Lecture 04

Design of T and L Beams in Flexure

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Lecture Contents

- Introduction to T and L Beams
- ACI Code provisions for T and L Beams
- Design Cases
- Design of Rectangular T-beam
- Design of True T-beam
- References

Learning Outcomes

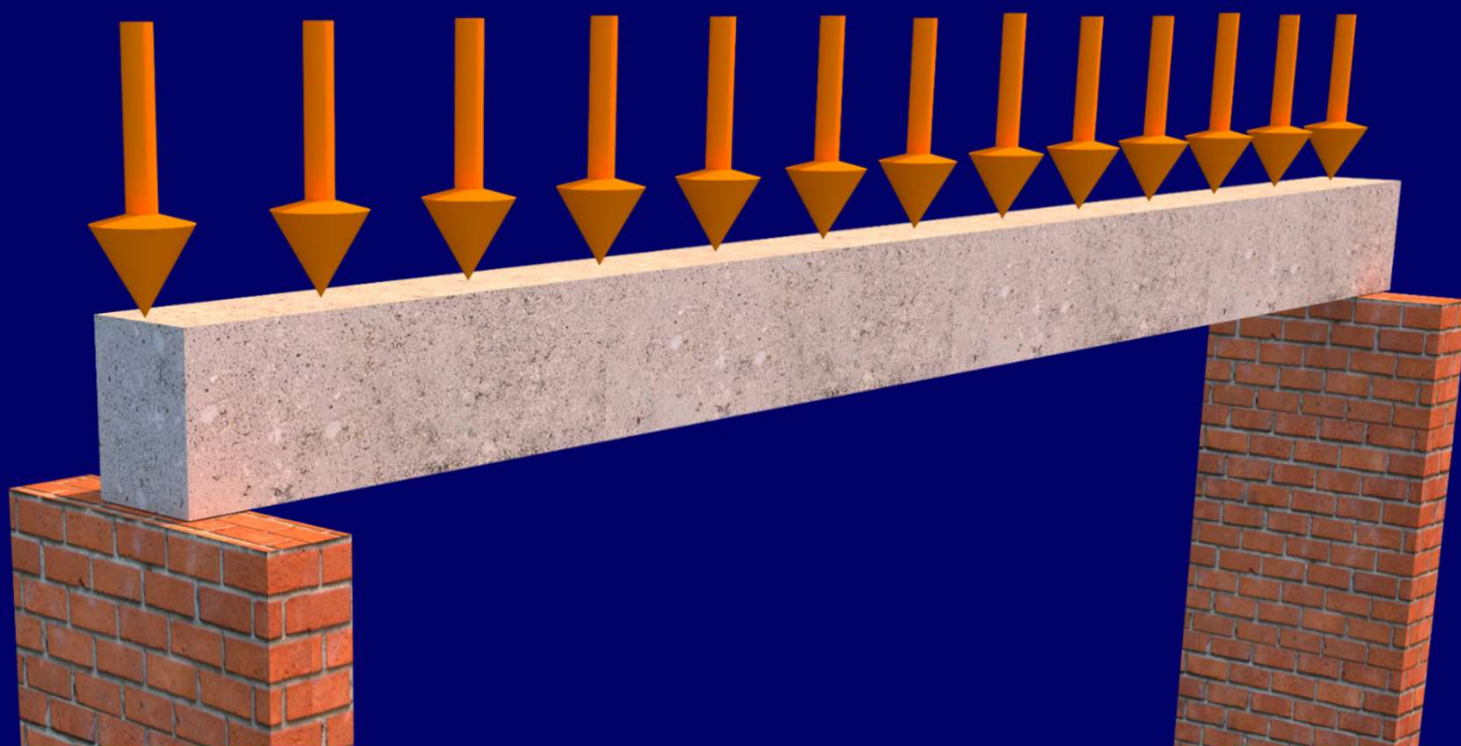
- **At the end of this lecture, students will be able to;**
  - ***Differentiate*** between T-beam and L-beam
  - ***Explain*** Mechanics of Rectangular T-beam and true T-beam
  - ***Design*** T- beam in flexure

Introduction to T and L Beam

- **T and L section beams**
  - The T or L Beam gets its name when the slab and beam produce the cross sections having the typical T and L shapes in a monolithic reinforced concrete construction.
  - In order to fully grasp the concept of T and L beams, carefully watch the animation on the following slides.

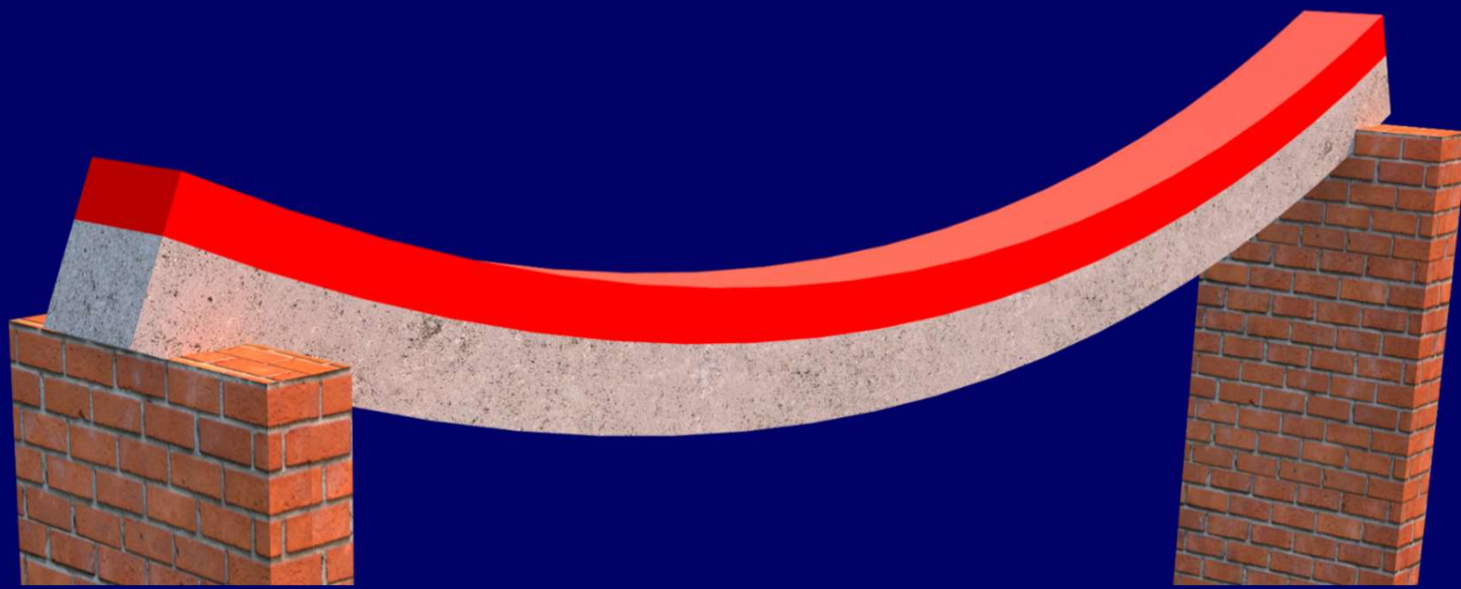
Introduction to T and L Beam

- **Behavior of a Rectangular section beam**



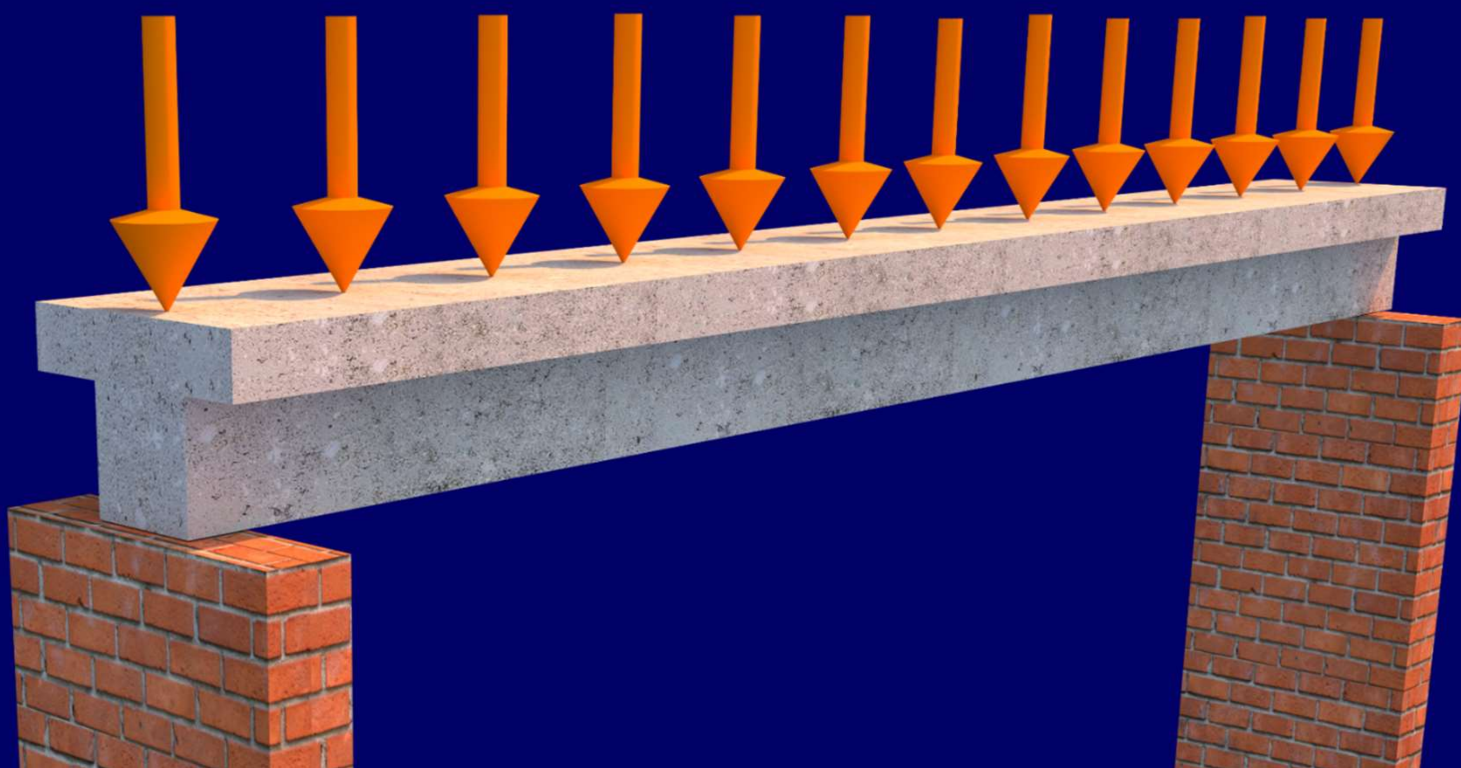
Introduction to T and L Beam

- **Behavior of a Rectangular section beam**



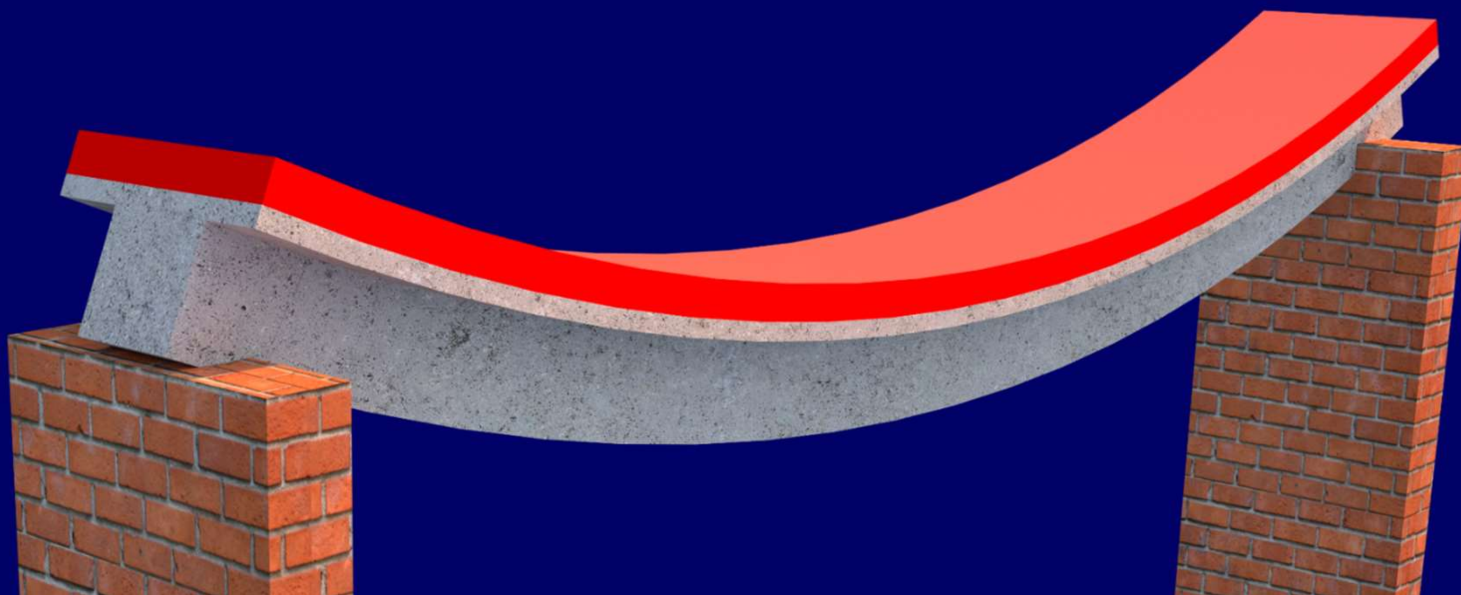
Introduction to T and L Beam

- **Behavior of a T section Beam**



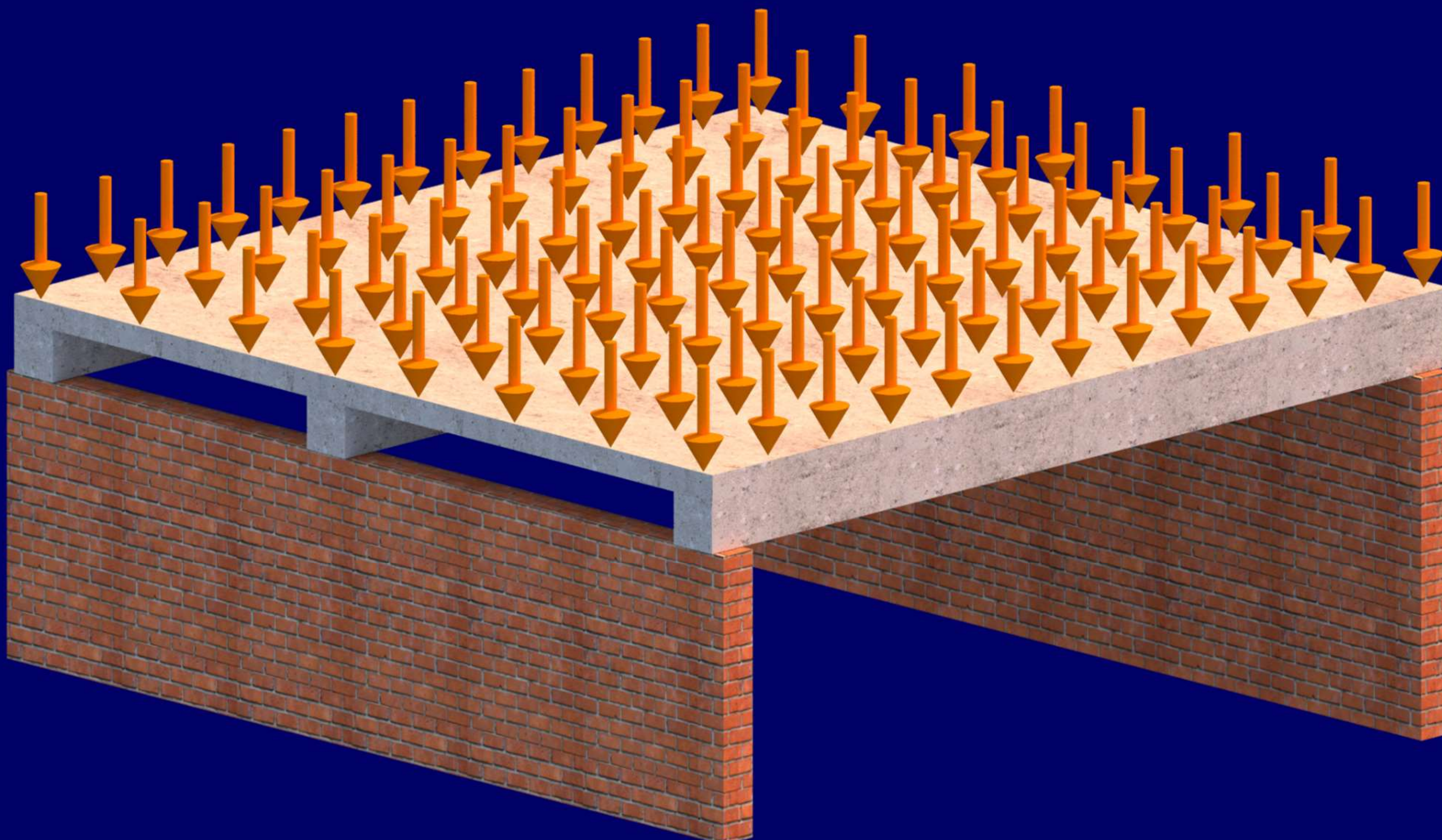
Introduction to T and L Beam

- **Behavior of a T section Beam**



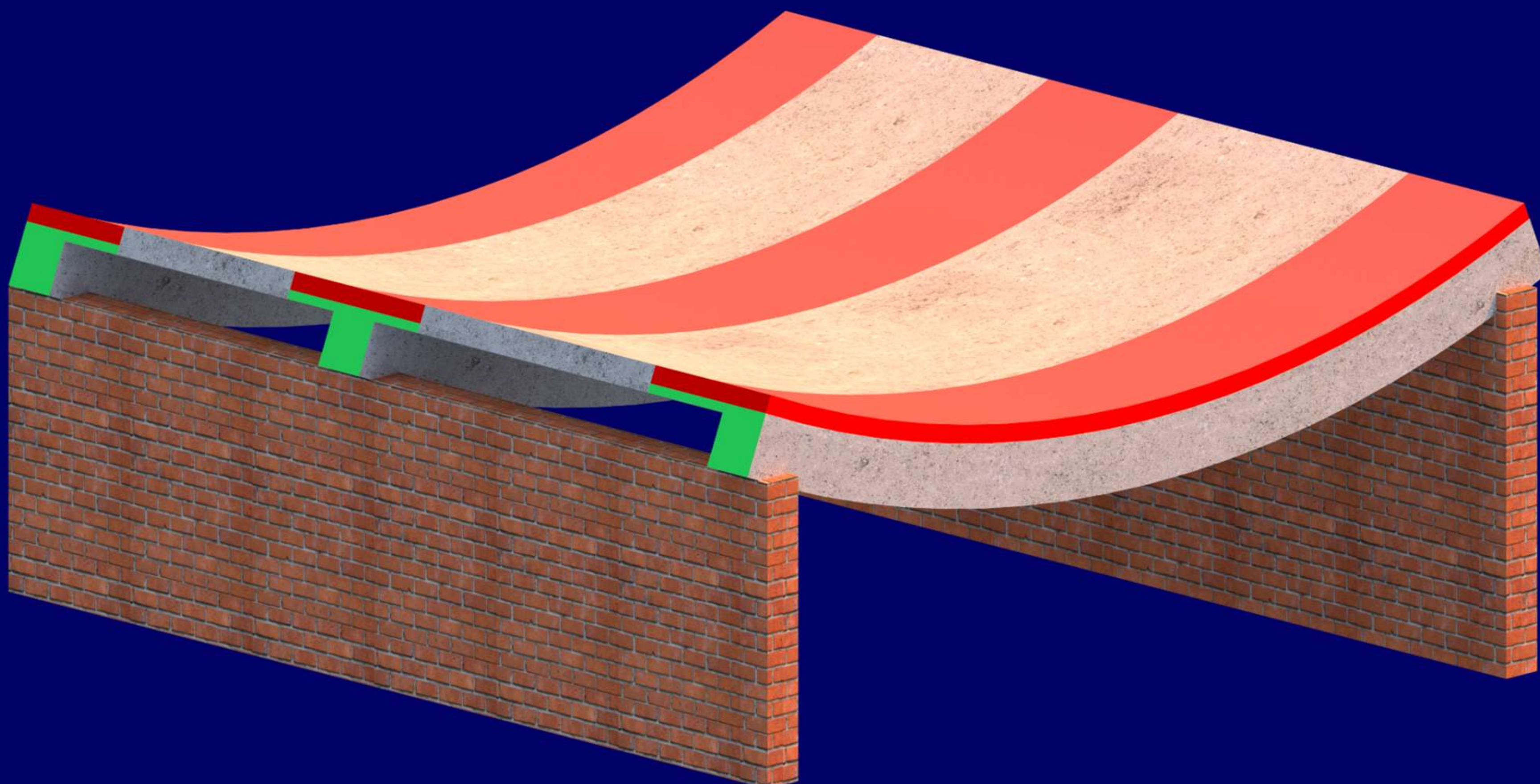
Introduction to T and L Beam

- **T and L section beams in a structure**



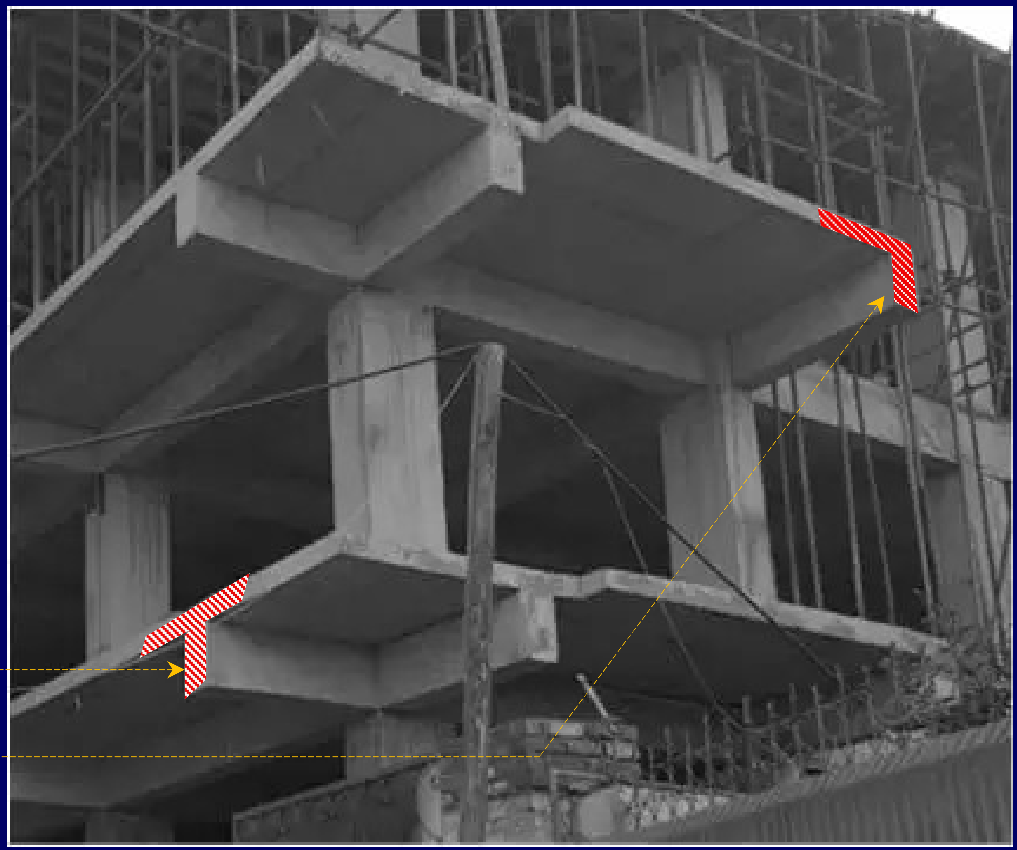
Introduction to T and L Beam

- T and L section beams in a structure



Introduction to T and L Beam

- **T and L section Beams**
  - T and L beams in a real structure



Introduction to T and L Beam

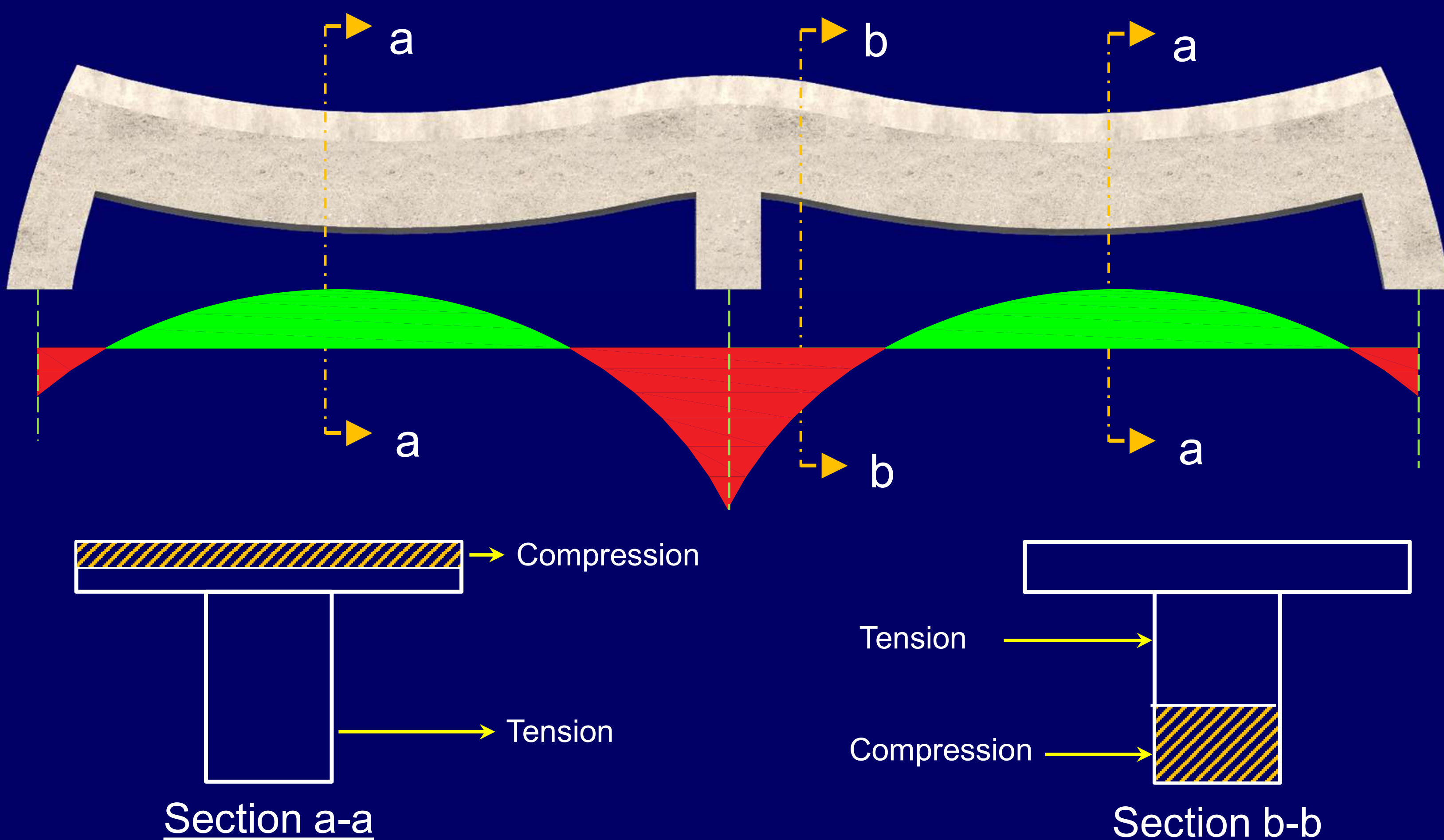
- **T and L section Beams**
  - In casting of reinforced concrete floors/roofs, forms are built for beam sides, the underside of slabs, and the entire concrete is mostly poured at once, from the bottom of the deepest beam to the top of the slab.

Introduction to T and L Beam

- **T and L section Beams**
  - Construction of T and L beam at site

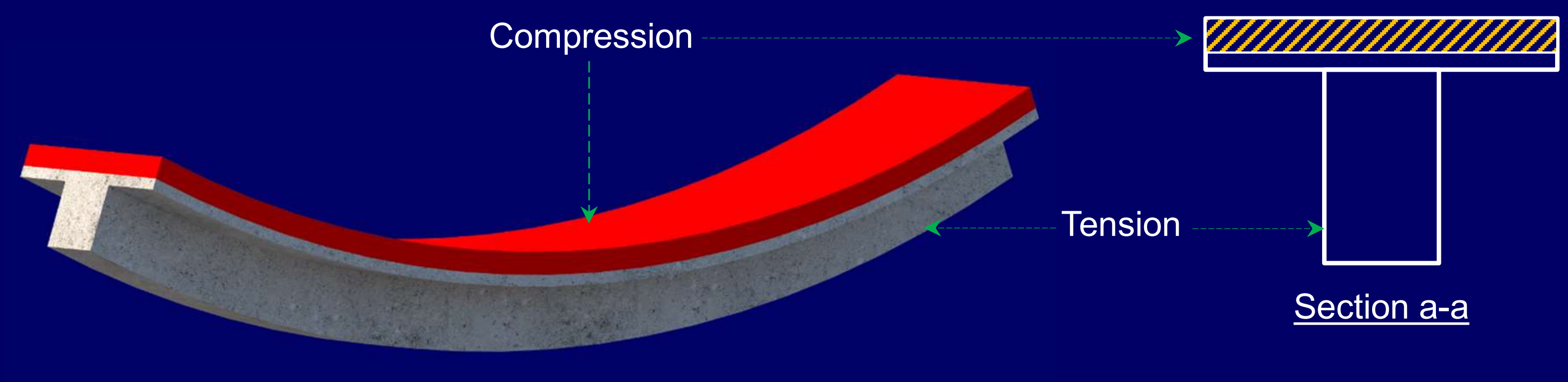
Introduction to T and L Beam

- **Behavior of T and L section Beams under gravity loading**



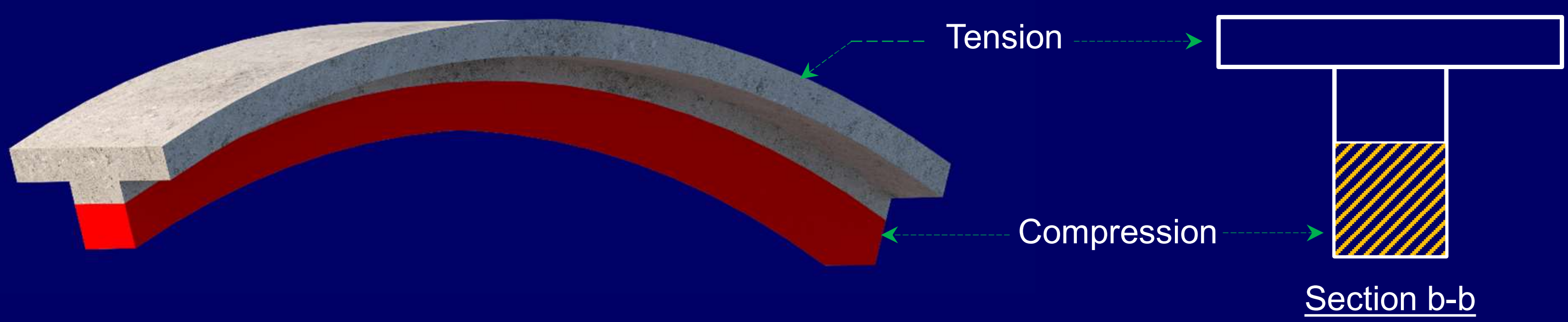
Introduction to T and L Beam

- **Positive Bending Moment**
  - In the analysis and design of floor and roof systems, it is common practice to assume that the monolithically placed slab and supporting beam interact as a unit in resisting the positive bending moment.
  - As shown, the slab acts as the compression flange, while the supporting beam becomes the web or stem.



Introduction to T and L Beam

- **Negative Bending Moment**
  - In the case of negative bending moment, the slab at the top of the stem (web) will be in tension, while the bottom of the stem will be in compression.
  - This usually occurs at interior support of continuous beam.



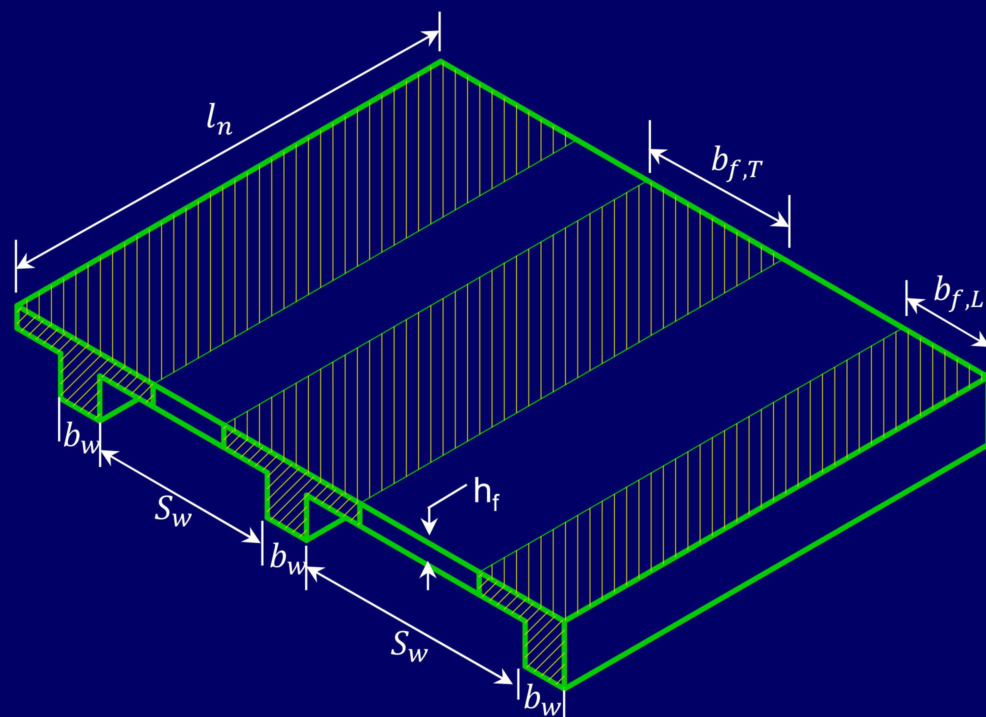
ACI 318 Code Provisions for T and L Beams

- **Calculation of Effective flange width**
  - As per ACI 318-19, the effective flange width b_f for T and L beams shall be calculated as per Table 6.3.2.1
  - **For T beam**

$$b_{f,T} = least\ of \begin{cases} b_w + 16h_f \\ b_w + S_w \\ b_w + l_n/4 \end{cases}$$

  - **For L beam**

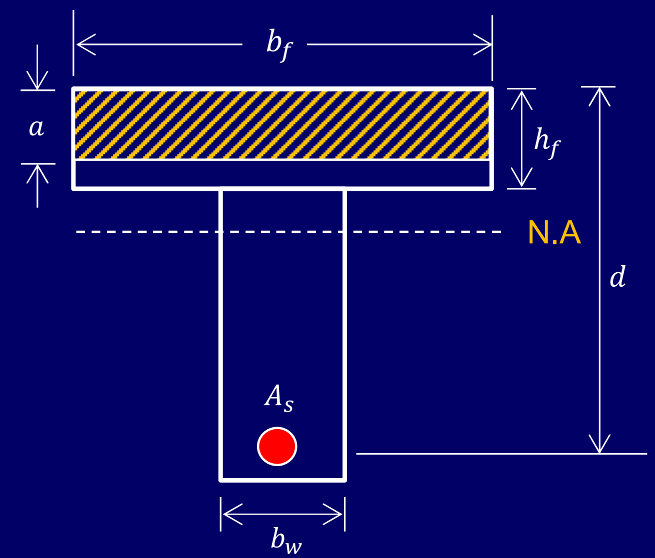
$$b_{f,L} = least\ of \begin{cases} b_w + 6h_f \\ b_w + S_w/2 \\ b_w + l_n/12 \end{cases}$$



Design Cases

- In designing a T-Beam for positive bending moment, there exist two conditions:

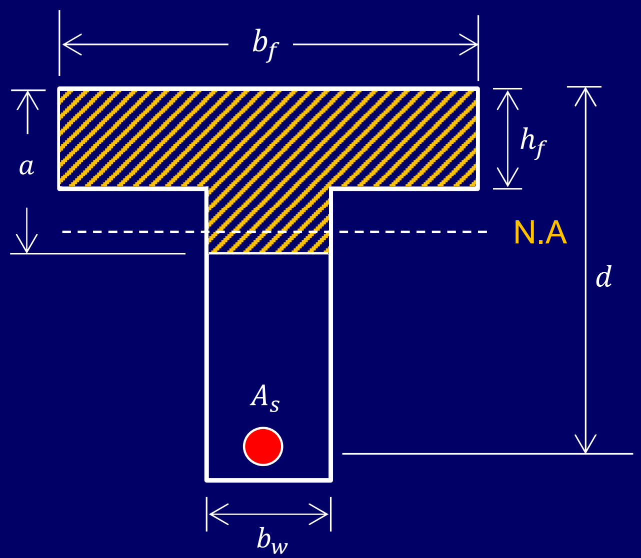
- **Case 1: Rectangular Compression Block**
  - When the value of compression block depth is less than or equal to flange thickness ($a \leq h_f$), it assumes a rectangular shape.
  - In such a case, T-beam should be designed as a rectangular beam with a compression block width of b_f.



Design Cases

- In designing a T-Beam for positive bending moment, there exist two conditions:

- **Case 2: T-shaped Compression Block**
  - When the compression block covers the whole flange and extends into the web portion i.e. ($a > h_f$) , the compression block becomes a T -shaped.
  - In such a condition, the T-Beam is designed as True T-beam.



Design of Rectangular T - beam

- **Flexural capacity (case 1)**

$$\sum F_x = 0 \quad \rightarrow C = T$$

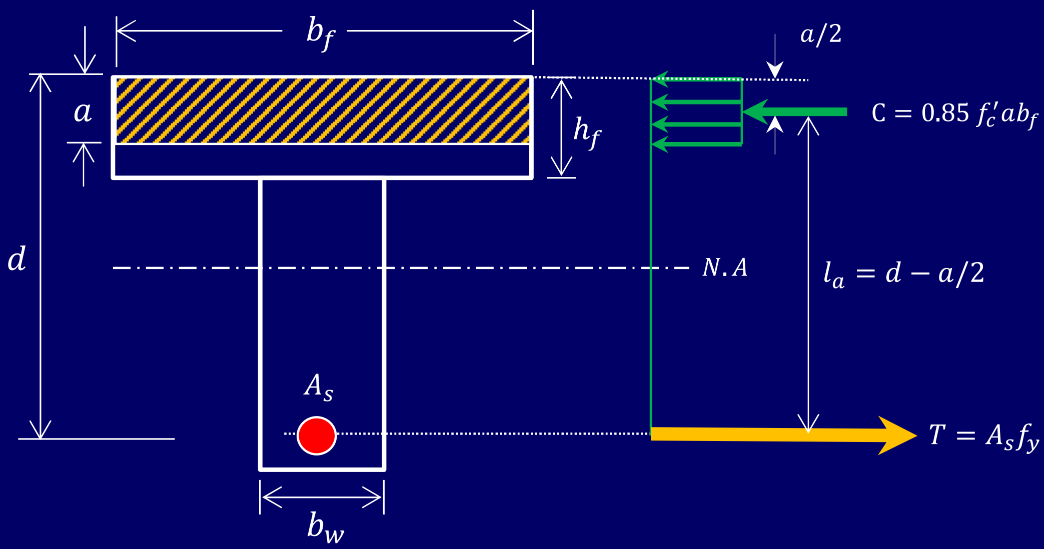
$$0.85 f_c' a b_f = A_s f_y$$

$$a = \frac{A_s f_y}{0.85 f_c' b_f}$$

$$M_n = T \times l_a = A_s f_y \left(d - \frac{a}{2}\right)$$

$$\emptyset M_n = \emptyset A_s f_y \left(d - \frac{a}{2}\right)$$

$$A_s = \frac{M_u}{\emptyset f_y \left(d - \frac{a}{2}\right)}$$



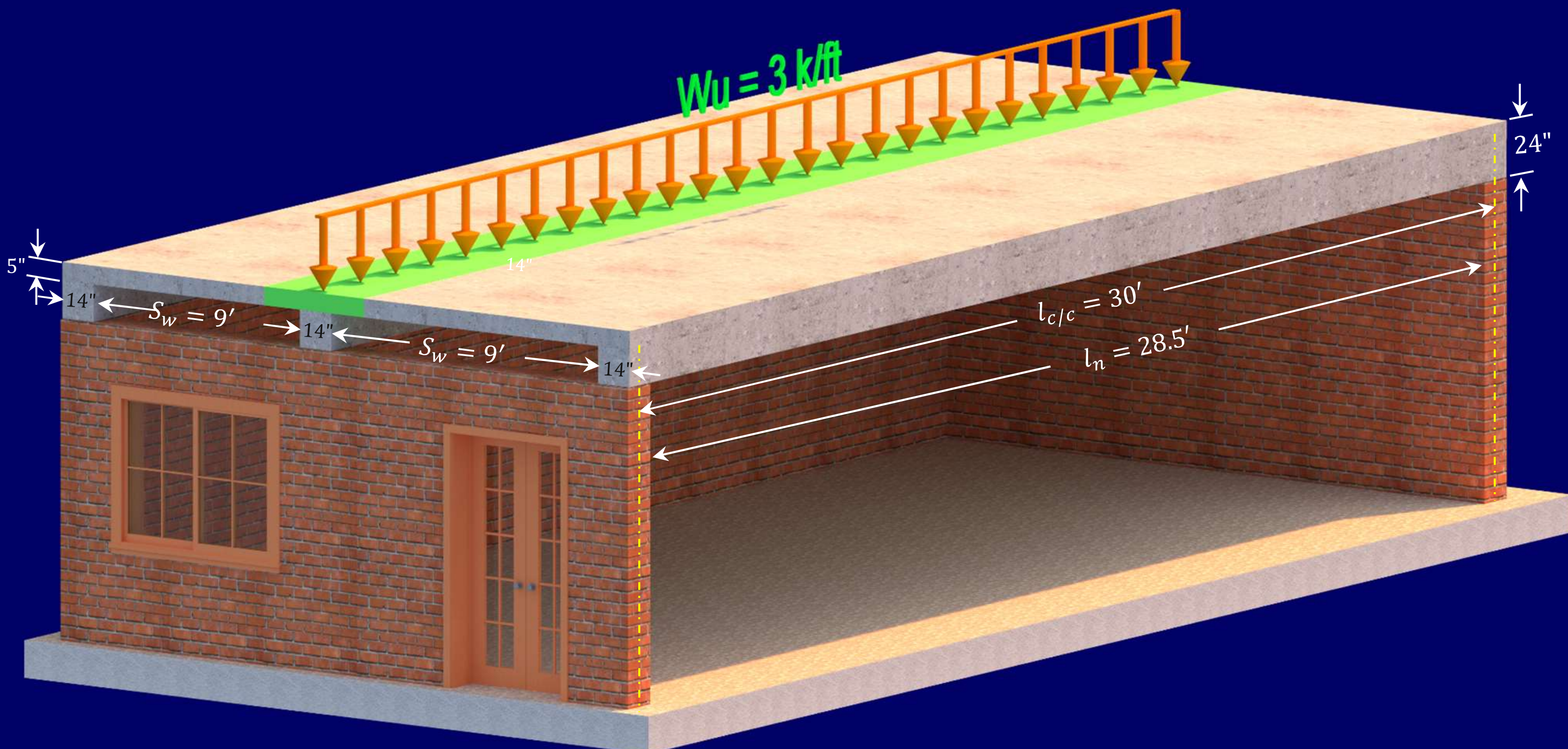
- For no failure, $\emptyset M_n = M_u$
- In calculating $A_{s,min}$ and $A_{s,max}$, use b_w not b_f
- Other checks remain same as that of rectangular beam design.

Design of Rectangular T - beam

- **Summary of design steps for a rectangular T-beam**
  - Step No.1,2 and 3: Sizes, Loads and Analysis
  - Step No.4: Checking the design case (whether rectangular or True T)
  - Step No.5: Determination of steel area
  - Step No.6: Applying reinforcement check
  - Step No.7: Drafting
  - Step No.8: Checking Flexural Capacity (optional)

Design of Rectangular T - beam

- **Example 4.1**
  - ***Design*** the highlighted T - beam for the data provided in figure using $f_c{}' = 3ksi$ and $f_y = 60ksi$.



Design of Rectangular T - beam

- **Solution**
  - **Given Data**

$b_w = 14", h = 24"\ \ and\ \ h_f = 5"$

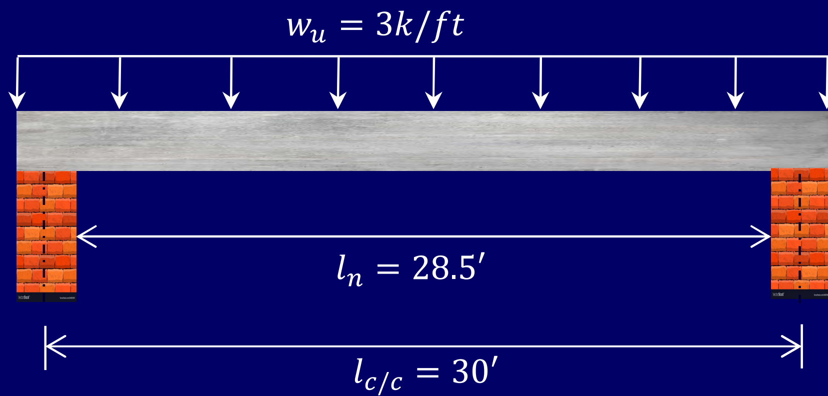
$l_{c/c} = 30', and\ \ l_n = 28.5'$

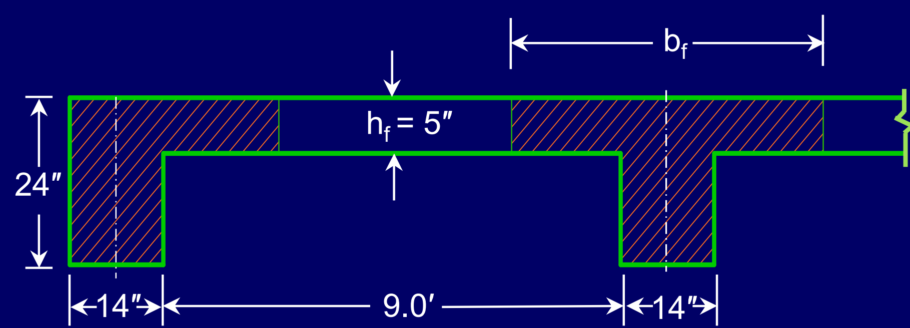
$S_w = 9'$

$w_u = 3k/ft$

$f_c' = 3ksi$

$f_y = 60ksi$





  - **Required Data**

Design the beam as per ACI 318 – 19

Design of Rectangular T - beam

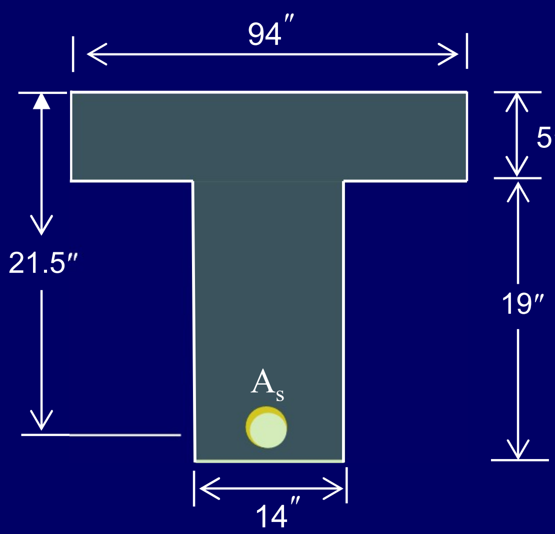
- **Solution**

  - **Step No.1: Selection of Sizes**

$b_w = 14"$, $h = 24"$, $h_f = 5"$ and assume $d = 24 - 2.5 = 21.5"$

Effective width of T- beam b_f is minimum of:

- $b_w + 16h_f = 14 + 16(5) = 94"$
- $b_w + s_w = 14 + 9 \times 12 = 122"$
- $b_w + \frac{l_n}{4} = 14 + \frac{28.5}{4} \times 12 = 99.5"$



Therefore, $b_f = 94"$

Design of Rectangular T - beam

- **Solution**
  - **Step No.2: Calculation of loads**

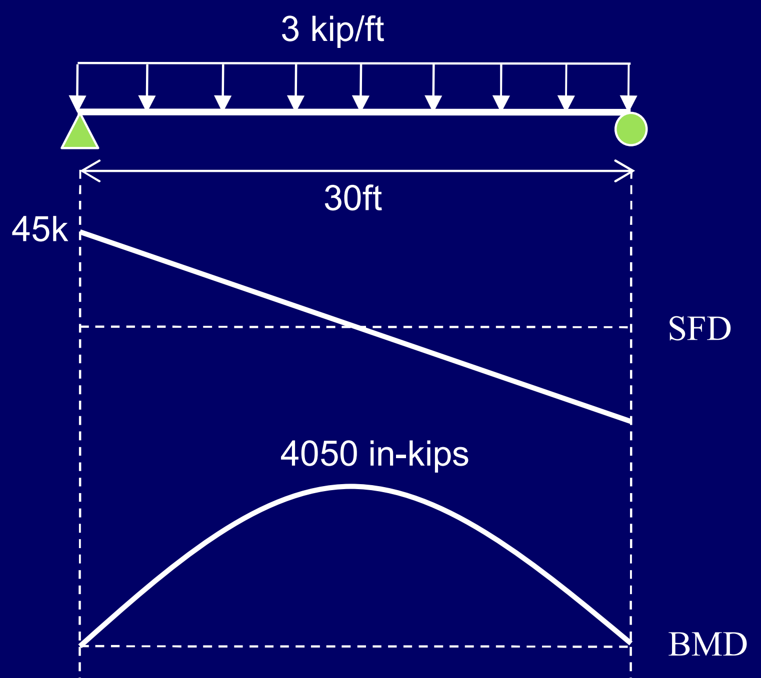
    Ultimate load including self weight is already given

    $w_u = 3k/ft$

  - **Step No.3: Analysis**

$$M_u = \frac{w_u\left(l_{c/c}\right)^2}{8} = \frac{3 \times 30^2}{8} = 337.5k.ft$$

$$M_u = 4050\ in.kip$$



Design of Rectangular T - beam

- **Solution**

  - **Step No.4: Checking behavior of section**

In order to check whether the section rectangular or True T-section we have;

$$a = d - \sqrt{d^2 - \frac{2.614M_u}{f_c{}'b_f}}$$

Note that for T and L beams, b_w is replaced by b_f in equation of a

$$a = 21.5 - \sqrt{21.5^2 - \frac{2.614 \times 4050}{3 \times 94}} = 0.892"$$

Since $a < h_f$, the section can be designed as rectangular beam

Design of Rectangular T - beam

- **Solution**
  - **Step No.5: Determination Steel area**

$$A_s = \frac{M_u}{0.9 f_y (d - a/2)} = \frac{4050}{0.9 \times 60(21.5 - 0.892/2)} = 3.56\ in^2$$

  - **Step No.6: Reinforcement Check**

$$A_{s,min} = \frac{200}{f_y} b_w d = \frac{200}{60000} \times 14 \times 21.5 = 1.0\ in^2$$

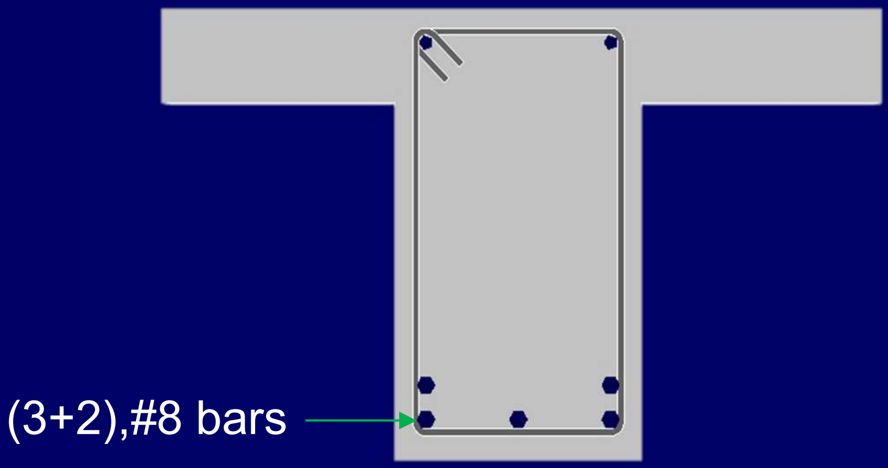
$$A_{s,max} = \frac{f_c' b_w d}{223} = \frac{3 \times 14 \times 21.5}{223} = 4.05 in^2$$

$$A_{s,min} < A_s < A_{s,max} \Rightarrow OK!$$

Design of Rectangular T - beam

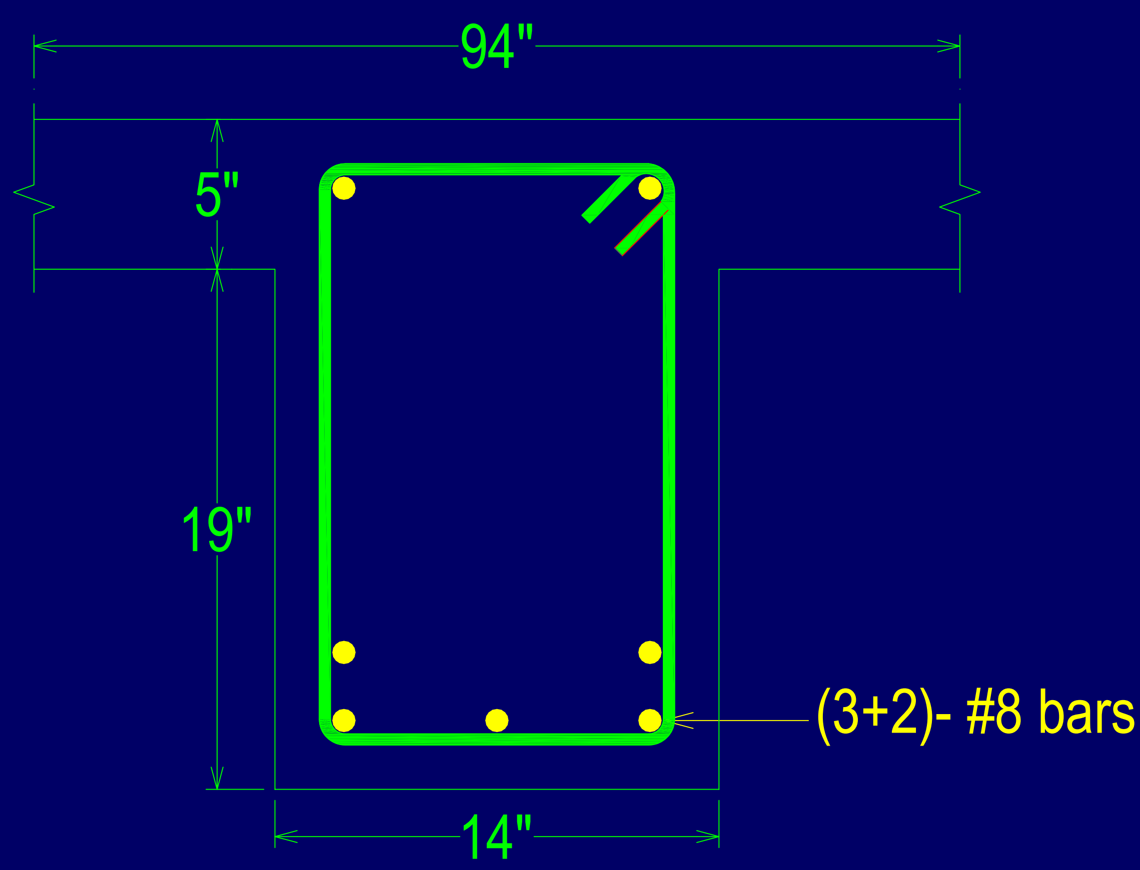
- **Solution**
  - **Step No.7: Detailing and Drafting**
    - Using #8 bars with bar area $A_b = 0.79in^2$

      Number of bars $= \frac{3.56}{0.79} = 4.5 \approx 5$
    - Provide 5 #8 bars in two layers
      - 3 in first layer and
      - 2 in second layer



Design of Rectangular T - beam

- **Solution**
  - **Step No.7: Detailing and Drafting**



Design of Rectangular T - beam

- **Solution**
  - **Step No.8: Flexural capacity Check (optional)**
    - Check flexural capacity of the beam as per actual effective depth and provided steel area

$$d = h - \bar{y} = 24 - 3.375 = 20.625''$$

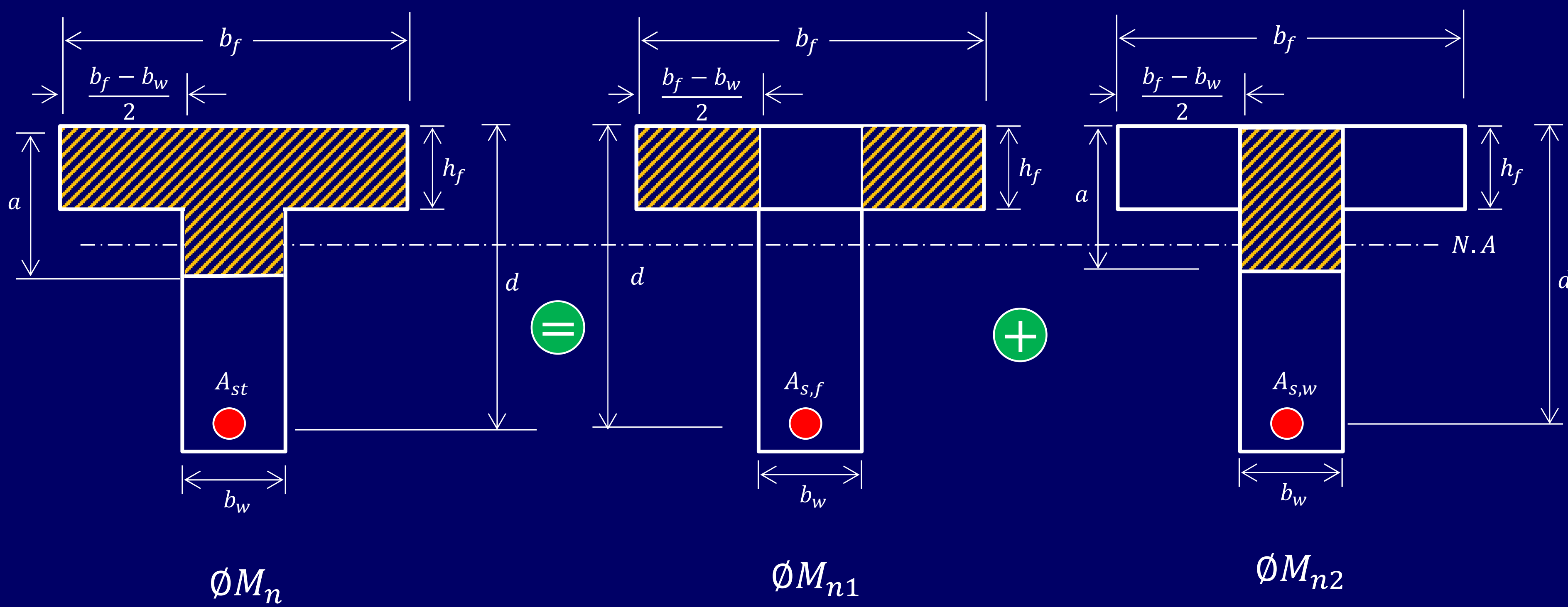
$$a = \frac{A_s f_y}{0.85 f_c' b_f} = \frac{3.95 \times 60}{0.85 \times 3 \times 94} = 0.989in$$

$$\emptyset M_n = 0.9 \times 3.95 \times 60 \left(20.625 - \frac{0.989}{2}\right)$$

$$\emptyset M_n = 4293.84 in.kip > M_u \Rightarrow OK!$$

Design of True T - beam

- **Flexural Capacity (case 2)**



$$\emptyset M_n = \emptyset M_{n1} + \emptyset M_{n2}$$

Design of True T - beam

- **Flexural Capacity (case 2)**
  - Calculation of $\emptyset M_{n1}$

From stress diagram

$$C_1 = T_1$$

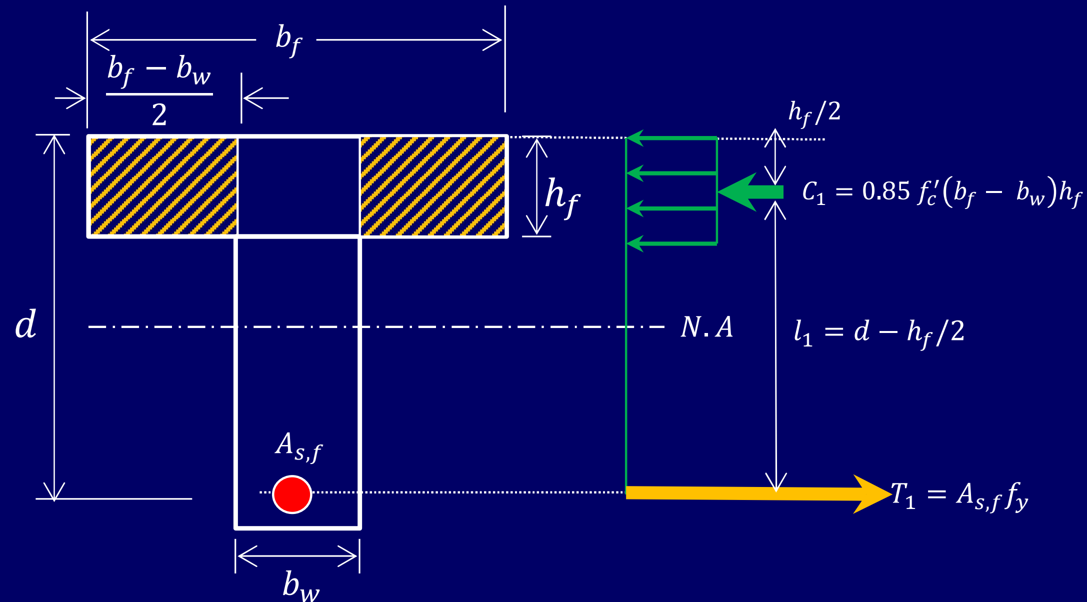
$$C_1 = 0.85 f_c'(b_f - b_w)h_f$$

$$T_1 = A_{s,f} f_y$$

$$0.85 f_c'(b_f - b_w)h_f = A_{s,f} f_y$$

$$A_{s,f} = \frac{0.85 f_c'(b_f - b_w)h_f}{f_y} \quad \text{----- (4.1)}$$

$$\emptyset M_{n1} = T_1 \times l_1 = \emptyset A_{s,f} f_y (d - h_f/2)$$

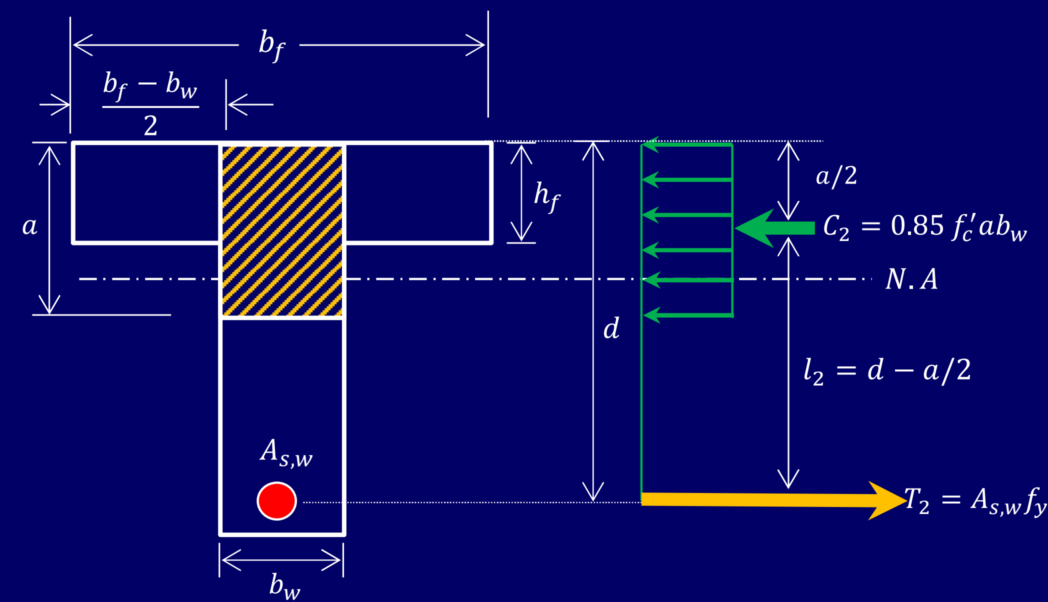


$A_{s,f}$ is the amount of steel to be resisted by flange part of the beam.

Design of True T - beam

- **Flexural Capacity (case 2)**
  - Calculation of $\emptyset M_{n2}$

Again, from stress diagram



$$C_2 = T_2$$

$$C_2 = 0.85\ f_c'ab_w$$

$$T_2 = A_{s,w}f_y$$

$$0.85\ f_c'ab_w = A_{s,w}f_y$$

$$a = \frac{A_{s,w}f_y}{0.85f_c'b_w}$$

$$\emptyset M_{n2} = T_2 \times l_2 = \emptyset A_{s,w}f_y(d - a/2)$$

$A_{s,w}$ is the amount of steel to be resisted by web part of the beam.

Design of True T - beam

- **Flexural Capacity (case 2)**
  - Calculation of steel area to be resisted by web part, $A_{s,w}$
    - As we have

$$\emptyset M_n \geq M_u$$

For $\emptyset M_n = M_u$

$$\emptyset M_{n1} + \emptyset M_{n2} = M_u$$

$$\emptyset M_{n1} + \emptyset A_{s,w} f_y \left(d - \frac{a}{2}\right) = M_u$$

$$A_{s,w} = \frac{M_{u,w}}{\emptyset f_y (d - a/2\ )} \text{ ----- (4.2)}$$

Where, $M_{u,w} = M_u - \emptyset M_{n1}$ is the extra demand moment to be resisted by web portion of T beam.

Design of True T - beam

- **Maximum Reinforcement Limit**

Equating horizontal forces

$$\sum F_x = 0$$

$$C_1 + C_2 = T$$

$$0.85\ f_c'(b_f - b_w)h_f + 0.85\ f_c' a b_w = A_{st} f_y$$

$$A_{s,f} f_y + 0.85\ f_c' a b_w = A_{st} f_y$$

$$A_{st,max} f_y = 0.85\ f_c' \beta_1 c b_w + A_{s,f} f_y$$

$$A_{st,max} = \frac{0.85 f_c' \beta_1 c b_w}{f_y} + \frac{A_{s,f} f_y}{f_y}$$

$$0.85\ f_c'(b_f - b_w)h_f = A_{s,f} f_y$$

For $a = \beta_1 c$ $A_{st} = A_{st,max}$

For $f_y = 40ksi$

$c = 0.41d$ and $a = 0.41\beta_1 d$

For $f_y = 60ksi$

$c = 0.38d$ and $a = 0.38\beta_1 d$

Design of True T - beam

- **Maximum Reinforcement Limit**

$$A_{st,max} = \frac{0.85 f_c' \beta_1 c b_w}{f_y} + A_{sf}$$

$$A_{st,max(TT)} = A_{s,max(SR)} + A_{sf}$$

$$\frac{0.85 f_c' \beta_1 c b_w}{f_y} = A_{s,max(SR)}$$

$$\beta_1 = 0.85 \text{ for } f_c' \leq 4000psi$$

which finally gives

$$A_{st,max(TT),40} = \frac{f_c' b_w d}{136} + A_{s,f} \quad \text{----- (4.3a)}$$

$$A_{st,max(TT),60} = \frac{f_c' b_w d}{223} + A_{s,f} \quad \text{----- (4.3b)}$$

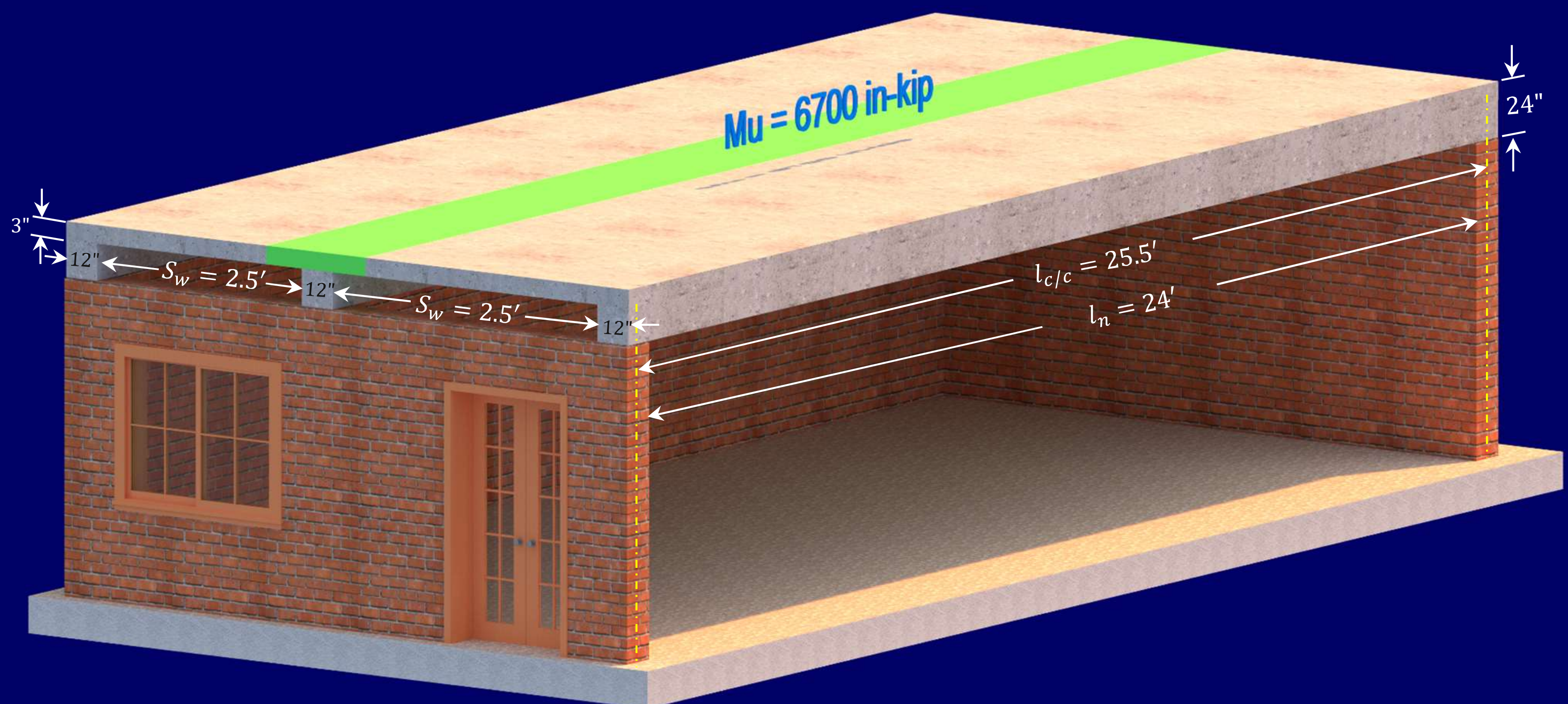
Only valid for $f_c' \leq 4000psi$

Design Procedure

- **Summary of design steps for a True T-beam**
  - Step No.1,2 and 3: Sizes, Loads and Analysis
  - Step No.4: Checking the design case (whether rectangular or True T)
  - Step No.5: Determination of $A_{s,f}$, $\emptyset M_{n,1}$ and $A_{s,w}$
  - Step No.6: Determination of total steel area $(A_{st} = A_{s,f} + A_{s,w})$
  - Step No.7: Applying maximum reinforcement check
  - Step No.8: Drafting
  - Step No.9: Check flexural capacity of section (optional)

Design of True T - beam

- **Example 4.2**
  - ***Design*** the highlighted beam for the data provided in figure using $f_c{}' = 3ksi$ and $f_y = 60ksi$.



Design of True T - beam

- **Solution**
  - **Given Data**

$b_w = 12", h = 24"\ \ and\ \ h_f = 3"$

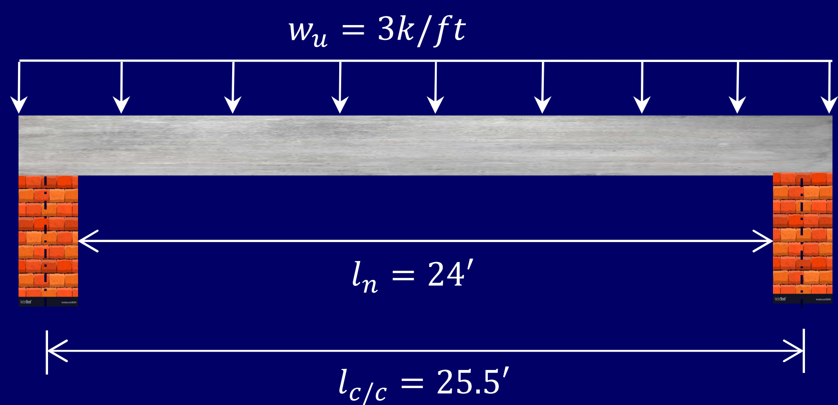
$l_{c/c} = 25.5', and\ \ l_n = 24'$

$S_w = 2.5'$

$M_u = 6700\ in.kip$

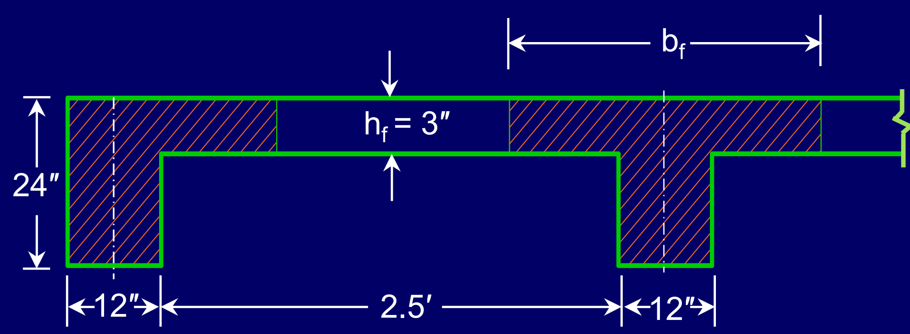
$f_c' = 3ksi$

$f_y = 60ksi$



  - **Required Data**

Design the beam as per ACI 318 – 19



Design of True T - beam

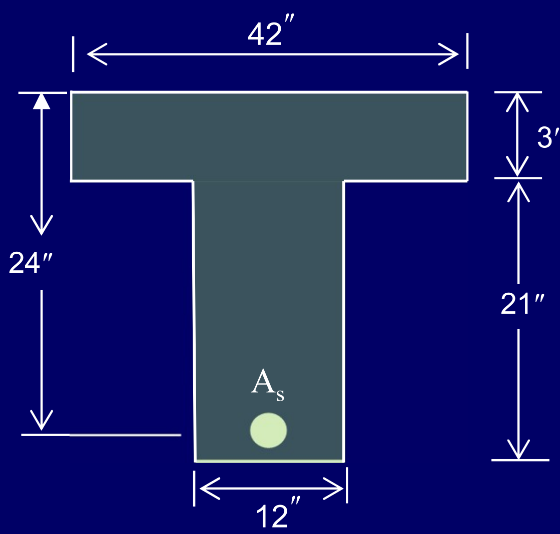
- **Solution**

  - **Step No.1: Selection of Sizes**

$b_w = 12"$, $h = 24"$, $h_f = 3"$ and assume $d = 24 - 2.5 = 21.5"$

Effective width of T- beam b_f is minimum of:

- $b_w + 16h_f = 12 + 16(3) = 60"$
- $b_w + s_w = 12 + 2.5 \times 12 = 42"$
- $b_w + \frac{l_n}{4} = 12 + \frac{24}{4} \times 12 = 84"$



Therefore, $b_f = 42"$

Design of True T - beam

- **Solution**
  - **Step No.2: Calculation of Loads**

    We have directly given the ultimate moment

  - **Step No.3: Analysis**

    $M_u = 6700\ in.kip$

  - **Step No.4: Checking the behavior of Section**

$$a = 21.5 - \sqrt{21.5^2 - \frac{2.614 \times 6700}{3 \times 42}} = 3.52''$$

Since $a > h_f$, the section can be designed as True T- beam

Design of True T - beam

- **Solution**

  - **Step No.5: Determination of $A_{s,f}$, $\emptyset M_{n,1}$ and $A_{s,w}$**

$$A_{s,f} = \frac{0.85 f_c' (b_f - b_w) h_f}{f_y}$$

$$= \frac{0.85 \times 3(42 - 12)3}{60} = 3.83\ in^2$$

And

$$\emptyset M_{n1} = \emptyset A_{s,f} f_y \left(d - \frac{h_f}{2} \right)$$

$$= 0.9 \times 3.83 \times 60 \left(21.5 - \frac{3}{2} \right) = 4136.4\ in.kip$$

Design of True T - beam

- **Solution**

  - **Step No.5: Determination of $A_{s,f}$, $\emptyset M_{n,1}$ and $A_{s,w}$**

Now,

$$M_{u,w} = M_u - \emptyset M_{n1} = 6700 - 4136.4 = 2563.6\ in.kip$$

$$a = d - \sqrt{d^2 - \frac{2.614 M_{u,w}}{f_c' b_w}} = 21.5 - \sqrt{21.5^2 - \frac{2.614(2563.6)}{3 \times 12}} = 4.88\ in.$$

Putting value of a in equation (4.2) gives;

$$A_{s,w} = \frac{M_{u,w}}{\emptyset f_y (d - a/2)} = \frac{2563.6}{0.9 \times 60(21.5 - 4.88/2)} = 2.49 in^2$$

Design of True T - beam

- **Solution**

  - **Step No.6: Determination of Total Steel Area**

    Total area of steel can be calculated as;

    $A_{st} = A_{s,f} + A_s$

    By Substituting values, we get

    $A_{st} = 3.83 + 2.49 = 6.32\ in^2$

    Using #8 bar with area of bar $A_b = 0.79\ in^2$

    Number of bars $= 6.32/0.79 = 8$ bars

    So, Provide 8- #8 bars in two layers (4+4)

Design of True T - beam

- **Solution**
  - **Step No.7: Reinforcement Check**

$$A_{s,min} = \frac{200}{f_y} b_w d \quad \text{(for } f_c{}' \leq 4500psi\text{)}$$

$$A_{s,min} = \frac{200}{60000} \times 12 \times 21.5 = 0.86\ in^2$$

And

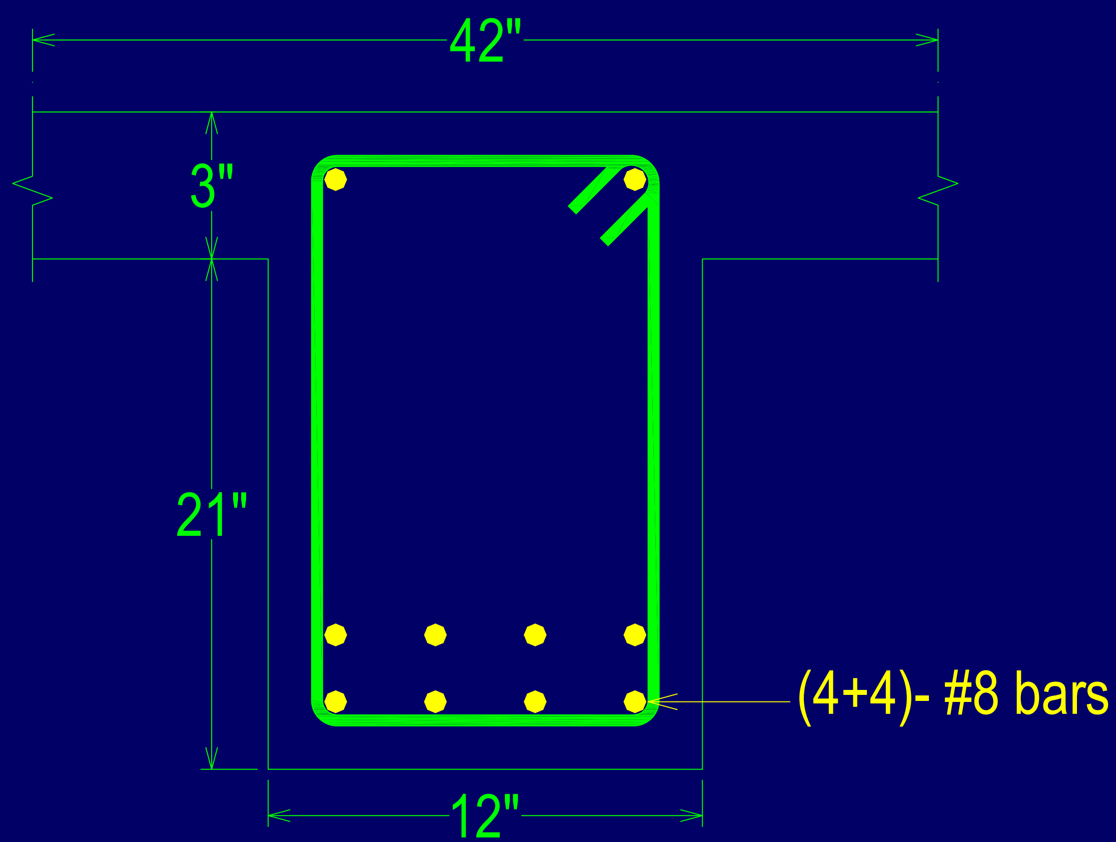
$$A_{st,max(TT),60} = \frac{f_c' b_w d}{223} + A_{sf} = \frac{3 \times 12 \times 21.5}{223} + 3.83$$

$$A_{st,max(TT),60} = 7.30 in^2$$

$$A_{s,min} < A_s < A_{st,max(TT),60} \Rightarrow OK!$$

Design of True T - beam

- **Solution**
  - **Step No.8: Drafting**



Design of True L - beam

- **Design Procedure:**
  - The design procedure for a True L-beam is identical to that of a True T-beam, with only one exception given below:
    - Calculate b_f of L section using the following equations

$$b_{f,L} = least\ of \begin{cases} b_w + 6h_f \\ b_w + S_w/2 \\ b_w + l_n/12 \end{cases}$$

Design of True L - beam

- **Example 4.3**

  Using the data from Example 4.2, design a simply supported L beam with a factored moment of 4500.

- **Solution**

  - **Given Data**

    $b_w = 12", h = 24"\ \ and\ \ h_f = 3"$

    $l_{c/c} = 25.5', and\ \ l_n = 24'$

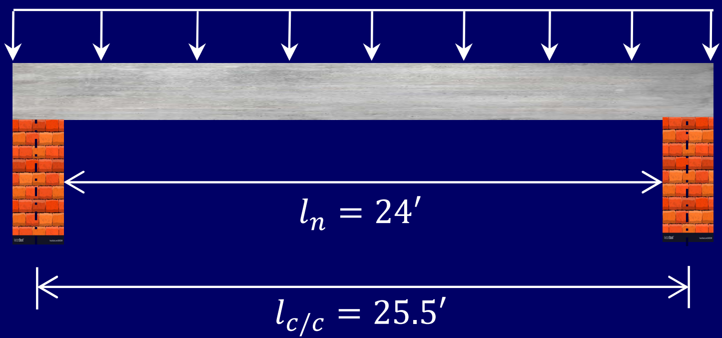
    $S_w = 2.5'$

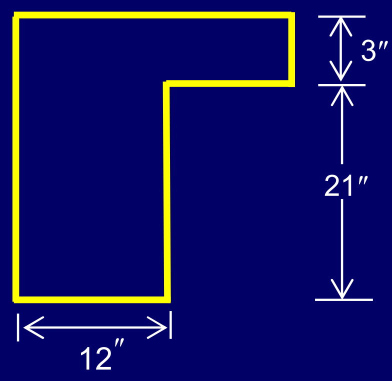
    $M_u = 4500 in.kip$

    $f_c' = 3ksi\ \ and\ \ f_y = 60ksi$

  - **Required Data**

    Design beam as per ACI 318-19





Design of True L - beam

- **Solution**

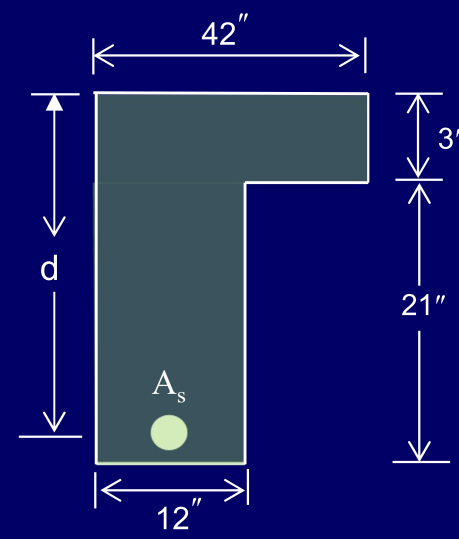
  - **Step No.1: Selection of Sizes**

$b_w = 12"$, $h = 24"$, $h_f = 3"$ and assume $d = 24 - 2.5 = 21.5"$

Effective width of T- beam b_f is minimum of:

- $b_w + 6h_f = 12 + 6(3) = 30"$
- $b_w + \frac{s_w}{2} = 12 + \frac{2.5}{2} \times 12 = 27"$
- $b_w + \frac{l_n}{12} = 12 + \frac{24}{12} \times 12 = 36"$

Therefore, $b_f = 27"$



Design of True L - beam

- **Solution**

  - **Step No.2: Calculation of loads**

    We have directly given the ultimate moment

  - **Step No.3: Analysis**

    $$M_u = 4500\ in.kip$$

  - **Step No.4: Checking the behavior of section**

    $$a = 21.5 - \sqrt{21.5^2 - \frac{2.614 \times 4500}{3 \times 27}} = 3.70"$$

    Since $a > h_f$, the section can be designed as True L- beam

Design of True L - beam

- **Solution**

  - **Step No.5: Determination of $A_{s,f}$, $\emptyset M_{n,1}$ and $A_{s,w}$**

$$A_{s,f} = \frac{0.85 f_c'(b_f - b_w)h_f}{f_y}$$

$$= \frac{0.85 \times 3(27 - 12) \times 3}{60} = 1.91\ in^2$$

And

$$\emptyset M_{n1} = \emptyset A_{s,f} f_y \left(d - \frac{h_f}{2}\right)$$

$$= 0.9 \times 1.91 \times 60 \left(21.5 - \frac{3}{2}\right) = 2062.8\ in.kip$$

Design of True L - beam

- **Solution**
  - **Step No.5: Determination of $A_{s,f}$, $\emptyset M_{n,1}$ and $A_{s,w}$**

Now,

$$M_{u,w} = M_u - \emptyset M_{n1} = 4500 - 2062.8 = 2437.2\ in.kip$$

$$a = d - \sqrt{d^2 - \frac{2.614 M_{u,w}}{f_c' b_w}} = 21.5 - \sqrt{21.5^2 - \frac{2.614(2437.2)}{3 \times 12}} = 4.61\ in.$$

Putting value of a in equation (4.2) gives;

$$A_{s,w} = \frac{M_{u,w}}{\emptyset f_y (d - a/2)} = \frac{2437.2}{0.9 \times 60(21.5 - 4.61/2)} = 2.35\ in^2$$

Design of True L - beam

- **Solution**

  - **Step No.6: Determination of Total Steel Area**

Total area of steel can be calculated as;

$A_{st} = A_{s,f} + A_s$

By Substituting values, we get

$A_{st} = 1.91 + 2.35 = 4.26\ in^2$

Using #6 bar with area of bar $A_b = 0.44\ in^2$

Number of bars $= 4.26/0.44 = 9.7$ say 10 bars

So, provide 10 - #8 bars in three layers (4+4+2)

Design of True L - beam

- **Solution**

  - **Step No.6: Reinforcement Check**

$$A_{s,min} = \frac{200}{f_y} b_w d \quad (\text{for } f_c{}' \leq 4500psi)$$

$$A_{s,min} = \frac{200}{60000} \times 12 \times 21.5 = 0.86\ in^2$$

And

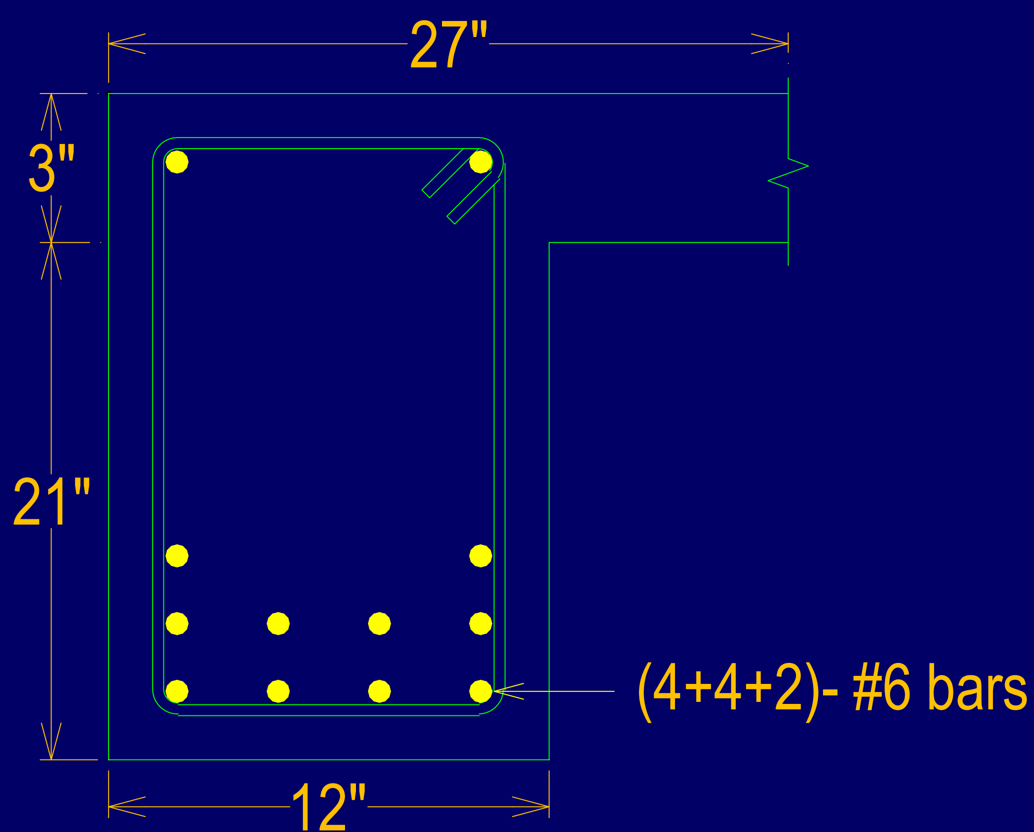
$$A_{st,max(TL),60} = \frac{f_c' b_w d}{223} + A_{sf} = \frac{3 \times 12 \times 21.5}{223} + 1.91$$

$$A_{st,max(TL),60} = 5.38\ in^2$$

$$A_{s,min} < A_s < A_{st,max(TL),60} \Rightarrow OK!$$

Design of True L - beam

- **Solution**
  - **Step No.7: Drafting**

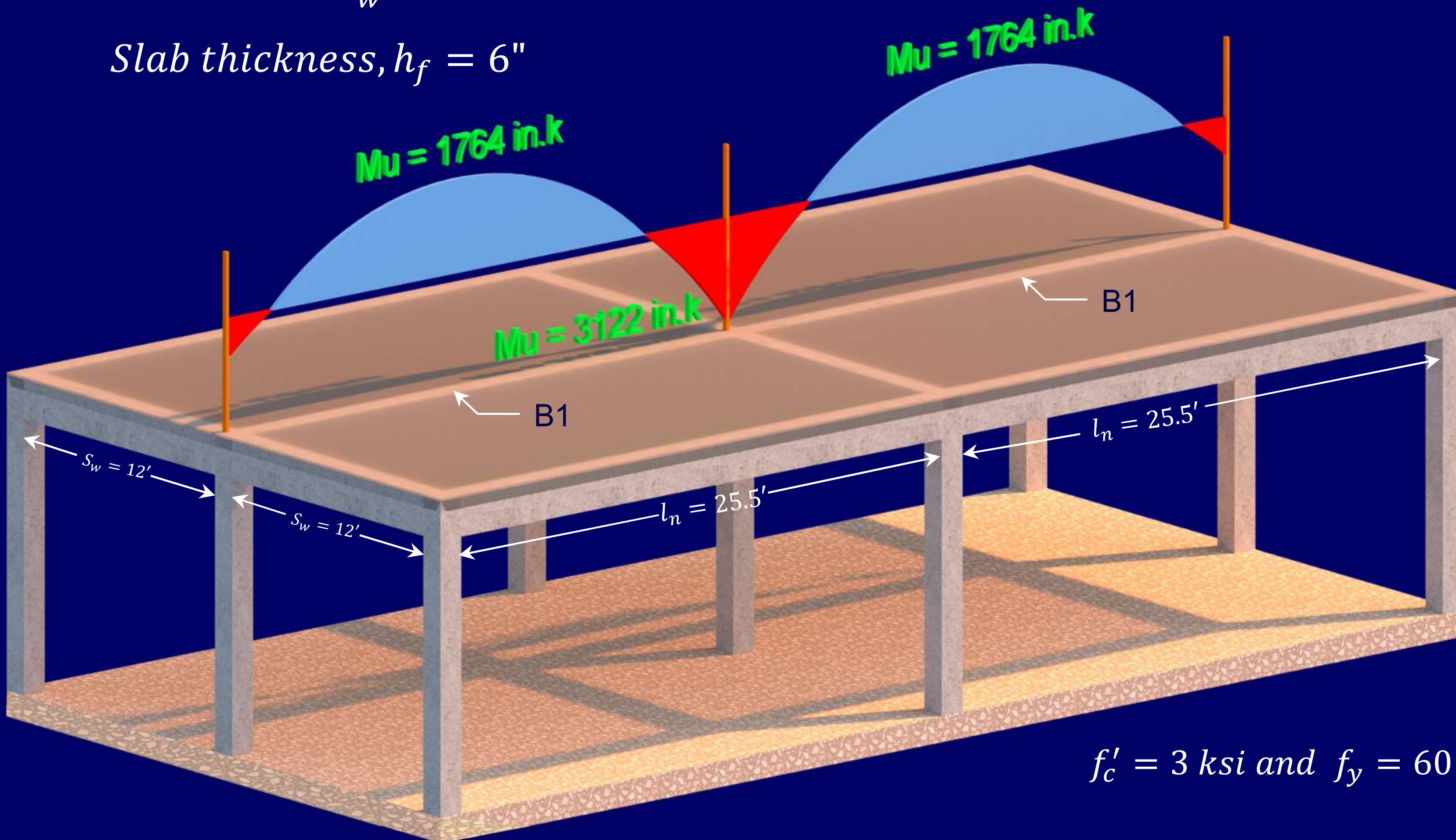


Homework

- ***Design*** Beam B1 for the following data

$Beams\ size: b_w \times h = 15" \times 24"$

$Slab\ thickness, h_f = 6"$



$f'_c = 3\ ksi\ and\ \ f_y = 60\ ksi$

References

- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-19)