Lecture 07

Design of Reinforced Concrete Slabs

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Lecture Contents

- Concrete Floor Systems
- Analysis and Design of slabs
- Strip Method of Analysis for One-way Slabs
- Basic Design Steps
- Example

Learning Outcomes

- **At the end of this lecture, students will be able to;**
  - ***Classify*** slab systems
  - ***Analyze*** one-way slabs using Strip Method of Analysis for flexure
  - ***Design*** one-way slab system for flexure

Concrete Floor Systems

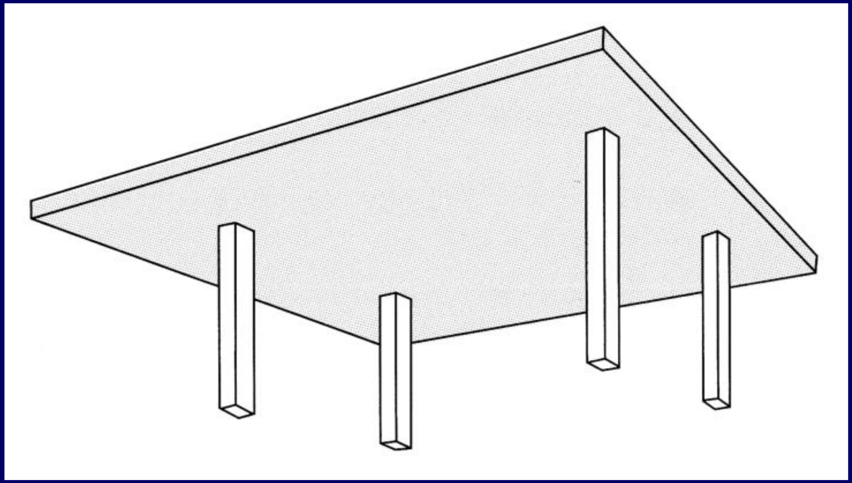
- **Introduction**
  - In reinforced concrete construction, Concrete Floor Systems are broad and flat plates, usually horizontal, with top and bottom surfaces parallel or nearly so.
  - They are used to provide flat and useful surfaces.
  - They may be supported by:
    - Reinforced concrete beams
    - Masonry or reinforced concrete walls
    - Structural steel members
    - Columns
    - Ground.

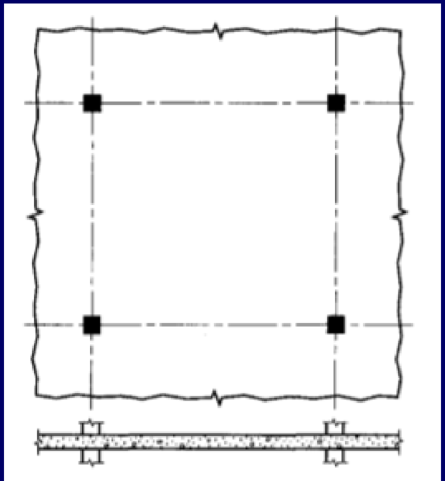
Concrete Floor Systems

- **Various types of concrete floor system**

  1. Flat Plate

  - Concrete floors directly supported on columns (with no beams) are called flat plates.
  - Punching shear is a typical problem in flat plates.



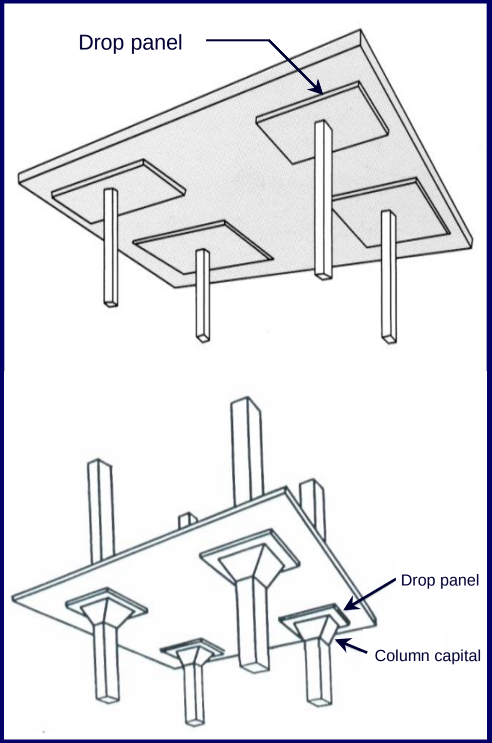


Concrete Floor Systems

- **Various types of concrete floor system**

  2. Flat Slab

  - A flat plate that has a column capital or a drop panel is referred to as a flat slab.
  - Drop panel is a projection of slab at the vicinity of column.
  - Column capital is a widening of the top of a support column where it meets the soffit of a slab.
  - The Drop panel and Column capital are intended to reduce the Punching shear problem.

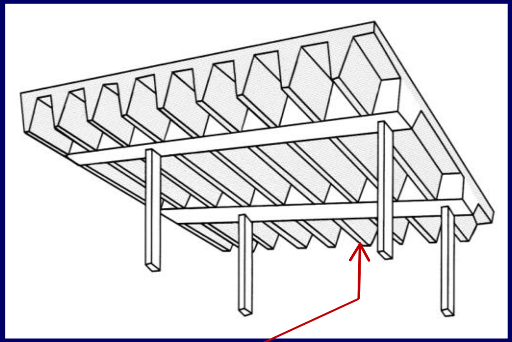


Concrete Floor Systems

- **Various types of concrete floor system**

3. **One-way Joist System**

   - Joist construction consists of a monolithic combination of regularly spaced ribs with clear spacing of 3′ and a top slab arranged to span in one direction or two orthogonal directions.



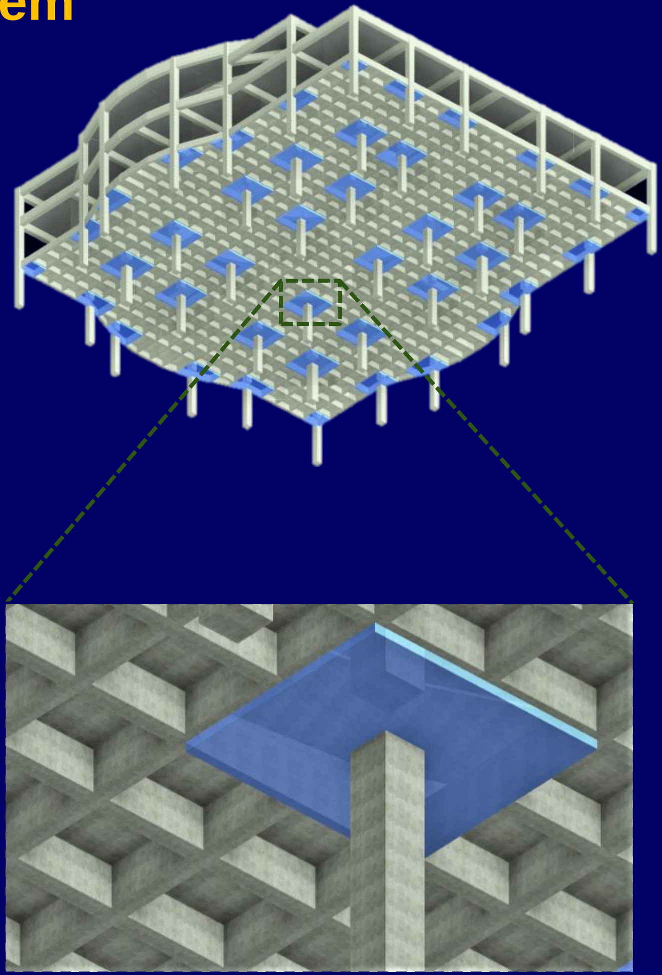
Rib

Peshawar University Auditorium

Concrete Floor Systems

- Various types of concrete floor system

  4. Two-way Joist System



Concrete Floor Systems

- **Various types of concrete floor system**

  5. Beam supported slab systems

  - Slabs may be supported on two opposite sides only, or on all four sides.
  - Based on the behavior of bending, there are two types of slab
    a. One – way slab (bending in one direction)
    b. Two – way slab (bending in two direction)

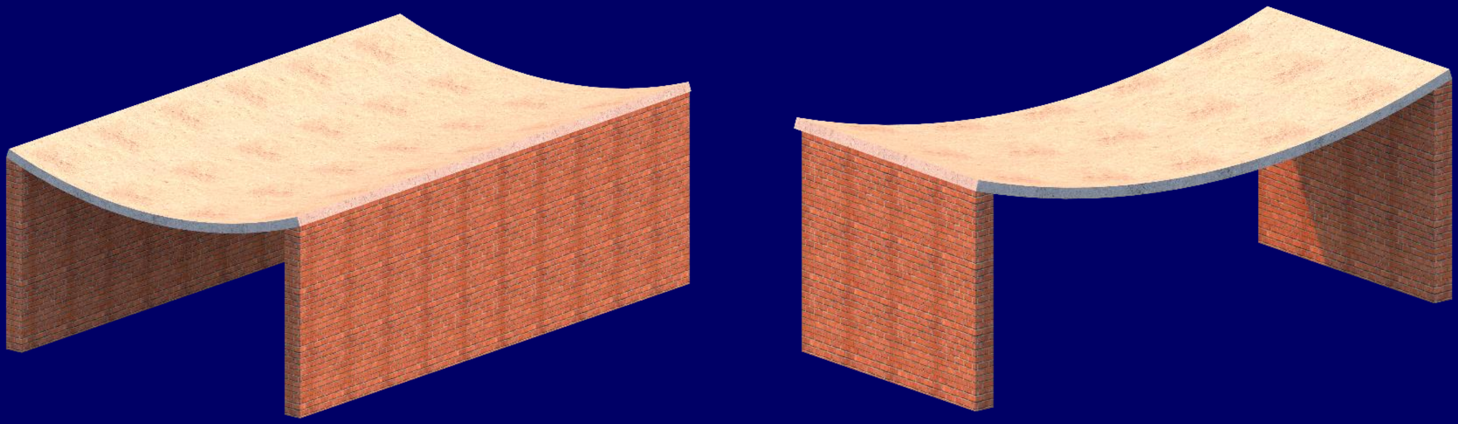
Concrete Floor Systems

- **Various types of concrete floor system**

  5. Beam supported slab systems

  a. One – way slab (Case I)

  - Slab supported on two opposing sides only is called one - way slab.
  - In such case, the bending occurs perpendicular to the supports as shown in the following figures.



Bending behavior of one-way slab

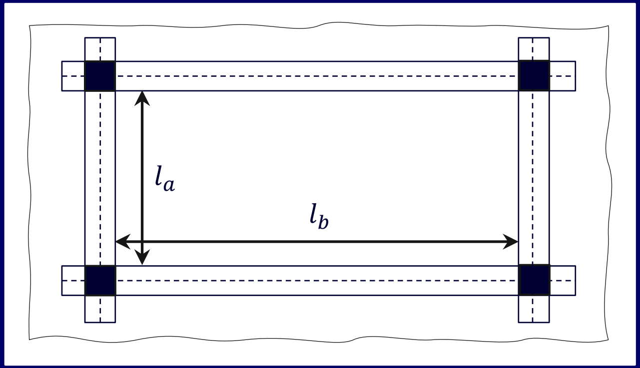
Concrete Floor Systems

- Various types of concrete floor system

5. Beam supported slab systems

a. One – way slab (Case II)

- When the ratio of long to short span in a slab supported on all sides is equal to or greater than 2 the slab is called one – way slab.
- Because the major bending moment is always along the short direction, these slabs are solely designed for the short direction.



Slab supported on all sides, but $\beta = l_b/l_a \geq 2$

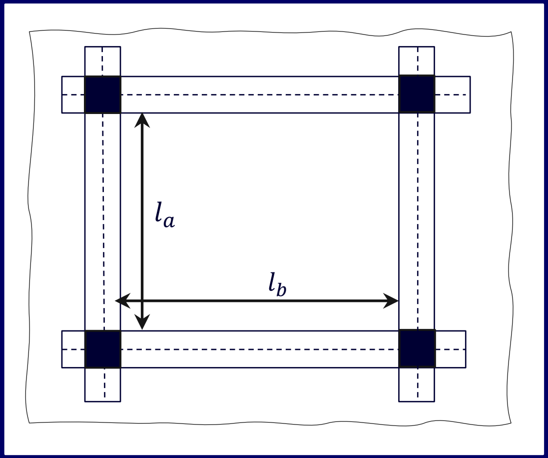
Concrete Floor Systems

- **Various types of concrete floor system**

  5. **Beam supported slab systems**

     b. **Two – way slab**

     - When the ratio of long to short span in a slab supported on all sides is less than 2, then the bending is in two directions. Such a slab is termed as a Two – way slab.



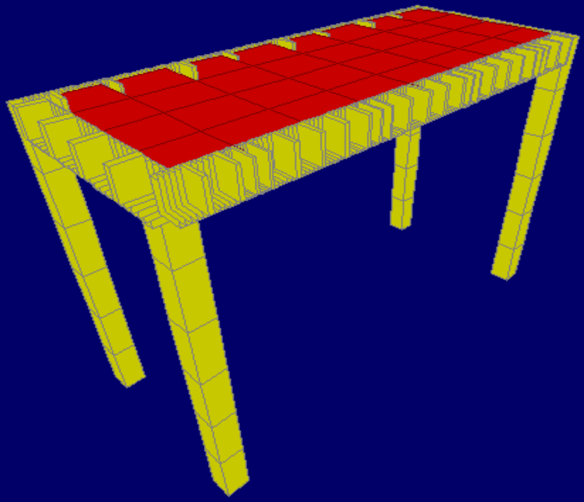
Slab supported on all sides, but $\beta = l_b/l_a < 2$

Concrete Floor Systems

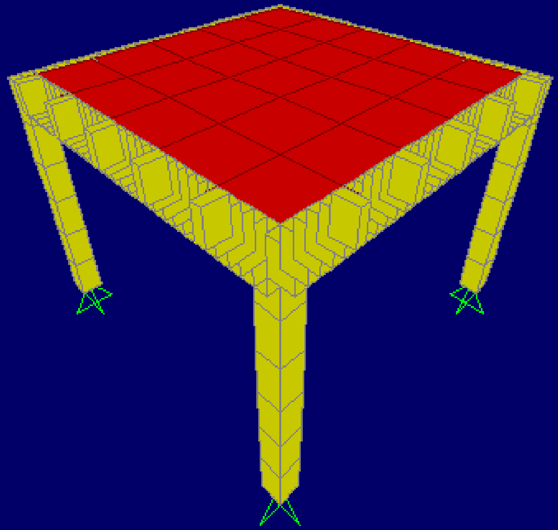
- **Various types of concrete floor system**

  5. Beam supported slab systems

  - Behavior of one – way and two – way slab



One – way Slab



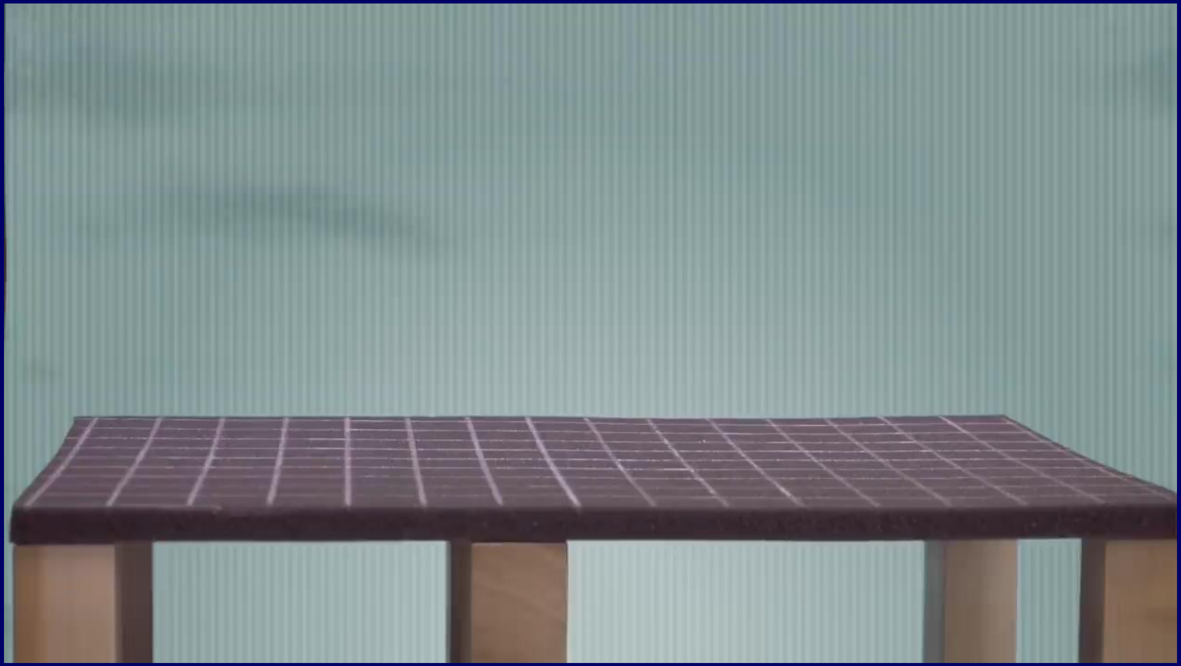
Two – way Slab

Concrete Floor Systems

- **Various types of concrete floor system**

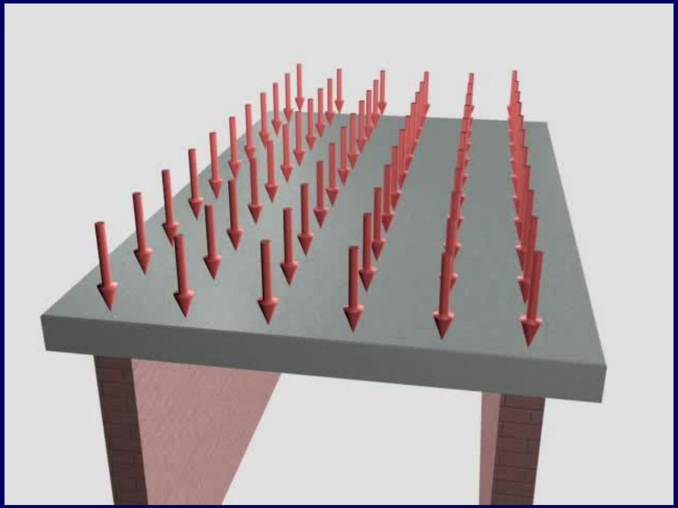
  5. **Beam supported slab systems**

     - Behavior of one – way and two – way slab

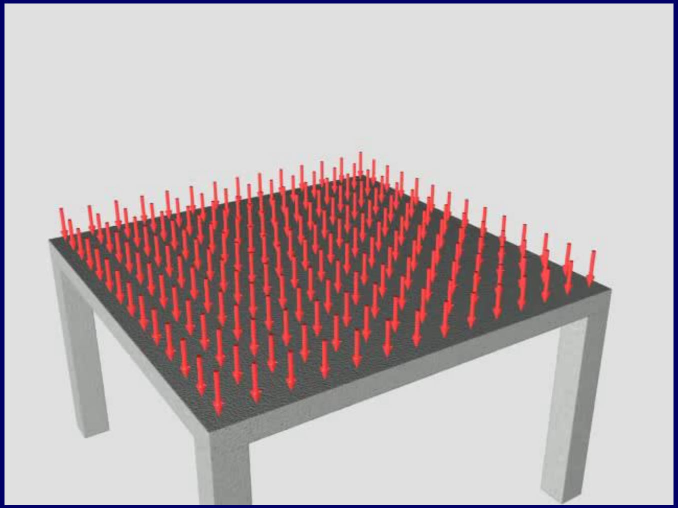


Concrete Floor Systems

- **Various types of slab system**
  - **Types of Beam Supported slabs**
    - Load Transfer Mechanism



One – way Slab



Two – way Slab

Analysis and Design of Slabs

- **Analysis**
  - Unlike beams and columns, slabs are two dimensional members. Therefore, their analysis except one-way slab systems is relatively difficult.
- **Design**
  - Once the analysis is done, the design is carried out in the usual manner. So, no problem in design, problem is only in analysis of slabs.

Analysis and Design of Slabs

- **Various Methods of slab Analysis**

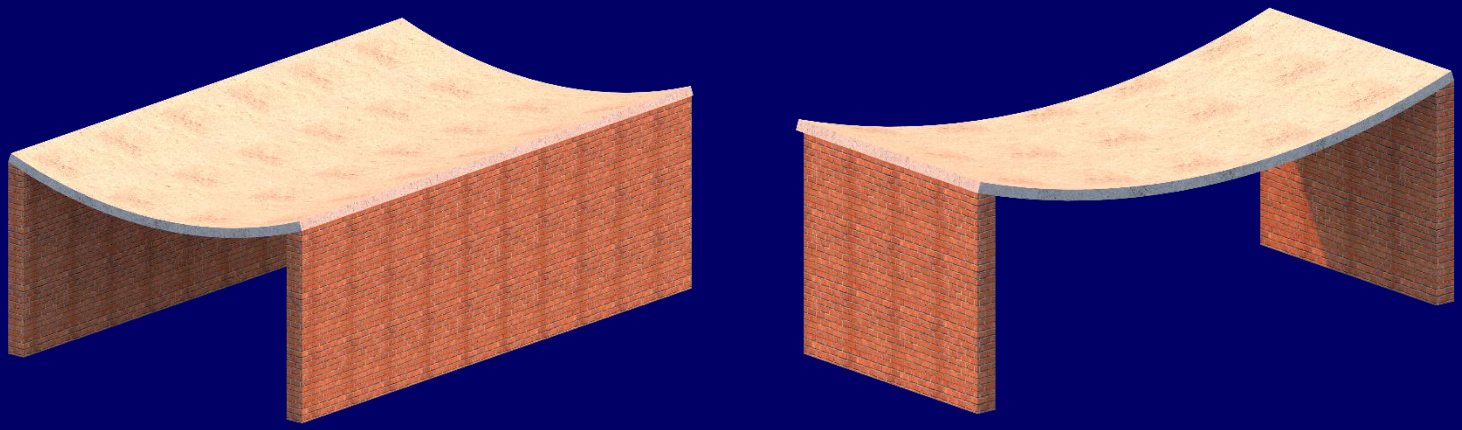
1. **Analysis using computer software (FEA)**
   - SAP 2000
   - ETABS
   - SAFE and so on.
2. **ACI Approximate Method of Analysis**
   - Strip Method for one-way slabs
   - Moment Coefficient Method for two way slabs
   - Direct Design Method (DDM) for two way slabs

Analysis and Design of

One – way Slabs

Introduction

- **Definition of One-way slab**
  - **Case I**
    - Slab supported on two opposing sides only is called one - way slab.
    - In such case, the bending occurs perpendicular to the supports as shown in the following figures.



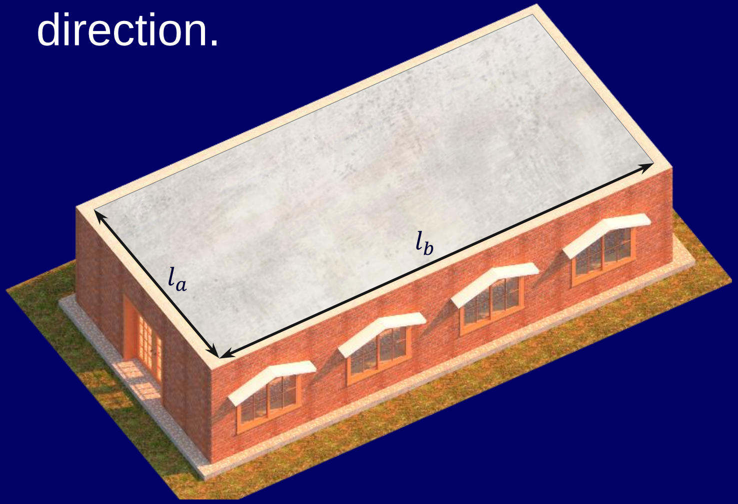
Bending behavior of one-way slab

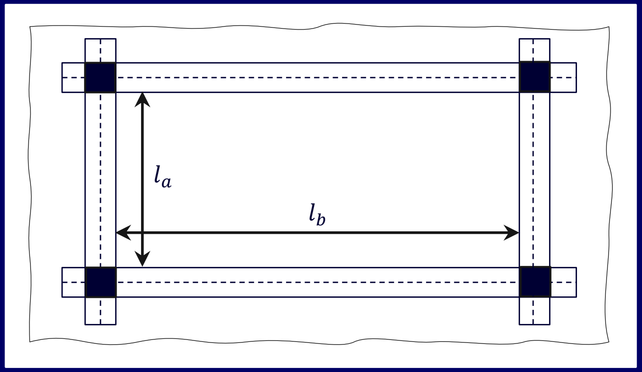
Introduction

- **Definition of One-way slab**

  - **Case II**

    - When the ratio of long to short span in a slab supported on all sides is equal to or greater than 2.
    - Because the principal/major bending moment is always along the short direction, these slabs are solely designed for the short direction.



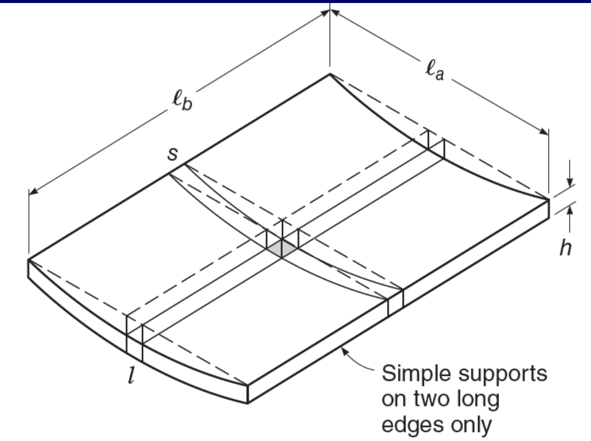


Slab supported on all sides, but $\beta = l_b/l_a \geq 2$

Analysis of One – way Slab

- **Reason for more Demand (Moment) in short direction**
  - Consider two strips along the long and short direction as shown in the figure. The deflection at the center of slab subjected to UDL is calculated as;

$$\Delta_{centre} = \frac{5wl^4}{384EI}$$



The deflection at any point, of the two orthogonal slab strips must be same

$$\Delta_a = \Delta_b \Rightarrow \frac{5w_a l_a^4}{384EI_a} = \frac{5w_b l_b^4}{384EI_b}$$

$w_a = w_b(l_b^4/l_a^4) \rightarrow$ larger share of load (demand) is taken by the short direction

Analysis of One – way Slab

- **Reason for more Demand (Moment) in short direction**

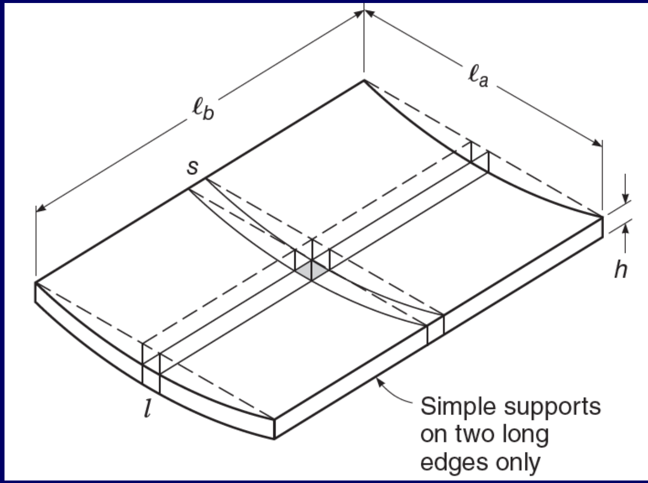
$$M_a = \frac{w_a {l_a}^2}{k} \quad \text{---- (a)}$$

$$M_b = \frac{w_b {l_b}^2}{k} \quad \text{---- (b)}$$

Substitute value of w_a in eq. (a)

$$M_a = \frac{w_b({l_b}^4/{l_a}^4){l_a}^2}{k}$$

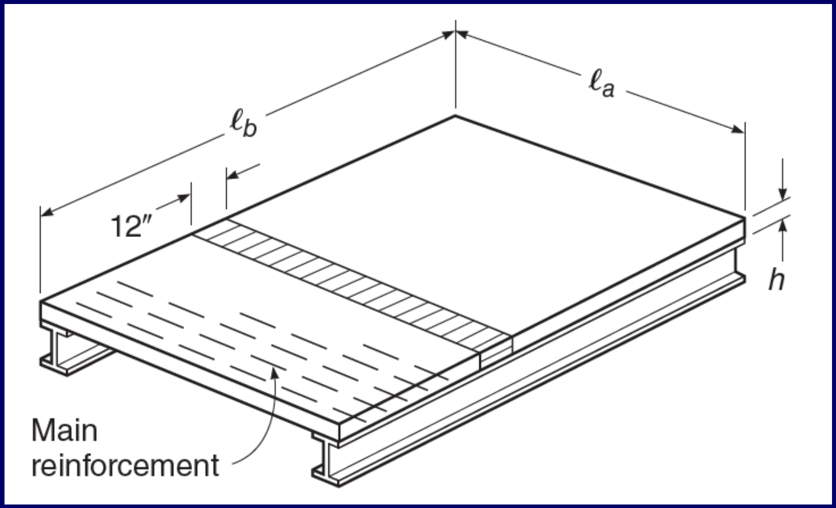
$$M_a = \frac{w_b(l_b)^2}{k}(l_b/l_a)^2$$



$M_a = M_b(l_b/l_a)^2$ → Bending in short direction is more than bending in long direction.

Analysis of One – way Slab

- **Strip method of analysis**
  - For the purpose of analysis and design, a unit strip of one way slab, cut out at right angles to the opposing beams, may be considered as a rectangular beam of unit width, with a depth h and a span l_a as shown in figure.
  - This method is called as strip method of analysis.



Analysis of One – way Slab

- **Limitations of Strip method of analysis**
  - The strip method of analysis for slabs having bending in one direction is applicable only when
    - Slab is supported on two opposing sides on stiff beams or walls,
    - Slab is supported on all sides on stiff beams or walls with ratio of long to short side is equal or greater than 2.
  - Strip method of analysis is not applicable to flat plates etc., even if bending is in one direction.

Design Procedure

- **Steps involved in design of One – way slabs**
  1. Selection of sizes (depth of slab)
  2. Calculation of Loads
  3. Analysis (Determining Ultimate moment)
  4. Determination of Steel Area
  5. Applying Minimum reinforcement Check
  6. Detailing and Drafting

Design Procedure

- **Steps involved in design of One – way slabs**

  1. **Selection of sizes**

     - As Per ACI 318-19, section 7.3.1.1, for solid nonprestressed slabs, h shall not be less than the limits in Table 7.3.1.1

Table 7.3.1.1— Minimum thickness of solid nonprestressed one-way slabs	
Support condition	**Minimum h [1]**
Simply supported	$l/20$
One end continuous	$l/24$
Both ends continuous	$l/28$
Cantilever	$l/10$

- l = Span length (Center to center length) of beam (Section 2.2)
- [1] For f_y other than 60,000 psi, the expressions in the table shall be multiplied by (0.4 + f_y /100,000)

Design Procedure

- **Steps involved in design of One – way slabs**

  2. **Calculation of loads**

     - One way slabs are usually designed for gravity loading.
     - Ultimate Load is calculated as follows:

     $$w_u = 1.2w_D + 1.6w_L$$

     Where;

     w_D = Surface dead load in psf or ksf

     w_L = Surface live load in psf or ksf

     w_u = Ultimate Surface load in psf or ksf

Design Procedure

- **Steps involved in design of One – way slabs**

  3. **Analysis**

     - The analysis is carried out for ultimate load including self weight obtained from size of the slab and the applied dead and live loads.
     - The maximum bending moment value is used for flexural design.

Design Procedure

- **Steps involved in design of One – way slabs**

  4. Determination of Flexural steel area

  - Area of steel can be calculated using direct method.

$$a = d - \sqrt{d^2 - \frac{2.614M_u}{f_c{'}b_w}}$$

$$A_s = \frac{M_u}{\emptyset f_y(d - \frac{a}{2})} \quad (in^2/ft)$$

  - This calculated area A_s is converted into bar spacing s using suitable bar size, as described in the next slide.

Design Procedure

- **Steps involved in design of One – way slabs**

4. Determination of Flexural steel area

In case of beam, we have;

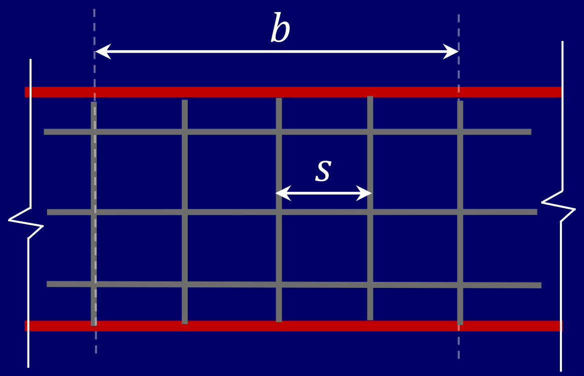
$$n = \frac{A_s}{A_b}$$

For a unit width $b = 12''$ with center to center spacing "s", the above equation becomes

$$\frac{b}{s} = \frac{A_s}{A_b} \quad \Rightarrow \frac{12}{s} = \frac{A_s}{A_b}$$

Which on solving for s gives,

$$s(in) = \frac{12A_b}{A_s}$$ ---- (7.1)



- The unit of A_s is in^2/ft

Design Procedure

- **Steps involved in design of One – way slabs**

  4. **Applying Necessary Checks**

     - Check for Flexural reinforcement

       - Minimum flexural reinforcement permitted by ACI 319-19, Section 7.6.1.1 for slabs is given by

         $A_{s,min} = 0.0018bh_f$

         Where,

         $b =$ Width of slab strip which is 12″ and

         $h_f =$ Thickness of slab

Design Procedure

- **Steps involved in design of One – way slabs**

  4. **Applying Necessary Checks**

     - Check for maximum bar spacing for Flexural reinforcement

       - Maximum allowed spacing S_{max} as per ACI 319-19, Section 7.7.2.3 is least of $3h_f$ or 18″.

Design Procedure

- **Steps involved in design of One – way slabs**

  5. **Shrinkage or Temperature reinforcement**

     - Concrete shrinks as it dries out.
     - Because slabs and other members are rigidly connected to other portions of the structure, they cannot contract freely, resulting in tension known as shrinkage stresses.
     - Since concrete is weak in tension, these temperature and shrinkage stresses are likely to result in cracking.

Design Procedure

- **Steps involved in design of One – way slabs**

  5. Shrinkage or Temperature reinforcement

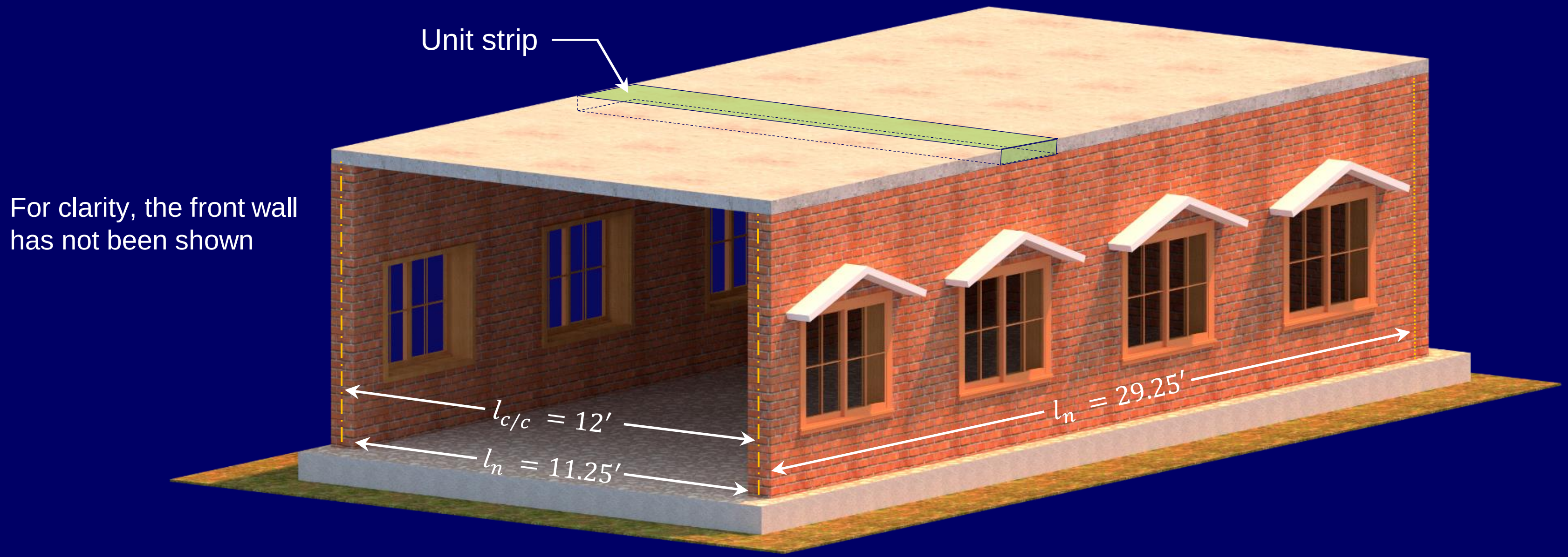
  - It is necessary to provide special reinforcement for shrinkage and temperature contraction in the direction perpendicular to the main reinforcement.
  - Shrinkage and temperature reinforcement is equal to the minimum flexural reinforcement i.e.

    $$A_{S+T} = A_{s,min} = 0.0018bh_f$$

  - Maximum spacing for shrinkage reinforcement permitted by ACI 318-19, Section 7.7.2.4 is least of $5h_f$ or 18"

Design of One way Slab

- **Example 7.1**
  - Design the given 12 feet simply supported slab carrying a uniform service dead load (excluding self weight) of 120 psf and a uniform service live load of 100 psf. Take $f_c{}' = 3ksi$ and $f_y = 60ksi$.



Design of One – way Slab

- **Solution**
  - **Given Data**

    $l_{c/c} = 12'$

    $l_a = 11.25'$

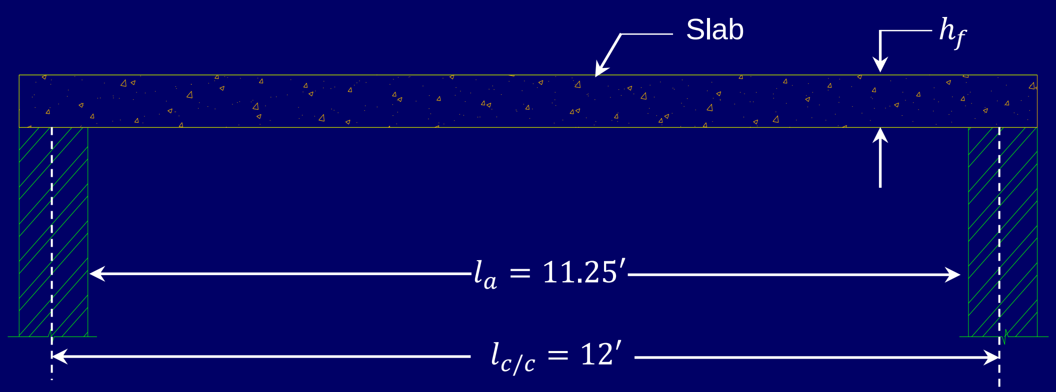
    $l_b = 29.25'$

    $SDL = 120psf$

    $LL = 100psf$

    $f_c{}' = 3ksi$

    $f_y = 60ksi$



  - **Required Data**

    Design the slab as per ACI 318 -19

Design of One – way Slab

- **Solution**
  - **Step No.1: Selection of sizes**

Check behavior of slab, whether one-way or two-way

$$\beta = \frac{Clear\ span\ of\ long\ side}{Clear\ span\ of\ short\ side} = \frac{l_b}{l_a}$$

$$\beta = \frac{29.25}{11.25} = 2.6 > 2 \rightarrow One - way\ slab$$

Now, minimum thickness for simply supported one way slab is

$h_{min} = \frac{l}{20} = \frac{12}{20} = 0.6' \ or\ 7.2" \rightarrow$ Take $h_f = 7.5"$

Design of One – way Slab

- **Solution**
  - **Step No.2: Calculation of loads**

Self weight of slab per unit area is calculated as

$$sw = \gamma_c h_f = 150 \times \frac{7.5}{12} = 93.75psf$$

Now,

$$w_u = 1.2w_D + 1.6w_L = 1.2(120 + 93.75) + 1.6(100) = 416.5psf$$

For unit width (1ft) strip, this ultimate load will be

$$W_u = w_u \times 1 = 416.5lb/ft \ or \ 0.4165k/ft$$

Design of One – way Slab

- **Solution**
  - **Step No.3: Analysis**

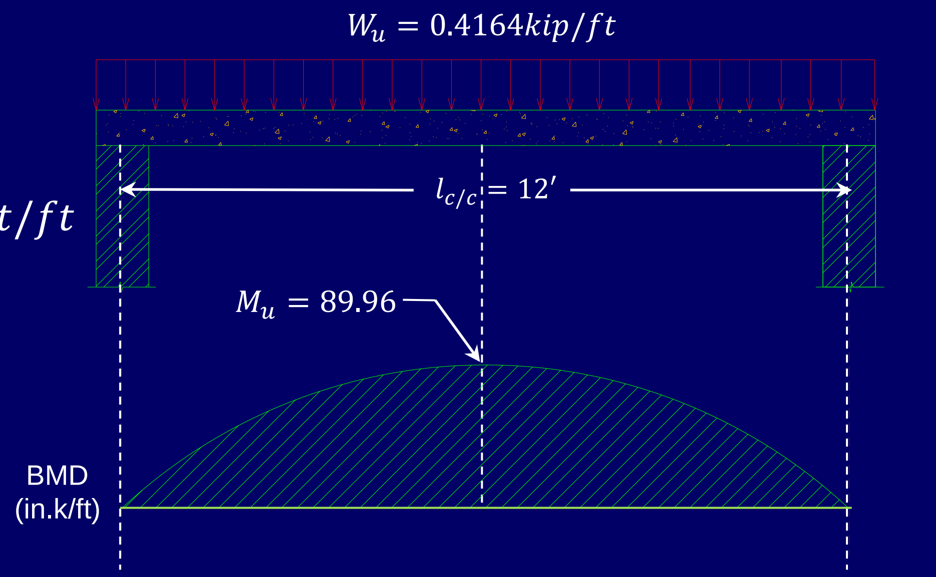
The maximum bending moment is given by

$$M_u = \frac{W_u {l_{c/c}}^2}{8}$$

$$M_u = \frac{0.4164 \times 12^2}{8} = 7.497k.ft/ft$$

OR

$$M_u = 89.96\, in.k/ft$$



Design of One – way Slab

- **Solution**
  - **Step No.4: Determination of Flexural steel area**

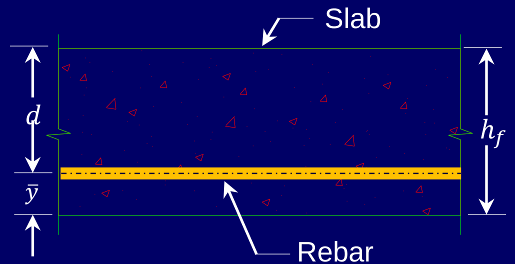
Assuming #4 bar to be used, then effective depth of slab is

$$d = h_f - \bar{y} = h_f - \left(C_c + \frac{d_b}{2}\right)$$

$$d = 7.5 - \left(0.75 + \frac{4}{16}\right) = 6.5"$$

$$a = 6.5 - \sqrt{6.5^2 - \frac{2.614(89.96)}{3 \times 12}} = 0.524"$$

$$A_s = \frac{89.96}{0.9 \times 60\left(6.5 - \frac{0.524}{2}\right)} = 0.267in^2/ft$$



$\bar{y} = C_c + d_b/2$

As per ACI 318-19, Section 20.6.1.3.1, Clear cover for slab is 0.75in

Design of One – way Slab

- **Solution**
  - **Step No.4: Minimum Reinforcement Check**

$$A_{s,min} = 0.0018bh_f = 0.0018(12)(7.5) = 0.162in^2/ft < A_s \rightarrow OK!$$

  - **Step No.5: Detailing of flexural reinforcement**

Bar spacing can be determined using eq (7.1)

$$S = \frac{12A_b}{A_s}$$

Using #4 bar with $A_b = 0.20in^2$

$$S = 12 \times \frac{0.20}{0.267} = 8.99"$$

Design of One – way Slab

- **Solution**
  - **Step No.5: Detailing of flexural reinforcement**
    - Check for maximum bar spacing

$$S_{max} = smaller\ of\ (3h_f\ , 18") = (3 \times 7.5\ , 18")$$

$$S_{max} = smaller\ of\ (22.5"\ , 18")$$

$$S_{max} = 18"$$

The calculated spacing of 8.99" is OK!

Finally, provide #4 @ 8.5" c/c

Design of One – way Slab

- **Solution**
  - **Step No.6: Detailing of Shrinkage & Temperature Reinforcement**

$$A_{S+T} = A_{s,min} = 0.162 in^2$$

Using #4 bar with $A_b = 0.20 in^2$

$$S = 12 \times \frac{0.20}{0.162} = 14.81''$$

- Check for maximum bar spacing

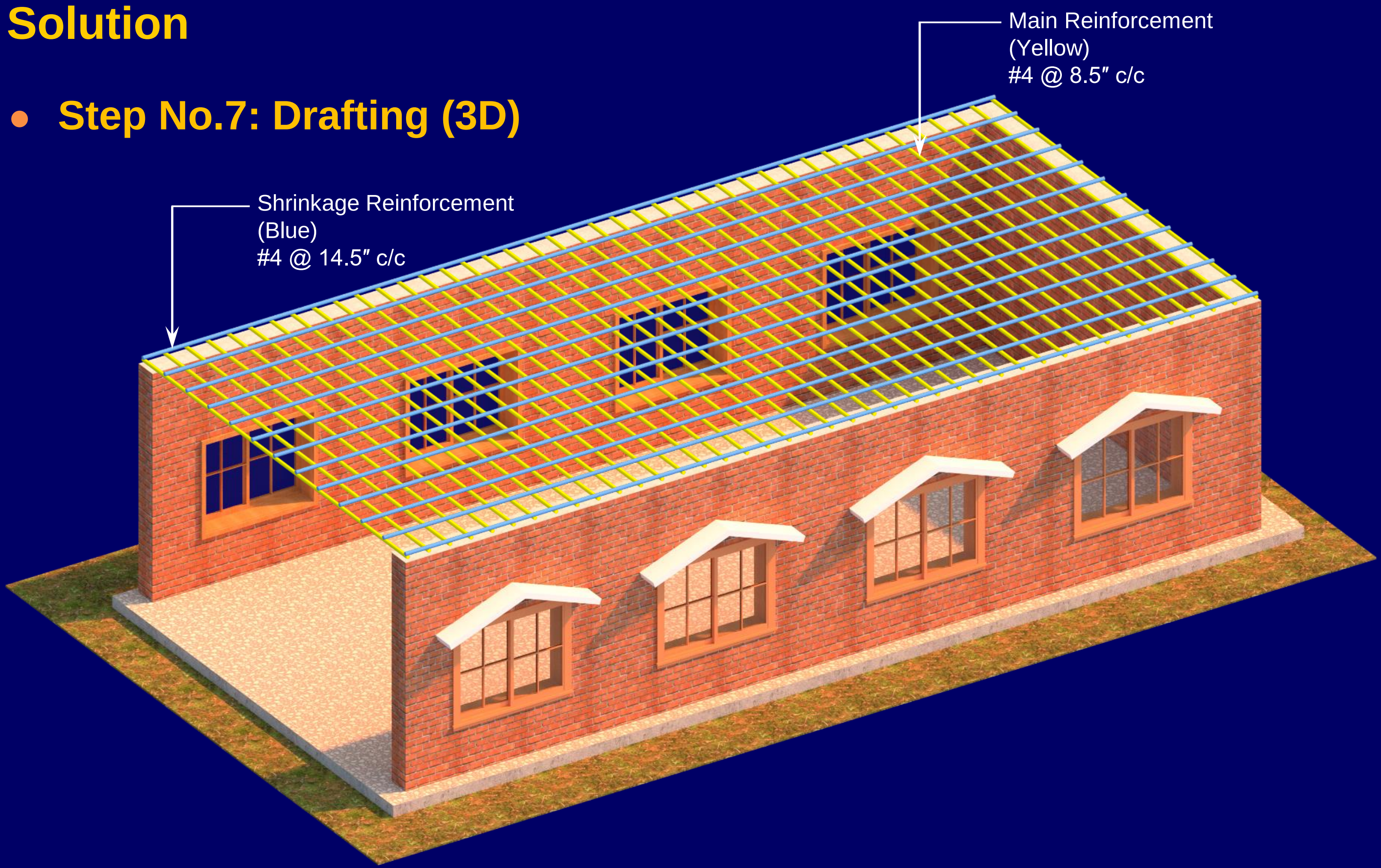
$$S_{max} = smaller\ of\ (5h_f\ , 18'') = (37.5''\ , 18'') = 18''$$

The calculated spacing of $14.81''$ is OK!

Finally, provide $\#4\ @\ 14.5''\ c/c$

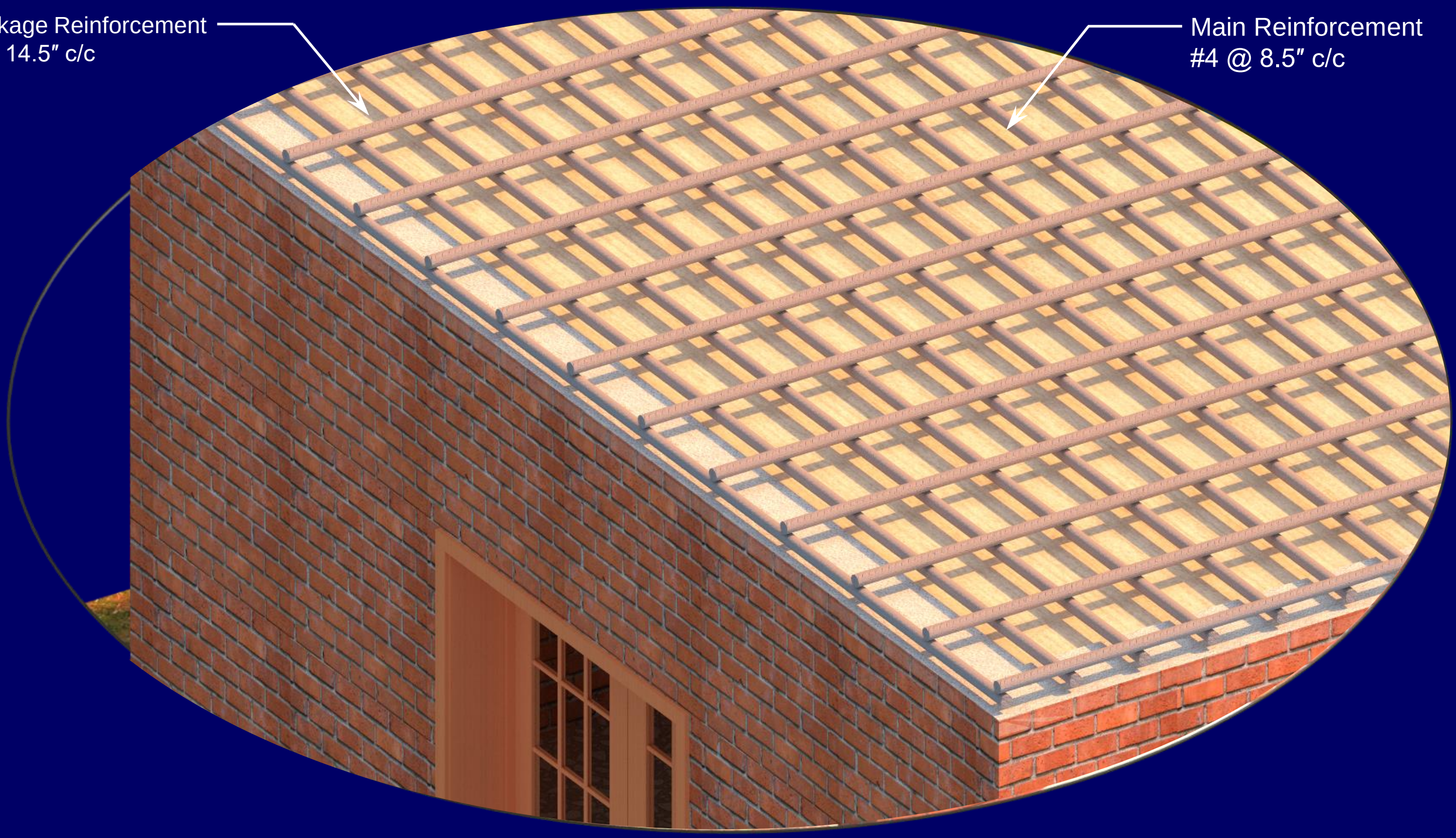
Design of One – way Slab

- **Solution**
  - **Step No.7: Drafting (3D)**



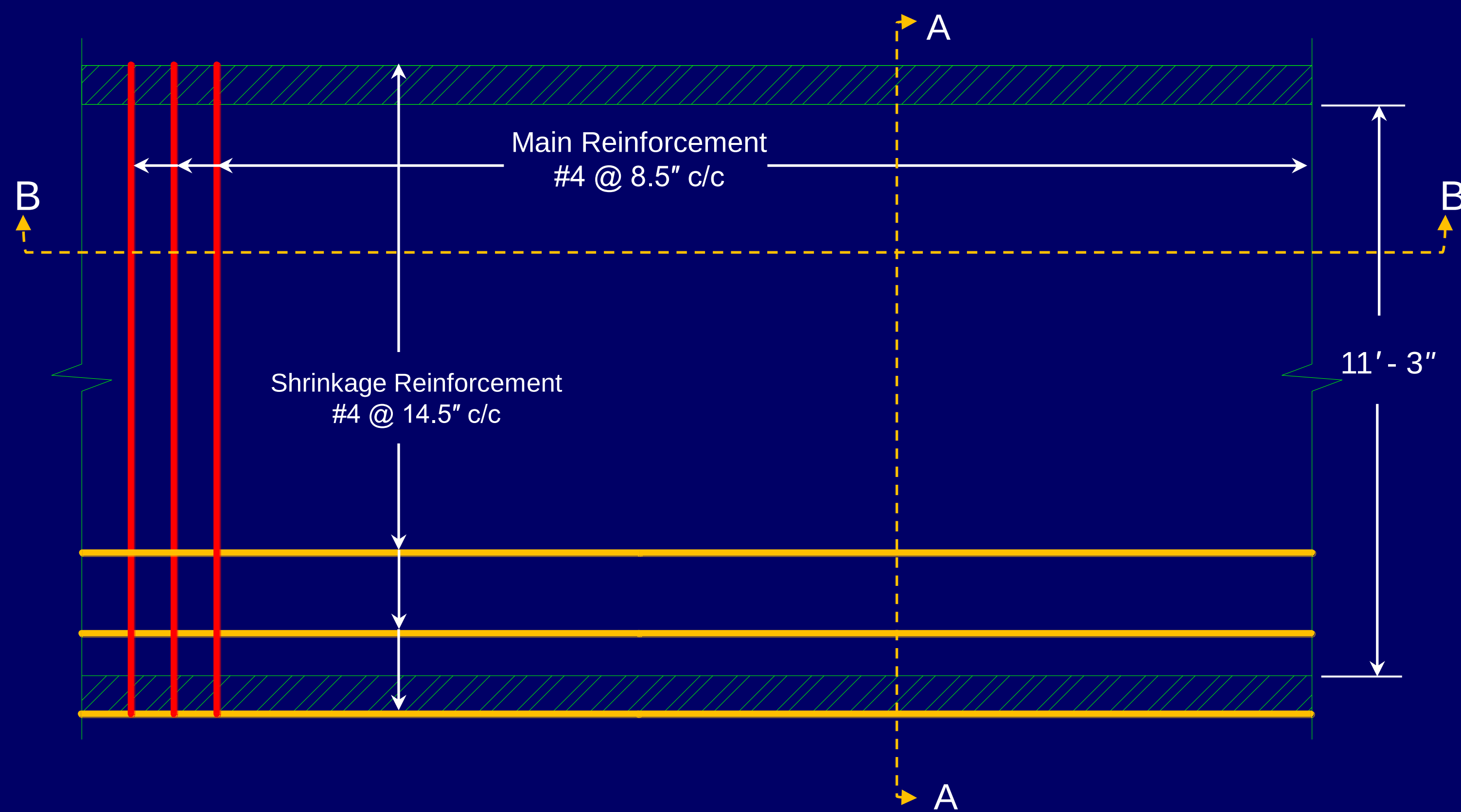
Design of One – way Slab

- **Solution**
  - **Step No.7: Drafting (3D)**



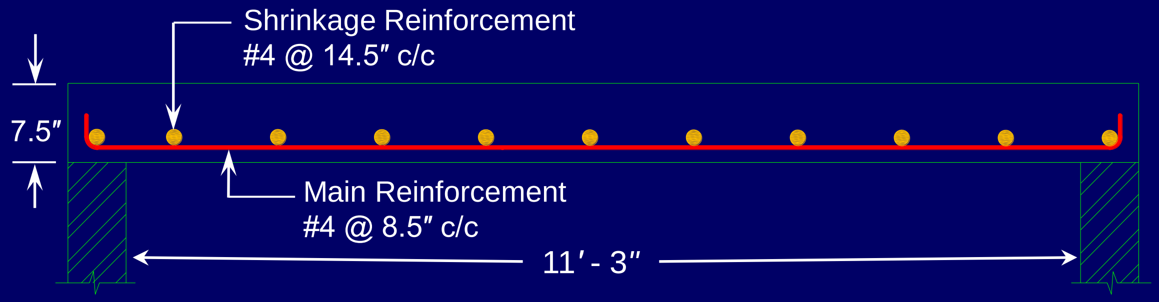
Design of One – way Slab

- **Solution**
  - **Step No.7: Drafting (Plan)**

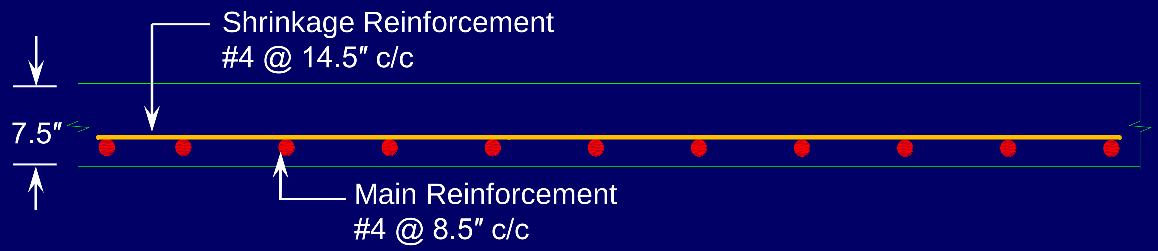


Design of One – way Slab

- **Solution**
  - **Step No.7: Drafting (Sectional details)**



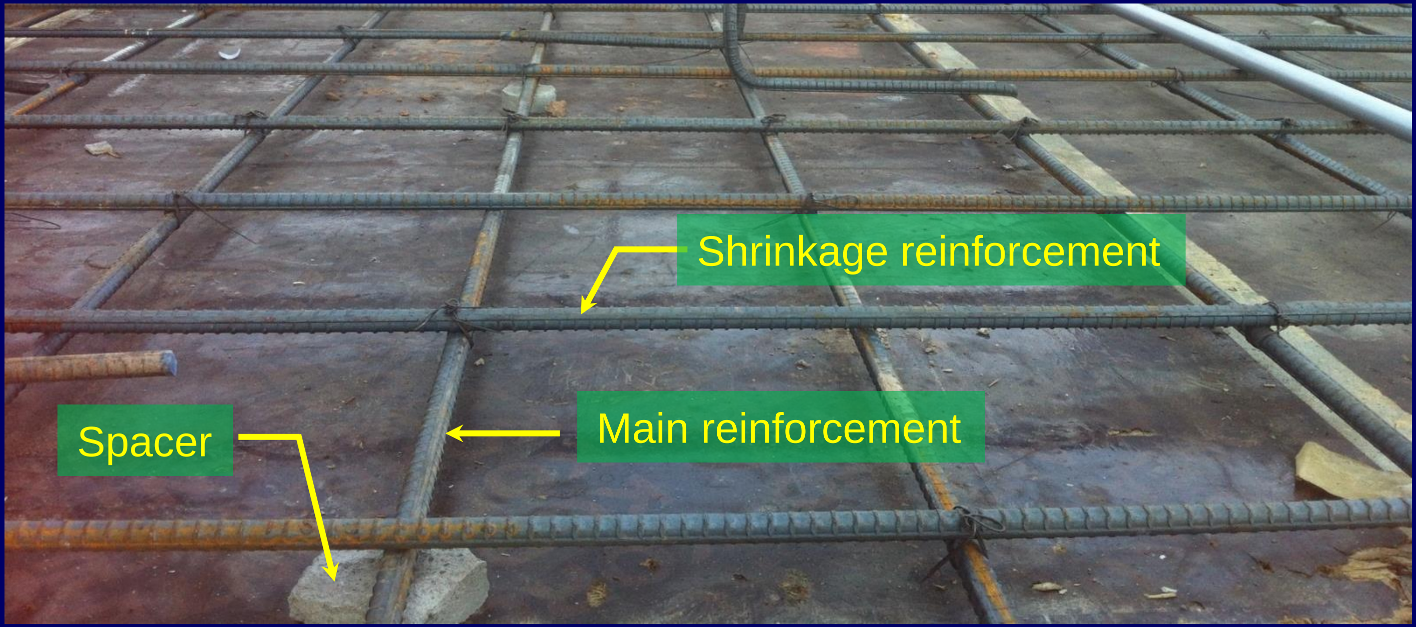
Section A-A



Section B-B

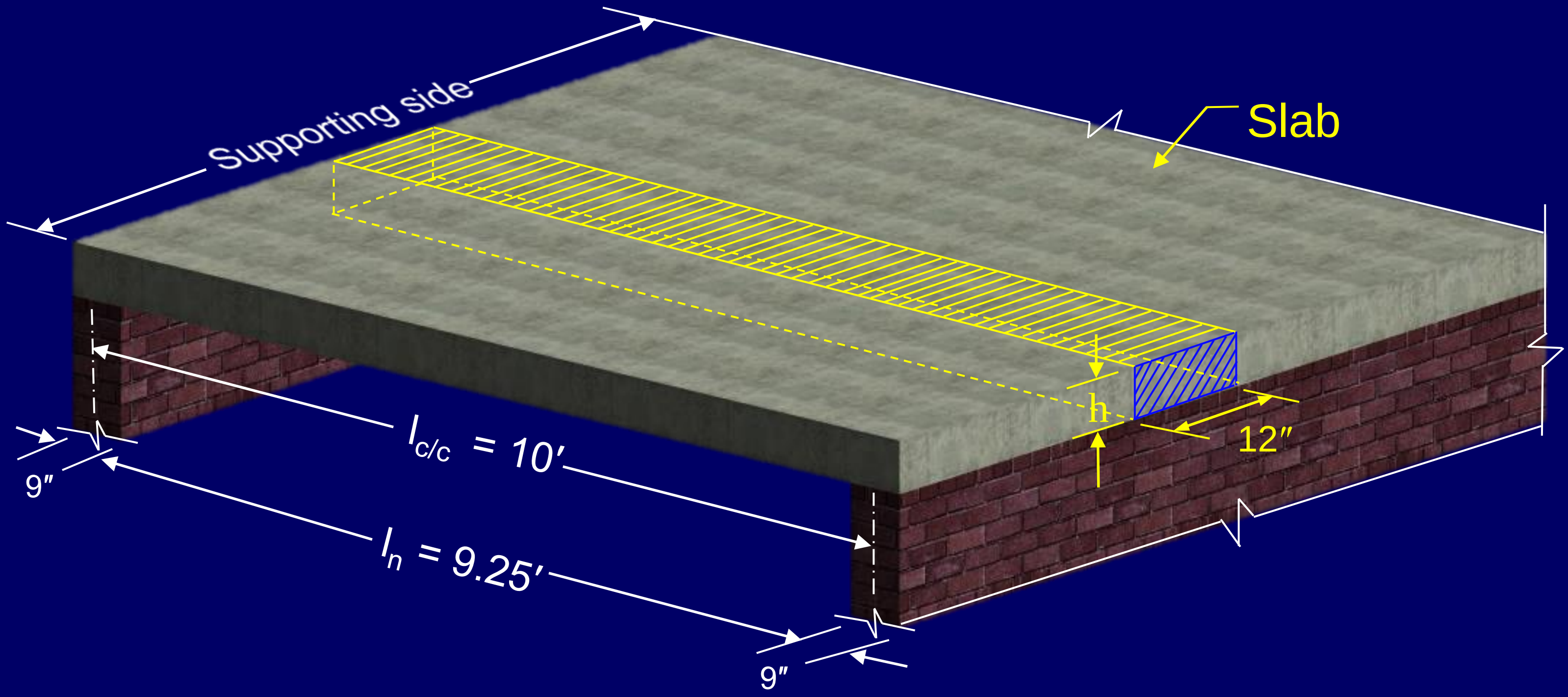
Design of One – way Slab

- **Slab rebar placement at site**
  - Main reinforcing bars are placed at the bottom in the direction of flexure stresses.
  - Shrinkage bars are perpendicularly placed above the main bars.



Design of One – way Slab

- **Example 7.2 (Class Activity)**
  - Design the given 10 feet simply supported slab carrying a uniform service dead load (excluding self weight) of 40 psf and a uniform service live load of 120 psf. Take $f_c' = 3ksi$ and $f_y = 40ksi$.



References

- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-19)