that the condition $f_s = (2/3)f_y$ is for full service-load condition. For loading less than that, $f_s$ shall be calculated.

# Maximum Spacing Requirement

- **Plot of Bar spacing vs Concrete cover**
  - For various values of concrete cover and $f_s$ of 24, 36 and 45 ksi,
  - The maximum center-to-center spacing between the reinforcing bars closest to the surface of a tension member to control crack width can be plotted as shown.



# References

- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-19)