Lecture 02

Design of Singly Reinforced Beam in Flexure

By:

Prof. Dr. Qaisar Ali

Civil Engineering Department

UET Peshawar

drqaisarali@uetpeshawar.edu.pk

www.drqaisarali.com

Lecture Contents

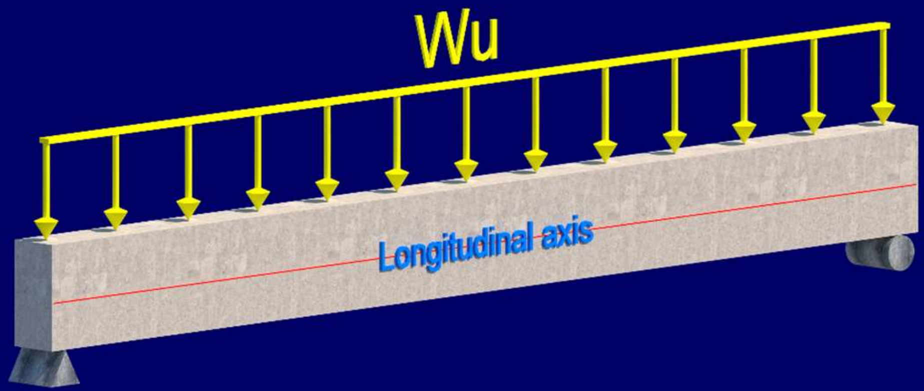
- General
- Behavior and mechanics of RC Beam under gravity load
- ACI 318 Code Provisions for RC beams
- Design Procedure
- Flexural Design of a Singly reinforced rectangular beam (Example)
- References
- Appendix

Learning Outcomes

- **At the end of this lecture, students will be able to;**
  - ***Define*** general terms related to the flexural design of RC beams
  - ***Explain*** behavior and mechanics of RC Beam under gravity load.
  - ***Identify*** relevant ACI 318 Code provisions for RC Beam.
  - ***Analyze*** and ***Design*** singly reinforced beam in flexure.

General

- **Beam**
  - A beam is generally a horizontal structural element/member that spans a distance between one or more supports and bears vertical loads transverse to its longitudinal axis.
  - Beam takes loads from the slab and transfers them safely to the columns.



General

- **Load effects on RC Beam**
  - A beam may be subjected to axial, flexural, shear, and torsional stresses.
  - Flexural stresses are the most prevalent of all, therefore the beam is classified as a "Flexural member".
  - Shear and Torsion can also be considered in some circumstances, although axial stresses are low and hence can be omitted.
  - This Lecture will go through the Design of a Rectangular Section Beam for Flexure.

General

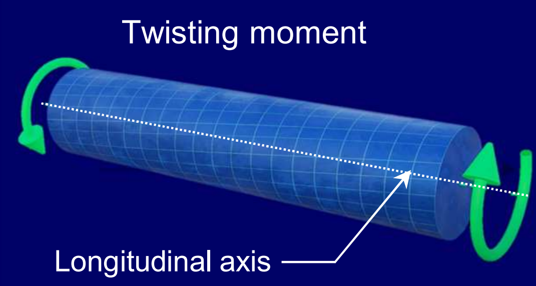
- **Moment**
  - The ability of a force to rotate a body is known as Moment.
  - Based on the axis of rotation, moment can be divided into two types

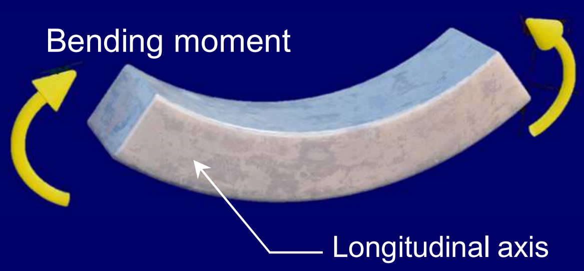
    i. Twisting Moment (Torque)

    - Moment about the longitudinal axis of a body is called twisting moment.

    ii. Bending Moment (Flexural Moment)

    - Moment transverse to the longitudinal axis of a body is known as bending moment.



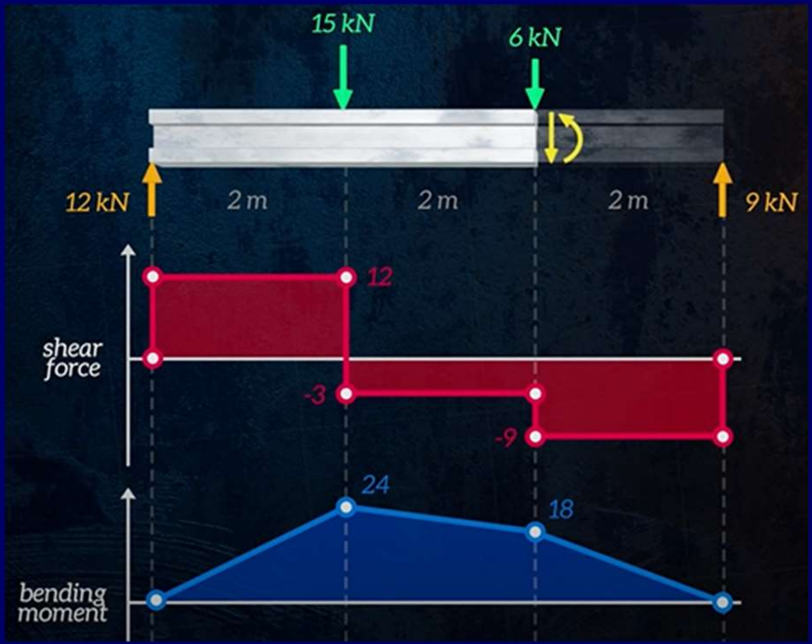


General

- **Moment**
  - Moment can also be classified as

    i. Applied Moment (M_A)

    - Moment produced in a body due to external loadings is known as Demand moment / External moment or Applied moment.
    - Depends on the loads, span length, and the support conditions of the member.
    - Can be obtained from Bending Moment Diagram.

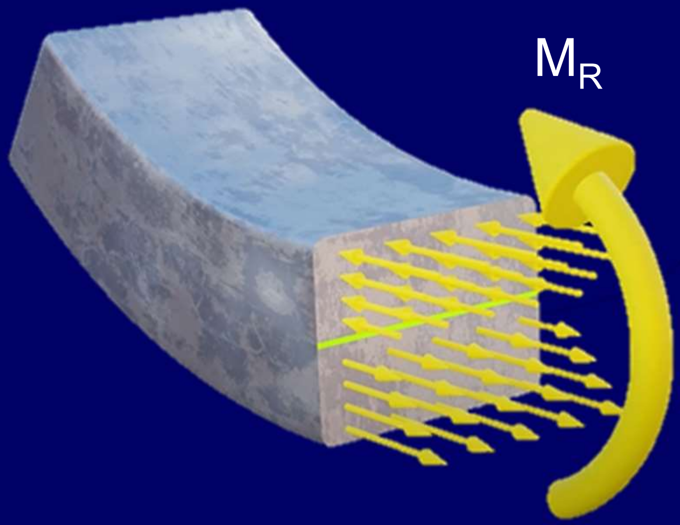


General

- **Moment**

ii. Resisting Moment (M_R)

- The resistance offered by a member to the applied moment is called the Resisting moment / Internal moment, or Flexural Capacity.
- Depends upon the Geometry (cross-section) of the member and the material strength.
- Can be obtained either from
  - Flexural Stress Formula or
  - Bending Stress Diagram.



General

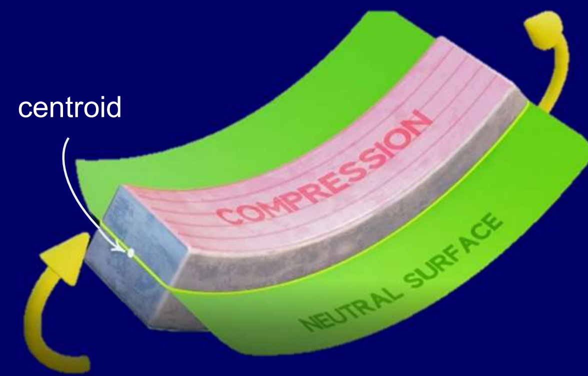
- **Flexural Stress Formula**
  - Stress caused by Bending moment is known as flexural or bending stress or flexural stress.
  - Flexural stress is a combination of compressive and tensile stresses.
  - Mathematically it is expressed as;

$$f = \frac{My}{I}$$

M = Applied moment

y = Depth of fiber from N.A

I = Moment of inertia of the section



General

- **Limitations of Flexural Stress Formula**
  - Flexural stress formula can be used to determine the Resisting moment of a member as;

$$M_R = \frac{fI}{y}$$

  - However, this formula assumes that the material is homogenous and linearly elastic.
  - The flexural formula is not applicable to reinforced concrete due to its nonhomogeneous and inelastic nature.
  - Therefore, the Resisting moment for RC beams will be determined from the Bending Stress Diagram of the section.

Behavior of RC Beam under Gravity Load

- **Behavior of RC Beam in Flexure**
  - Before moving on to the discussion of computing "Resisting moment" and for a better understanding of the behavior of an RC beam under gravity load, the next slide shows an experimental test of a beam subjected to progressive point load.
  - Students should carefully watch the video and take notes on the various stages that the beam goes through with progressively increasing load until it reaches the ultimate condition.

Behavior of RC Beam under Gravity Load

- **Experimental Test on RC Beam Subjected to Point Load**

Behavior of RC Beam under Gravity Load

- **Concluding Remarks on the Beam Test**
  - The beam test demonstrates that the beam passes through numerous stages from the start of loading until it collapses.
  - Initially, small unseen cracks form under load; as load increases, they become visible, spread, and multiply.
  - First crack in tension zone depletes concrete's tensile strength, transferring stresses to steel bars.
  - Eventually, cracks widen, indicating steel yielding and finally, the concrete in compression region crushes.

Behavior of RC Beam under Gravity Load

- **Concluding Remarks on the Beam Test**
  - The Demand Moment due to applied point load can easily be determined as; $M_A = PL/4$
  - The Resisting Moment will be calculated for three specific stages of the beam (although it can be determined at any stage).
    1. Uncracked Concrete – Elastic Stage
    2. Cracked Concrete (tension zone) – Elastic Stage
    3. Cracked Concrete (tension zone) – Inelastic (Ultimate Strength) Stage

Behavior of RC Beam under Gravity Load

- **Description of Stages**

1. **Uncracked Concrete – Elastic Stage**
   - At loads much lower than the ultimate, concrete remains uncracked in compression as well as in tension and the behavior of steel and concrete both is elastic.
2. **Cracked Concrete (tension zone) – Elastic Stage**
   - With the increase in load, concrete cracks in tension but remains uncracked in compression.
   - Concrete in compression and steel in tension both behave in an elastic manner.

Behavior of RC Beam under Gravity Load

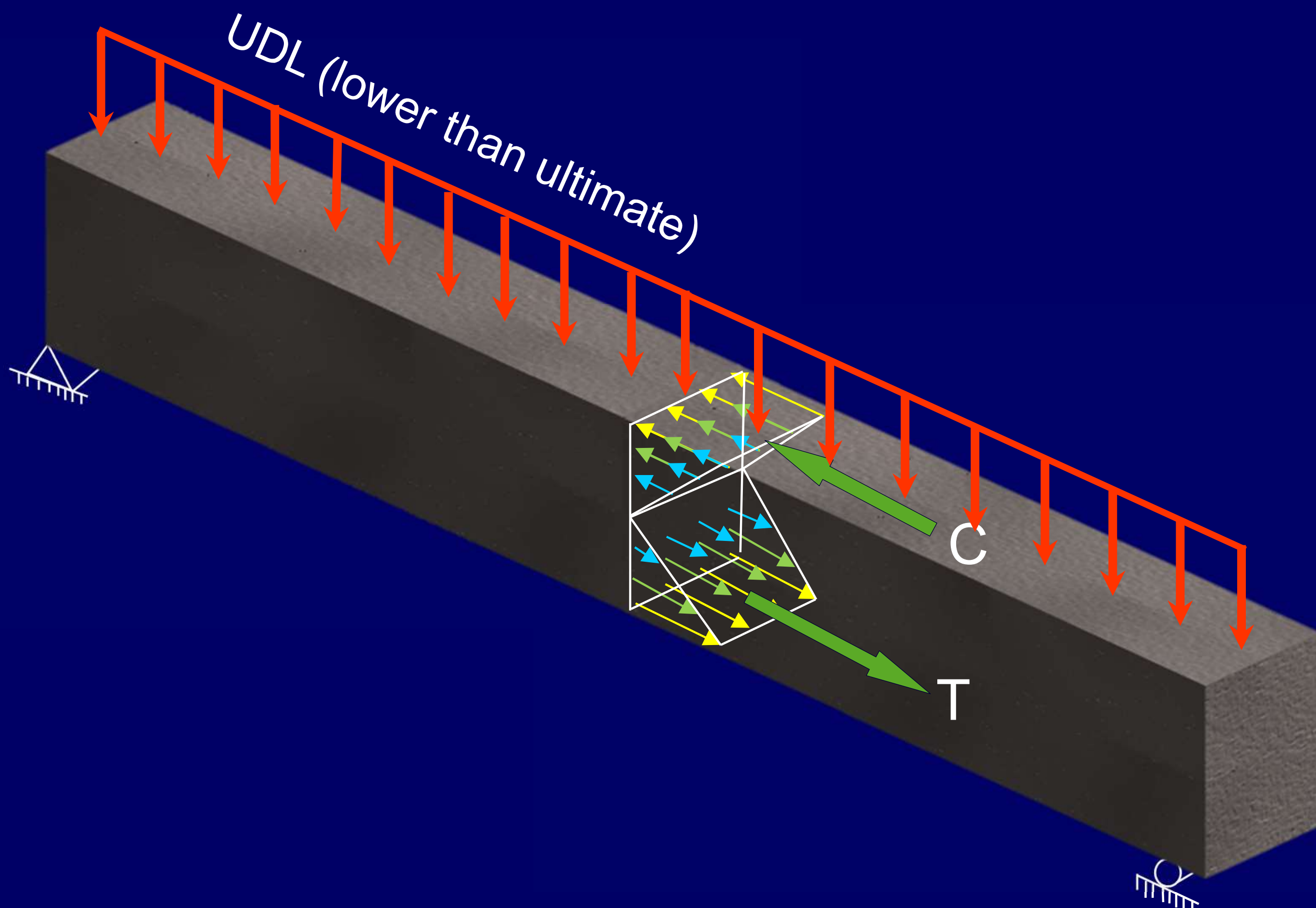
- **Description of Stages**

  3. **Cracked Concrete (tension zone) –(Ultimate Strength) Stage**

     - Concrete is cracked in tension. Concrete in compression and steel in tension both enter the inelastic range.
     - At collapse, steel yields and concrete in compression crushes.

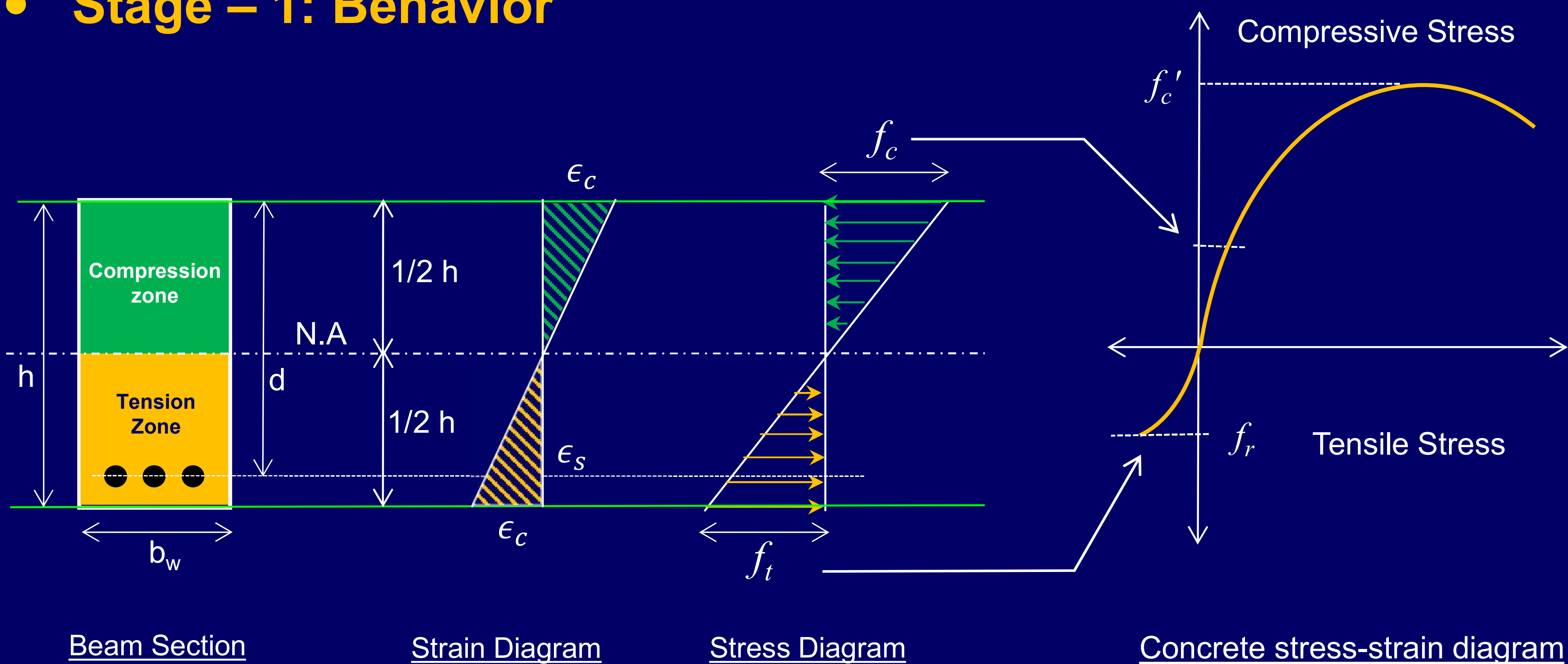
Behavior of RC Beam under Gravity Load

- **Stage – 1: Behavior**



Behavior of RC Beam under Gravity Load

- **Stage – 1: Behavior**

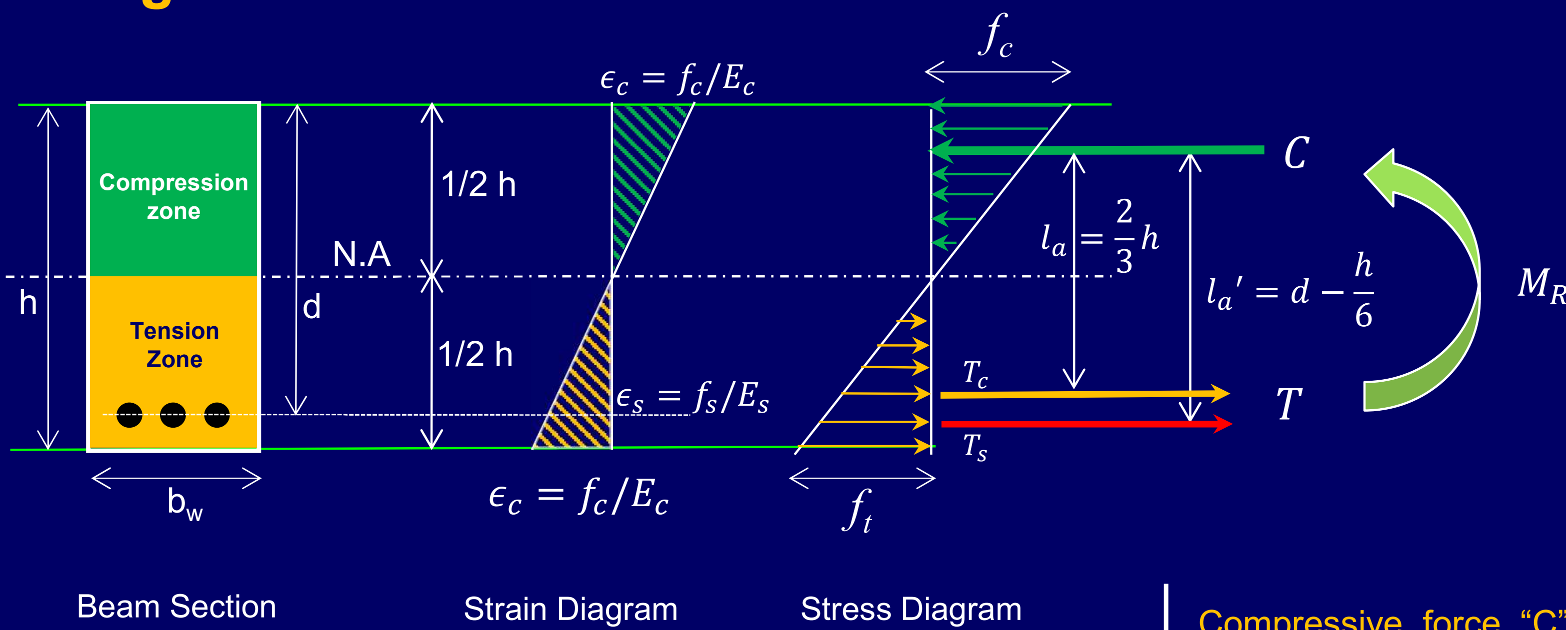


At this stage, the loading condition is such that the concrete in the tension zone reaches its tensile strength, that is $f_t = f_r$

while in the compression zone ; $f_c \ll f_c'$

Behavior of RC Beam under Gravity Load

- **Stage – 1: Calculations**



Compressive force "C" is balanced by T_c and T_s such that:

$C = T_c + T_s$

C = Compressive strength of concrete

T = Tensile strength of steel

M_R = Resisting moment produced by C and T

l_a = Perpendicular distance between C and T (Lever Arm)

Behavior of RC Beam under Gravity Load

- **Stage – 1: Calculations**
  - **Determination of Resisting Moment (M_R)**
    - Resisting Moment offered by both concrete and steel is given by;

$$M_R = M_c + M_s$$

$$M_R = T_c \times l_a + T_s \times l_a'$$

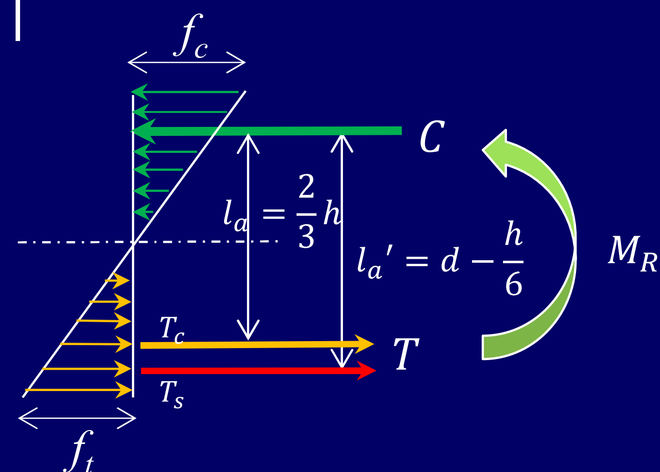
Here,
Mc = Moment due to Concrete and
M_s = Moment due to Steel

Putting

$$l_a = \frac{2}{3}h \;\; and \; l_a' = d - \frac{1}{6}h$$

We get,

$$M_R = \frac{2h}{3}T_c + \left(d - \frac{1}{6}h\right)T_s \qquad \text{------ (2.1)}$$



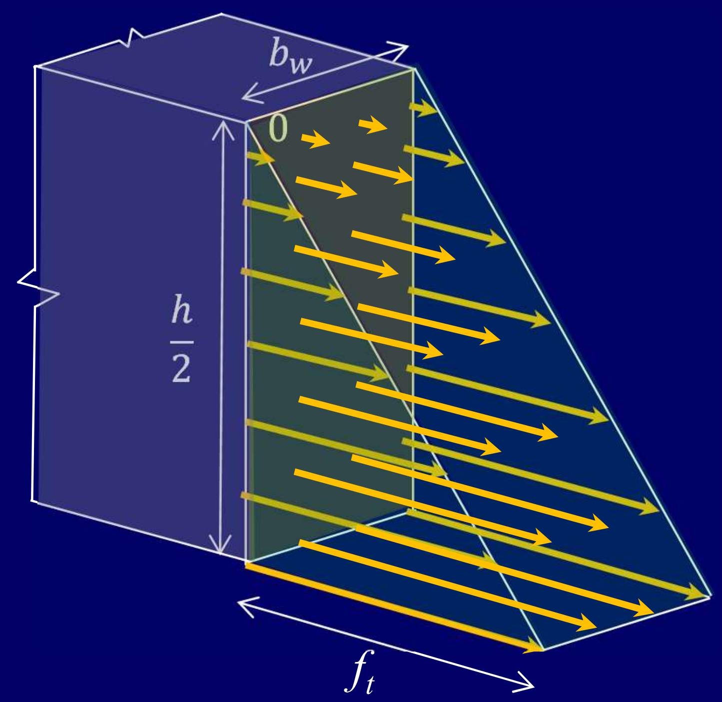
Behavior of RC Beam under Gravity Load

- **Stage – 1: Calculations**
  - **Determination of Resisting Moment (M_R)**
    - "T_c" is calculated as;

$$T_c = Average\ Stress \times \text{Area}$$

For triangular distribution we get

$$T_c = \underbrace{\left(\frac{0 + f_t}{2}\right)}_{\text{Average Stress}} \times \underbrace{\left(b_w \times \frac{h}{2}\right)}_{\text{Area}} = \frac{b_w h f_t}{4}$$



Therefore,

$$T_c = \frac{b_w h f_t}{4}$$

Behavior of RC Beam under Gravity Load

- **Stage – 1: Calculations**
  - **Determination of Resisting Moment (M_R)**
    - Tensile force of steel is

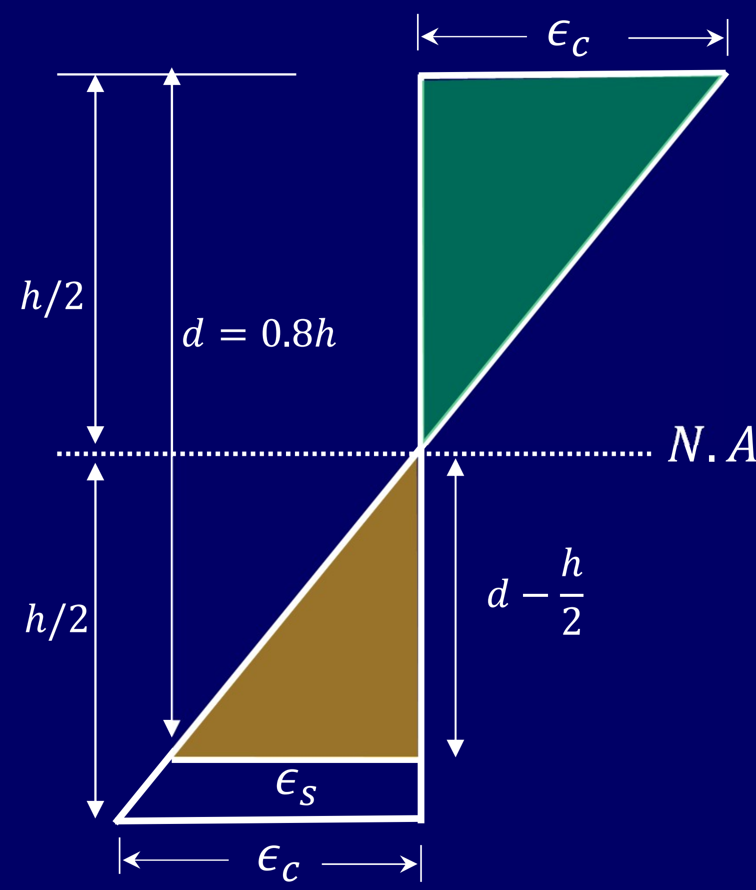
$$T_s = A_s \times f_s = A_s \times E_s \times \epsilon_s$$

ϵ_s can be calculated from the strain diagram as follows

$$\frac{\epsilon_s}{\epsilon_c} = \frac{d - h/2}{h/2}$$

$$\frac{\epsilon_s}{\epsilon_c} = \frac{0.8h - 0.5h}{0.5h} = 0.6$$

$$\epsilon_s = 0.6\epsilon_c$$



Behavior of RC Beam under Gravity Load

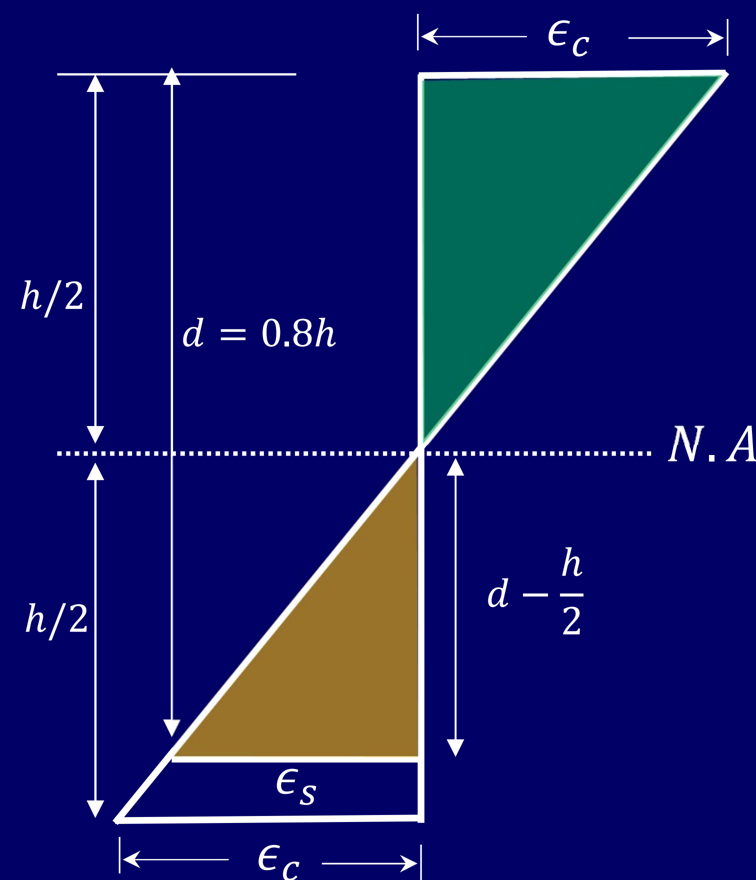
- **Stage – 1: Calculations**
  - **Determination of Resisting Moment (M_R)**
    - Concrete stain ϵ_c is given by

$$\epsilon_c = \frac{f_t}{E_c} = \frac{7.5\sqrt{f_c'}}{57000\sqrt{f_c'}} = \frac{1}{7600}$$

$$\epsilon_s = 0.6\epsilon_c = \frac{0.6}{7600}$$

    - So finally, T_s is;

$$T_s = A_s \times 29000 \times \frac{0.6}{7600} = 2.3A_s$$



Behavior of RC Beam under Gravity Load

- **Stage – 1: Calculations**
  - **Determination of Resisting Moment (M_R)**
    - Putting values of T_c and T_s in eq. (2.1), gives

$$M_R = \frac{2h}{3} \times \frac{b_w h f_t}{4} + \left(d - \frac{h}{6}\right) \times 2.3A_s$$

$$M_R = \frac{b_w h^2}{6} f_t + 2.3A_s\left(d - \frac{h}{6}\right)$$

$$f_t = f_r = 7.5\sqrt{f_c{}'}$$

$$M_R = \frac{b_w h^2}{6} \times 7.5\sqrt{f_c}{}' + 2.3A_s\left(d - \frac{h}{6}\right)$$

Behavior of RC Beam under Gravity Load

- **Stage – 1: Calculations**
  - **Determination of Resisting Moment (M_R)**

$$M_R = \underbrace{1.25\sqrt{f_c{}'}b_w h^2}_{M_c} + \underbrace{2.3A_s\left(d - \frac{h}{6}\right)}_{M_s} \quad \text{------ (2.2)}$$

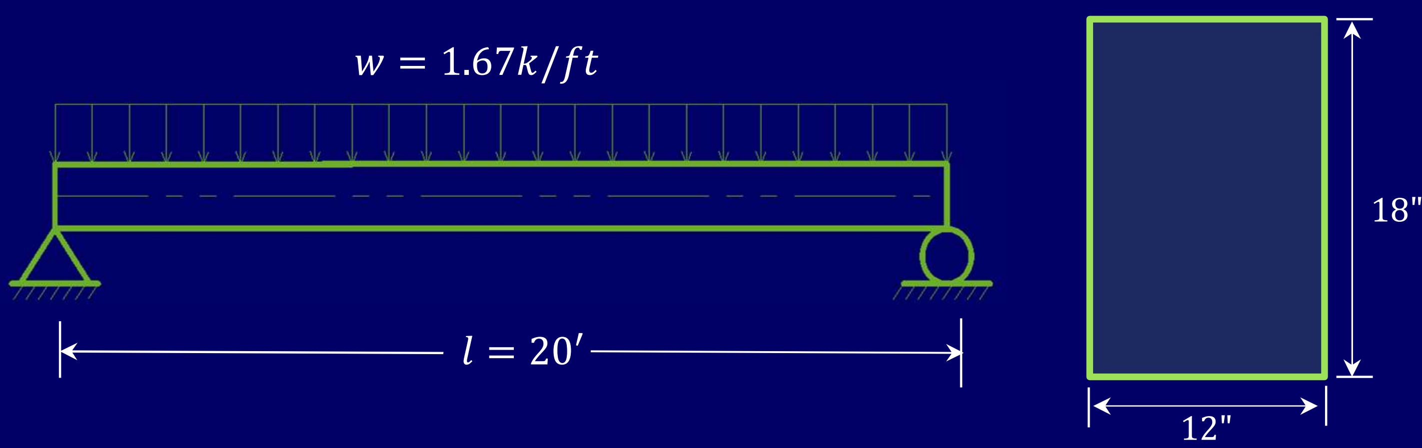
If the beam is treated as "Plain concrete", then $M_s = 0$ and eq. 2.2 reduces to,

$$M_R = M_c = 1.25\sqrt{f_c{}'}b_w h^2$$

- Eq. 2.2 is the required Resisting moment / Design moment or Flexural capacity of the beam.
- Any applied moment greater than this moment will crack the beam, Therefore, it can also be called the "cracking Moment".

Behavior of RC Beam under Gravity Load

- **Stage – 1: Example 2.1**
  - A simply supported beam having a span length of 20ft is subjected to a uniformly distributed load of 1.67k/ft as shown in the figure below. Material properties are; $f_c' = 3000psi \quad and \quad f_y = 40{,}000psi$



Behavior of RC Beam under Gravity Load

- **Stage – 1: Example 2.1**

A. Neglecting the contribution of reinforcing steel,

i. *Calculate* the Demand and Resisting moments and check whether the beam fails or not.

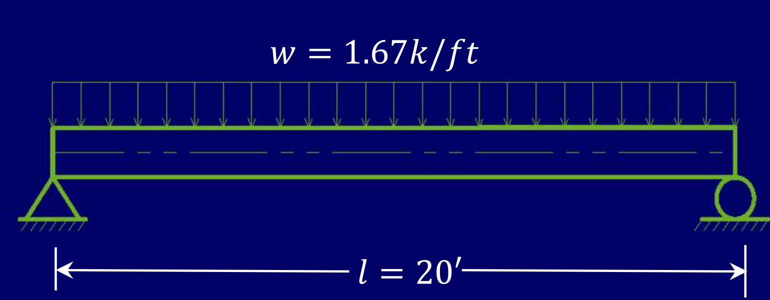
ii. *Determine* how much compressive strength of concrete will be required to resist the given demand if the beam cross-sections are restricted?.

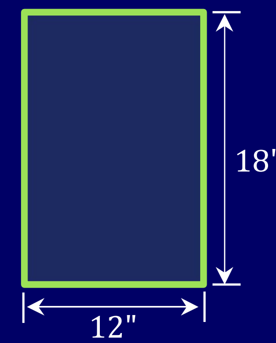
iii. *Compute* the minimum depth "h" of beam required to meet the given demand, Keeping the concrete strength constant.

B. If the contribution of steel is considered, then calculate the area of steel required for the applied moment.

Behavior of RC Beam under Gravity Load

- **Stage – 1: Example 2.1**
  - **Solution:**
    - Part (A)(i)





- Applied moment

$$M_A = \frac{wl^2}{8} = \frac{1.67 \times 20^2}{8}$$

$$M_A = 83.5 kip.ft \quad or\ 1002\ in.kip$$

- Resisting moment

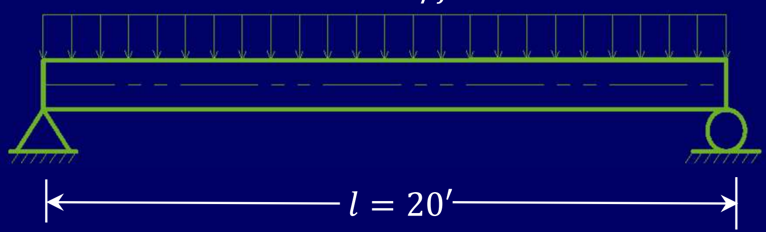
$$M_R = 1.25\sqrt{f_c{}'}b_w h^2 = 1.25\sqrt{3000} \times 12 \times 18^2 = 266193.16\ lb.in$$

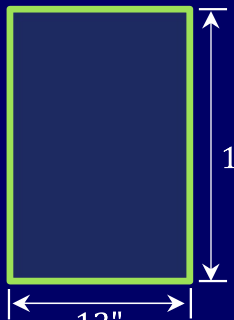
$$M_R = 266.19\ in.kip$$

Since $M_R \ll M_A \rightarrow The\ beam\ will\ fail$

Behavior of RC Beam under Gravity Load

- **Stage – 1: Example 2.1**
  - **Solution:**





- Part (A)(ii)

$$M_R = 1.25\sqrt{f_c{}'}b_w h^2$$

- For No failure, $M_R \geq M_A$
- Taking $M_R = M_A$ (FS is ignored for the sake of simplicity)

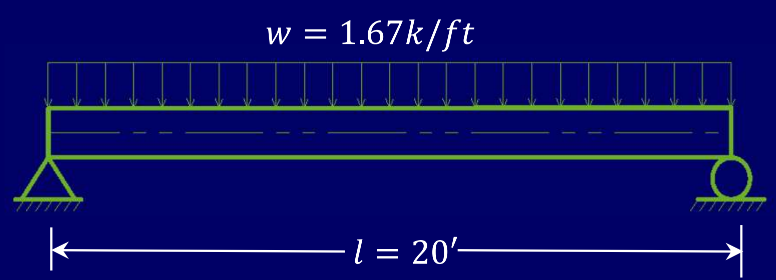
$$\Rightarrow f_c{}' = \left(\frac{M_A}{1.25 b_w h^2}\right)^2$$

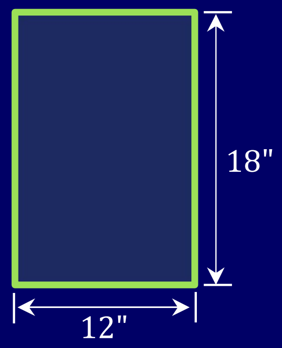
$$\Rightarrow f_c' = \left(\frac{1002 \times 1000}{1.25 \times 12 \times 18^2}\right) = 42507.24psi \dashrightarrow$$

Imagine this much compressive strength of concrete with a typical strength of 3000psi !

Behavior of RC Beam under Gravity Load

- **Stage – 1: Example 2.1**
  - **Solution:**





- Part (A)(iii)

$$M_R = 1.25\sqrt{f_c'}b_w h^2$$

- For No failure, $M_R \geq M_A$
- Taking $M_R = M_A$ (FS is ignored for the sake of simplicity)

$$\Rightarrow \ h = \sqrt{\frac{M_A}{1.25 b_w \sqrt{f_c'}}}$$

$$\Rightarrow h = \sqrt{\frac{1002 \times 1000}{1.25 \times 12\sqrt{3000}}} = 34.92" \approx 3'$$

Behavior of RC Beam under Gravity Load

- **Stage – 1: Example 2.1**
  - **Solution:**
    - Part (B)

$$M_R = 1.25\sqrt{f_c{}'}b_w h^2 + 2.3A_s\left(d - \frac{1}{6}h\right)$$

- For No failure, $M_R \geq M_A$
- Taking $M_R = M_A$

$$1.25\sqrt{f_c{}'}b_w h^2 + 2.3A_s\left(d - \frac{1}{6}h\right) = M_A$$

$$266.19 + 2.3A_s\left(15.5 - \frac{18}{6}\right) = 1002$$

$$A_s = 25.59in^2$$

FS is ignored for the sake of simplicity

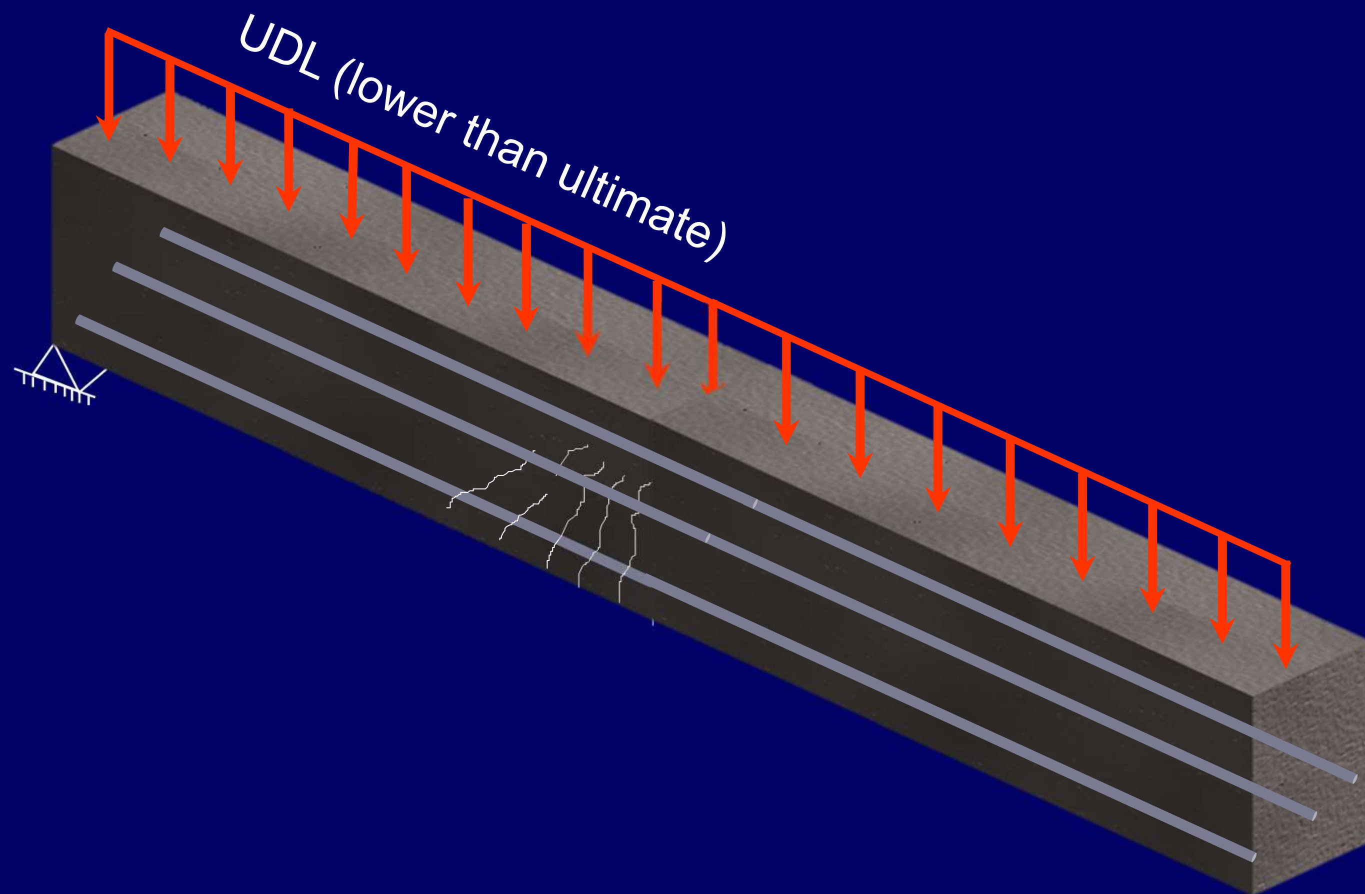
$$M_c = 1.25\sqrt{f_c{}'}b_w h^2 = 266.19in.kip$$

$$M_A = 1002\ in.kip$$

$$d = h - 2.5 = 18 - 2.5 = 15.5"$$

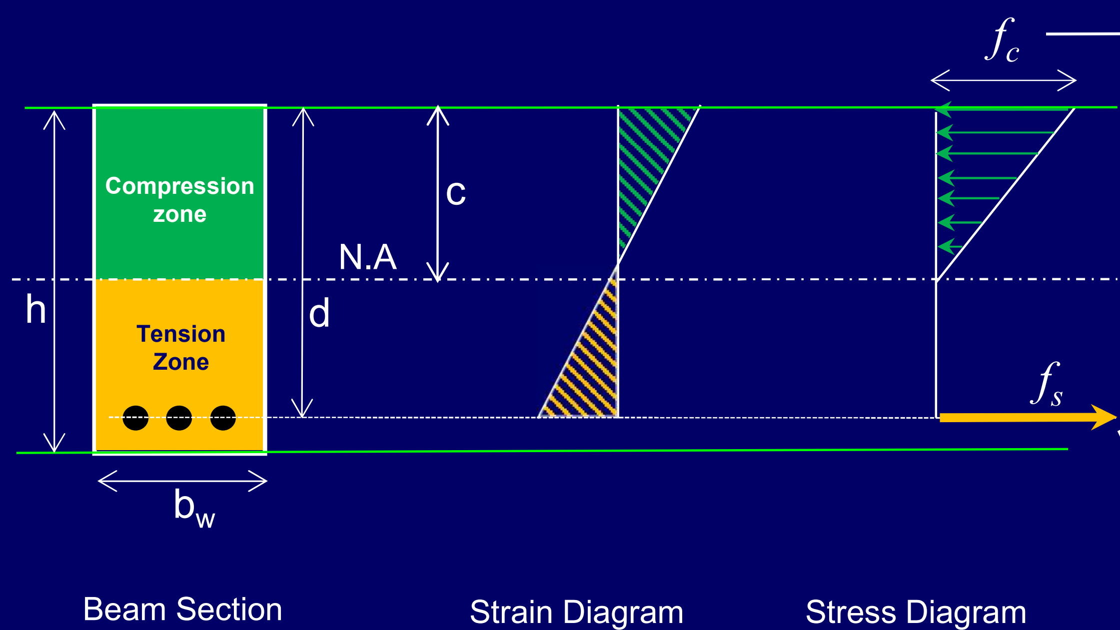
Behavior of RC Beam under Gravity Load

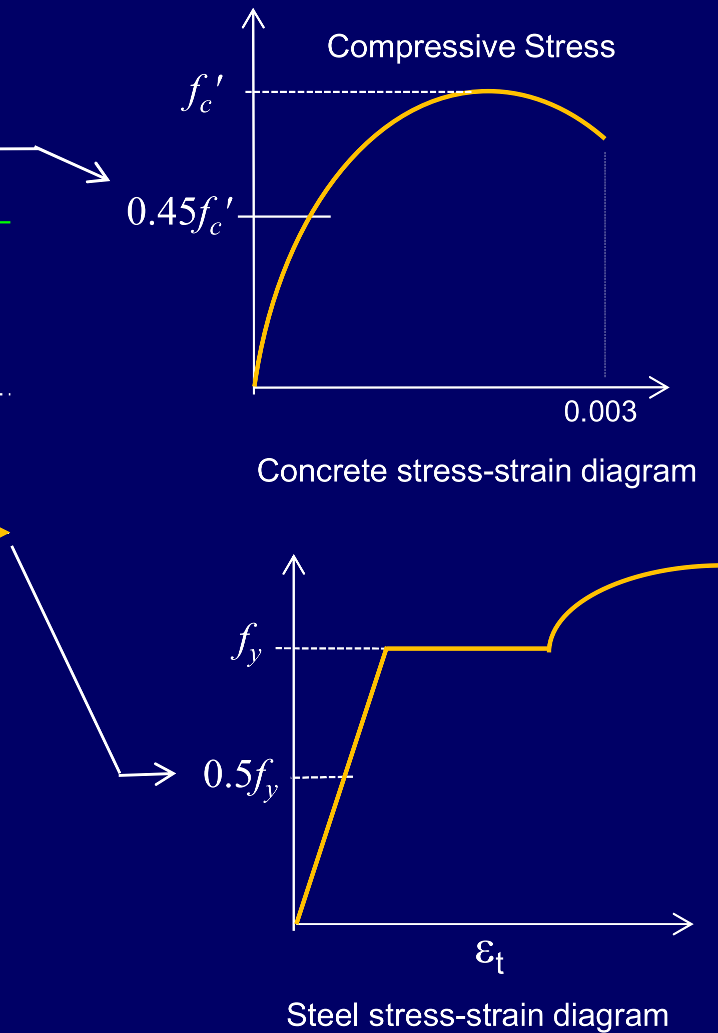
- **Stage – 2: Behavior**



Behavior of RC Beam under Gravity Load

- **Stage – 2: Behavior**





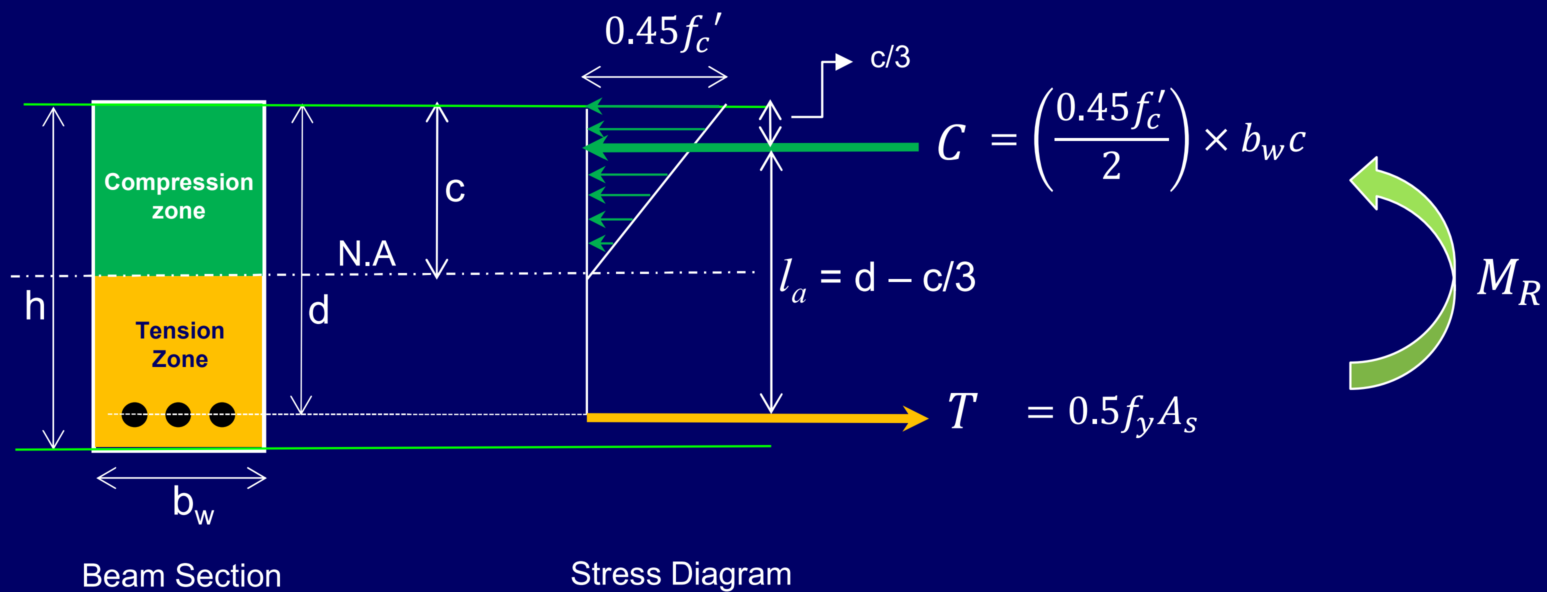
At this stage, the loading condition is such that;

$f_c = 0.45f_c'$ and

$f_s = 0.5f_y$

Behavior of RC Beam under Gravity Load

- **Stage – 2: Calculations**



Calculating Resisting moment

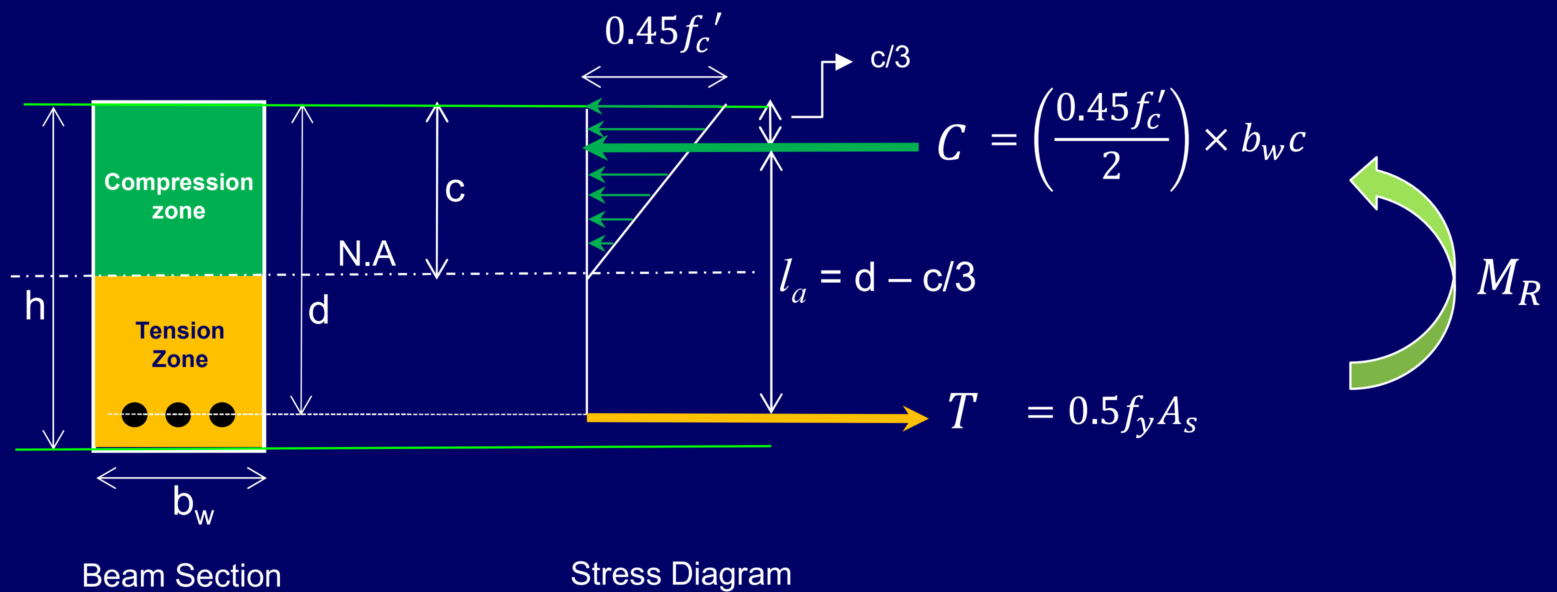
$$M_R = \cancelto{0}{M_c} + M_s = T \times l_a = (0.5 f_y A_s) \times \left(d - \frac{c}{3}\right)$$

$$M_R = 0.5 A_S f_y \left(d - \frac{c}{3}\right) \quad \text{------ (2.3)}$$

M_c shall be neglected as per ACI 318, 22.2

Behavior of RC Beam under Gravity Load

- **Stage – 2: Calculations**



Equating horizontal forces;

$$C = T \;\; \Rightarrow \;\; \frac{0.45 f_c{}'}{2} \times (b_w \, c) \; = A_s 0.5 f_y$$

Which on simplifying gives, $c = \dfrac{A_s f_y}{0.45 f_c' b_w}$

Behavior of RC Beam under Gravity Load

- **Stage – 2: Calculations**
  - Putting the value of “c” in eq. (2.3) gives

$$M_R = 0.5A_S f_y\left(d - \frac{c}{3}\right) = 0.5A_S f_y\left(d - \frac{A_s f_y}{3 \times 0.45 f_c' b_w}\right)$$

  - The resisting moment is calculated assuming the area of steel A_s which is then compared with the applied moment to check whether the beam fails or not.
  - However, instead of assuming, it is preferable to compute the area of steel required for a given demand by equating the resisting and applied moments, $M_R = M_A$, as discussed on the next slide.

Behavior of RC Beam under Gravity Load

- **Stage – 2: Calculations**
  - From eq. (2.3), we have

$$M_R = 0.5A_S f_y\left(d - \frac{c}{3}\right)$$

equating $M_R = M_A$

$$0.5A_S f_y\left(d - \frac{c}{3}\right) = M_A$$

which on solving for A_s gives

$$\boxed{A_s = \frac{M_A}{0.5f_y\left(d - \frac{c}{3}\right)}}$$ ------ (2.4)

Behavior of RC Beam under Gravity Load

- **Stage – 2: Calculations**
  - Area of steel A_s can be determined by the Trial and Success method as described below.

1. Assume the value of “c”
2. Calculate the area of steel using eq.(2.3)

$$A_s = \frac{M_A}{0.5f_y(d - c/3)}$$

3. Confirm the value of “c” using

$$c = \frac{A_s f_y}{0.45 f_c' b_w}$$

4. Repeat the process until the same A_s value is obtained from the two consecutive trials.

Behavior of RC Beam under Gravity Load

- **Stage – 2: Example 2.2**
  - Using the data from Example 2.1, calculate the area of steel required for the beam corresponding to stage 2.
  - **Solution**
    - Trial 1: Choosing c = h/2 = 9" and d = h – 2.5 = 15.5"

$$A_s = \frac{1002}{0.5(40)(15.5 - 9/3)} = 4\ in^2$$

$$\Rightarrow \quad c = \frac{4 \times 40}{0.45 \times 3 \times 12} = 9.88''$$

    - Trial 2: Choosing c = 9.88"

$$A_s = \frac{1002}{0.5(40)(15.5 - 9.88/3)} = 4.10\ in^2$$

Behavior of RC Beam under Gravity Load

- **Stage – 2: Example 2.2**
  - **Solution**
    - Trial 2:

$$\Rightarrow \quad c = \frac{4.10 \times 40}{0.45 \times 3 \times 12} = 10.12''$$

    - Trial 3: Choosing c = 10.12"

$$A_s = \frac{1002}{0.5(40)(15.5 - 10.12/3)} = 4.13\ in^2$$

$$\Rightarrow \quad c = \frac{4.13 \times 40}{0.45 \times 3 \times 12} = 10.2''$$

Trial 4: Choosing c = 10.2" and $A_s = 4.14\text{in}^2$

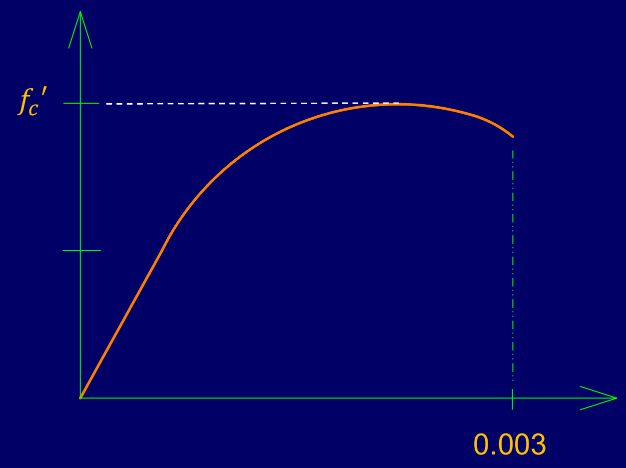
Hence the required area of steel is 4.14in^2

Behavior of RC Beam under Gravity Load

- **Stage – 3**
  - Stage 3 is the ultimate or final stage in which both concrete in compression and steel in tension enter the inelastic state.
  - Because of the several possible situations of failure in this stage, defining the "Ultimate stage" is quite difficult.
  - Furthermore, due to severe concrete cracking and the complexity of the stress-strain relationship at this point, calculating the resisting moment without making some key assumptions is extremely challenging.
  - Therefore, the definition of the “ultimate stage” and the “basic assumptions” as per the ACI Code are discussed next.

Behavior of RC Beam under Gravity Load

- **Stage – 3**
  - **Definition of the Ultimate Stage**
    - As per ACI 318-19, R21.2.2, "the ultimate stage is said to be reached when the concrete strain at the extreme fiber in the compression zone reaches a value of 0.003".



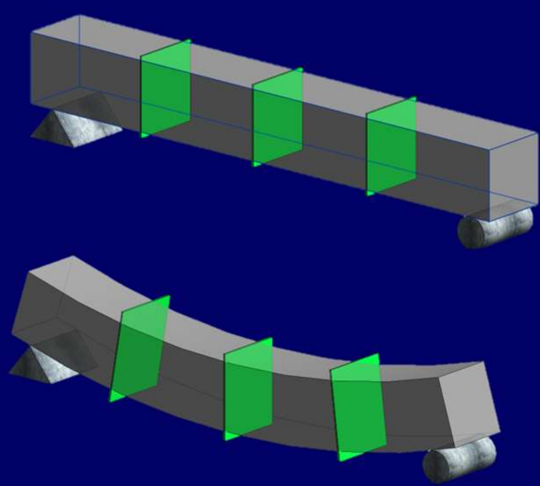
Stress-strain curve of concrete

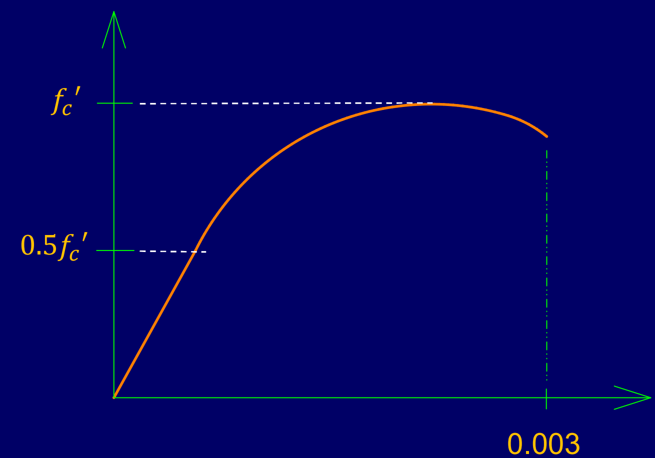
Behavior of RC Beam under Gravity Load

- **Stage – 3**
  - **Fundamental Assumptions (ACI 318-19, section 22.2)**
    - A plane section before bending remains plane after bending.
    - Stress and strain in concrete are approximately proportional up to moderate loads (concrete stress ≤ $0.5f_c'$). When the load is increased, the variation in the concrete stress is no longer linear.

Before bending

After bending

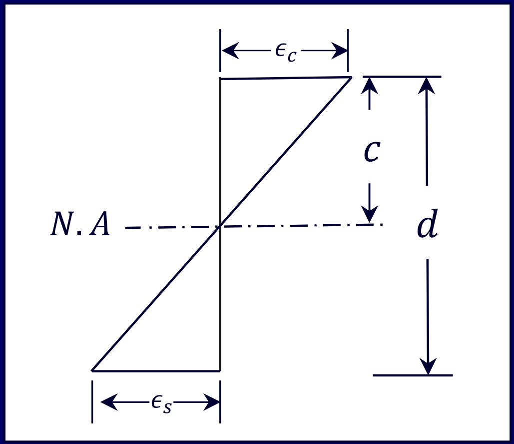


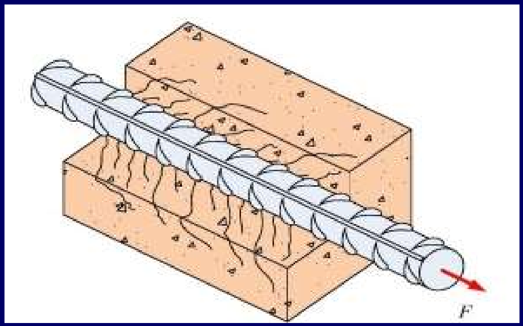


Stress-strain curve of concrete

Behavior of RC Beam under Gravity Load

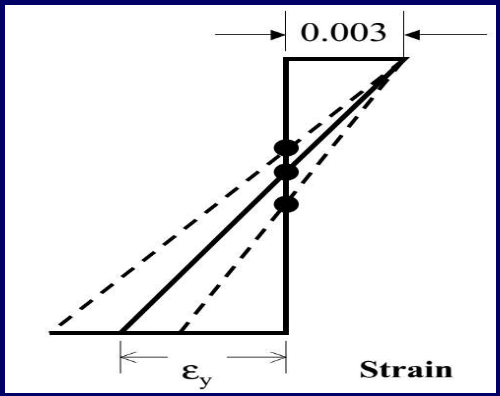
- **Stage – 3**
  - **Fundamental Assumptions (ACI 318-19, section 22.2)**
    - Strain in concrete and reinforcement shall be assumed proportional to the distance from the neutral axis.
    - Tensile strength of concrete is neglected in the design of reinforced concrete beams.
    - The bond between the steel and concrete is PERFECT and NO SLIP occurs.

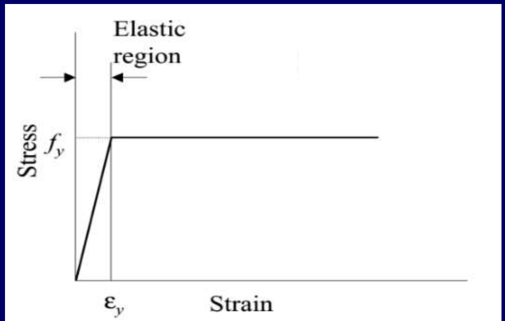




Behavior of RC Beam under Gravity Load

- **Stage – 3**
  - **Fundamental Assumptions (ACI 318-19, section 22.2)**
    - The maximum usable concrete compressive strain at the extreme fiber is assumed to be 0.003.
    - The steel is assumed to be uniformly strained to the strain that exists at the level of the centroid of the steel.
      - If $\epsilon_s < \epsilon_y$ $then$ $f_s = \epsilon_s E_s$
      - If $\epsilon_s > \epsilon_y$ $then$ $f_s = f_y$

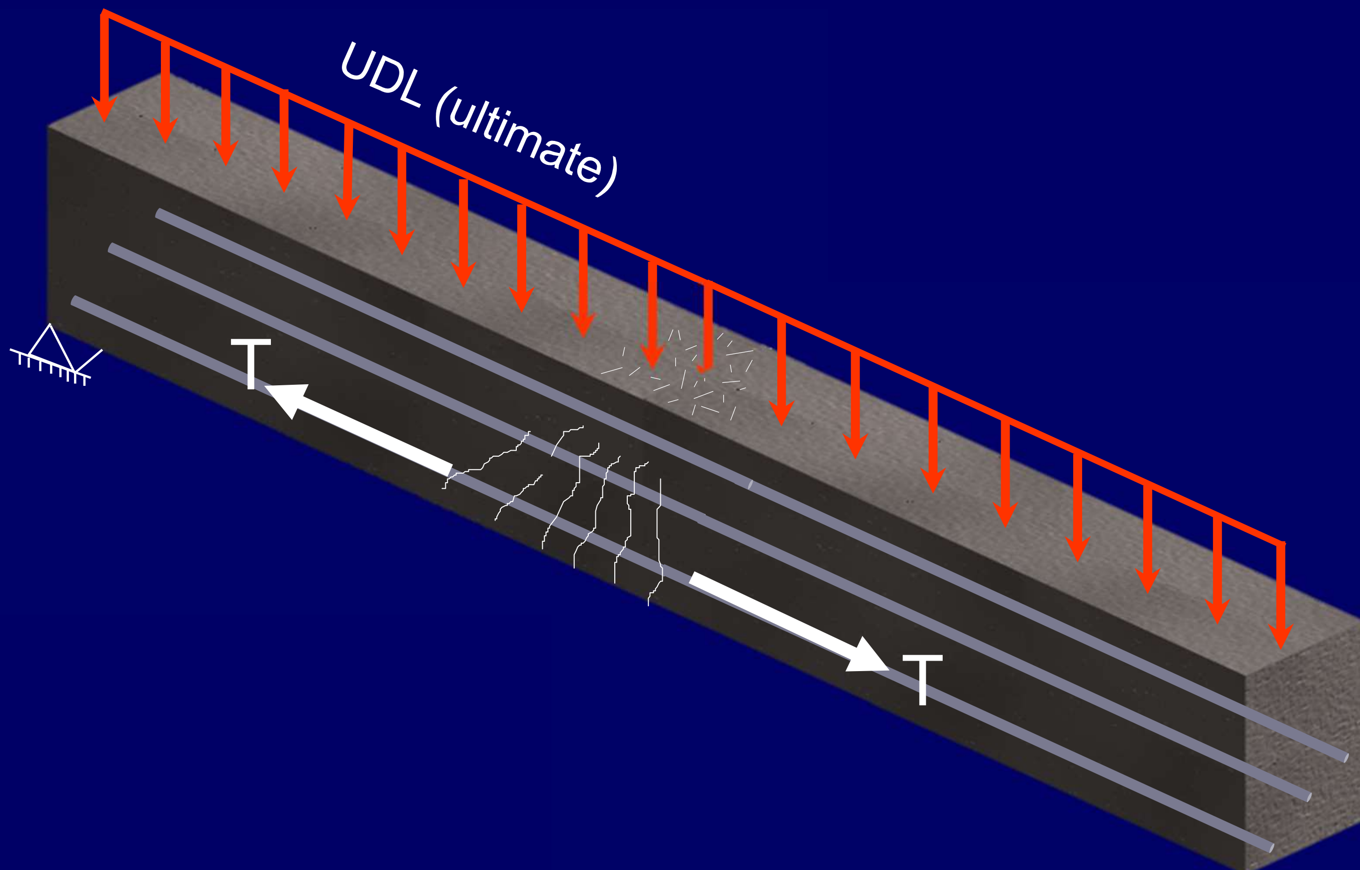




Idealized Stress-Strain Curve

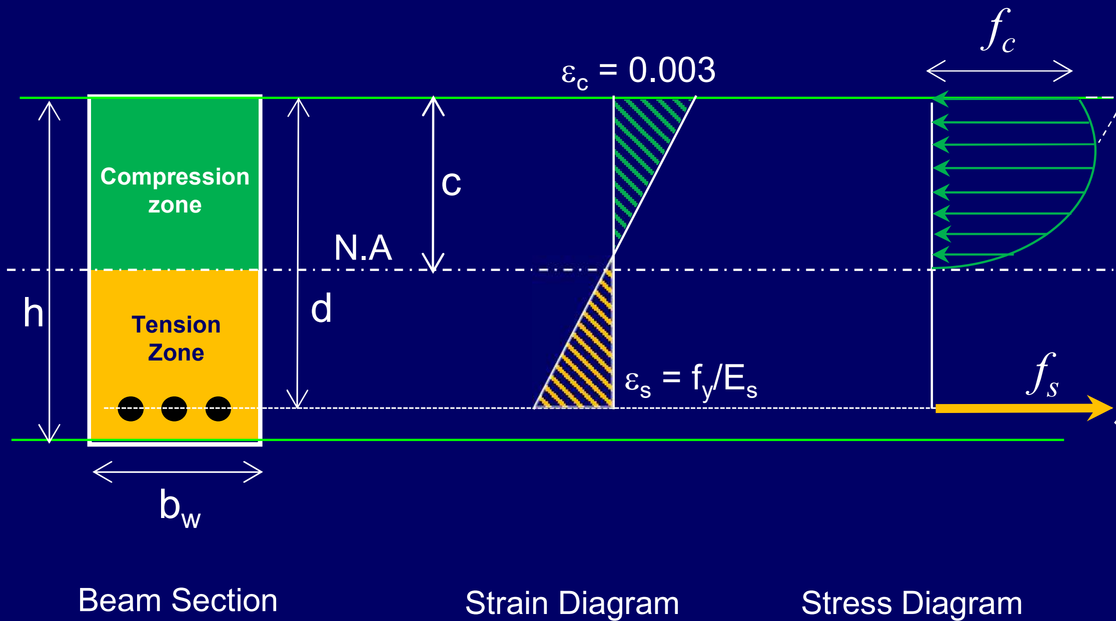
Behavior of RC Beam under Gravity Load

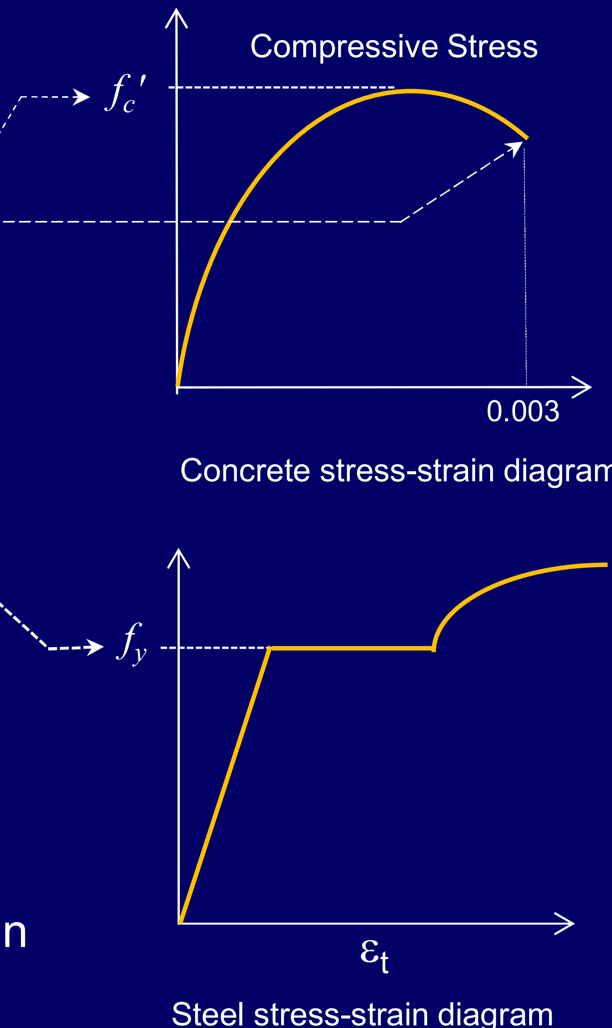
- **Stage – 3: Behavior**



Behavior of RC Beam under Gravity Load

- **Stage – 3: Behavior**





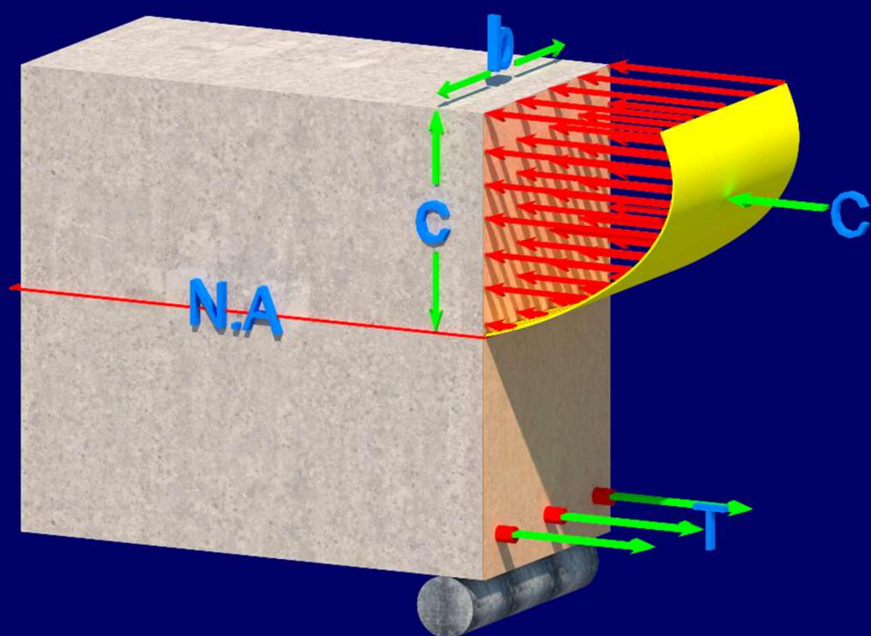
At this stage, the loading condition is such that the concrete crushes in compression, and the steel yields in tension. $f_c = slightly\ less\ than\ f_c'$ $\quad and \quad$ $f_s = f_y$

Behavior of RC Beam under Gravity Load

- **Stage – 3: Calculations**
  - As the stress distribution in this stage is parabolic, therefore calculating the compressive force and its position is extremely challenging.
  - The actual complex stress distribution can be transformed into a simple geometric shape, that gives the same results as the original.
  - C. S. Whitney proposed a rectangular distribution known as the "Whitney Stress Block" which has gained widespread acceptance and is included in the ACI Code.

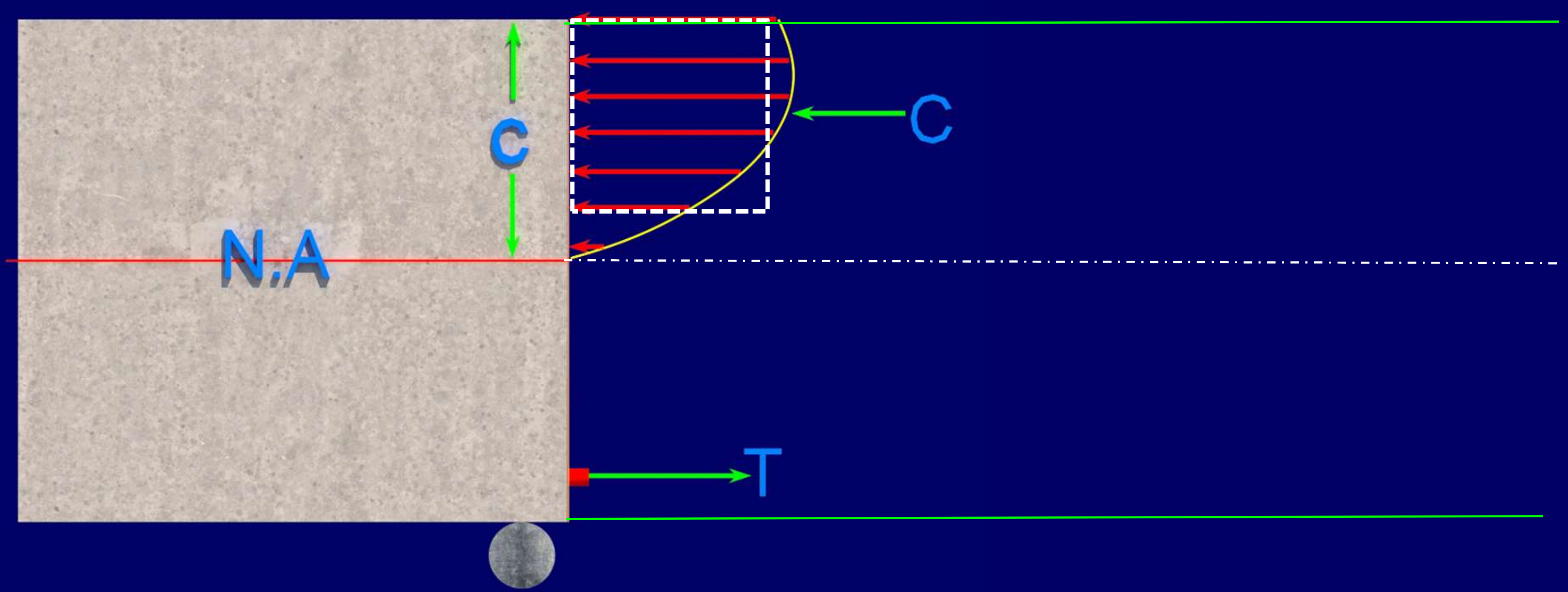
Behavior of RC Beam under Gravity Load

- **Stage – 3: Calculations**
  - **Whitney Stress Block**



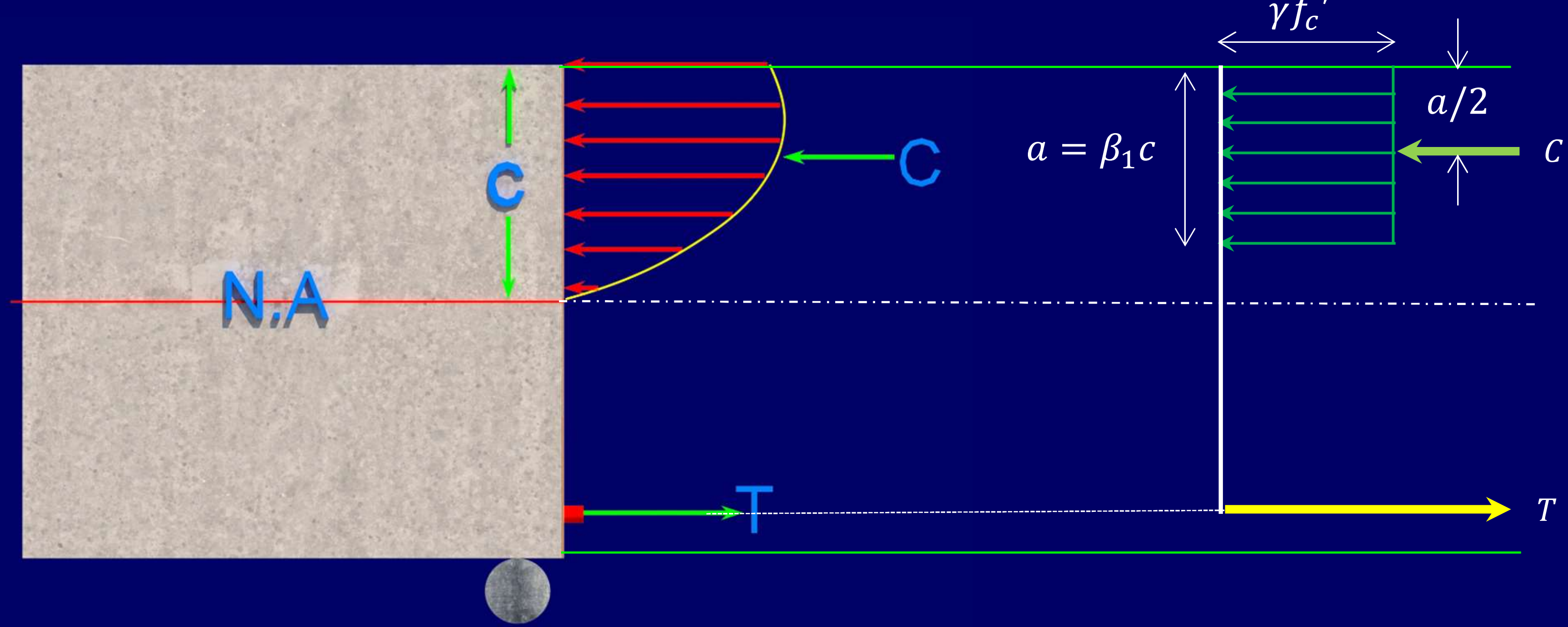
Behavior of RC Beam under Gravity Load

- **Stage – 3: Calculations**
  - **Whitney Stress Block**



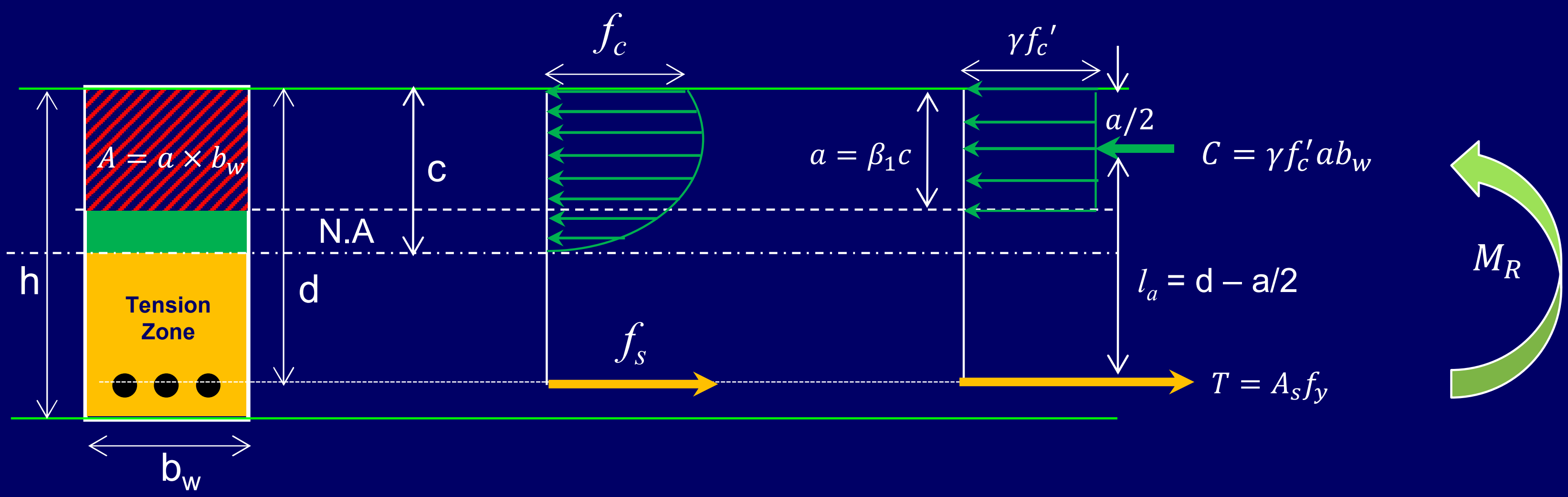
Behavior of RC Beam under Gravity Load

- **Stage – 3: Calculations**
  - **Whitney Stress Block**



Behavior of RC Beam under Gravity Load

- **Stage – 3: Calculations**



Beam Section

Actual Stress Diagram

Equivalent Stress Diagram

Calculating Resisting moment

$$M_R = \cancel{M_c}^{\,0} + M_s = T \times l_a = (A_s \times f_y) \times \left(d - \frac{a}{2}\right)$$

$$M_R = A_s f_y \left(d - \frac{a}{2}\right) \quad \text{------ (2.5)}$$

$\gamma = 0.85$ (ACI 318 -19, 22.2.2.4)

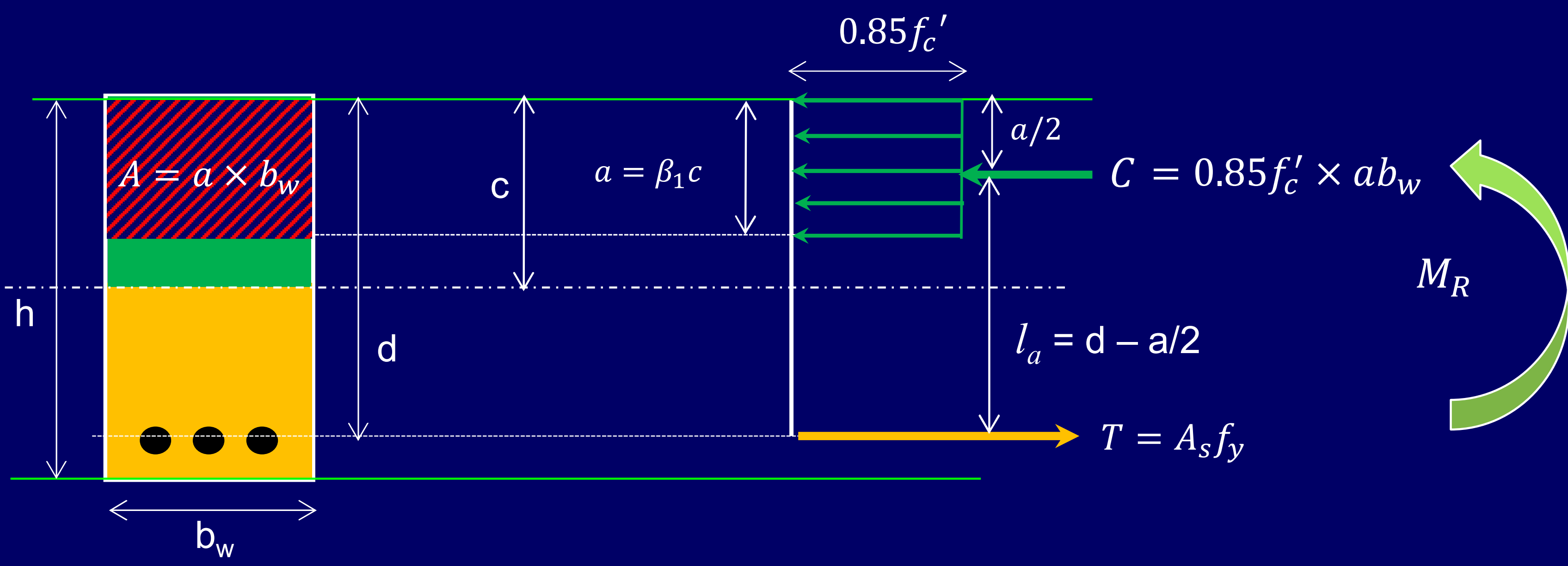
and

$\beta_1 = 0.85 \;\; for \; f_c' \leq 4000psi$

For strengths above 4000 psi, refer to ACI 318-19, 22.2.2.4.3

Behavior of RC Beam under Gravity Load

- **Stage – 3: Calculations**



Beam Section

Equivalent Stress Diagram

Equating Horizontal forces; C = T $\Rightarrow 0.85f_c' \times ab_w = A_s f_y$

Solving for "a", we get

$$\Rightarrow \quad a = \frac{A_s f_y}{0.85 f_c' b_w}$$

Behavior of RC Beam under Gravity Load

- **Stage – 3: Calculations**
  - From eq. (2.5) we have

$$M_R = A_S f_y \left(d - \frac{a}{2}\right)$$

  - Equating $M_R = M_A$

$$A_S f_y \left(d - \frac{a}{2}\right) = M_A$$

  - Which on solving for A_s gives

$$A_s = \frac{M_A}{f_y \left(d - \frac{a}{2}\right)} \quad \text{------ (2.6)}$$

Behavior of RC Beam under Gravity Load

- **Stage – 3: Calculations**
  - The same Trial and Success method that was discussed in stage 2 can be used to determine the area of steel A_s.

1. Assume the value of "a"
2. Calculate the area of steel using eq. (2.6)

$$A_s = \frac{M_A}{f_y(d - a/2)}$$

3. Confirm the value of "a" using

$$a = \frac{A_s f_y}{0.85 f_c' b_w}$$

4. Repeat the process until the same A_s value is obtained from the two consecutive trials.

Behavior of RC Beam under Gravity Load

- **Stage – 3: Example 2.3**
  - Using the data from Example 2.1, calculate the area of steel required for the beam corresponding to stage 3.
  - **Solution**
    - Trial 1: Choosing a = 2" and d = h – 2.5 = 15.5"

$$A_s = \frac{1002}{(40)(15.5 - 2/2)} = 1.73in^2$$

$$\Rightarrow \quad a = \frac{1.73 \times 40}{0.85 \times 3 \times 12} = 2.26"$$

    - Trial 2: Choosing a = 2.26"

$$A_s = \frac{1002}{(40)(15.5 - 2.26/2)} = 1.74\ in^2$$

Behavior of RC Beam under Gravity Load

- **Stage – 3: Example 2.3**
  - **Solution**

$$\Rightarrow \quad a = \frac{1.74 \times 40}{0.85 \times 3 \times 12} = 2.27"$$

- Trial 3: Choosing a = 2.27”

$$A_s = \frac{1002}{(40)(15.5 - 2.27/2)} = 1.74\ in^2$$

$$\Rightarrow \quad a = \frac{1.74 \times 40}{0.85 \times 3 \times 12} = 2.27" \qquad \text{(OK!)}$$

- Hence the required area of steel is 1.74in^2

Behavior of RC Beam under Gravity Load

- **Concluding Remarks on the Three Stages**
  - **Stage 1**
    - The beam based on stage 1, which does not allow for any cracking, requires an abnormally deep depth and a very large amount of steel.
    - As a result, designing based on this stage is both uneconomical and impractical.

Behavior of RC Beam under Gravity Load

- **Concluding Remarks on the Three Stages**
  - **Stage 2**
    - It is basically a working stress approach where the strength has been divided by 2 in order to achieve the factor of safety in the design.
    - Designing beam at this stage is uneconomical as compared to that of stage 3.

Behavior of RC Beam under Gravity Load

- **Concluding Remarks on the Three Stages**
  - **Stage 3**
    - Stage 3 corresponds to the Strength Design Method.
    - Designing based on this stage is the most cost-effective among all stages.
    - The recommendations of the ACI Code related to the strength design method are discussed next.

ACI 318 Code Provisions for Beams

- **Design Method**
  - According to ACI 318-19, section 4.6, RC members shall be designed by the Strength Design method.
  - In the strength design method Demand is amplified by load factors and capacity is reduced by strength reduction factors, that is
    - Factored Applied moment or Ultimate moment $= M_u = \gamma M_A$
    - Reduced flexural capacity or Design moment $= M_d = \emptyset M_R$
  - The load factors and strength reduction factors are shown on the following slides.

ACI 318 Code Provisions for Beams

- **Load Combinations**
  - According to ACI code section 5.3, the following load combinations should be used in determining the ultimate load for beam analysis.

$$W_u = 1.2W_D + 1.6W_L$$

$$M_u = 1.2M_D + 1.6M_L$$

Where;

W_D = Service Dead load

W_L = Service Live load

W_U = Amplified load or Ultimate load

M_U = Amplified moment or Ultimate moment

ACI 318 Code Provisions for Beams

- **Strength Reduction Factors**
  - ACI Code recommends the following strength reduction factors to be used in the design of RC members.
  - The reduction factor ∅ shall be taken as;
    - 0.90 for tension-controlled regions (flexure)
    - 0.75 for shear and torsion.
    - 0.65 for compression – controlled regions

(ACI 318-19,Table 21.2.1)

ACI 318 Code Provisions for Beams

- **Nominal and Design Strength**
  - According to strength design method, the resisting member flexural capacity calculated from specified dimension (size of members) and specified material strength called as the Nominal flexural capacity M_n. (Note that $M_n = M_R$)
  - The Design Strength M_d is obtained by multiplying Nominal flexural strength by strength reduction factor ∅ = 0.9

$$\boxed{M_d = \emptyset M_n = \emptyset A_s f_y\left(d - \frac{a}{2}\right)} \quad \text{------ (2.5)}$$

  - For no failure;

$\emptyset M_n \geq M_u$ (M_u is the factored applied moment coming from the loads)

ACI 318 Code Provisions for Beams

- **Nominal Flexural Capacity of RC Members**
  - The Nominal Flexural Strength M_n of an RC member is reached when the strain in the extreme compression fiber reaches the assumed strain limit of 0.003.

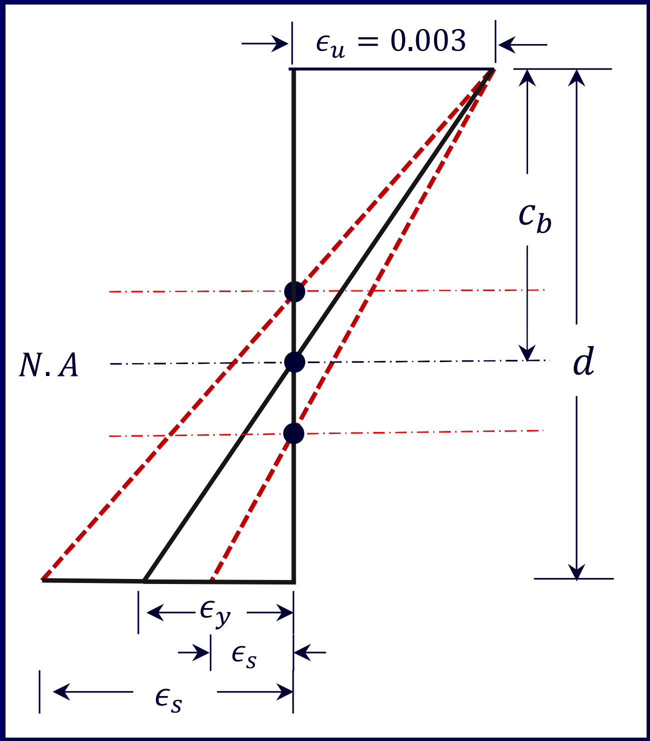
(ACI 318-19, R21.2.2)

ACI 318 Code Provisions for Beams

- **Mode of Flexural Failure**
  - The ACI Code requires that the beam designed using the strength design method should fail, if ever, in a ductile rather than brittle manner to allow for adequate evacuation time.
  - The ductile failure mode can be ensured only when steel on the tension side yields well before the concrete crushes on the compression side.
  - Yielding of steel will only be possible if tension steel is less than a certain amount, otherwise steel will not yield before the crushing of concrete, and the beam will fail in a brittle manner.

ACI 318 Code Provisions for Beams

- **Mode of Flexural Failure**
  - When the concrete strain ϵ_u in the extreme fiber of the compression zone reaches 0.003, depending on the amount of tension reinforcement, Steel stain ϵ_s may exhibit one of the following conditions,



1. $\epsilon_s = \epsilon_y$ (Balanced condition)
2. $\epsilon_s < \epsilon_y$ (Over reinforced condition)
3. $\epsilon_s > \epsilon_y$ (Under reinforced condition)

ACI 318 Code Provisions for Beams

- **Mode of Flexural Failure**
  - From the preceding discussion, it is clear that for any beam with given material properties and cross-sectional dimensions, there exists a specific amount of steel at which yielding and crushing occur simultaneously.
  - This amount of steel is known as Balanced steel $A_{s,b}$, and the beam is said to be in "Balanced condition".
  - If $A_s < A_{s,b}$ the steel yields before the concrete crushes and the beam is said to be in "Under reinforced condition".
  - If $A_s > A_{s,b}$ the concrete crushes before the steel yields and the beam is said to be in "Over reinforced condition".

ACI 318 Code Provisions for Beams

- Mode of Flexural Failure
  - Experimental Test on an Over Reinforced Beam

ACI 318 Code Provisions for Beams

- **Reinforcement Limits**

  - Both the balanced condition ($\epsilon_s = \epsilon_y$) and over-reinforced condition ($\epsilon_s < \epsilon_y$) results in a brittle mode of failure.
  - Hence to achieve ductility, the value of strain must be sufficiently greater than the yield strain ($\epsilon_s > \epsilon_y$) How much greater?
  - This condition can be satisfied by imposing a maximum limit on the amount of steel.
  - Similarly, there is also a minimum reinforcement limit which will be explained later.

ACI 318 Code Provisions for Beams

- **Reinforcement Limits**

1. **Maximum Flexural Reinforcement Limit**

   - To determine the maximum amount of reinforcement, it is necessary to establish a correlation between the steel area and its corresponding strain.

   - Once this relationship has been derived, the area of steel for any given amount of steel strain can be calculated.

   - The next slide illustrates the process of deriving this relationship.

ACI 318 Code Provisions for Beams

- **Reinforcement Limits**

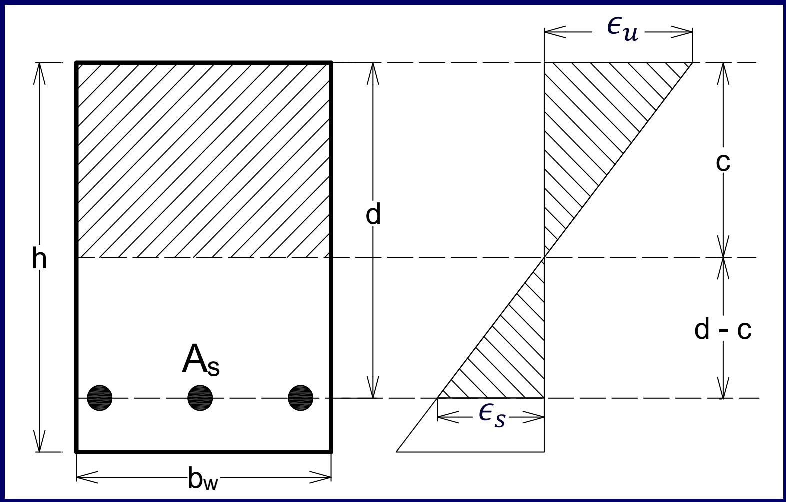
1. **Maximum Flexural Reinforcement Limit**

- For equilibrium of internal forces,

$$\sum F_X = 0 \quad \rightarrow \quad C = T$$

$$\Rightarrow \; 0.85 f_c' \, a \, b_w = A_s f_y$$

$$\Rightarrow \; 0.85 f_c' \, \boxed{\beta_1 c} \, b_w = A_s f_y \text{ ------ (2.7)}$$



- From the similarity of triangles

$$\frac{c}{\epsilon_u} = \frac{d-c}{\epsilon_s}$$

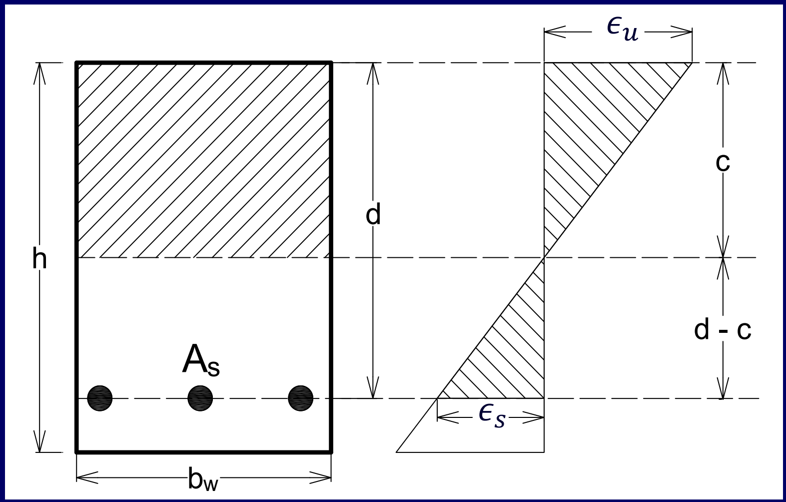
$$\Rightarrow c\epsilon_s = \epsilon_u (d-c) \quad OR \quad c\epsilon_s - \epsilon_u c = \epsilon_u d$$

ACI 318 Code Provisions for Beams

- **Reinforcement Limits**

1. **Maximum Flexural Reinforcement Limit**

$$\Rightarrow c = \left(\frac{\epsilon_u}{\epsilon_u + \epsilon_s}\right) d$$



Put this value in eq (2.7), gives

$$\Rightarrow 0.85 f_c' \beta_1 \left(\frac{\epsilon_u}{\epsilon_u + \epsilon_s}\right) b_w d = A_s f_y$$

$$\Rightarrow A_s = \frac{0.85 f_c' \beta_1}{f_y} \left(\frac{0.003}{0.003 + \epsilon_s}\right) b_w d \text{ ------ (2.8)}$$

Eq.(2.8) is the required relation between strain and the area of steel.

ACI 318 Code Provisions for Beams

- **Reinforcement Limits**

  1. **Maximum Flexural Reinforcement Limit**

     - The ACI code recommends that for tension-controlled sections (Beams). $\varepsilon_s = \varepsilon_t = \varepsilon_{ty} + 0.003$

     - Putting this value in eq. (2.8), we get

$$\Rightarrow A_{s,max} = \frac{0.85 f_c' \beta_1}{f_y} \left(\frac{0.003}{0.006 + \epsilon_{ty}} \right) b_w d \quad \text{------ (2.9)}$$

- This is the maximum flexural steel area that will ensure ductility.
- Any value of A_s greater than $A_{s,max}$ will lead to brittle failure.

ACI 318 Code Provisions for Beams

- **Reinforcement Limits**

  1. **Maximum Flexural Reinforcement Limit**

     - $For\ f_c' \leq 4ksi\ and\ f_y = 40ksi$
     - Taking $\beta_1 = 0.85$, $\epsilon_{ty} = f_y/E_s = 40/29000 = 0.001379$

       and putting these values in Eq. (2.9), we get

$$\Rightarrow A_{s,max} = \frac{0.85 f_c' \times 0.85}{40}\left(\frac{0.003}{0.006 + 0.001379}\right) b_w d$$

$$\boxed{A_{s,max,40} = \frac{f_c'}{136} b_w d}$$ ------ (2.10a)

ACI 318 Code Provisions for Beams

- **Reinforcement Limits**

1. **Maximum Flexural Reinforcement Limit**

- $For\ f_c' \le 4ksi\ and\ f_y = 60ksi$
- Taking $\beta_1 = 0.85\,, \epsilon_{ty} = f_y/E_s = 60/29000 = 0.002069$

  and putting these values in Eq. (2.9), we get

$$\Rightarrow\ A_{s,max} = \frac{0.85 f_c' \times 0.85}{60}\left(\frac{0.003}{0.006 + 0.002069}\right) b_w d$$

$$\boxed{A_{s,max,60} = \frac{f_c'}{223} b_w d}$$ ------ (2.10b)

ACI 318 Code Provisions for Beams

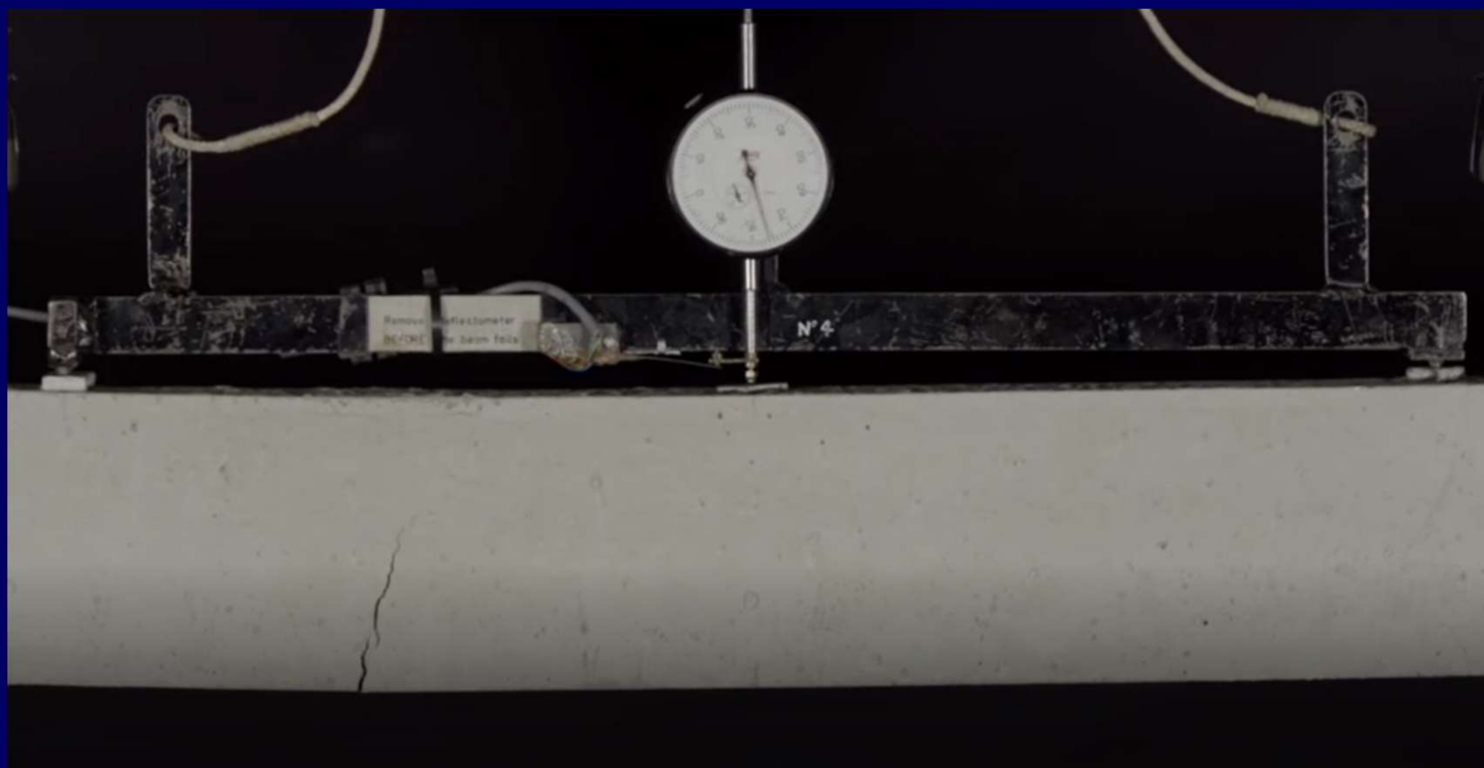
- **Reinforcement Limits**

  2. **Minimum Flexural Reinforcement Limit**

     - In some circumstances, the demand moment is so small that the resulting calculated area of steel provides flexural capacity (resisting moment) less than the cracking moment of plain concrete beam.
     - As a result, the beam will fail in a brittle manner.
     - To avoid this, the ACI Code has imposed a minimum limit on the amount of steel which will ensure ductility requirements.

ACI 318 Code Provisions for Beams

- **Mode of Flexural Failure**
  - **Experimental Test on a Reinforced Beam having $A_s < A_{s,min}$**



ACI 318 Code Provisions for Beams

- **Reinforcement Limits**

  2. **Minimum Flexural Reinforcement Limit**

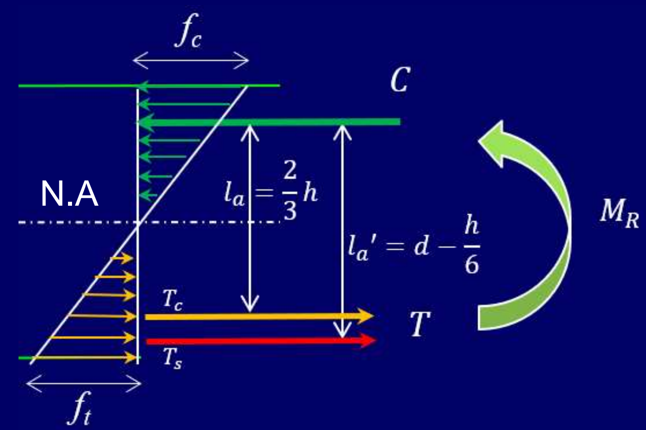
     - Moment provided by steel ≥ Cracking moment of PC beam

$$M_s \geq M_c$$

$$A_{s,min} f_y \left(d - \frac{h}{6}\right) \geq 1.25\sqrt{f_c{'}}\, b_w h^2$$

$$A_{s,min} \geq \frac{1.25\sqrt{f_c{'}}\, b_w h^2}{f_y\left(d - \frac{h}{6}\right)} = \frac{1.25\sqrt{f_c{'}}\, b_w (d/0.8)^2}{f_y\left(d - \frac{d}{6 \times 0.8}\right)}$$

$$\Rightarrow\ A_{s,min} \geq \frac{2.47\sqrt{f_c{'}}\, b_w d}{f_y}$$



$d = 0.8h$

ACI Code suggests a value of 3 rather than 2.47.

ACI 318 Code Provisions for Beams

- **Reinforcement Limits**

  2. **Minimum Flexural Reinforcement Limit**

     - As per ACI 318-19, section 9.6.1.2 the minimum flexural reinforcement $A_{s,min}$ for $f_y \leq 80ksi$ shall be larger of (a) and (b)

$$(a)\quad \frac{3\sqrt{f_c{}'}}{f_y}\, b_w d$$

$$(b)\quad \frac{200}{f_y}\, b_w d$$

     - For $f_c' \leq 4500psi,$ eq.(b) will always govern.

ACI 318 Code Provisions for Beams

- **Reinforcement Limits**
  - **Reinforcement Ratio:**
    - The reinforcement limits can also be expressed in the form of reinforcement ratio, which is the amount of reinforcement per unit effective area ($b_w d$) of the concrete section.

$$\rho = \frac{A_s}{b_w d}$$

For example, minimum reinforcement ratio for $f_c' = 3ksi\ and\ f_y = 40ksi$ is

$$\rho_{min} = \frac{A_{s,min}}{b_w d} = \frac{200 b_w d}{f_y\ b_w d} = 0.005$$

$$\rho_{max} = \frac{A_{s,max}}{b_w d} = \frac{f_c' b_w d}{136\ b_w d} = 0.0221$$

ACI 318 Code Provisions for Beams

- **Reinforcement Limits**
  - **Reinforcement Ratio:**
    - The table below provides Minimum and maximum reinforcement ratio for various values of f_c' and f_y.

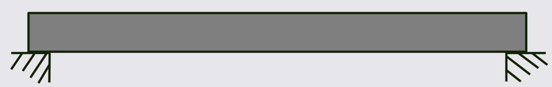
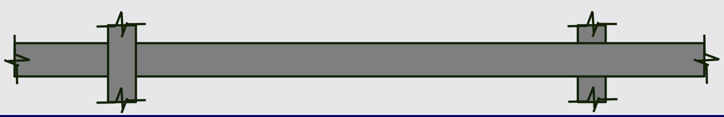
Minimum and Maximum reinforcement ratios				
$f_c'\ (psi)$	3000		4000	
$f_y\ (psi)$	40,000	60,000	40,000	60,000
ρ_{min}	0.005	0.0033	0.005	0.0033
ρ_{max}	0.0221	0.0135	0.0294	0.0181

Design Procedure

- **Steps Involved in Flexural Design of Beam**
  - Step No.1: Selection of Sizes
  - Step No.2: Calculation of Loads
  - Step No.3: Analysis (calculation of maximum bending moment)
  - Step No.4: Determination of steel area
  - Step No.5: Reinforcement Check
  - Step No.6: Detailing of reinforcement
  - Step No.7: Drafting
  - Step No.8: Design flexural capacity Check

Design Procedure

- **Steps Involved in Flexural Design of Beam**
  - **Step No.1: Selection of Sizes**
    - Minimum depth of beams for various support conditions are given below.

Support Conditions	Depiction	Minimum h (f_y = 60 ksi)
Simply supported		$l/16$
One end continuous		$l/18.5$
Both ends continuous		$l/21$
Cantilever		$l/8$

(Ref: ACI 318 -19, Table 9.3.1.1)

- *For f_y other than 60 ksi, the expressions in Table shall be multiplied by* $\left(0.4+\frac{f_y}{100000}\right)$

Design Procedure

- **Steps Involved in Flexural Design of Beam**
  - **Step No.1: Selection of Sizes**
    - Width of beam is selected based on accommodation of steel bars. Generally, the minimum width is taken as 12″.
    - Effective depth of beam is calculated as

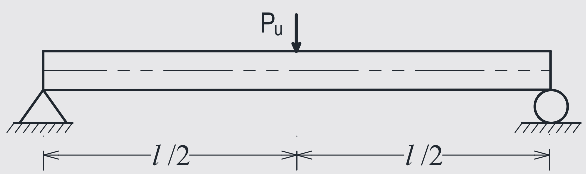
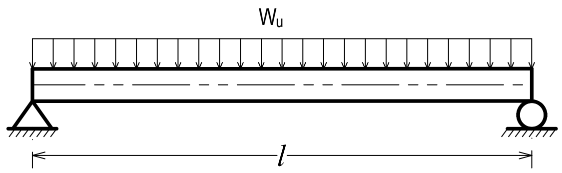
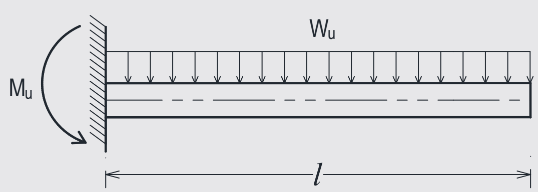
      $d = h - \bar{y}$, where $\bar{y} =$ centroid of steel area.
    - Typically, $\bar{y}$ is assumed to be 2.5 to 3 inches, and finally the actual "d" is calculated based on the provided steel area.
    - The final selection of beam size is determined by several factors, including architectural constraints and availability of formwork.

Design Procedure

- **Steps Involved in Flexural Design of Beam**
  - **Step No.2: Calculation of Loads**
    - The factored loads can be calculated using the following combination: $W_u = 1.2W_D + 1.6W_L$
  - **Step No.3: Analysis**
    - The analysis of the member is carried out for ultimate load including self weight obtained from size of the member and the applied dead and live loads.
    - The maximum bending moment value is used for flexural design.
    - Maximum moments for beams having different support conditions are tabulated on the next slide.

Design Procedure

- **Steps Involved in Flexural Design of Beam**
  - **Step No.3: Analysis**
    - Maximum Bending Moment equations for beams

Beam Configuration	Maximum Bending Moment
	$M_u = \frac{P_u l}{4}$
	$M_u = \frac{w_u l^2}{8}$
	$M_u = \frac{w_u l^2}{2}$

Design Procedure

- **Steps Involved in Flexural Design of Beam**

  - **Step No.4: Determination of Steel Area**

    - We have

$$A_s = \frac{M_u}{\emptyset f_y(d - \frac{a}{2})} \quad \dashrightarrow (1)$$

$$a = \frac{A_s f_y}{0.85 f_c{}' b_w} \quad \dashrightarrow (2)$$

    - Area of steel can be computed either by

      i. Performing Trial and Success Method (already discussed), or

      ii. Direct Method

Design Procedure

- **Steps Involved in Flexural Design of Beam**
  - **Step No.4: Determination of Steel Area**
    - **Direct method**

i. Calculate value of a using

$$a = d - \sqrt{d^2 - \frac{2.614 M_u}{f_c{}' b_w}}$$

ii. Determine area of steel using

$$A_s = \frac{M_u}{\emptyset f_y (d - \frac{a}{2})}$$

This equation can be derived by putting eq. (1) in eq.(2).

The proof is provided in the Appendix.

Design Procedure

- **Steps Involved in Flexural Design of Beam**

  - **Step No.5: Reinforcement Check**

    - The calculated area of steel should be within the reinforcement limits provided by ACI 318.

      - Minimum Limit

$$A_{s,min} = \frac{3\sqrt{f_c{}'}}{f_y} bd \geq \frac{200}{f_y} b_w d$$

These equations are valid only for $f_y \leq 80ksi$

      - Maximum Limit

$$A_{s,max,40} = \frac{f_c' b_w d}{136}$$

$$A_{s,max,60} = \frac{f_c' b_w d}{223}$$

These equations are valid only for $f_c' \leq 4000psi$

For strength above 4000psi, calculate $A_{s,max}$ by putting the relevant value of β_1 in eq. (2.9)

Design Procedure

- **Steps Involved in Flexural Design of Beam**
  - **Step No.6: Detailing of Reinforcement**
    - Conversion of steel area into Number of bars

$$Number\ of\ bars = \frac{A_s}{A_b}$$

- A_b is the area of reinforcing bar to be used.
- The calculated no. of bars must be placed according to the ACI code criteria which is discussed next.

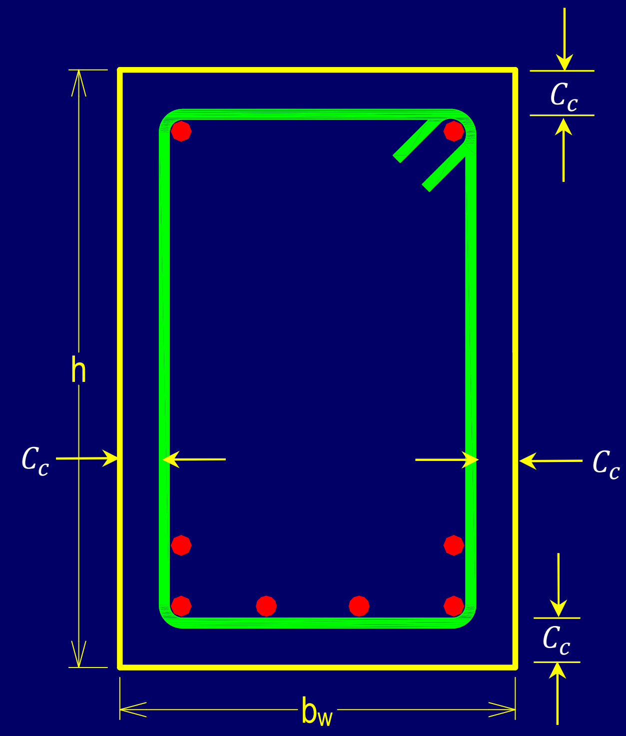
Bar Designation	Area (in^2)
#3	0.11
#4	0.20
#5	0.31
#6	0.44
#7	0.60
#8	0.79
#9	1.00
#10	1.27

Design Procedure

- **Steps Involved in Flexural Design of Beam**
  - **Step No.6: Detailing of Reinforcement**
    - Placement of bars
      - Bars should be placed in the tension zone as close to the extreme end as practicable.
      - It is preferable to arrange as many bars as possible on the bottom most side to increase the effective depth "d".
      - The detailing criteria as per ACI 318 are discussed in the next slides.

Design Procedure

- **Steps Involved in Flexural Design of Beam**
  - **Step No.6: Detailing of Reinforcement**
    - Placement of bars
      i. Concrete Cover
        - Minimum concrete clear cover for RC beams reinforcement shall be 1-1.5in. (ACI 318-19, Section 20.5.1.3)
        - Usually, concrete clear cover is taken as 1.5 in.

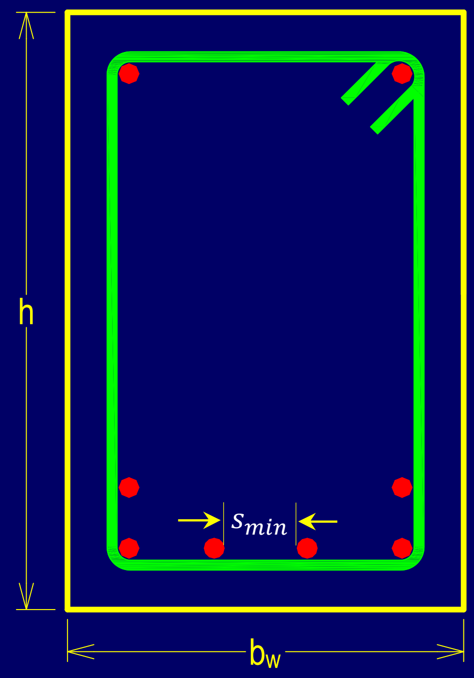


Design Procedure

- **Steps Involved in Flexural Design of Beam**
  - **Step No.6: Detailing of Reinforcement**
    - Placement of bars

      ii. Minimum spacing b/w adjacent bars

      - ACI 318-19, Section 25.2 specifies that the minimum clear distance between adjacent bars shall be at least the greatest of;
        - Diameter of the bar, d_b
        - 1 in. and
        - $(4/3)d_{agg}$.



Design Procedure

- **Steps Involved in Flexural Design of Beam**
  - **Step No.6: Detailing of Reinforcement**
    - Placement of bars

      iii. Maximum distance between adjacent bars (for crack control)

      - Maximum spacing among adjacent bars is restricted by specifying the minimum number of bars in a single layer.

Bar size	Beam width	Minimum no of bars
#3 to #14	12" - 15"	2
	16" - 25"	3
	26" - 36"	4

(Ref: Table A.8, Design of concrete structures, 15th edition, Nilson)

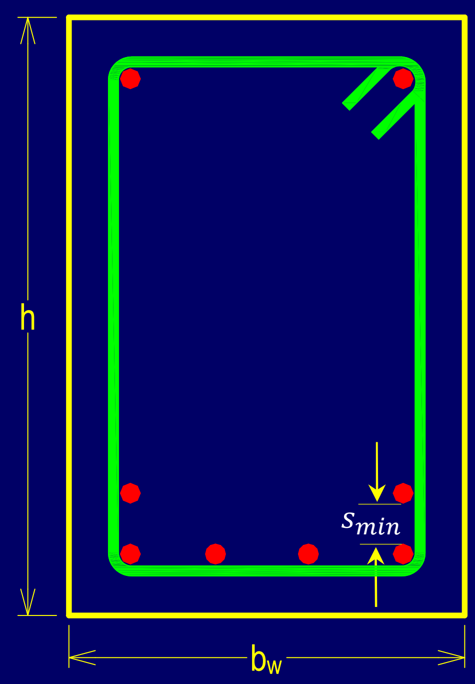
Design Procedure

- **Steps Involved in Flexural Design of Beam**
  - **Step No.6: Detailing of Reinforcement**
    - Placement of bars

      iv. Spacing between adjacent layers

      - The bars in the upper layer should be placed directly above those in the bottom layer with a clear spacing between layers of at least 1in.

(ACI 318-19, Section 25.2.2)



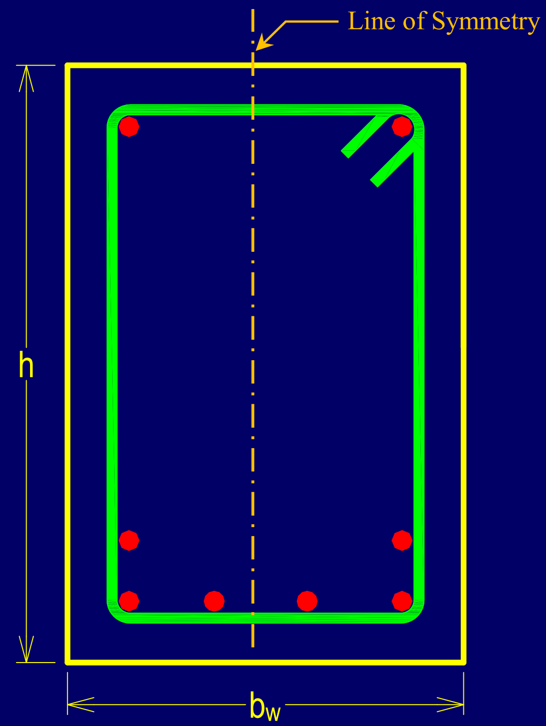
Design Procedure

- **Steps Involved in Flexural Design of Beam**
  - **Step No.6: Detailing of Reinforcement**
    - Placement of bars

      vi. Symmetry of bars

      - Although it is not explicitly stated in the code, many designers recommend that the bars should be arranged symmetrically around the vertical centerline.

(Ref: Design of Concrete Structures 15th edition, Chapter 4, Section 4.5)



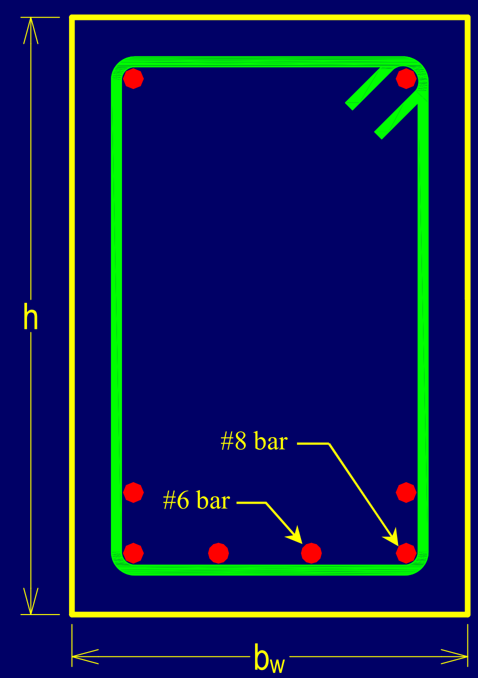
Design Procedure

- **Steps Involved in Flexural Design of Beam**
  - **Step No.6: Detailing of Reinforcement**
    - Placement of bars

v. Variation in diameter of bars

- Some engineers also suggest that the variation in diameter of bars in a single layer shall be limited to two bar sizes.
- For example, No. 8 and No. 6 bars together, but not Nos. 5 and 8.

(Ref: Design of Concrete Structures 15th edition, Chapter 4, Section 4.5)



Design Procedure

- **Steps Involved in Flexural Design of Beam**
  - **Step No.7: Drafting**
    - Based on the design, drawings of the structural members are prepared showing the dimensions of member and detail of reinforcing bars.

Design Procedure

- **Steps Involved in Flexural Design of Beam**
  - **Step No.8: Design Flexural Capacity Check**
    - After placement of bars, check the flexural capacity from the actual *"d"* and provided amount of reinforcement.

$$\emptyset M_n = \emptyset A_s f_y (d - \frac{a}{2})$$

$$\emptyset M_n \geq M_u$$

    - The procedure for calculating the actual "d" is given on the next slide.

Design Procedure

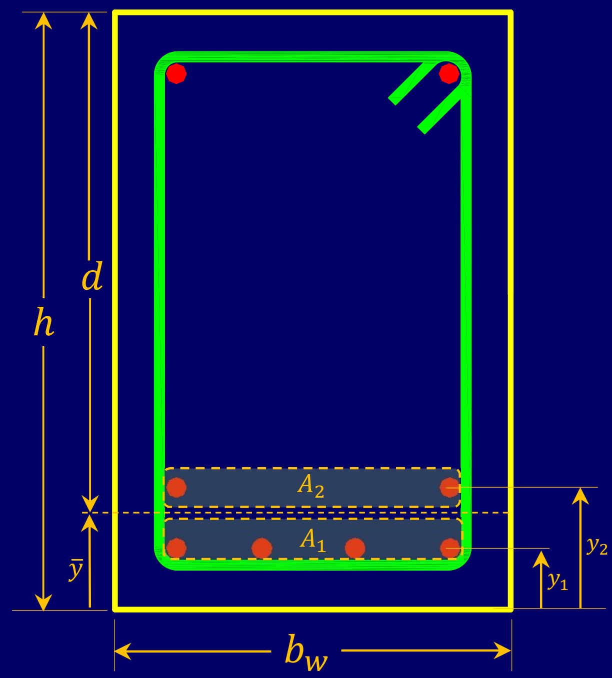
- **Steps Involved in Flexural Design of Beam**
  - **Step No.8: Design Flexural Capacity Check**
    - Calculation of effective depth

$$d = h - \bar{y}$$

Where,

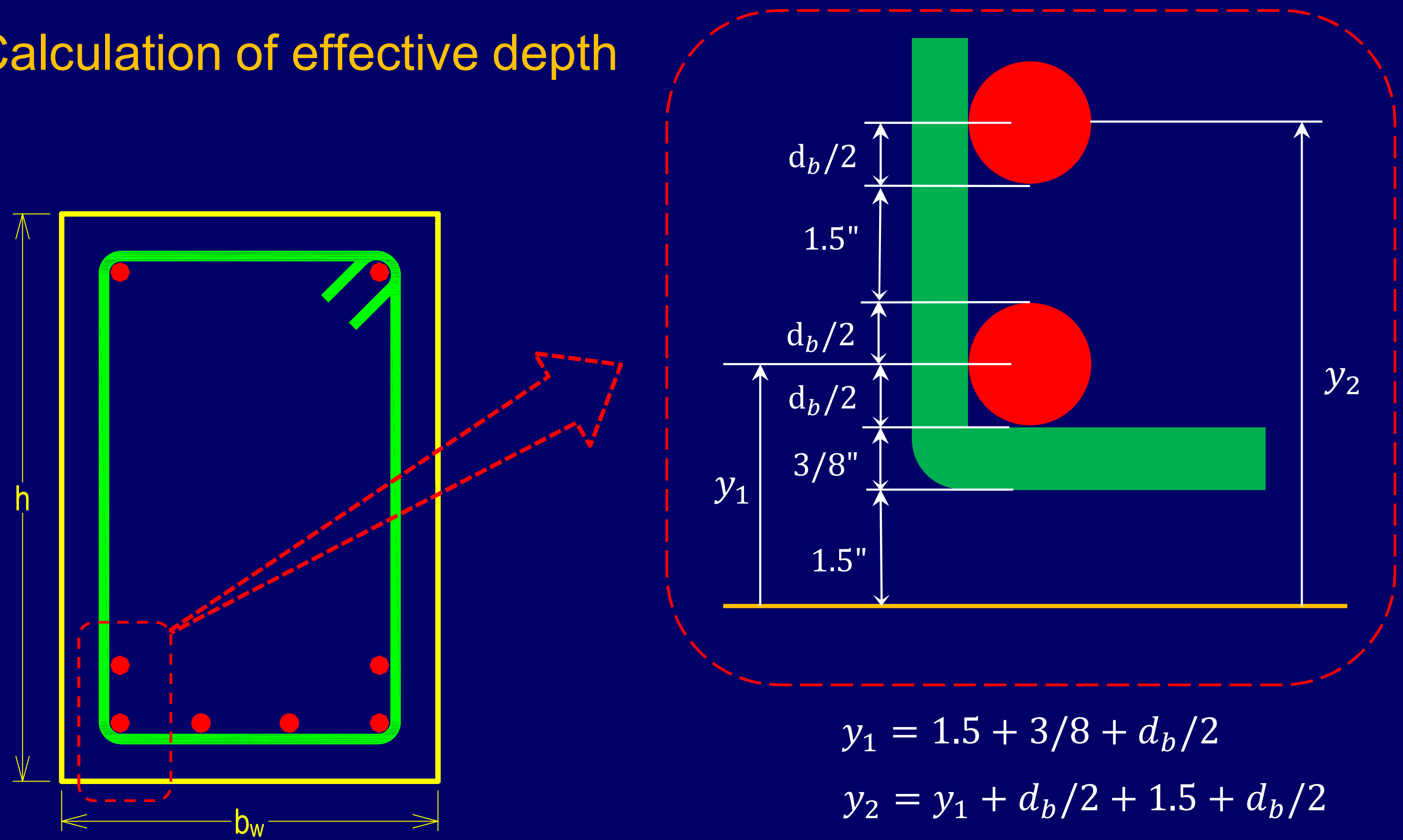
$$\bar{y} = centroid\ of\ steel\ area$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + \cdots}{A_1 + A_2 + \cdots}$$



Design Procedure

- **Steps Involved in Flexural Design of Beam**
  - **Step No.8: Design Flexural Capacity Check**
    - Calculation of effective depth



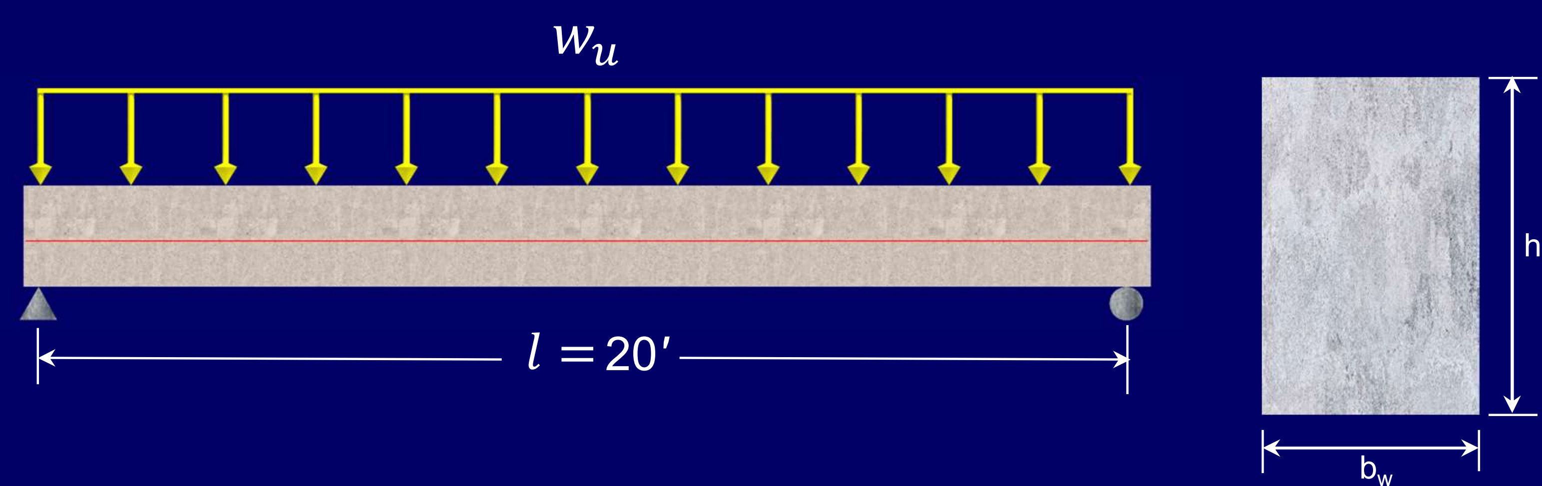
$$y_1 = 1.5 + 3/8 + d_b/2$$

$$y_2 = y_1 + d_b/2 + 1.5 + d_b/2$$

Flexural Design of Singly Reinforced Rectangular Beam

- **Example 2.4**
  - A simply supported beam subjected to a uniformly distributed load as shown in the figure below. The service dead load (excluding the self-weight) and live load are both 0.5kip/ft.

Analyze and ***Design*** the beam for flexure in accordance with ACI 318-19. Take $f_c{}' = 3ksi$ and $f_y = 40ksi$



Flexural Design of Singly Reinforced Rectangular Beam

- **Solution**
  - **Given Data**

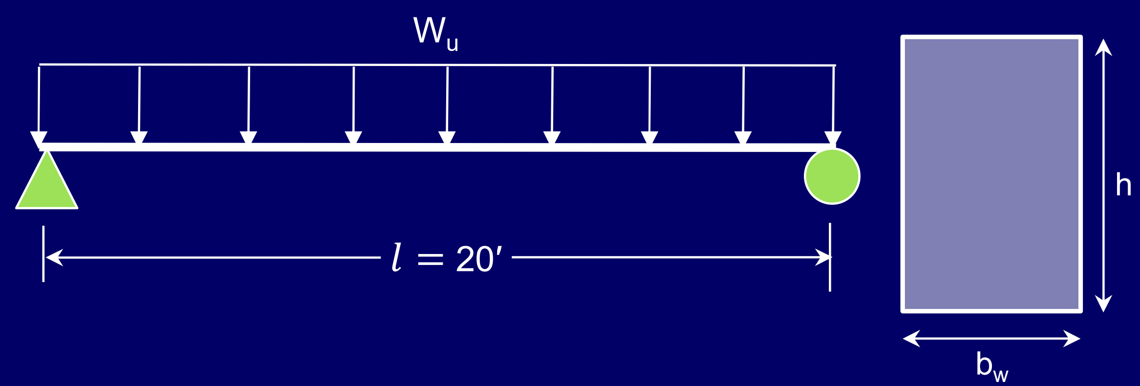
$l = 20'$

$SDL = 0.5k/ft$

$LL = 0.5k/ft$

$f_c' = 3ksi$

$f_y = 40ksi$



  - **Required Data**

Analyze and ***Design*** the beam for flexure in accordance with ACI 318-19.

Flexural Design of Singly Reinforced Rectangular Beam

- **Solution**

  - **Step No.1: Selection of Sizes**

    - The minimum depth for simply supported beam is;

$$h_{min} = \frac{l}{16}\left(0.4 + \frac{f_y}{100000}\right)$$ (for f_y other than 60ksi)

$$h_{min} = 20/16(0.4 + 40{,}000/100000) = 1' \text{ or } 12''$$

    - This is the minimum requirement of the code for depth of beam.
    - However, we select 18″ deep beam.
    - Generally, the minimum beam width is 12″, therefore, width of the beam is taken as 12″

Flexural Design of Singly Reinforced Rectangular Beam

- **Solution**
  - **Step No.1: Selection of Sizes**
    - The effective depth is calculated as,

      $$d = h - \bar{y}$$

    - Assuming $\bar{y} = 2.5"$

      $$d = 18 - 2.5 = 15.5"$$

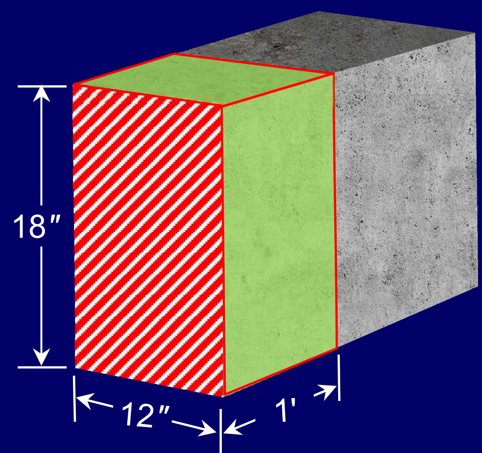
Flexural Design of Singly Reinforced Rectangular Beam

- **Solution**
  - **Step No.2: Calculation of Loads**
    - Self weight of beam is given by;

$$sw = Volume \times Unitweitht\ of\ concrete$$

$$sw = b_w hL \times \gamma_c = \frac{12 \times 18 \times 1}{12 \times 12} \times 0.15$$

$$sw = 0.225k/ft$$



    - The ultimate load can be determined as ;

$$w_u = 1.2w_D + 1.6w_L$$

$$w_u = 1.2(0.5 + 0.225) + 1.6(0.5) = 1.67k/ft$$

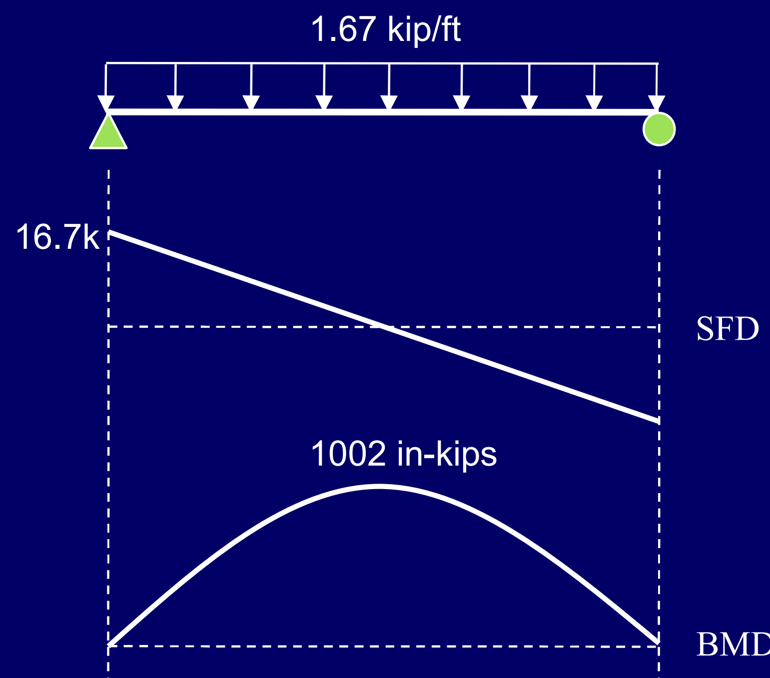
Flexural Design of Singly Reinforced Rectangular Beam

- **Solution**

  - **Step No.3: Analysis**

    - The maximum bending moment for a simply supported beam is

$$M_u = \frac{w_u l^2}{8}$$



- Putting $w_u = 1.67$ and $l = 20$

$$M_u = \frac{1.67 \times 20^2}{8} = 83.5kip.ft$$

$$M_u = 1002\ in.kip$$

Flexural Design of Singly Reinforced Rectangular Beam

- **Solution**
  - **Step No.4: Determination of Steel Area**
    - Using direct method, we have

$$a = d - \sqrt{d^2 - \frac{2.614M_u}{f_c'b_w}} = 15.5 - \sqrt{15.5^2 - \frac{2.614 \times 1002}{3 \times 12}} = 2.56"$$

    - Putting $a = 2.56$ and $\emptyset = 0.90$, we get

$$A_s = \frac{M_u}{\emptyset f_y(d - \frac{a}{2})} = \frac{1002}{0.9 \times 40(15.5 - \frac{2.56}{2})}$$

$$A_s = 1.96in^2$$

Flexural Design of Singly Reinforced Rectangular Beam

- **Solution**
  - **Step No.5: Reinforcement Check**
    - Minimum reinforcement limit

$$A_{s,min} = \frac{200}{f_y} b_w d \qquad (\text{ for } f_c{}' \leq 4500psi)$$

By substituting values, we get

$$A_{s,min} = \frac{200}{40000} \times 12 \times 15.5$$

$$A_{s,min} = 0.93in^2$$

Flexural Design of Singly Reinforced Rectangular Beam

- **Solution**
  - **Step No.5: Reinforcement Check**
    - Maximum reinforcement limit

$$A_{s,max} = \frac{f_c' b_w d}{136} \qquad (\text{ for } f_y \leq 40ksi)$$

$$A_{s,max} = \frac{3 \times 12 \times 15.5}{136}$$

$$A_{s,max} = 4.1in^2$$

Since,

$$A_{s,min} < A_s < A_{s,max} \Rightarrow OK!$$

Food for Thought:

What would you do if you encounter either of these situations??

a) When A_s is less than $A_{s,min}$

b) When A_s greater than $A_{s,max}$

Flexural Design of Singly Reinforced Rectangular Beam

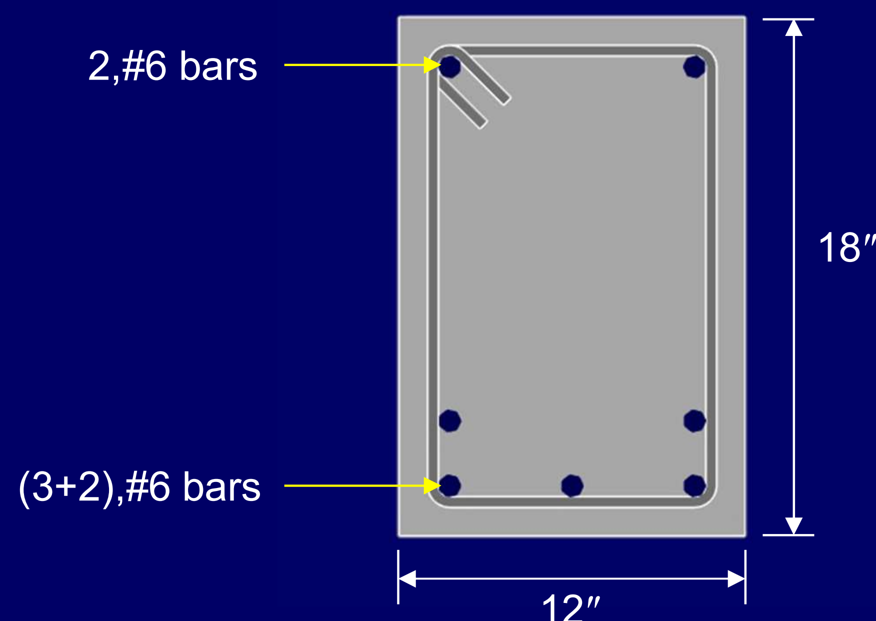
- **Solution**
  - **Step No.6: Detailing of Reinforcement**
    - Conversion of steel area into number of bars
      - Using #6 bar with $A_b = 0.44 in^2$

$$Number\ of\ bars = \frac{A_s}{A_b} = \frac{1.96}{0.44} = 4.45 \approx 5$$

      - Other options can be explored. For example,
        - 4, #7 bars (2.4 in2),
        - 3, #8 bars (2.37 in2),
        - or combination of two different size bars.

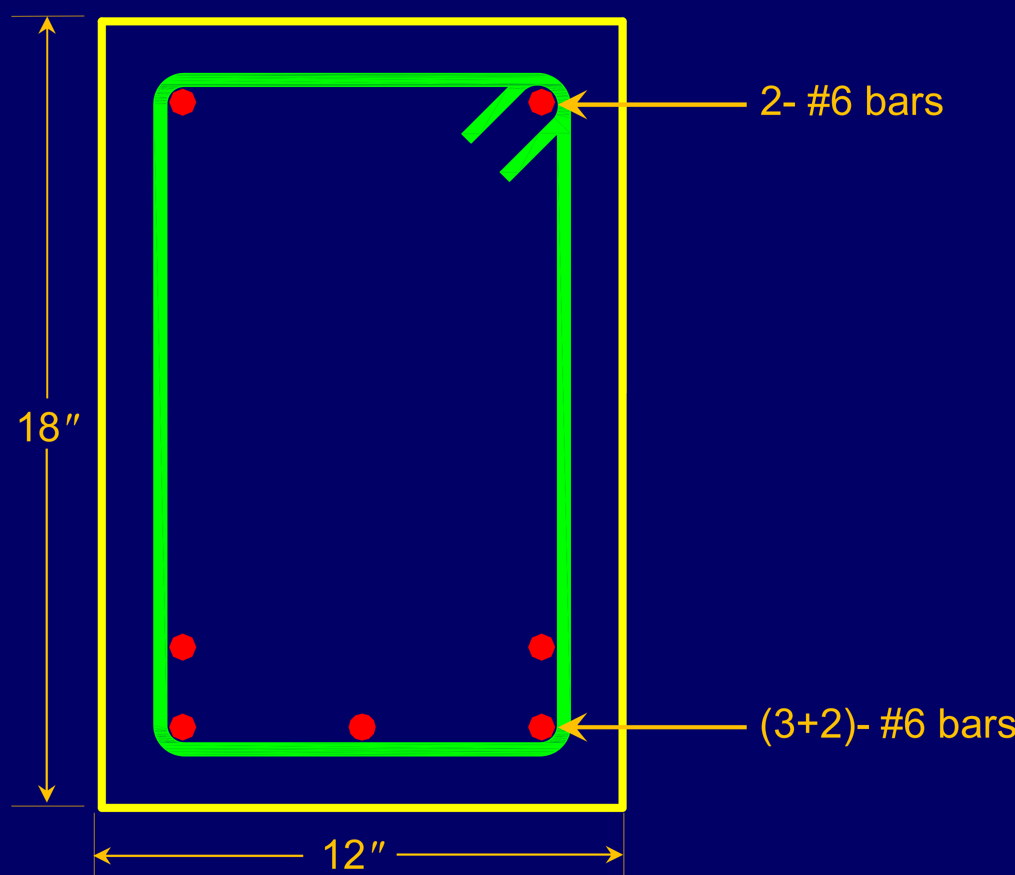
Flexural Design of Singly Reinforced Rectangular Beam

- **Solution**
  - **Step No.6: Detailing of Reinforcement**
    - Bar placement
      - Provide 5 #6 bars in two layers;
        - 3 in lower layer and
        - 2 in upper layer



Flexural Design of Singly Reinforced Rectangular Beam

- **Solution**
  - **Step No.7: Drafting**



Flexural Design of Singly Reinforced Rectangular Beam

- **Solution**

  - **Step No.8: Check Design Flexural Capacity**

$$\emptyset M_n = \emptyset A_S f_y \left(d - \frac{a}{2}\right)$$

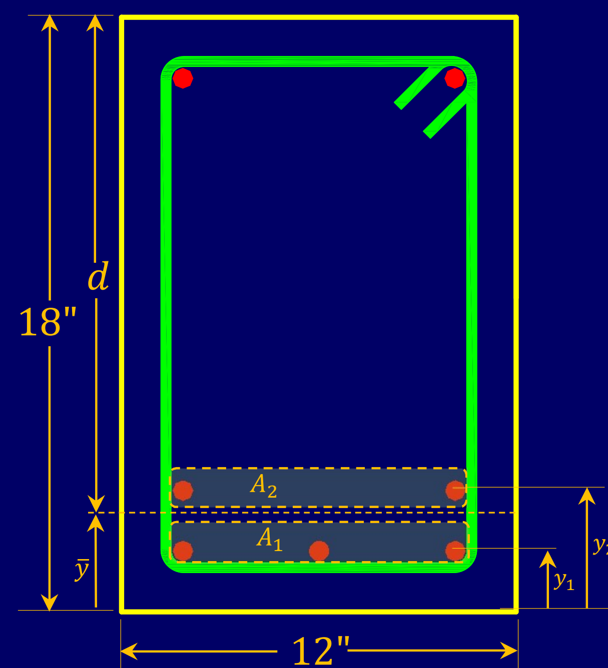
$$d = h - \bar{y}$$

Where;

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$y_1 = Cc + d_{st} + d_b/2$$

$$y_1 = 1.5 + \frac{3}{8} + \frac{6}{8 \times 2} = 2.25"$$



Flexural Design of Singly Reinforced Rectangular Beam

- **Solution**

  - **Step No.8: Check Design Flexural Capacity**

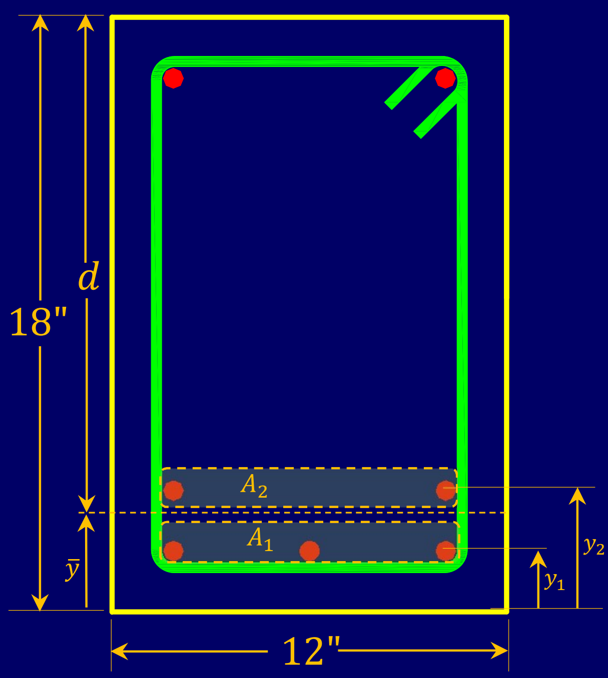
$$y_2 = y_1 + \left(\frac{d_b}{2}\right)_{layer\ 1} + 1.5 + \left(\frac{d_b}{2}\right)_{layer\ 2}$$

$$y_2 = 2.25 + \frac{6}{16} + 1.5 + \frac{6}{16} = 4.5"$$

$$A_1 = 3(0.44) = 1.32 in^2 \text{ and}$$

$$A_2 = 2(0.44) = 0.88 in^2$$

$$\bar{y} = \frac{1.32 \times 2.25 + 0.88 \times 4.5}{1.32 + 0.88} = 3.15"$$



Flexural Design of Singly Reinforced Rectangular Beam

- **Solution**
  - **Step No.8: Check Design Flexural Capacity**

$$d = 18 - 3.15 = 14.85"$$

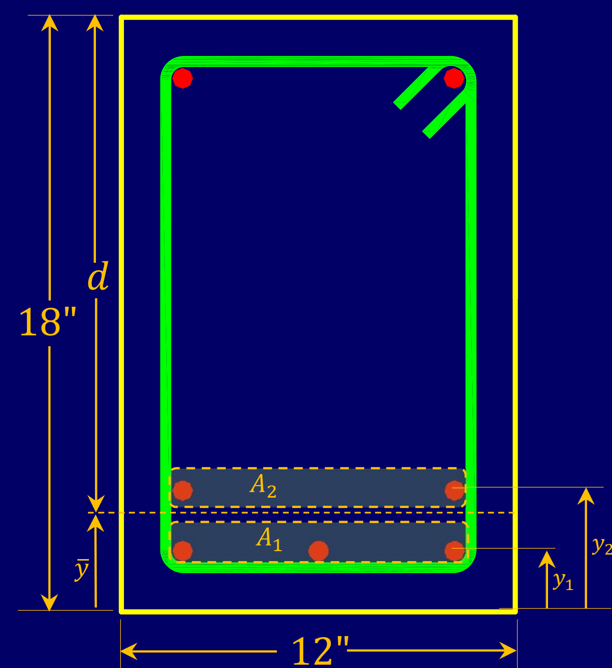
$$a = \frac{A_s f_y}{0.85 f_c' b_w} = \frac{2.2 \times 40}{0.85 \times 3 \times 12} = 2.88"$$

$$\emptyset M_n = 0.9(2.2) \times 40 \left(14.85 - \frac{2.88}{2}\right)$$

Now,

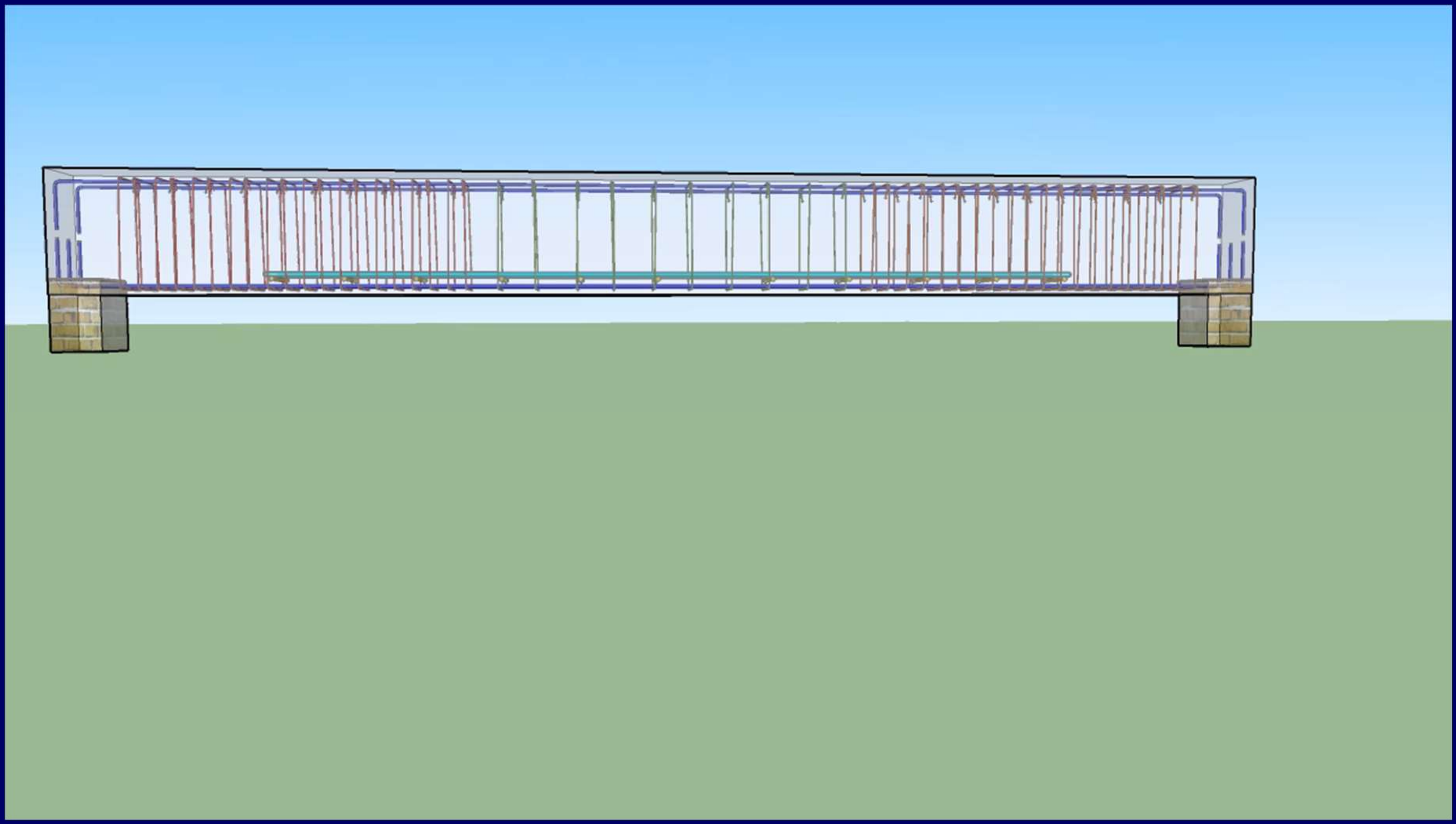
$$\emptyset M_n = 1062.1 in.kip > 1002 in.kip$$

$$\emptyset M_n > M_u \Rightarrow OK!$$



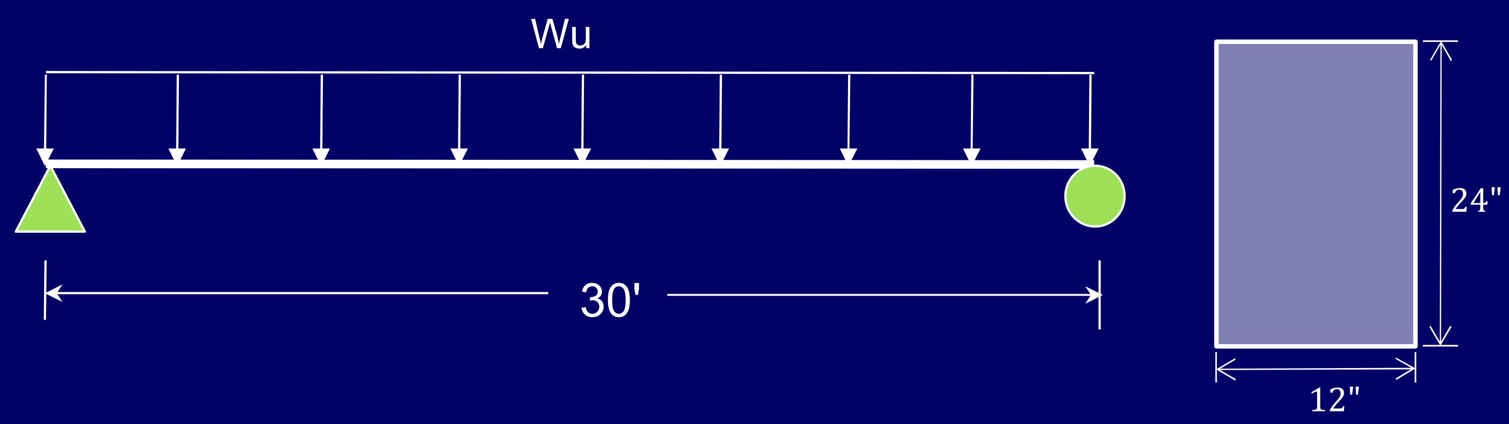
Flexural Design of Singly Reinforced Rectangular Beam

- 3D Detailing Animation



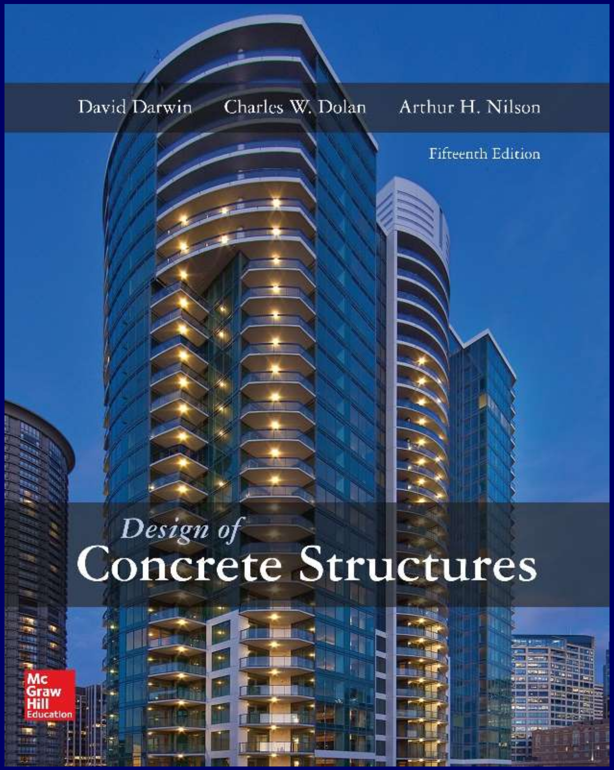
Flexural Design of Singly Reinforced Rectangular Beam

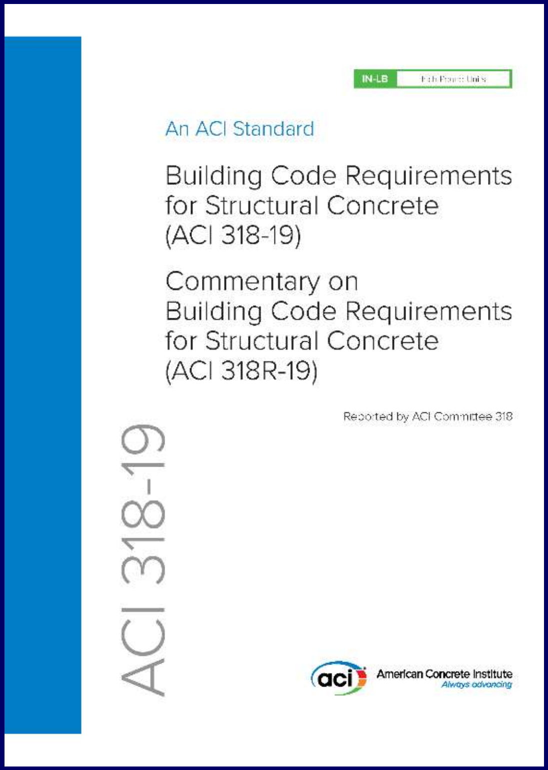
- **Class Activity**
  - ***Design*** a reinforced concrete simply supported beam having a span length of 30 ft and supporting a service dead load of 1.2 kip/ft and a uniform service live load of 1.0 kip/ft in addition to its self-weight. Take $f_c' = 3$ ksi and $f_y = 40$ ksi.



References

- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-19)





Appendix

- **Beam Rebar cage on site**

Appendix

- **Direct Method**
  - **Derivation of** $a = d - \sqrt{d^2 - \frac{2.614M_u}{f_c{}'b_w}}$

We have

$$A_s = \frac{M_u}{\emptyset f_y(d - \frac{a}{2})} \quad \dashrightarrow (1)$$

and

$$a = \frac{A_s f_y}{0.85 f_c' b_w} \quad \dashrightarrow (2)$$

Putting eq. (1) in eq. (2)

$$a = \frac{A_s f_y}{0.85 f_c' b_w} = \frac{M_u}{\emptyset f_y \left(d - \frac{a}{2}\right)} \times \frac{f_y}{0.85 f_c' b_w}$$

Appendix

- **Direct Method**

$$\Rightarrow a\left(d-\frac{a}{2}\right)=\frac{M_u}{f_y}\times\frac{f_y}{0.85f_c'b_w}$$

$$\Rightarrow ad-0.5a^2=\frac{M_u}{0.85f_c'\emptyset b_w}$$

$$\Rightarrow 0.5a^2-ad+\frac{M_u}{0.85f_c'\emptyset b_w}=0$$

This is a quadratic equation and can be sovled by quadratic formula

Comparing this with $Ax^2+Bx+C=0$

$$\Rightarrow A=0.5\,,\qquad B=-d\qquad and\qquad C=\frac{M_u}{0.85f_c'\emptyset b_w}$$

Appendix

- **Direct Method**

Using quadratic formula

$$a = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{d \pm \sqrt{d^2 - 4 \times 0.5 \times \frac{M_u}{0.85 f_c' \emptyset b_w}}}{2 \times 0.5}$$

$$a = d \pm \sqrt{d^2 - \frac{2M_u}{0.85 f_c' \emptyset b_w}}$$

This gives two roots

$$a = d + \sqrt{d^2 - \frac{2M_u}{0.85 f_c' \emptyset b_w}} \quad and \quad a = d - \sqrt{d^2 - \frac{2M_u}{0.85 f_c' \emptyset b_w}}$$

Appendix

- **Direct Method**

Neglecting the first root (meaningless), we get

$$a = d - \sqrt{d^2 - \frac{2M_u}{0.85 f_c' \emptyset b_w}}$$

This can be further simplified by as;

$$\boxed{a = d - \sqrt{d^2 - \frac{2.614 M_u}{f_c' b_w}}}$$