Lecture 03

Design of RC Members for Flexural and Axial Loads (Part – I)

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Learning Outcomes

- **At the end of this lecture, students will be able to;**
  - ***Understand*** the behavior and mechanics of flexural member behavior.
  - ***Design*** solid rectangular, T, and L sections, and extend the mechanical principles to encompass hollow rectangular sections.
  - ***Understand*** the behavior and Mechanics of RC members under axial and combined loads.
  - ***Design*** RC members under axial compressive loads as well as axial loads with uniaxial and biaxial bending.

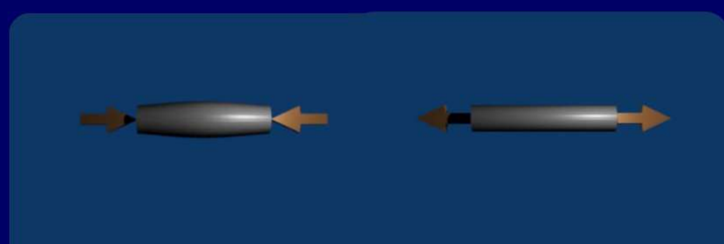
Lecture Contents

- General
- **Section – I : RC Members Under Flexural Loads Only**
  - Behavior of Flexural Members
  - Design of Solid Rectangular Sections
  - Design of Solid T and L Sections
  - Design of Hollow Rectangular Sections
- **Section – II : RC Members Under Axial and Combined Loads**
  - General
  - RC Members Under Compressive Loads with Uniaxial Bending
  - RC Members Under Axial Compressive Loads with Biaxial Bending
- References

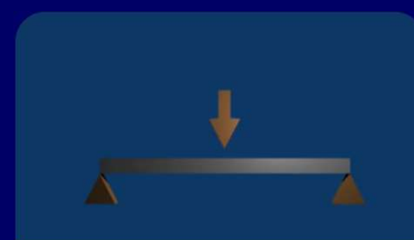
General

❑ Load Effects

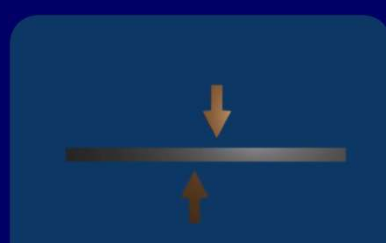
- While transmitting load from floors and roof to the foundations, frame members (beams and columns) of a RC frame structure are subjected to one or more of the following load effects:



Axial Stress (compression and tension)



Flexural / Bending Stress



Shear Stress

Torsion Stress

General

❑ Load Effects

- If all these effects exist together in a RC frame member,
  - Axial and Flexural loads are considered as one set of effects in the design process; whereas
  - Shear and Torsion are considered as another set of load effects.
- This means that the design for Axial + Flexure is not affected by Shear + Torsion and vice versa.

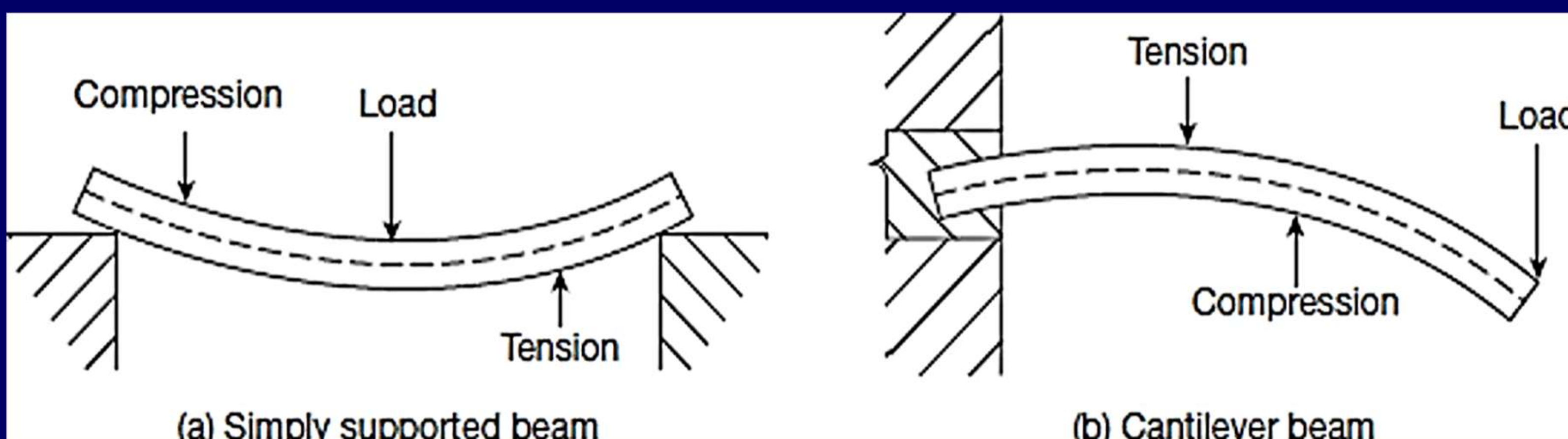
General

Design of Frame Members for Load Effects

- When frame members are designed for the effects of Axial and Flexural loads (with or without shear + torsion) , following cases are possible:

1. **Members Under Flexural Loads Only**

   - Normal beam design procedures are followed.

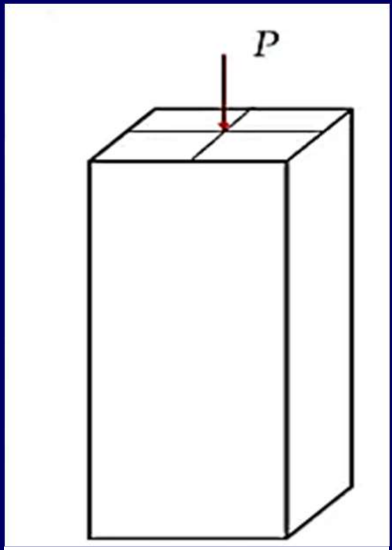


General

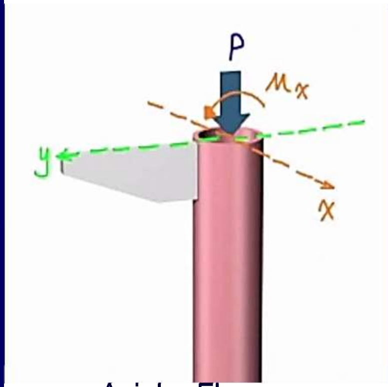
❑ Design of Frame Members for Load Effects

2. **Members Under Axial Loads Only**
   - Pure compression member design procedures are used.
3. **Members Under Combined Axial and Flexural Loads**
   - Interaction diagram procedures, considering Axial and Flexure effects together, are used.

- These cases will be discussed one by one in the next slides.



Pure compression



Axial + Flexure

Section – I

RC Members Under Flexural Loads Only (Beams)

Behavior of Flexural Members

Behavior of Flexural Members Under Gravity Load

- To gain a more comprehensive insight into how an RC beam responds to gravity loads, please carefully watch the following video, showing an experimental test of a beam subjected to a progressively applied point load..

Experimental Test on RC Beam Subjected to Point Load

Behavior of Flexural Members

❑ Concluding Remarks on the Beam Test

- The beam test demonstrates that the beam passes through numerous stages from the start of loading until it collapses.
- Initially, small unseen cracks form under load; as load increases, they become visible, spread, and multiply.
- First crack in tension zone depletes concrete's tensile strength, transferring stresses to steel bars.
- Eventually, cracks widen, indicating steel yielding and finally, the concrete in compression region crushes.

Behavior of Flexural Members

❑ Concluding Remarks on the Beam Test

- The Demand Moment due to applied point load can easily be determined which in this case is $M_A = PL/4$
- The Resisting Moment will be calculated for three specific stages of the beam (although it can be determined at any stage).
  1. Uncracked Concrete – Elastic Stage
  2. Cracked Concrete (tension zone) – Elastic Stage
  3. Cracked Concrete (tension zone) – Inelastic (Ultimate Strength) Stage

Behavior of Flexural Members

1. **Uncracked Concrete – Elastic Stage**

   - At loads much lower than the ultimate, concrete remains uncracked in compression as well as in tension and the behavior of steel and concrete both is elastic.

2. **Cracked Concrete (tension zone) – Elastic Stage**

   - With the increase in load, concrete cracks in tension but remains uncracked in compression.
   - Concrete in compression and steel in tension both behave in an elastic manner.

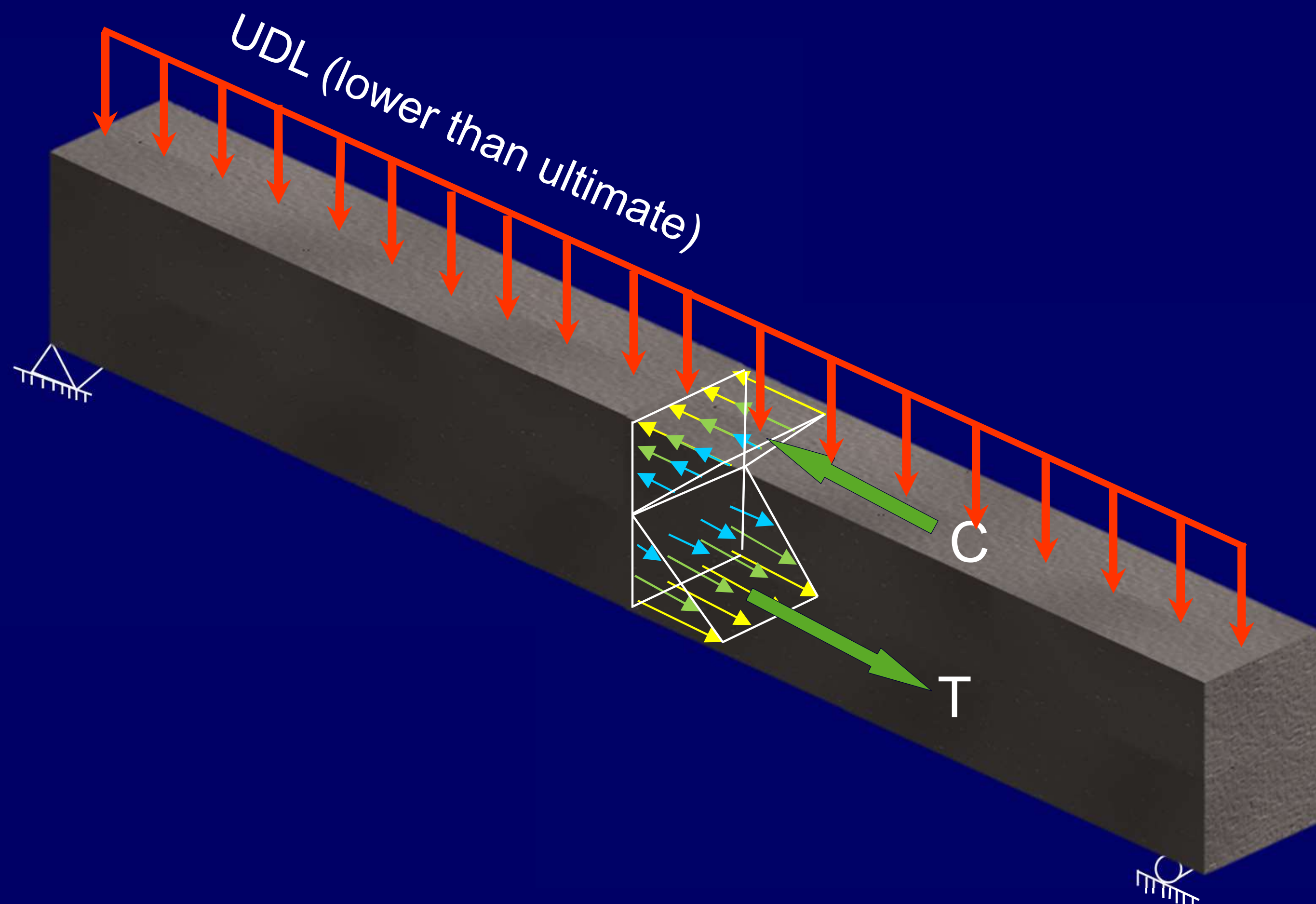
Behavior of Flexural Members

3. **Cracked Concrete (tension zone) –(Ultimate Strength) Stage**

- Concrete is cracked in tension. Concrete in compression and steel in tension both enter the inelastic range.
- At collapse, steel yields and concrete in compression crushes.

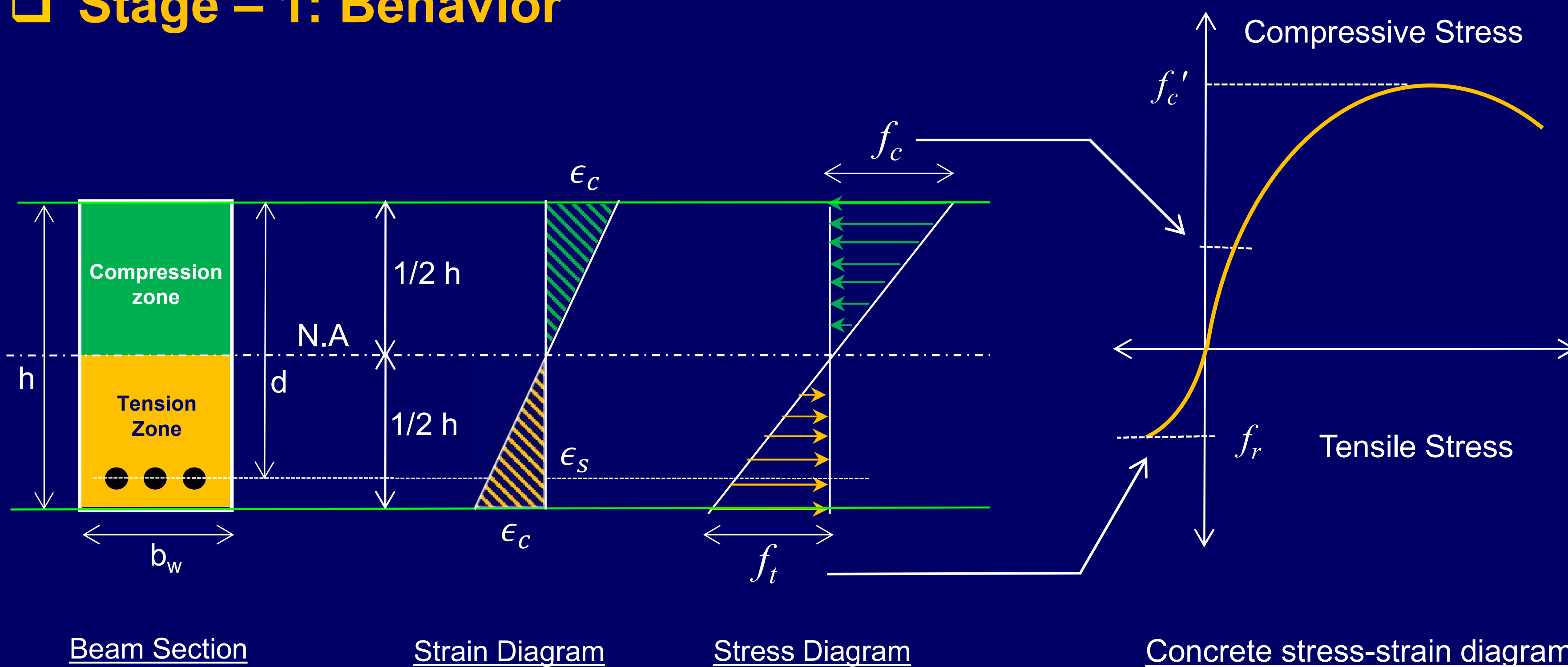
Behavior of Flexural Members

❑ Stage – 1: Behavior



Behavior of Flexural Members

❑ Stage – 1: Behavior

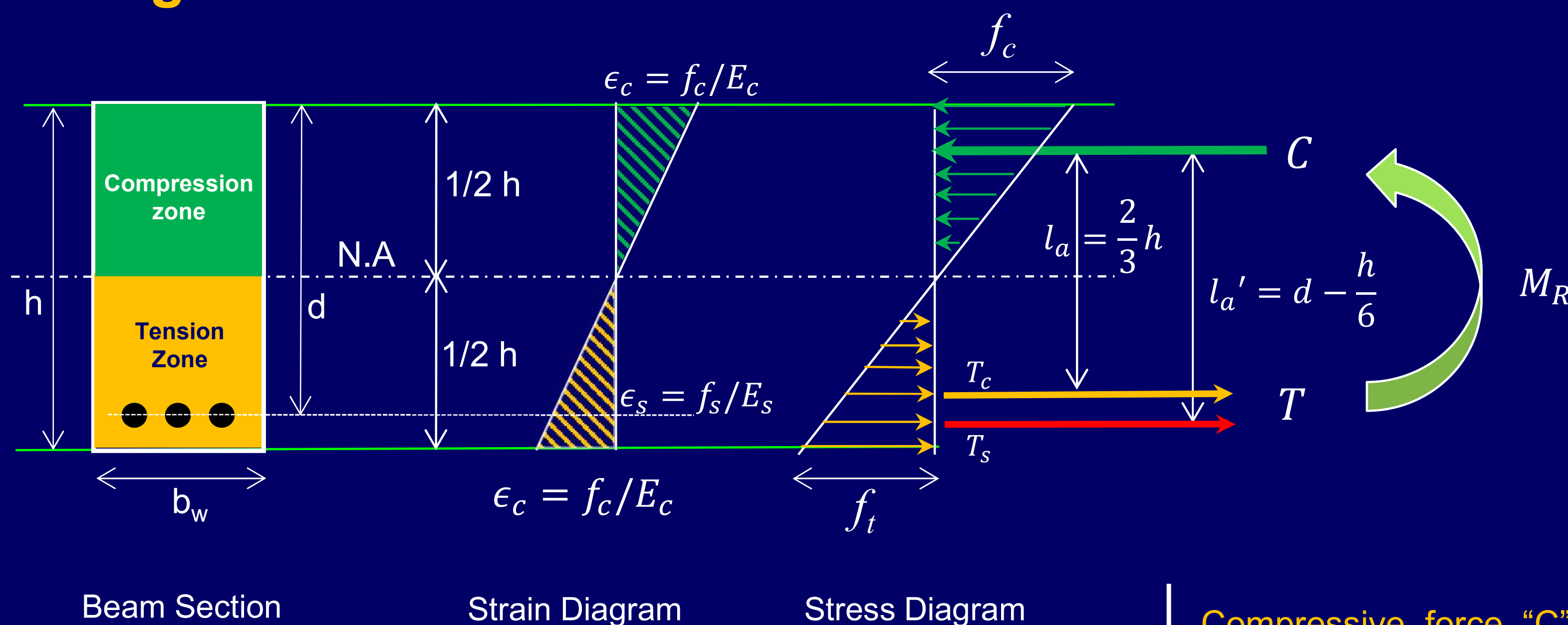


At this stage, the loading condition is such that the concrete in the tension zone reaches its tensile strength, that is $f_t = f_r$

while in the compression zone ; $f_c \ll f_c'$

Behavior of Flexural Members

❑ Stage – 1: Calculations



Compressive force "C" is balanced by T_c and T_s such that:

$C = T_c + T_s$

C = Compressive strength of concrete

T = Tensile strength of steel

$\mathrm{M_R}$ = Resisting moment produced by C and T

l_a = Perpendicular distance between C and T (Lever Arm)

Behavior of Flexural Members

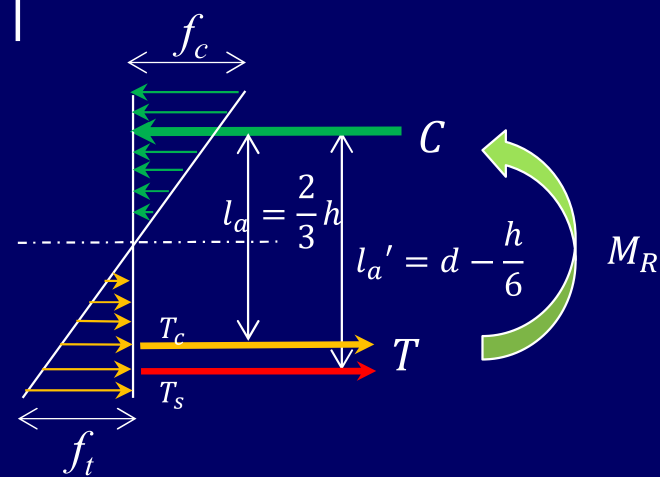
Stage – 1: Calculations

- **Determination of Resisting Moment (M_R)**
  - Resisting Moment offered by both concrete and steel is given by;

$$M_R = M_c + M_s$$

$$M_R = T_c \times l_a + T_s \times l_a'$$

Here,
Mc = Moment due to Concrete and
M_s = Moment due to Steel



Putting

$$l_a = \frac{2}{3}h \ \ and \ l_a' = d - \frac{1}{6}h$$

We get,

$$M_R = \frac{2h}{3}T_c + \left(d - \frac{1}{6}h\right)T_s \quad \text{------ (3.1)}$$

Behavior of Flexural Members

❑ Stage – 1: Calculations

- **Determination of Resisting Moment (M_R)**
  - "T_c" is calculated as;

$$T_c = Average\ Stress \times \text{Area}$$

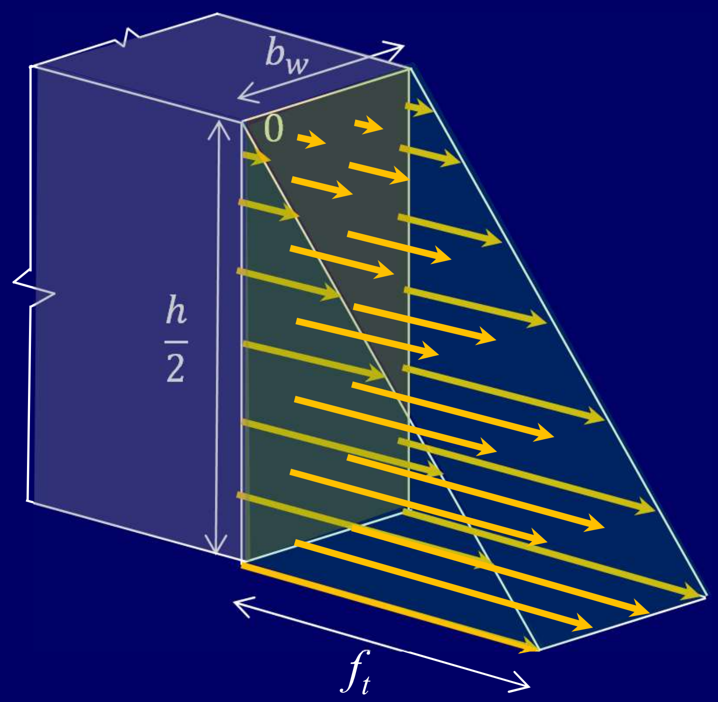
For triangular distribution we get

$$T_c = \left(\frac{0 + f_t}{2}\right) \times \left(b_w \times \frac{h}{2}\right) = \frac{b_w h f_t}{4}$$

Average Stress — Area

Therefore,

$$T_c = \frac{b_w h f_t}{4}$$



Behavior of Flexural Members

Stage – 1: Calculations

- **Determination of Resisting Moment (M_R)**
  - Tensile force of steel is

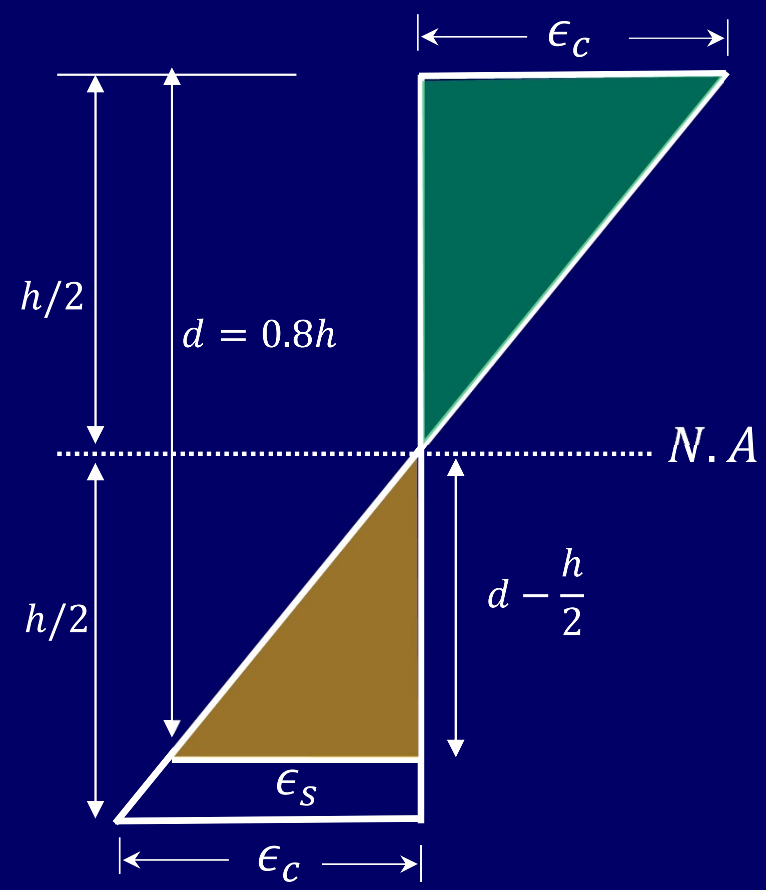
$$T_s = A_s \times f_s = A_s \times E_s \times \epsilon_s$$

ϵ_s can be calculated from the strain diagram as follows

$$\frac{\epsilon_s}{\epsilon_c} = \frac{d - h/2}{h/2}$$

$$\frac{\epsilon_s}{\epsilon_c} = \frac{0.8h - 0.5h}{0.5h} = 0.6$$

$$\epsilon_s = 0.6\epsilon_c$$



Behavior of Flexural Members

Stage – 1: Calculations

Determination of Resisting Moment (M_R)

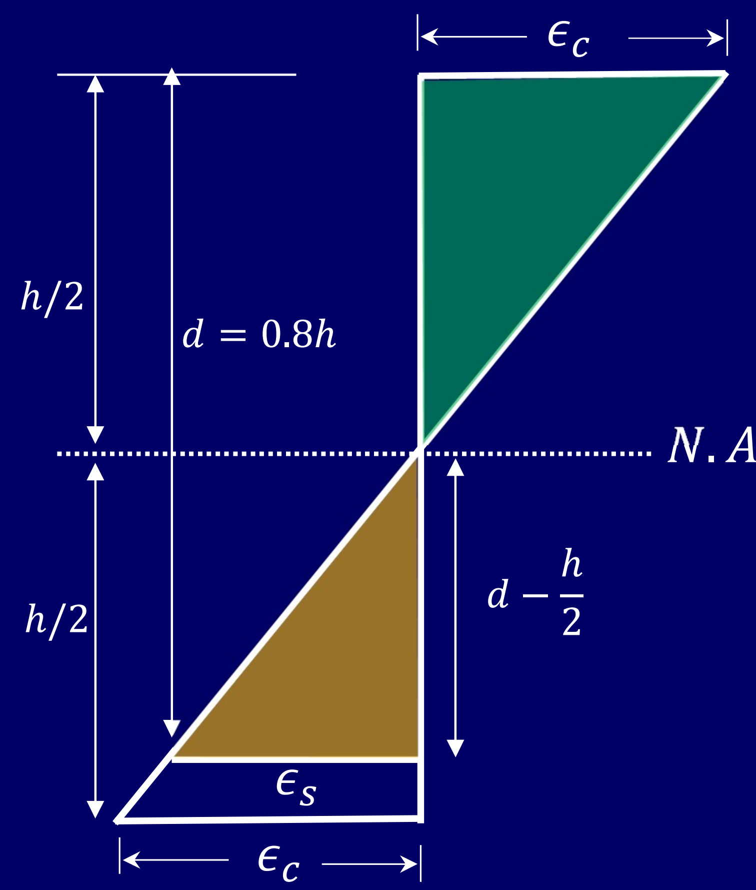
- Concrete stain ϵ_c is given by

$$\epsilon_c = \frac{f_t}{E_c} = \frac{7.5\sqrt{f_c{}'}}{57000\sqrt{f_c{}'}} = \frac{1}{7600}$$

$$\epsilon_s = 0.6\epsilon_c = \frac{0.6}{7600}$$

- So finally, T_s is;

$$T_s = A_s \times 29000 \times \frac{0.6}{7600} = 2.3A_s$$



Behavior of Flexural Members

❑ Stage – 1: Calculations

- **Determination of Resisting Moment (M_R)**
  - Putting values of T_c and T_s in eq. (3.1), gives

$$M_R = \frac{2h}{3} \times \frac{b_w h f_t}{4} + \left(d - \frac{h}{6}\right) \times 2.3A_s$$

$$M_R = \frac{b_w h^2}{6} f_t + 2.3A_s\left(d - \frac{h}{6}\right)$$

$$f_t = f_r = 7.5\sqrt{f_c{}'}$$

$$M_R = \frac{b_w h^2}{6} \times 7.5\sqrt{f_c}' + 2.3A_s\left(d - \frac{h}{6}\right)$$

Behavior of Flexural Members

Stage – 1: Calculations

- **Determination of Resisting Moment (M_R)**

$$\underbrace{M_R = 1.25\sqrt{f_c{}'}b_w h^2}_{M_c} + \underbrace{2.3A_s\left(d - \frac{h}{6}\right)}_{M_s} \quad \text{------ (3.2)}$$

If the beam is treated as "Plain concrete", then $M_s = 0$ and eq. 3.2 reduces to,

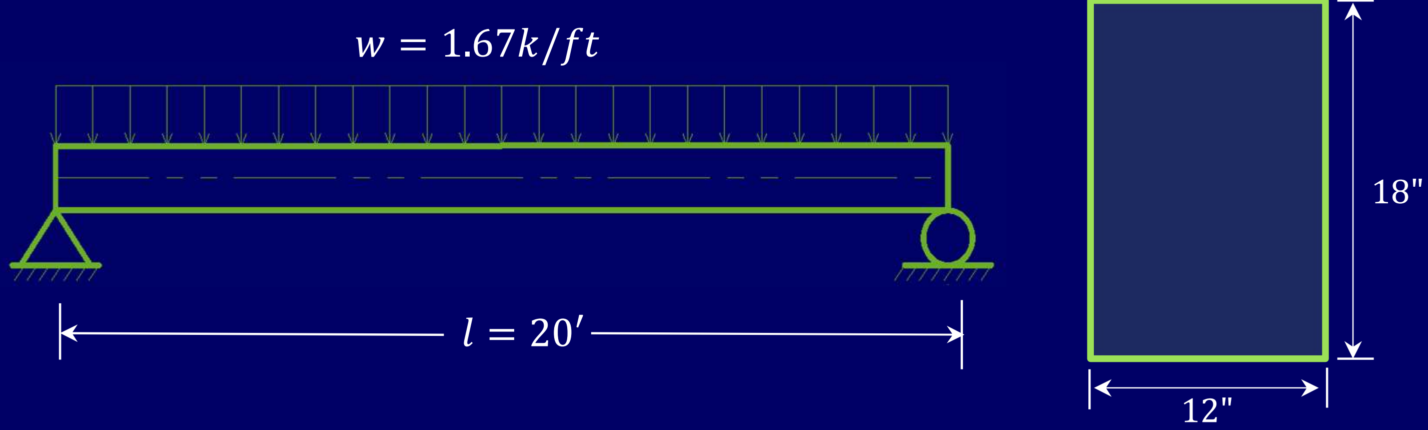
$$M_R = M_c = 1.25\sqrt{f_c{}'}b_w h^2$$

- Eq. 3.2 is the required Resisting moment / Design moment or Flexural capacity of the beam.
- Any applied moment greater than this moment will crack the beam, Therefore, it can also be called the "cracking Moment".

Behavior of Flexural Members

❑ Stage – 1: Example 3.1

- A simply supported beam having a span length of 20ft is subjected to a uniformly distributed load of 1.67k/ft as shown in the figure below. Material properties are; $f_c' = 3000psi \quad and \quad f_y = 40{,}000psi$



Behavior of Flexural Members

Stage – 1: Example 3.1

A. Neglecting the contribution of reinforcing steel,

   i. *Calculate* the Demand and Resisting moments and check whether the beam fails or not.

   ii. *Determine* how much compressive strength of concrete will be required to resist the given demand if the beam cross-sections are restricted?.

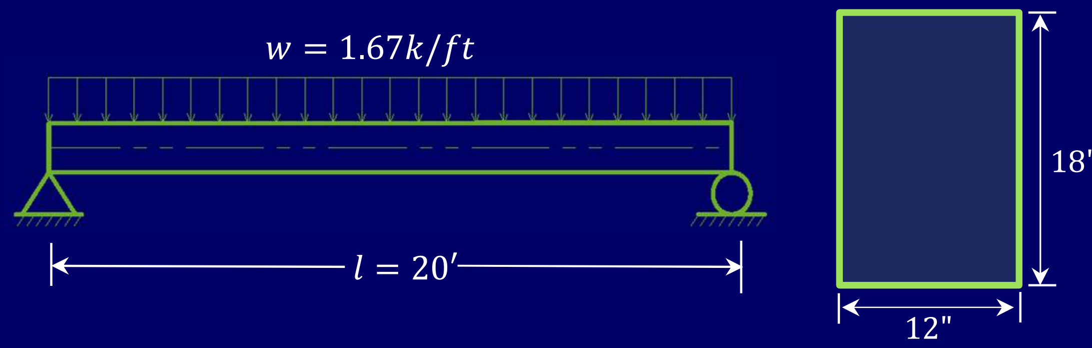
   iii. *Compute* the minimum depth "h" of beam required to meet the given demand, Keeping the concrete strength constant.

B. If the contribution of steel is considered, then calculate the area of steel required for the applied moment.

Behavior of Flexural Members

Stage – 1: Example 3.1

Solution



Part (A)(i)

- Applied moment

$$M_A = \frac{wl^2}{8} = \frac{1.67 \times 20^2}{8}$$

$$M_A = 83.5kip.ft \quad or\ 1002\ \text{in.kip}$$

- Resisting moment

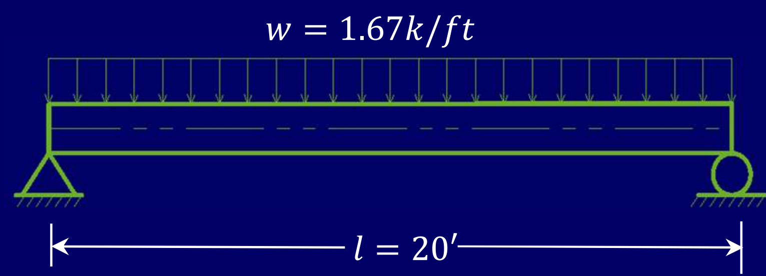
$$M_R = 1.25\sqrt{f_c{}'}b_w h^2 = 1.25\sqrt{3000} \times 12 \times 18^2 = 266193.16\ lb.in$$

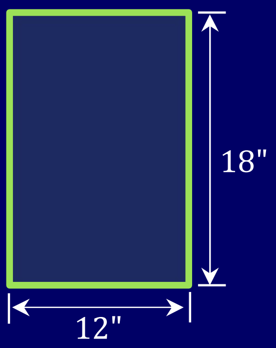
$$M_R = 266.19\ in.kip$$

Since $M_R \ll M_A \rightarrow The\ beam\ will\ fail$

Behavior of Flexural Members

❑ Stage – 1: Example 3.1





❖ Solution

- **Part (A)(ii)**

$M_R = 1.25\sqrt{f_c'}b_w h^2$

For no failure, $M_R \geq M_A$

Taking $M_R = M_A$

$$\Rightarrow \quad f_c' = \left(\frac{M_A}{1.25 b_w h^2}\right)^2$$

$$\Rightarrow \quad f_c' = \left(\frac{1002 \times 1000}{1.25 \times 12 \times 18^2}\right) = 42507.24 \text{ psi} \longrightarrow$$

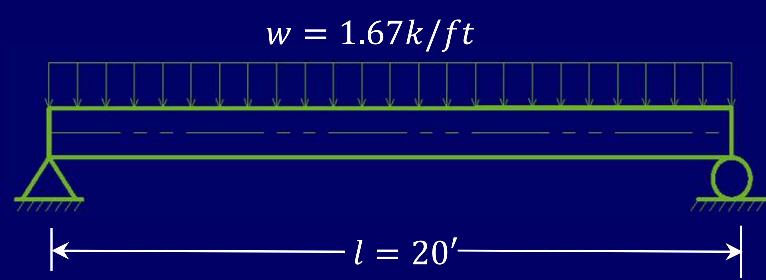
Imagine this much compressive strength of concrete with a typical strength of 3000psi !

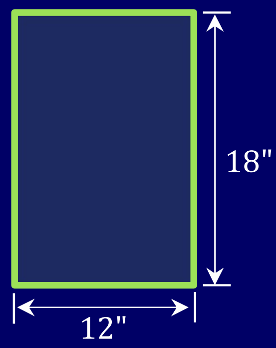
Behavior of Flexural Members

Stage – 1: Example 3.1

Solution

- Part (A)(iii)





$$M_R = 1.25\sqrt{f_c{}'}b_w h^2$$

Taking $M_R = M_A$

$$\Rightarrow \ h = \sqrt{\frac{M_A}{1.25 b_w \sqrt{f_c{}'}}}$$

$$\Rightarrow h = \sqrt{\frac{1002 \times 1000}{1.25 \times 12\sqrt{3000}}} = 34.92" \approx 3'$$

Behavior of Flexural Members

Stage – 1: Example 3.1

Solution

- **Part (B)**

$$M_R = 1.25\sqrt{f_c'}b_w h^2 + 2.3A_s\left(d - \frac{1}{6}h\right)$$

Taking $M_R = M_A$

$$1.25\sqrt{f_c'}b_w h^2 + 2.3A_s\left(d - \frac{1}{6}h\right) = M_A$$

$$266.19 + 2.3A_s\left(15.5 - \frac{18}{6}\right) = 1002$$

$$A_s = 25.59\ in^2$$

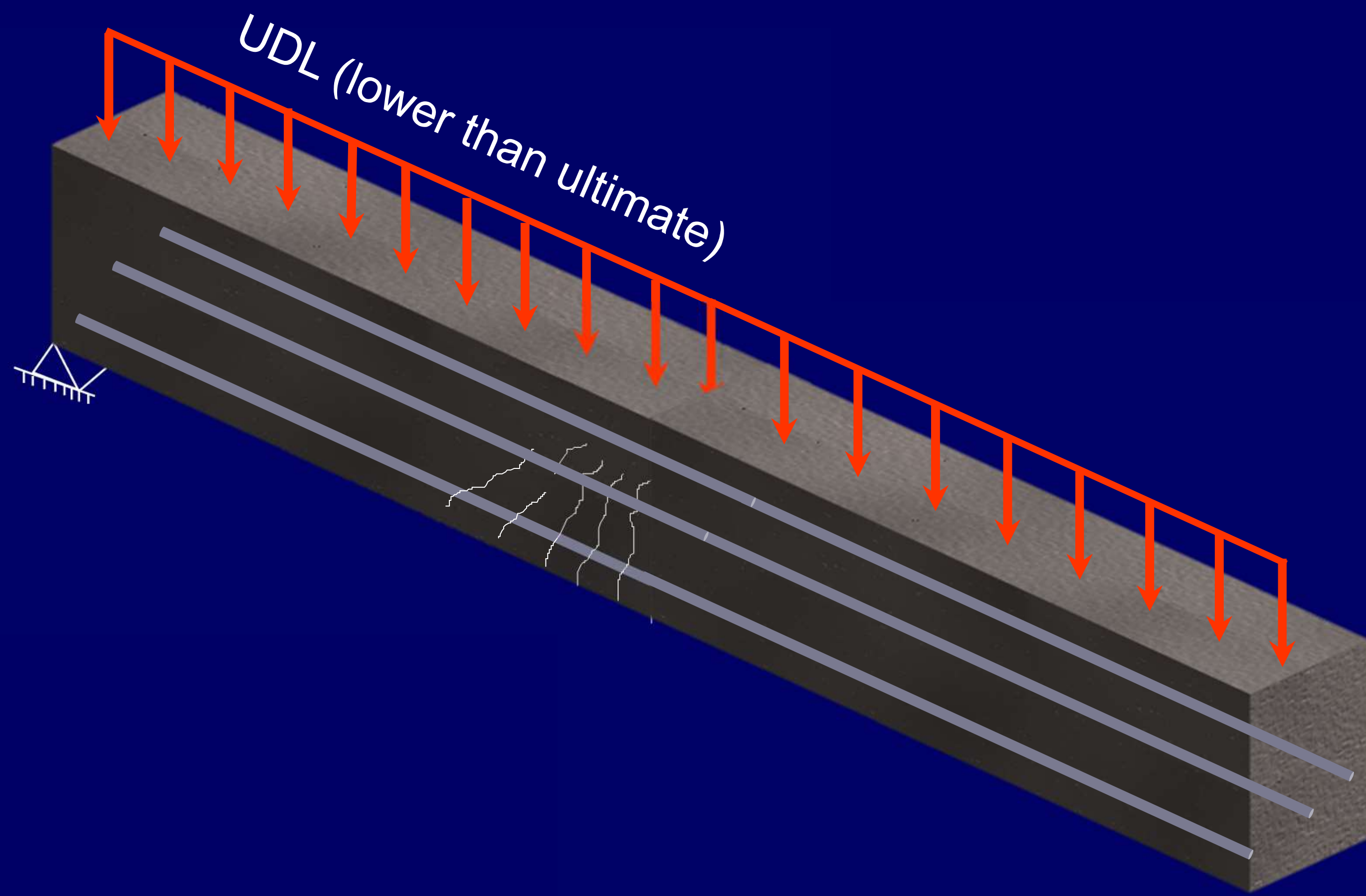
$$M_c = 1.25\sqrt{f_c'}b_w h^2 = 266.19\ in.kip$$

$$M_A = 1002\ in.kip$$

$$d = h - 2.5 = 18 - 2.5 = 15.5"$$

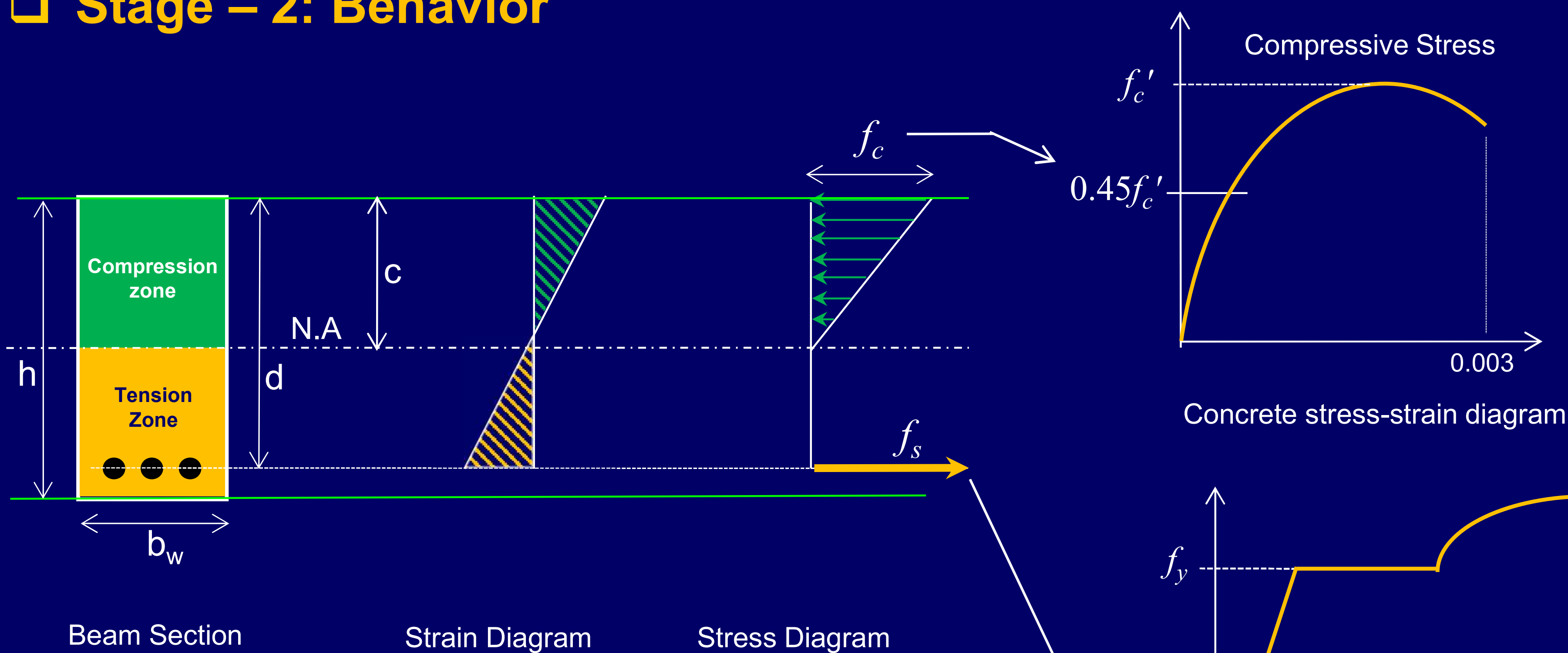
Behavior of Flexural Members

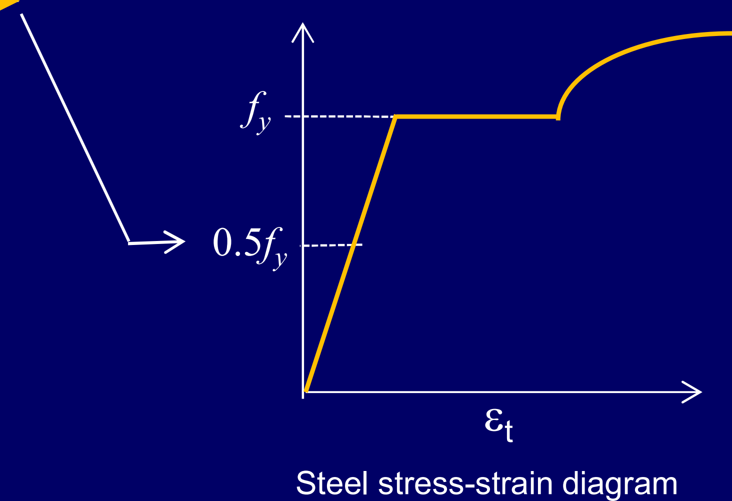
- Stage – 2: Behavior



Behavior of Flexural Members

❑ Stage – 2: Behavior





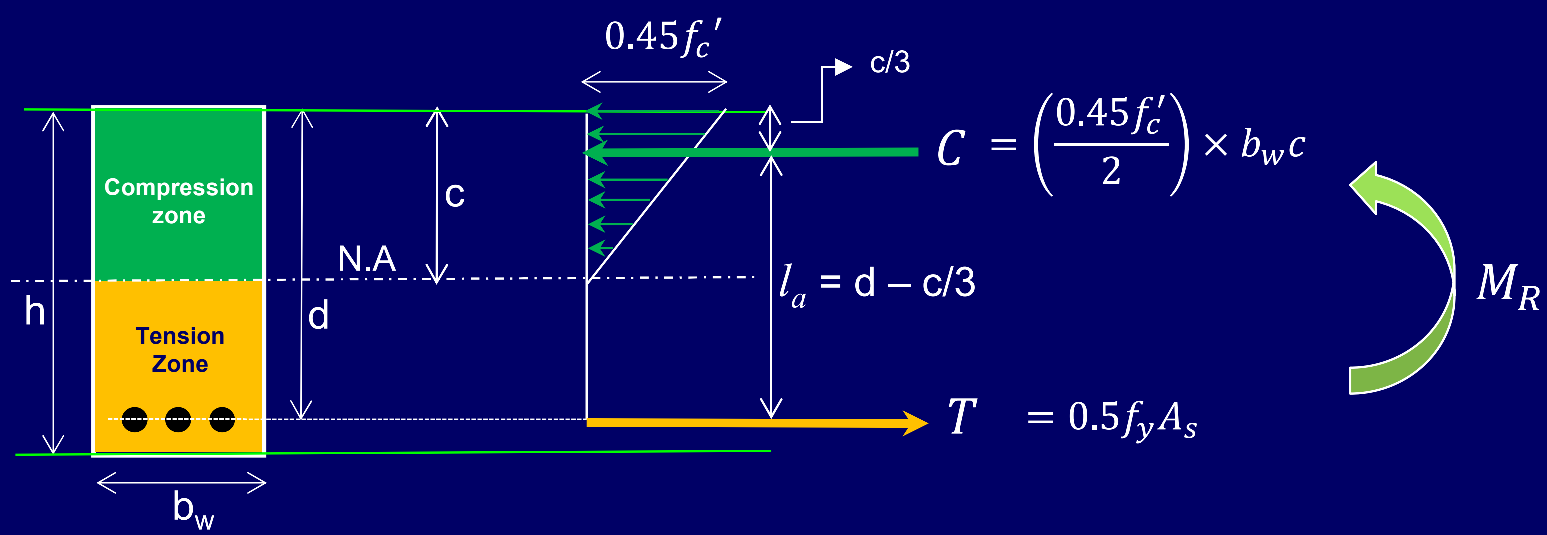
At this stage, the loading condition is such that;

$f_c = 0.45f_c'$ and

$f_s = 0.5f_y$

Behavior of Flexural Members

❑ Stage – 2: Calculations



Beam Section

Stress Diagram

Calculating Resisting moment

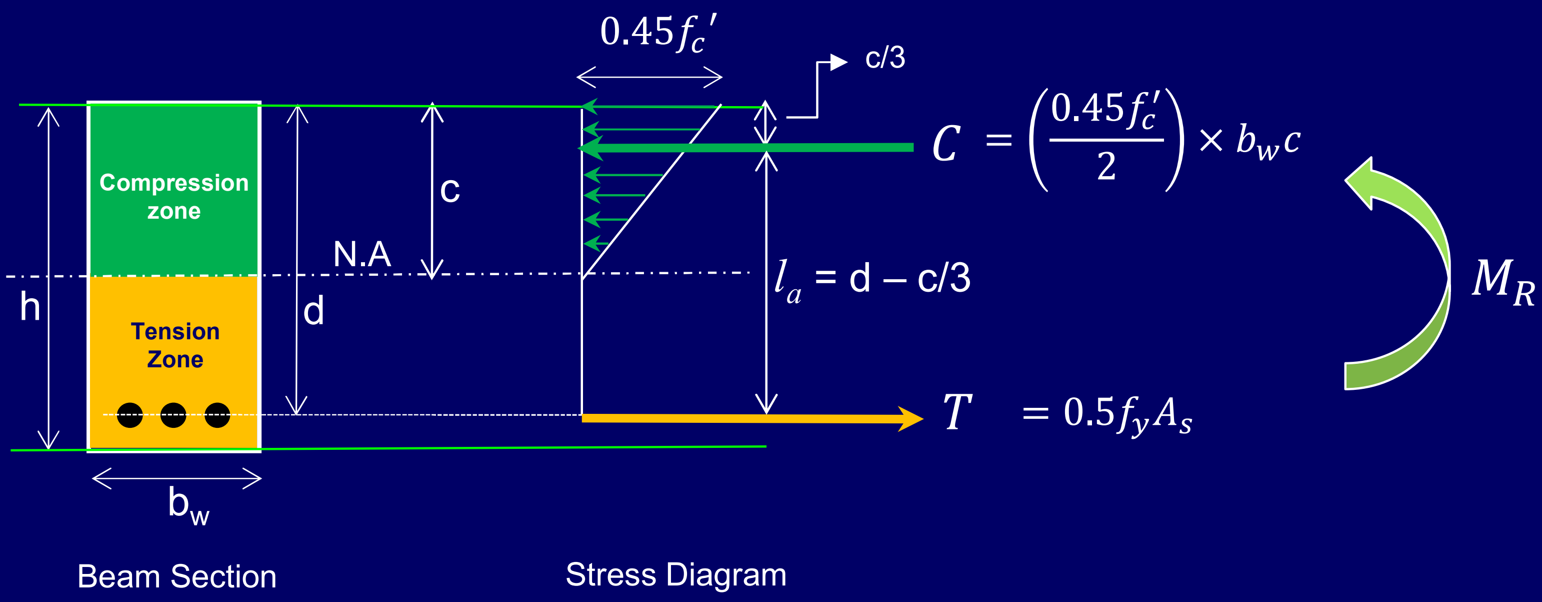
$$M_R = \cancelto{0}{M_c} + M_s = T \times l_a = (0.5 f_y A_s) \times \left(d - \frac{c}{3}\right)$$

$$M_R = 0.5 A_S f_y \left(d - \frac{c}{3}\right)$$

M_c shall be neglected as per ACI 318, 22.2

Behavior of Flexural Members

❑ Stage – 2: Calculations



Equating horizontal forces;

$$C = T \Rightarrow \frac{0.45f_c'}{2} \times (b_w\ c) = A_s 0.5 f_y$$

Which on simplifying gives, $c = \dfrac{A_s f_y}{0.45 f_c' b_w}$

Behavior of Flexural Members

Stage – 2: Calculations

- Putting the value of "c" in equation of M_R gives

$$M_R = 0.5A_S f_y\left(d - \frac{c}{3}\right) = 0.5A_S f_y\left(d - \frac{A_s f_y}{3 \times 0.45 f_c' b_w}\right)$$

- The resisting moment is calculated assuming the area of steel A_s which is then compared with the applied moment to check whether the beam fails or not.

- However, instead of assuming, it is preferable to compute the area of steel required for a given demand by equating the resisting and applied moments, $M_R = M_A$, as discussed on the next slide.

Behavior of Flexural Members

Stage – 2: Calculations

- We have

$$M_R = 0.5A_S f_y\left(d - \frac{c}{3}\right)$$

equating $M_R = M_A$

$$0.5A_S f_y\left(d - \frac{c}{3}\right) = M_A$$

which on solving for A_s gives

$$A_s = \frac{M_A}{0.5f_y\left(d - \frac{c}{3}\right)}$$

Behavior of Flexural Members

Stage – 2: Calculations

- Area of steel A_s can be determined by the Trial and Success method as described below.

1. Assume the value of "c"
2. Calculate the area of steel using

$$A_s = \frac{M_A}{0.5 f_y (d - c/3)}$$

3. Confirm the value of "c" using

$$c = \frac{A_s f_y}{0.45 f_c' b_w}$$

4. Repeat the process until the same A_s value is obtained from the two consecutive trials.

Behavior of Flexural Members

Stage – 2: Example 3.2

- Using the data from Example 3.1, calculate the area of steel required for the beam corresponding to stage 2.

- **Solution**

  - Trial 1: Choosing c = h/2 = 9" and d = h – 2.5 = 15.5"

$$A_s = \frac{1002}{0.5(40)(15.5 - 9/3)} = 4\ in^2$$

$$\Rightarrow \quad c = \frac{4 \times 40}{0.45 \times 3 \times 12} = 9.88''$$

  - Trial 2: Choosing c = 9.88"

$$A_s = \frac{1002}{0.5(40)(15.5 - 9.88/3)} = 4.10\ in^2$$

Behavior of Flexural Members

Stage – 2: Example 3.2

- **Solution**
  - Trial 2:

$$\Rightarrow \quad c = \frac{4.10 \times 40}{0.45 \times 3 \times 12} = 10.12''$$

  - Trial 3: Choosing c = 10.12"

$$A_s = \frac{1002}{0.5(40)(15.5 - 10.12/3)} = 4.13\ in^2$$

$$\Rightarrow \quad c = \frac{4.13 \times 40}{0.45 \times 3 \times 12} = 10.2''$$

Trial 4: Choosing c = 10.2" and $A_s = 4.14\text{in}^2$

Hence the required area of steel is 4.14in^2

Behavior of Flexural Members

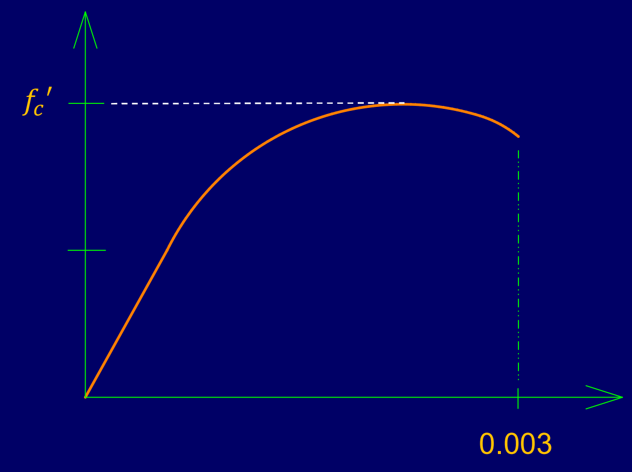
Stage – 3

- Stage 3 is the ultimate or final stage in which both concrete in compression and steel in tension enter the inelastic state.
- Because of the several possible situations of failure in this stage, defining the Ultimate stage is quite difficult.
- Furthermore, due to severe concrete cracking and the complexity of the stress-strain relationship at this point, calculating the resisting moment without making some key assumptions is extremely challenging.
- Therefore, the definition of the "ultimate stage" and the "basic assumptions" as per the ACI Code are discussed next.

Behavior of Flexural Members

❑ Stage – 3

- **Definition of the Ultimate Stage**
  - As per ACI 318-19, R21.2.2, "the ultimate stage is said to be reached when the concrete strain at the extreme fiber in the compression zone reaches a value of 0.003".

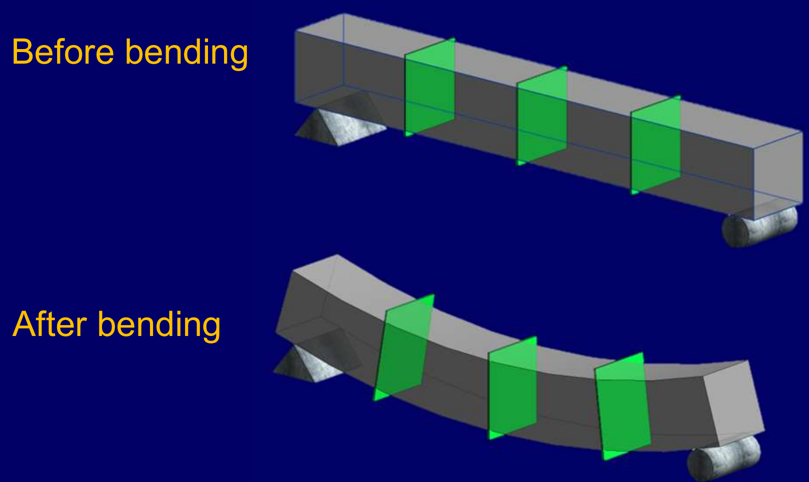


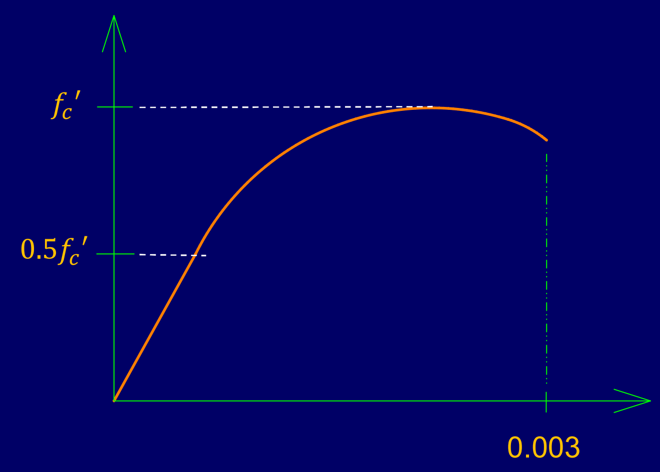
Stress-strain curve of concrete

Behavior of Flexural Members

❑ Stage – 3

- **Fundamental Assumptions (ACI 318-19, section 22.2)**
  - A plane section before bending remains plane after bending.
  - Stress and strain in concrete are approximately proportional up to moderate loads (concrete stress ≤ $0.5f_c'$). When the load is increased, the variation in the concrete stress is no longer linear.



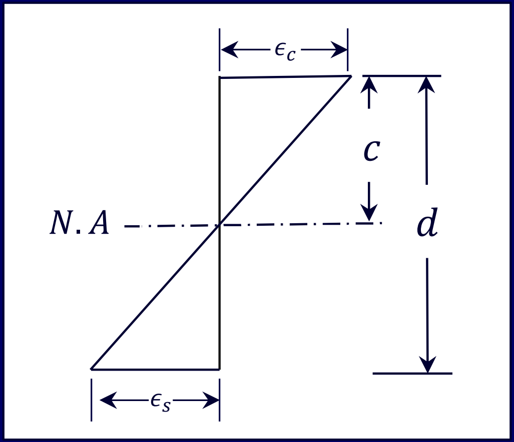


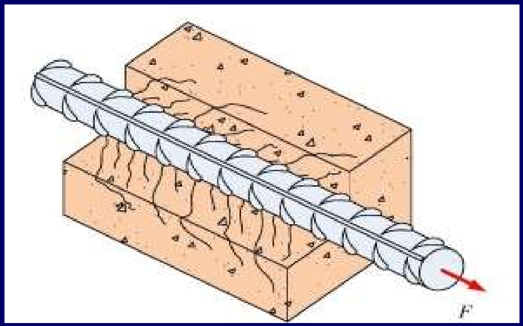
Stress-strain curve of concrete

Behavior of Flexural Members

Stage – 3

- **Fundamental Assumptions (ACI 318-19, section 22.2)**
  - Strain in concrete and reinforcement shall be assumed proportional to the distance from the neutral axis.
  - Tensile strength of concrete is neglected in the design of reinforced concrete beams.
  - The bond between the steel and concrete is PERFECT and NO SLIP occurs.

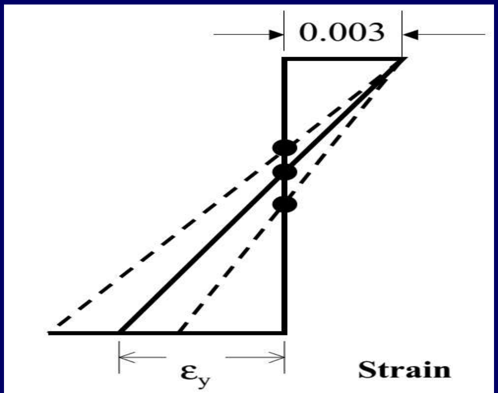


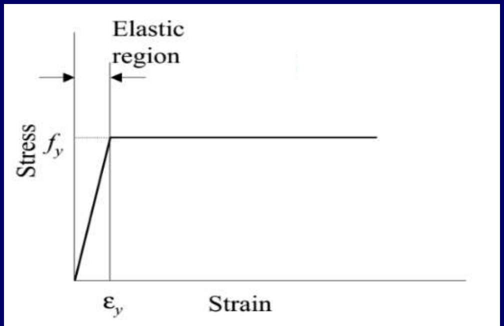


Behavior of Flexural Members

Stage – 3

- **Fundamental Assumptions (ACI 318-19, section 22.2)**
  - The maximum usable concrete compressive strain at the extreme fiber is assumed to be 0.003.
  - The steel is assumed to be uniformly strained to the strain that exists at the level of the centroid of the steel.
    - If $\epsilon_s < \epsilon_y$ $then$ $f_s = \epsilon_s E_s$
    - If $\epsilon_s > \epsilon_y$ $then$ $f_s = f_y$

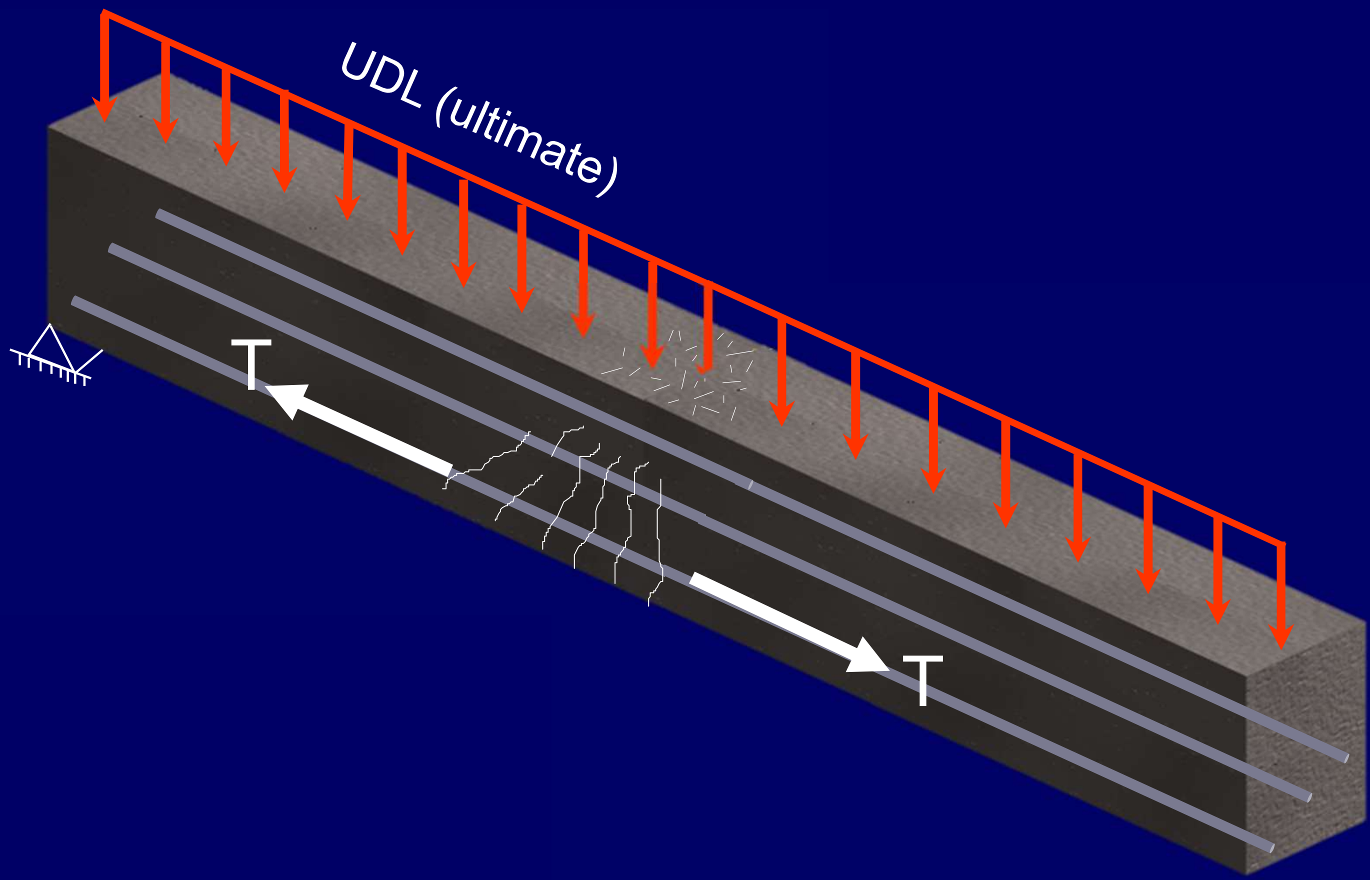




Idealized Stress-Strain Curve

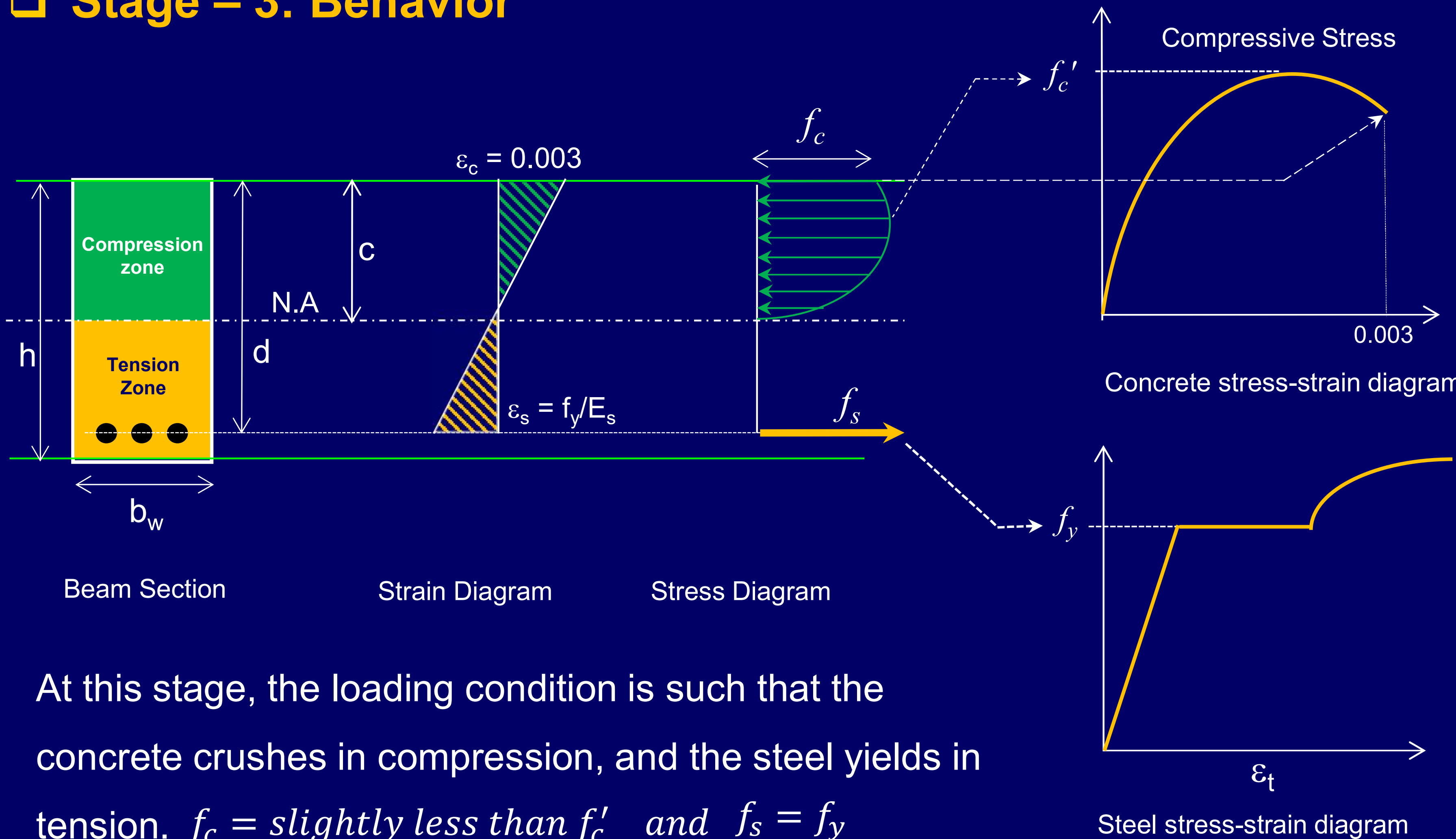
Behavior of Flexural Members

- Stage – 3: Behavior



Behavior of Flexural Members

❑ Stage – 3: Behavior



At this stage, the loading condition is such that the concrete crushes in compression, and the steel yields in tension. $f_c = slightly\ less\ than\ f_c' \quad and \quad f_s = f_y$

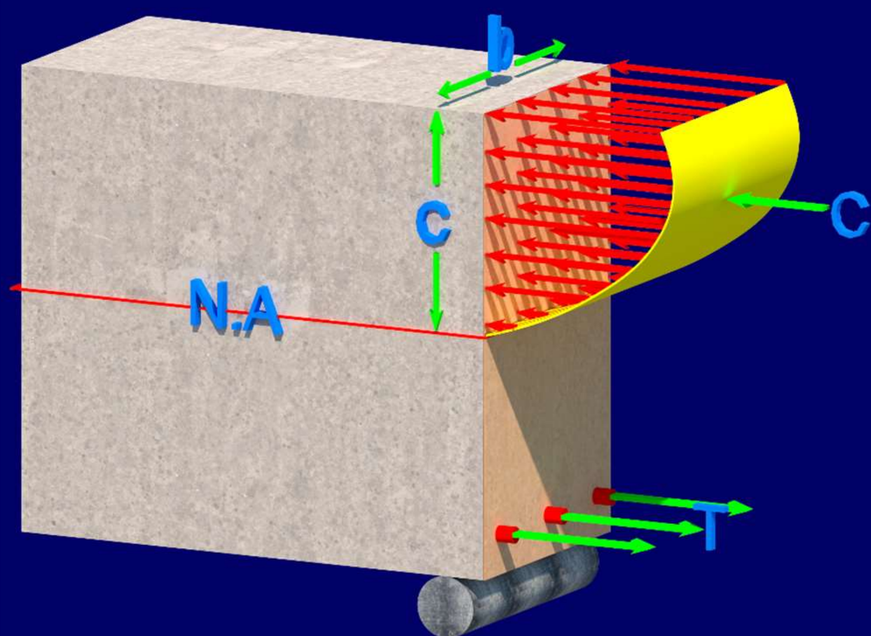
Behavior of Flexural Members

Stage – 3: Calculations

- As the stress distribution in this stage is parabolic, therefore calculating the compressive force and its position is extremely challenging.
- The actual complex stress distribution can be transformed into a simple geometric shape, that gives the same results as the original.
- C. S. Whitney proposed a rectangular distribution known as the "Whitney Stress Block" which has gained widespread acceptance and is included in the ACI Code.

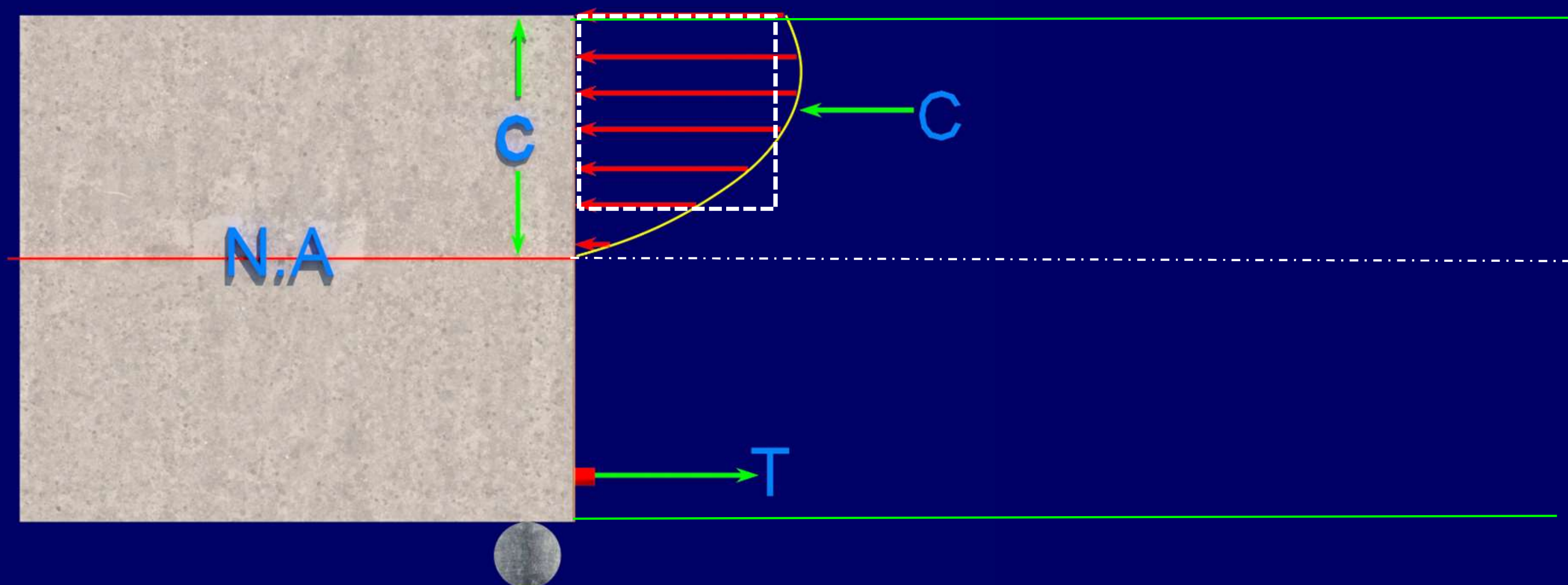
Behavior of Flexural Members

- **Stage – 3: Calculations**
  - **Whitney Stress Block**



Behavior of Flexural Members

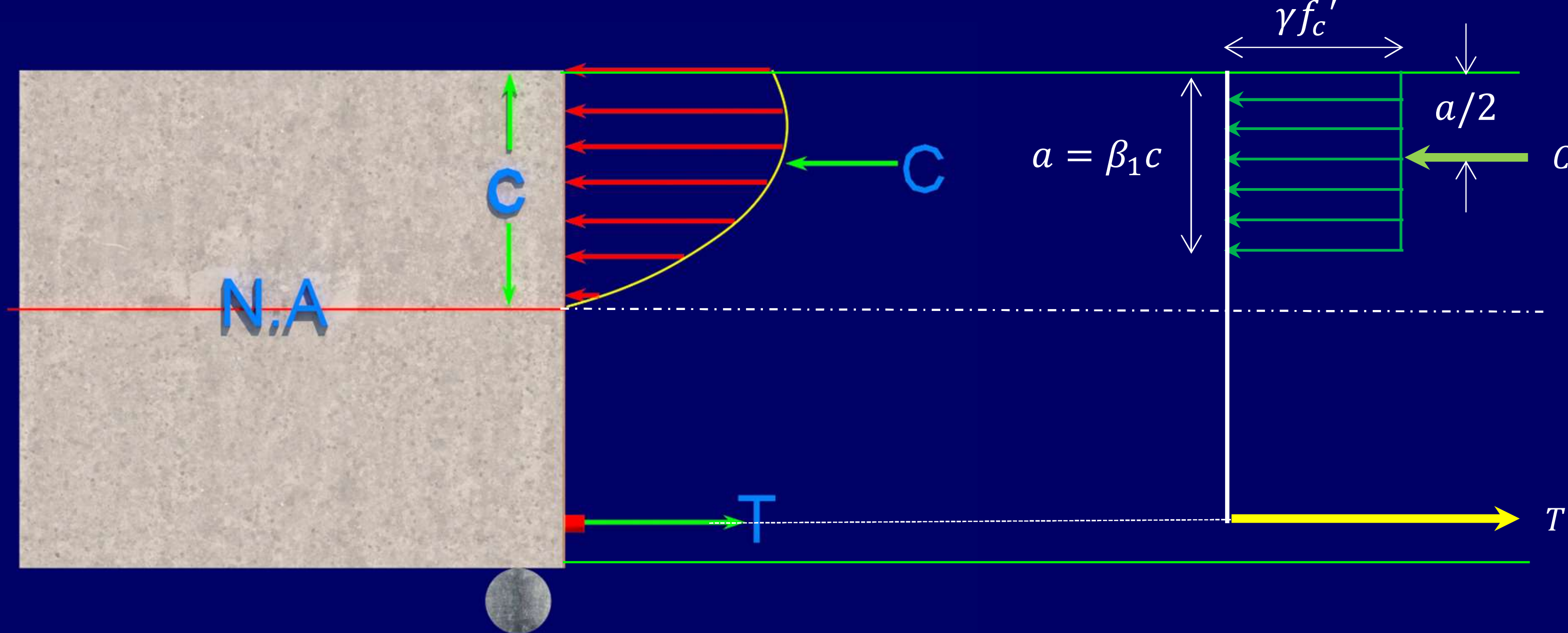
- Stage – 3: Calculations
  - Whitney Stress Block



Behavior of Flexural Members

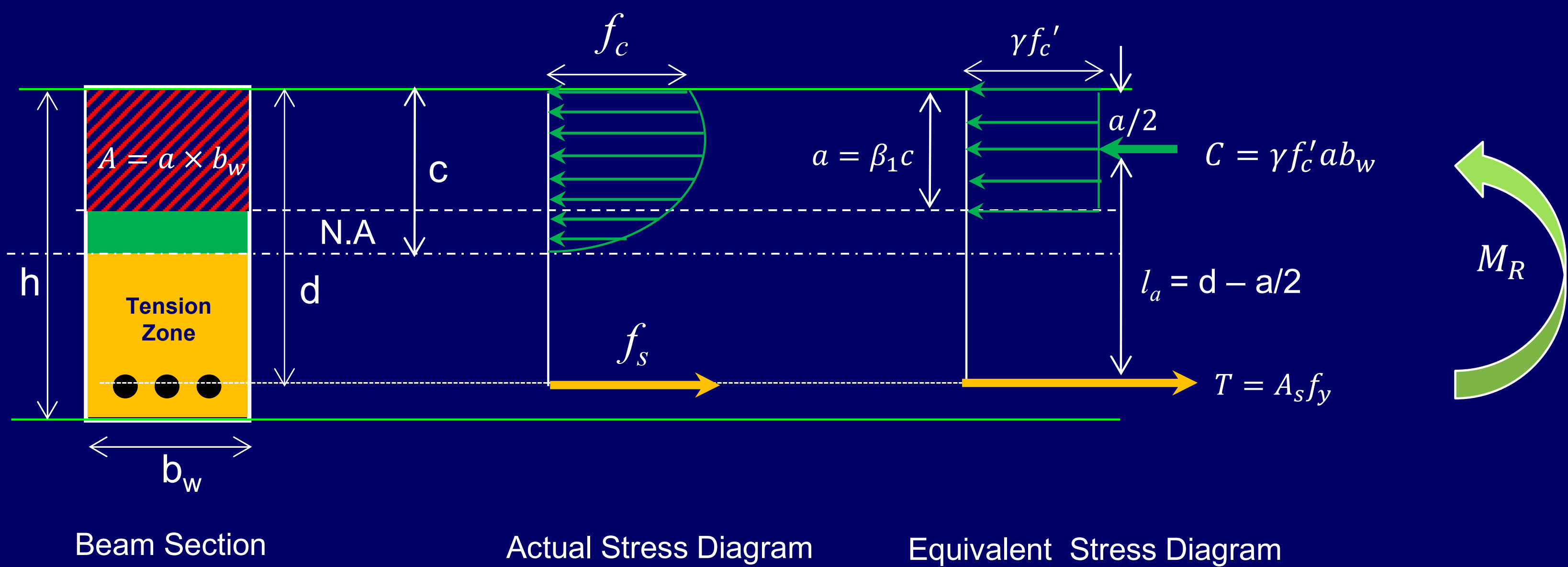
❑ Stage – 3: Calculations

- **Whitney Stress Block**



Behavior of Flexural Members

Stage – 3: Calculations



Calculating Resisting moment

$$M_R = M_c^{\nearrow 0} + M_s = T \times l_a = (A_s \times f_y) \times \left(d - \frac{a}{2}\right)$$

$$M_R = A_s f_y \left(d - \frac{a}{2}\right)$$

$\gamma = 0.85$ (ACI 318 -19, 22.2.2.4)

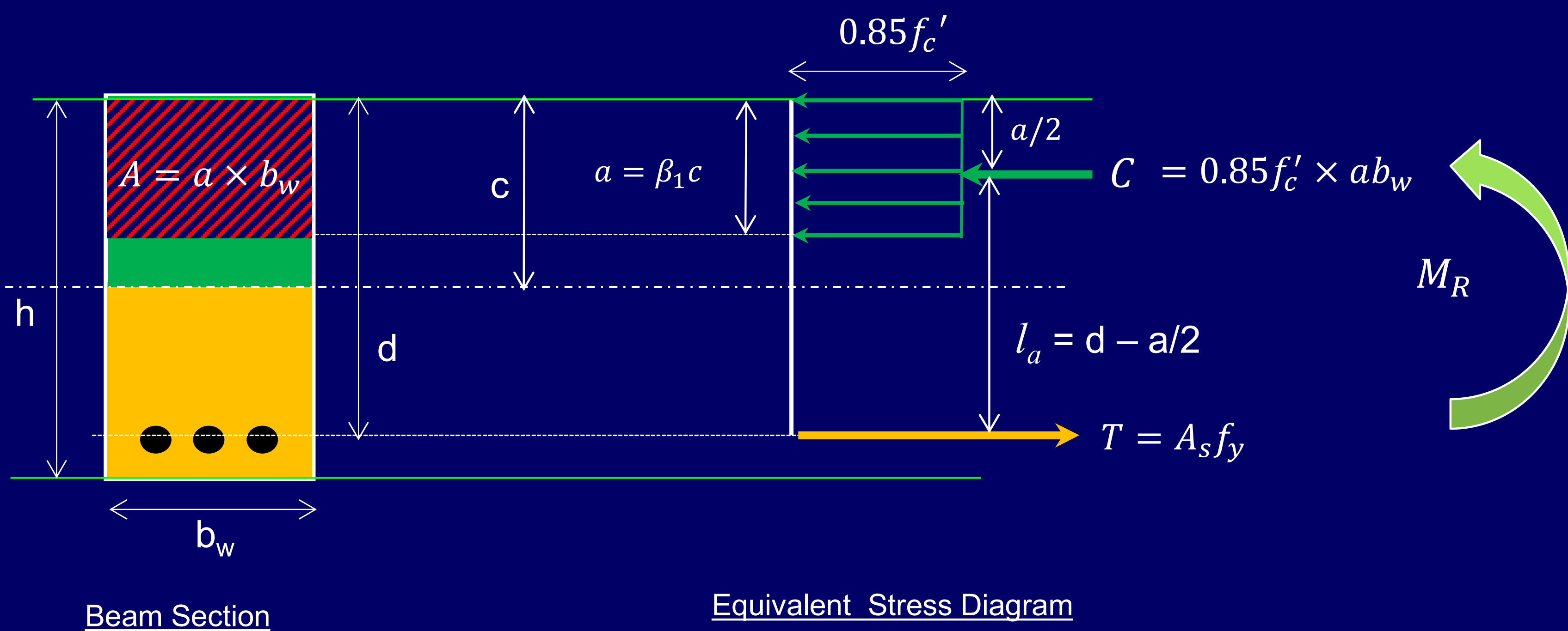
and

$\beta_1 = 0.85 \;\; for \; f_c' \leq 4000psi$

For strengths above 4000 psi, refer to ACI 318-19, 22.2.2.4.3

Behavior of Flexural Members

Stage – 3: Calculations



Equating Horizontal forces; C = T $\Rightarrow 0.85f_c' \times ab_w = A_s f_y$

Solving for "a", we get

$$\Rightarrow \quad a = \frac{A_s f_y}{0.85 f_c' b_w}$$

Behavior of Flexural Members

❑ Stage – 3: Calculations

We have

$$M_R = A_S f_y \left(d - \frac{a}{2}\right)$$

Equating $M_R = M_A$

$$A_S f_y \left(d - \frac{a}{2}\right) = M_A$$

Which on solving for A_s gives

$$A_s = \frac{M_A}{f_y \left(d - \frac{a}{2}\right)}$$

Behavior of Flexural Members

Stage – 3: Calculations

- The same Trial and Success method that was discussed in stage 2 can be used to determine the area of steel A_s.

1. Assume the value of "a"
2. Calculate the area of steel using

$$A_s = \frac{M_A}{f_y(d - a/2)}$$

3. Confirm the value of "a" using

$$a = \frac{A_s f_y}{0.85 f_c' b_w}$$

4. Repeat the process until the same A_s value is obtained from the two consecutive trials.

Behavior of Flexural Members

Stage – 3: Example 3.3

- Using the data from Example 3.1, calculate the area of steel required for the beam corresponding to stage 3.

- **Solution**

  - Trial 1: Choosing a = 2" and d = h – 2.5 = 15.5"

$$A_s = \frac{1002}{(40)(15.5 - 2/2)} = 1.73 in^2$$

$$\Rightarrow \quad a = \frac{1.73 \times 40}{0.85 \times 3 \times 12} = 2.26"$$

  - Trial 2: Choosing a = 2.26"

$$A_s = \frac{1002}{(40)(15.5 - 2.26/2)} = 1.74\ in^2$$

Behavior of Flexural Members

Stage – 3: Example 3.3

- **Solution**

$$\Rightarrow \quad a = \frac{1.74 \times 40}{0.85 \times 3 \times 12} = 2.27''$$

- Trial 3: Choosing a = 2.27"

$$A_s = \frac{1002}{(40)(15.5 - 2.27/2)} = 1.74\ in^2$$

$$\Rightarrow \quad a = \frac{1.74 \times 40}{0.85 \times 3 \times 12} = 2.27'' \qquad \text{(OK!)}$$

- Hence the required area of steel is 1.74in^2

Behavior of Flexural Members

Concluding Remarks

Stage 1

- The beam based on stage 1, which does not allow for any cracking, requires an abnormally deep depth and a very large amount of steel.
- As a result, designing based on this stage is both uneconomical and impractical.

Behavior of Flexural Members

❑ Concluding Remarks

Stage 2

- It is basically a working stress approach where the strength has been divided by 2 in order to achieve the factor of safety in the design.
- Designing beam at this stage is uneconomical as compared to that of stage 3.

Behavior of Flexural Members

Concluding Remarks

Stage 3

- Stage 3 corresponds to the Strength Design Method.
- Designing based on this stage is the most cost-effective among all stages.

Behavior of Flexural Members

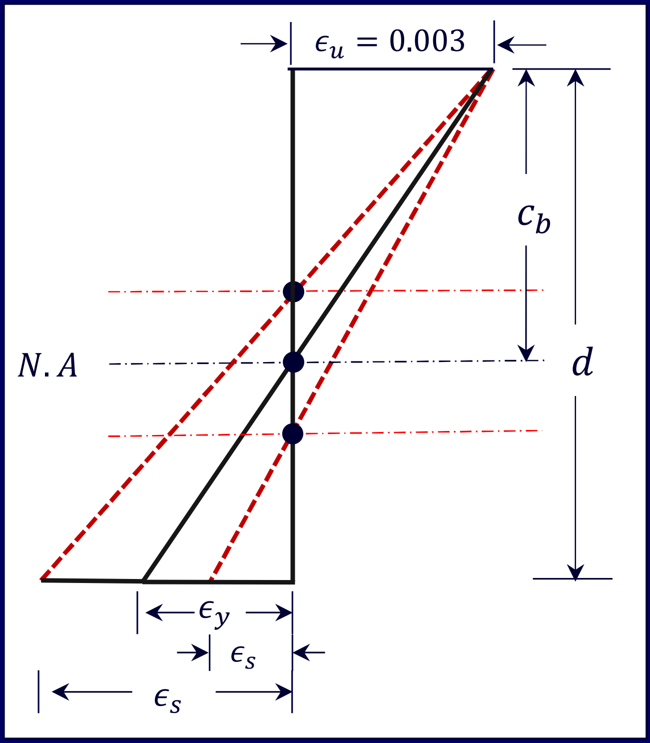
Mode of Flexural Failure

- The ACI Code requires that the beam designed using the strength design method should fail, if ever, in a ductile rather than brittle manner to allow for adequate evacuation time.
- The ductile failure mode can be ensured only when steel on the tension side yields well before the concrete crushes on the compression side.
- Yielding of steel will only be possible if tension steel is less than a certain amount, otherwise steel will not yield before the crushing of concrete, and the beam will fail in a brittle manner.

Behavior of Flexural Members

Mode of Flexural Failure

- When the concrete strain ϵ_u in the extreme fiber of the compression zone reaches 0.003, depending on the amount of tension reinforcement, Steel stain ϵ_s may exhibit one of the following conditions,



1. $\epsilon_s = \epsilon_y$ (Balanced condition)
2. $\epsilon_s < \epsilon_y$ (Over reinforced condition)
3. $\epsilon_s > \epsilon_y$ (Under reinforced condition)

Behavior of Flexural Members

Mode of Flexural Failure

- For any beam with given material properties and cross-sectional dimensions, there exists a specific amount of steel at which yielding and crushing occur simultaneously.
- This amount of steel is known as Balanced steel $A_{s,b}$, and the beam is said to be in Balanced condition.
- If $A_s < A_{s,b}$ the steel yields before the concrete crushes and the beam is said to be in Under reinforced condition.
- If $A_s > A_{s,b}$ the concrete crushes before the steel yields and the beam is said to be in Over reinforced condition.

Behavior of Flexural Members

❑ Reinforcement Limits

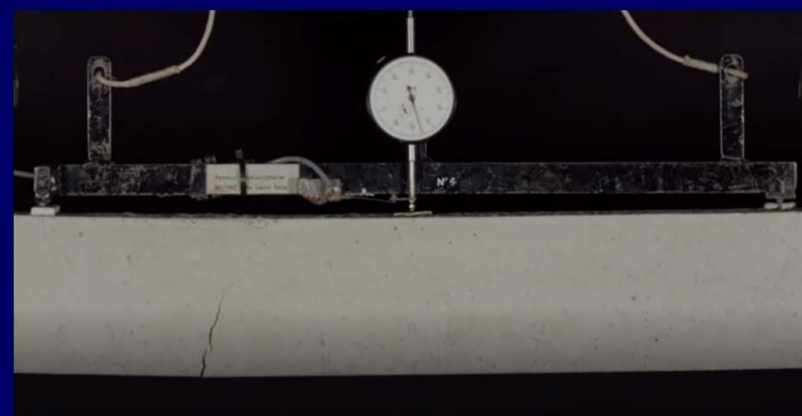
- Both the balanced condition ($\epsilon_s = \epsilon_y$) and over-reinforced condition ($\epsilon_s < \epsilon_y$) result in a brittle mode of failure.
- Hence to achieve ductility, the value of strain must be sufficiently greater than the yield strain ($\epsilon_s > \epsilon_y$) How much greater?
- This condition can be satisfied by imposing a maximum limit on the amount of steel.
- Similarly, there is also a minimum reinforcement limit to prevent the flexural member from behaving as plain concrete.

Behavior of Flexural Members

Mode of Flexural Failure

- **Tests on Beams with Non-Compliant Reinforcement per ACI 318.**

Beam having $A_s > A_{s,max}$



Beam having $A_s < A_{s,min}$

Design of Solid Rectangular Sections

General

- Solid rectangular sections, whether singly or doubly reinforced, are thoroughly discussed in Lectures 02 and 03 of the RCD – I Course. Here a summary of is provided.

Singly Reinforced Sections

Flexural Capacity

$$\emptyset M_n = \emptyset A_s f_y \left(d - \frac{a}{2}\right)$$

Where;

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

Design of Solid Rectangular Sections

Singly Reinforced Sections

Reinforcement Limits

- **Maximum Reinforcement**

$$A_{s,max} = \frac{0.85 f_c' \beta_1}{f_y}\left(\frac{0.003}{0.006+\epsilon_y}\right) b_w d$$

(refer to Lecture 02 of RCD – I for complete derivation)

$$A_{s,max,40} = \frac{f_c'}{136} b_w d$$

$$A_{s,max,60} = \frac{f_c'}{223} b_w d$$

(these equations are applicable for $\beta_1 = 0.85$)

- **Minimum Reinforcement (9.6.1.2)**

$$A_{s,min} = \text{larger}\left(\frac{3\sqrt{f_c'}}{f_y}, \frac{200}{f_y}\right) b_w d$$

(for $f_y \leq 80$ ksi)

Design of Solid Rectangular Sections

Singly Reinforced Sections

Maximum Flexural Capacity, $M_{n,max(SR)}$

Table 1: Maximum Factored flexural capacity of singly reinforced RC rectangular beam (f'_c = 3 ksi and f_y = 40 ksi)			
Depth h (in.)	Width b (in.)		
	b =12	b = 15	b = 18
12	802 (2.565)	1003 (3.206)	1204 (3.848)
18	2137 (4.185)	2671 (5.231)	3205 (6.278)
20	2724 (4.725)	3405 (5.906)	4086 (7.088)
24	4111 (5.805)	5139 (7.256)	6167 (8.708)
30	6726 (7.425)	8408 (9.281)	10089 (11.138)

- Effective depth is taken assuming d′ = 2.5″
- Values in brackets show maximum reinforcement in in^2

Design of Solid Rectangular Sections

❑ Singly Reinforced Sections

❖ Flexural Capacity at other strains

- We know that the ductility requirement of ACI code does not allow us to utilize the beam flexural capacity beyond ΦM_{nmax}. The code wants to ensure that steel in tension yield before concrete crushes in compression.

- However, if we ignore ACI code restriction, let see what happens.

  We know that

  $c = d\varepsilon_u/(\varepsilon_u + \varepsilon_s)$; $a = 0.85c$; $A_s = 0.85f_c'ab/ f_s$; $M_n = A_sf_s(d – a/2)$; $f_s = E\varepsilon_s \le f_{y;}$

  For ε_u = 0.003 and assuming various values of ε_s , we can determine A_s and M_n

Design of Solid Rectangular Sections

❑ Singly Reinforced Sections

❖ Flexural Capacity at other Strains

Table 2: Flexural Capacity of 12 x 24 inch [d=21.5″] RC beam at different tensile strain condition (f'_c = 3 ksi and f_y = 40 ksi)

ε_s (in/in)	0.0005	0.001	0.00137*	0.0021	0.003	0.004	0.005**	0.007
c (in)	18.43	16.13	14.76*	12.65	10.75	9.21	8.06**	6.46
A_s (in²)	33.06	14.46	9.66*	8.22	6.99	5.99	5.24**	4.19
f_s (ksi)	14.5	29	39.73*	40	40	40	40**	40
M_n (in-kips)	6551	6143	5846*	5304	4734	4214	3790**	3147

- * Yield strain for steel
- ** ACI Code limit for strain

Design of Solid Rectangular Sections

Singly Reinforced Sections

Flexural Capacity at other Strains

- **Conclusions**

  - At balance condition (Yield strain = 0.00137, M = 5846), there is no significant capacity increase with further steel reinforcement or strain reduction.
  - At the ACI code limit (strain = 0.005, M = 3790), there's a noticeable gap between moment capacity at balance and the ACI limit. Without ductility requirements, capacity can be increased up to the balanced point.
  - For ductility, moment capacity can only increase (without altering dimensions) by adding compression reinforcement (doubly reinforced).

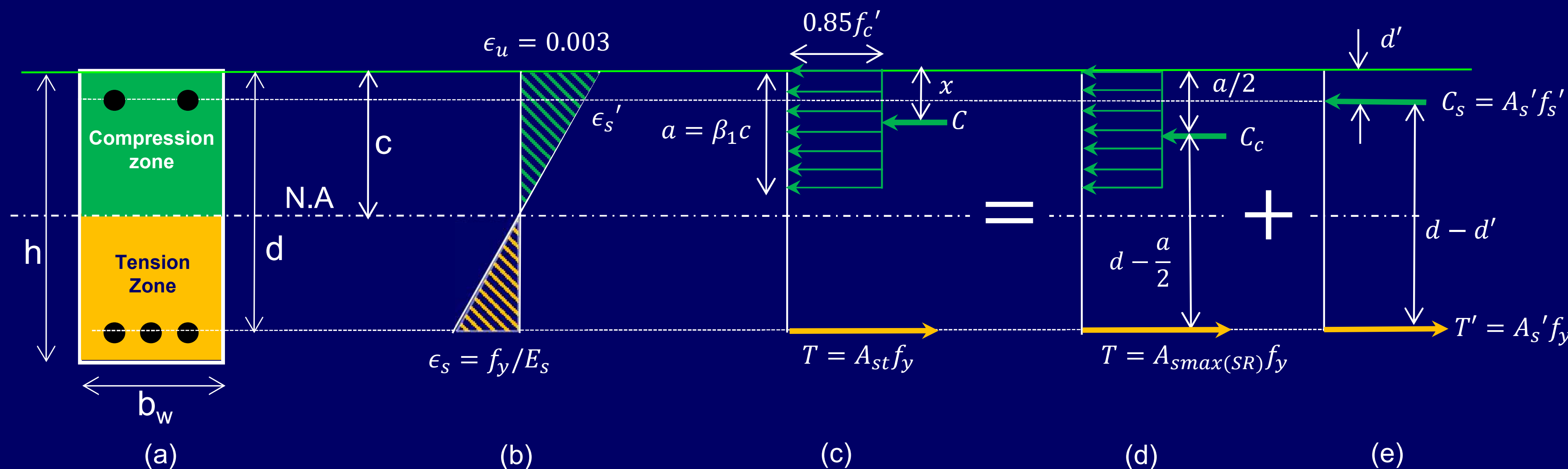
Design of Solid Rectangular Sections

❑ Doubly Reinforced Sections

❖ Flexural Capacity

$$\emptyset M_{n(DR)} = \emptyset M_{n1} + \emptyset M_{n2}$$

$$\emptyset M_{n(DR)} = \emptyset A_{s,max(SR)}\left(d - \frac{a_{max}}{2}\right) + \emptyset A_s' f_s'(d - d')$$



Design of Solid Rectangular Sections

❑ Doubly Reinforced Sections

❖ Flexural Capacity

$$\emptyset M_{n(DR)} = \emptyset M_{n,max(SR)} + \emptyset A_s' f_s'(d - d')$$

Where;

$$\emptyset M_{n,max(SR),40} = 0.219 f_c' b_w d^2$$

(refer to Lecture 03 of RCD – I for complete derivation)

$$\emptyset M_{n,max(SR),60} = 0.204 f_c' b_w d^2$$

$$f_s' = 87 - (174 + f_y)\frac{d'}{d} \le f_y$$

Design of Solid Rectangular Sections

❑ Doubly Reinforced Sections

Maximum Reinforcement

$$A_{s,max(DR),40} = \frac{f_c'}{136} b_w d + \frac{f_s'}{f_y} (A_s')_{pvd}$$

(refer to Lecture 03 of RCD – I for complete derivation)

$$A_{s,max(DR),60} = \frac{f_c'}{223} b_w d + \frac{f_s'}{f_y} (A_s')_{pvd}$$

Design of Solid Rectangular Sections

Doubly Reinforced Sections

Condition for Yielding of Compression Steel

- **For Grade 40 Steel**

$$f_s' = 87 - (174 + f_y)d'/d$$

Setting $f_s' = f_y = 40$, we get;

$$d'/d = 0.22$$

- **For Grade 60 Steel**

$$f_s' = 87 - (174 + f_y)d'/d$$

Substituting $f_s' = f_y = 60$, we get;

$$d'/d = 0.12$$

Design of Solid Rectangular Sections

Doubly Reinforced Sections

Condition for Yielding of Compression Steel

- The following table presents the ratios (d′/d) and minimum beam effective depths (d) required to achieve compression reinforcement yield for various steel grades.

Minimum beam depths for compression reinforcement to yield		
f_y, psi	Maximum d'/d	Minimum d for d' = 2.5" (in.)
40,000	0.22	11.5
60,000	0.12	21.5
80,000	0.03	83.33

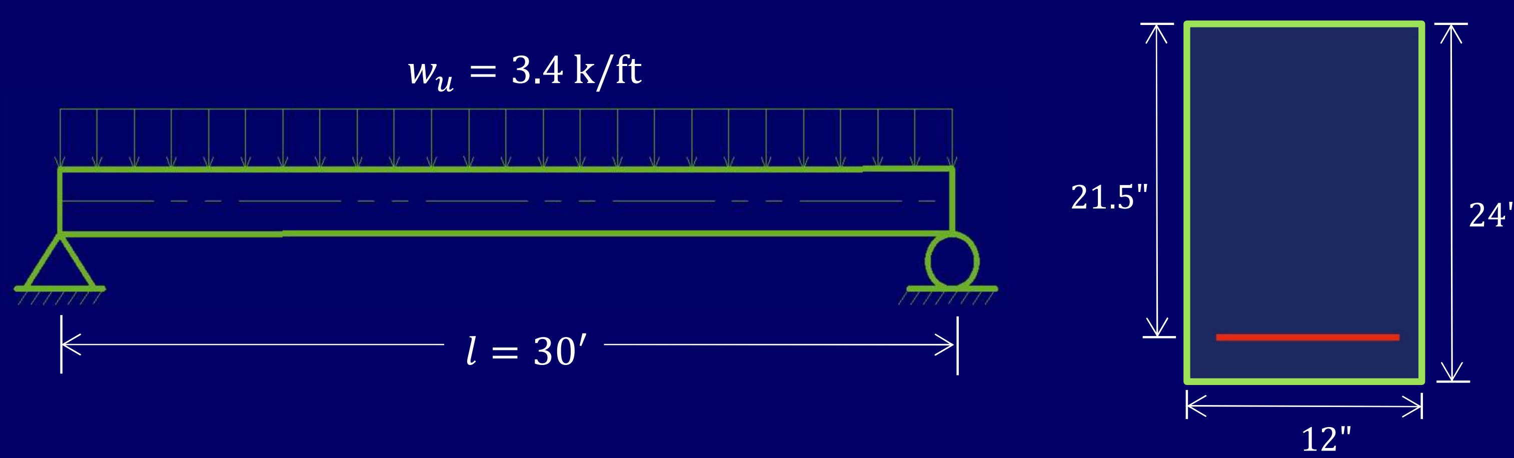
Design of Solid Rectangular Sections

❑ Example 3.4

- A simply supported reinforced concrete beam having span length of 30 ft., subjected to ultimate load of 3.4 k/ft is shown below. Material strengths to be used are; $f_c' = 3\ ksi$ and $f_y = 40\ ksi$. *Determine* flexural reinforcement for the given beam if:

  a) There is restriction on material properties of beam.

  b) There is restriction on both material and geometry of beam.



Design of Solid Rectangular Sections

❑ Solution

- **Given Data**

  $l = 30'$

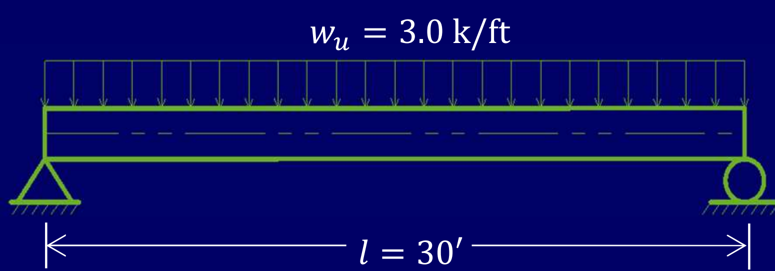
  $b_w = 12''$

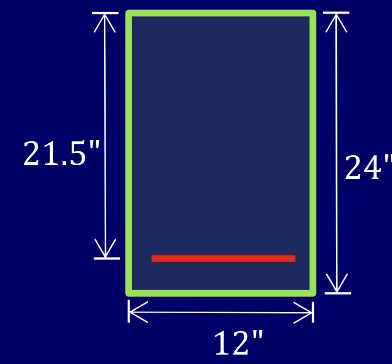
  $h = 24''$

  $d = 21.5''$

  $f_c' = 3$ ksi and

  $f_y = 60$ ksi





- **Required Data**
  - Calculate required flexural reinforcement, $A_s = ?$

Design of Solid Rectangular Sections

Solution

- **Step 1: Selection of Sizes**
  - Sizes are given
- **Step 2: Calculation of Loads**
  - Factored load is given
- **Step 3: Analysis**

$$M_u = \frac{w_u l^2}{8} = \frac{3.0 \times 30^2}{8} \times 12 = 4050 \text{ in. kip}$$

Design of Solid Rectangular Sections

❑ Solution

❖ Part (a)

➢ **Step 4: Determination of Reinforcement**

$$a = d - \sqrt{d^2 - \frac{2.614 M_u}{f_c' b}} = 21.5 - \sqrt{21.5^2 - \frac{2.614 \times 4050}{3 \times 12}} = 8.53"$$

$$A_s = \frac{M_u}{0.9 f_y \left(d - \frac{a}{2}\right)} = \frac{4050}{0.9 \times 60 \left(21.5 - \frac{8.53}{2}\right)} = 4.35 \text{ in}^2$$

Step 5: Reinforcement Checks

$$A_{s,max(SR),60} = \frac{f_c' b_w d}{223} = \frac{3 \times 12 \times 21.5}{223} = 3.47 \text{ in}^2 < \mathrm{A_s} \rightarrow \text{Not OK!}$$

Design of Solid Rectangular Sections

Solution

Part (a)

Step 5: Reinforcement Checks

- Since there is not restriction on beam's geometry, we have the flexibility to increase both its width and depth. By doing so,
  - Maximum reinforcement range can be increased.
  - Required area of steel can be reduced.
- Let increase the depth to 27″ and re-calculate steel area.

$a = 7.0" \quad and\ A_s = 3.57\ \text{in}^2$

$A_{s,max(SR),60} = 3.96\ \text{in}^2 > \text{A}_\text{s} = 3.57 \rightarrow$ OK!

- Hence, provide (3+2)-#8 bars at bottom face of beam.

Design of Solid Rectangular Sections

Solution

Part (b)

- **Step 5: Reinforcement Checks**
  - As there are restrictions on the beam's geometry and material properties, the only viable option is to design the section as doubly reinforced.

- **Step 6: Determine Area of Compression Steel**

$$A_s' = \frac{M_u - \emptyset M_{n,max(SR)}}{\emptyset f_s'(d - d')}$$

$$A_s' = \frac{4050 - 0.204 \times 3 \times 12 \times 21.5^2}{0.9 \times 59.8(21.5 - 2.5)}$$

$$A_s' = 0.64 \text{ in}^2$$

$$\emptyset M_{n,max(SR),60} = 0.204 f_c' b_w d^2$$

$$f_s' = 87 - (174 + f_y)d'/d \leq f_y$$

$$f_s' = 87 - \frac{(174 + 60)2.5}{21.5} = 59.8\ ksi$$

Design of Solid Rectangular Sections

❑ Solution

❖ Part (b)

➢ **Step 7: Determine Area of Tensile Steel**

$$A_{st} = A_{s,max(SR)} + A_s{}' = 3.47 + 0.64 = 4.11 \text{ in}^2$$

➢ **Step 8: Reinforcement Checks**

Using #8 bar for tension steel and #6 for compression steel:

No. of bars on tension side $= \dfrac{A_{st}}{A_b} = \dfrac{4.11}{0.79} = 5.2 \approx 6$

No. of bars on compression side $= \dfrac{A'_s}{A_b} = \dfrac{0.64}{0.44} = 1.5 \approx 2$

Design of Solid Rectangular Sections

Solution

Part (b)

Step 8: Reinforcement Checks

$$A_{s,max(DR),60} = \frac{f_c'}{223} b_w d + \frac{f_s'}{f_y} (A_s')_{pvd}$$

$$A_{s,max(DR),60} = 3.47 + \frac{59.8}{60} (2 \times 0.44) = 4.35 \text{ in}^2$$

Provided area of tension steel is,

$$A_{st,pvd} = 6(0.79) = 4.74 \text{ in}^2 \rightarrow \text{Not OK!}$$

What to do now?

Design of Solid Rectangular Sections

❑ Solution

❖ Part (b)

➢ **Step 8: Reinforcement Checks**

- Adding one extra #6 bar on compression side and re-calculate $A_{s,max(DR)}$

$$A_{s,max(DR),60} = 3.47 + \frac{59.8}{60}(3 \times 0.44) = 4.79 \text{ in}^2$$

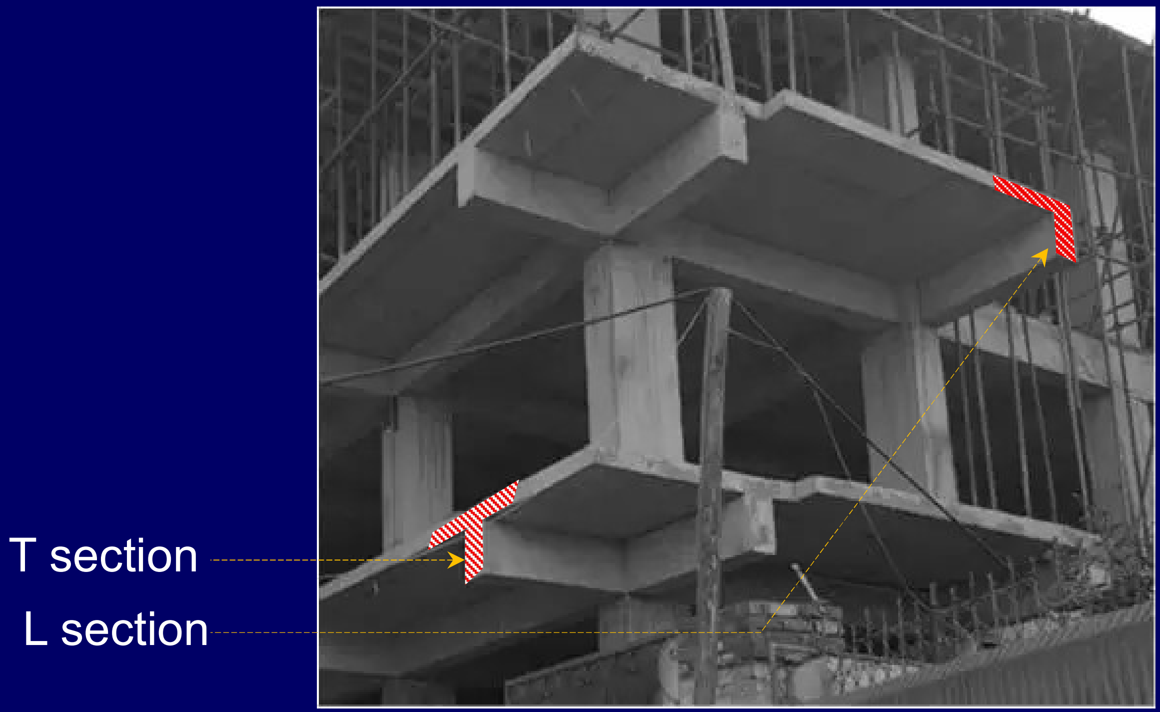
$$A_{st,pvd} = 6(0.79) = 4.74 \text{ in}^2 \rightarrow \text{OK!}$$

- Hence provide,
  - (3+3) - #8 bars on tension side and
  - 3-#6 bars on compression side.

Design of Solid T and L Sections

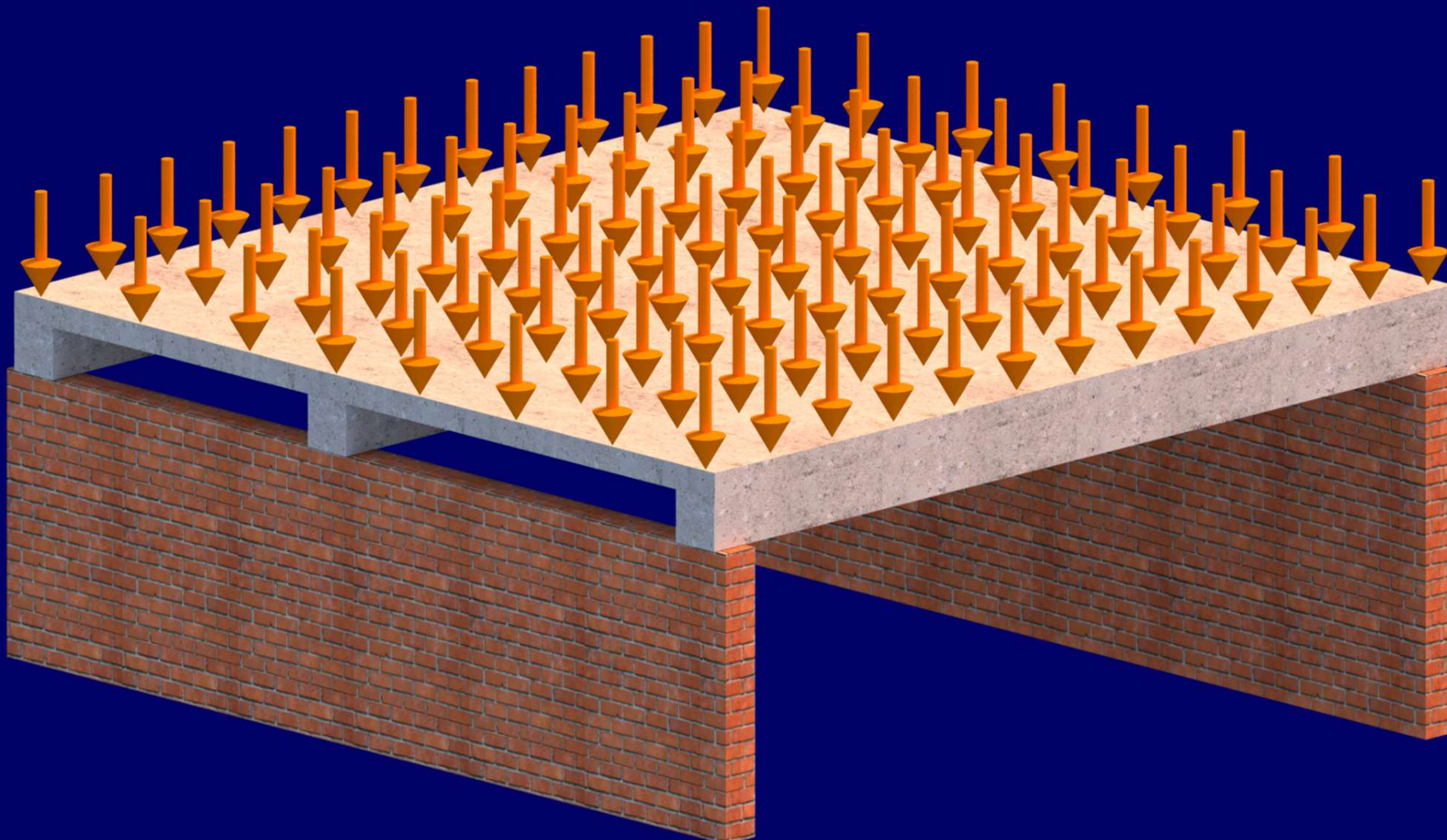
❑ Introduction

- The T or L Beam gets its name when the slab and beam produce the cross sections having the typical T and L shapes in a monolithic reinforced concrete construction.



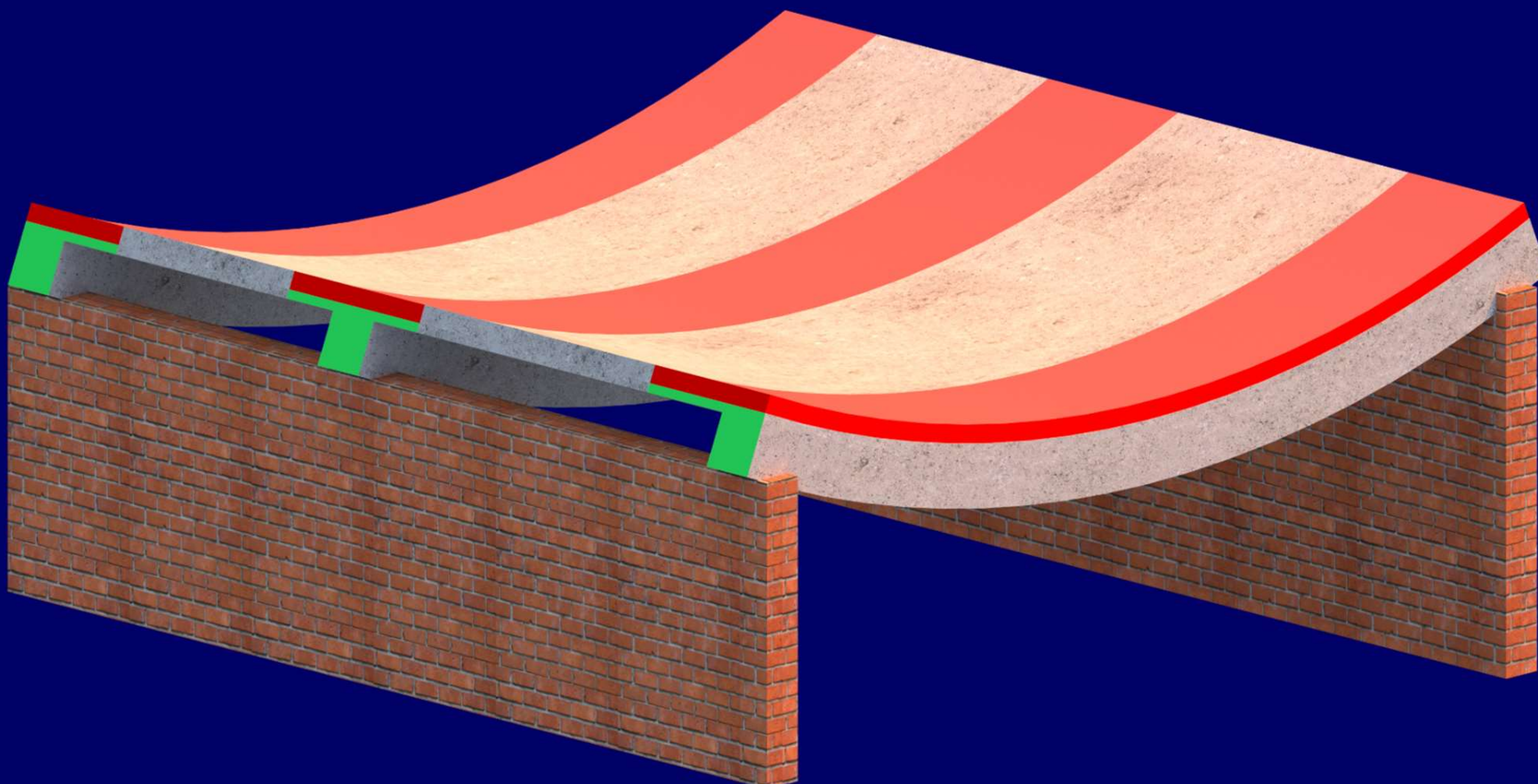
Design of Solid T and L Sections

- Introduction



Design of Solid T and L Sections

❑ Introduction



Design of Solid T and L Sections

Introduction

- In casting of reinforced concrete floors/roofs, forms are built for beam sides, the underside of slabs, and the entire concrete is mostly poured at once, from the bottom of the deepest beam to the top of the slab.

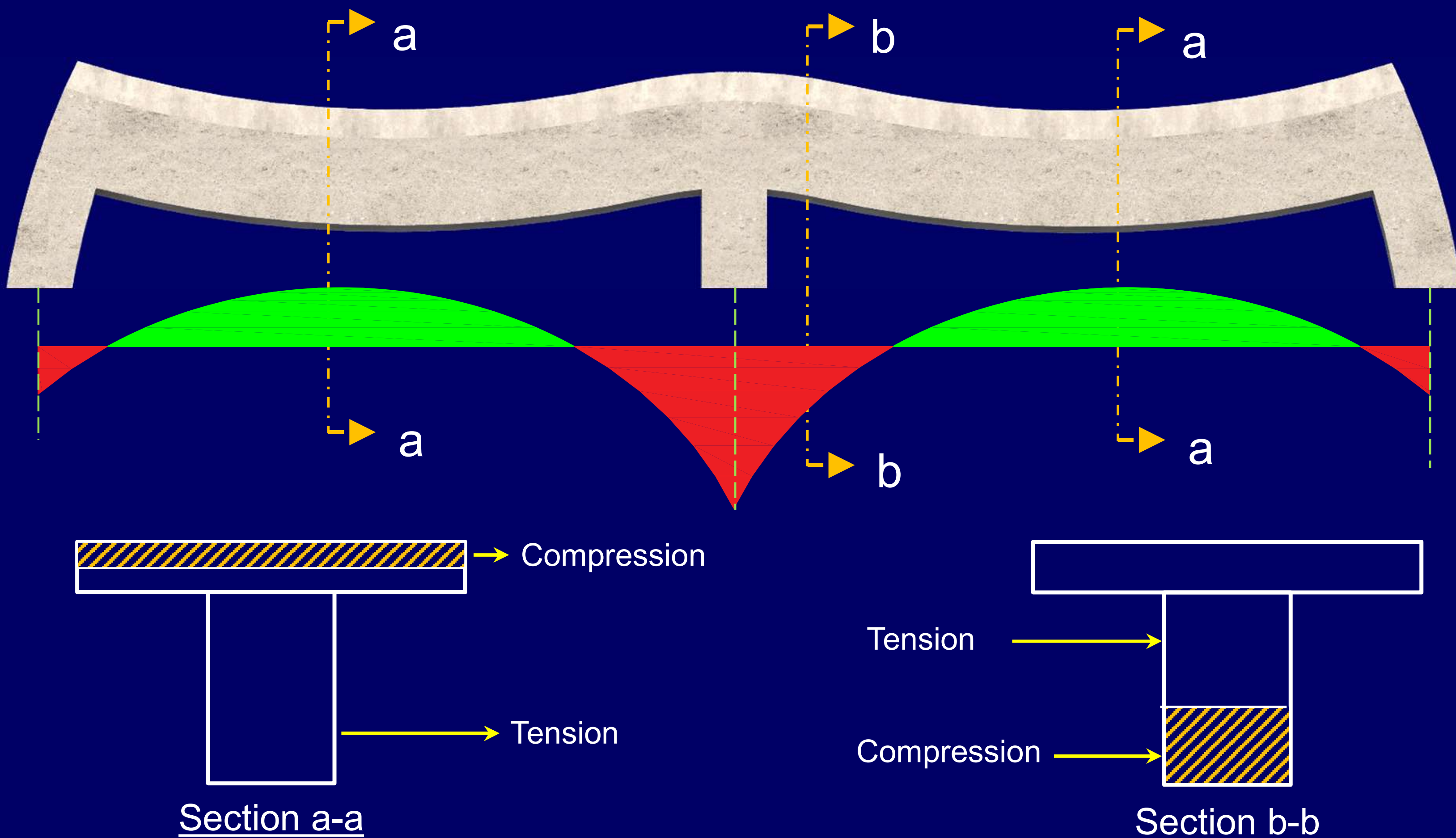
Design of Solid T and L Sections

Introduction

- Construction of T and L beam at site

Design of Solid T and L Sections

- Behavior of T and L section Beams under gravity Loading

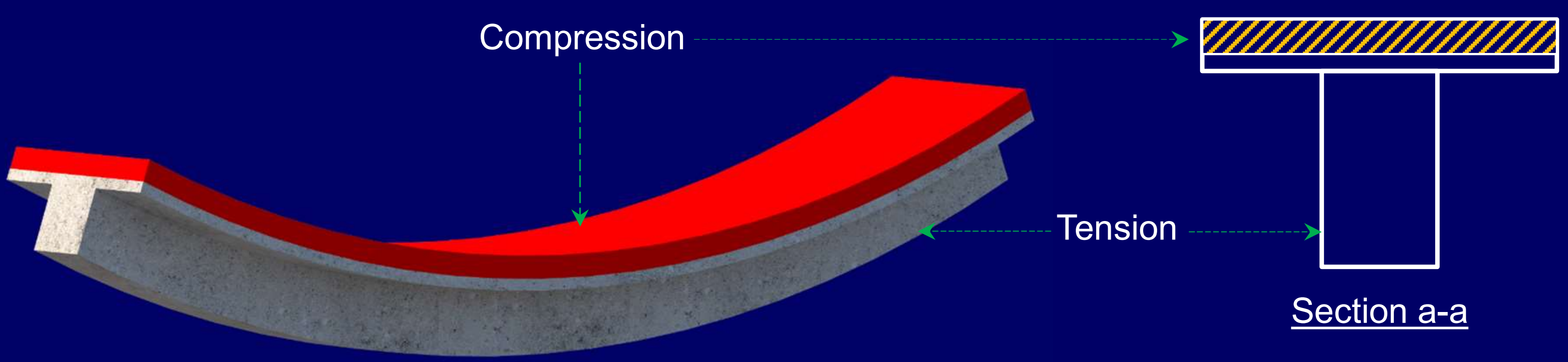


Design of Solid T and L Sections

Behavior of T and L section Beams under gravity Loading

Positive Bending Moment

- It is common practice to assume that the monolithically placed slab and supporting beam interact as a unit in resisting the positive bending moment.
- As shown, the slab acts as the compression flange, while the supporting beam becomes the web or stem.

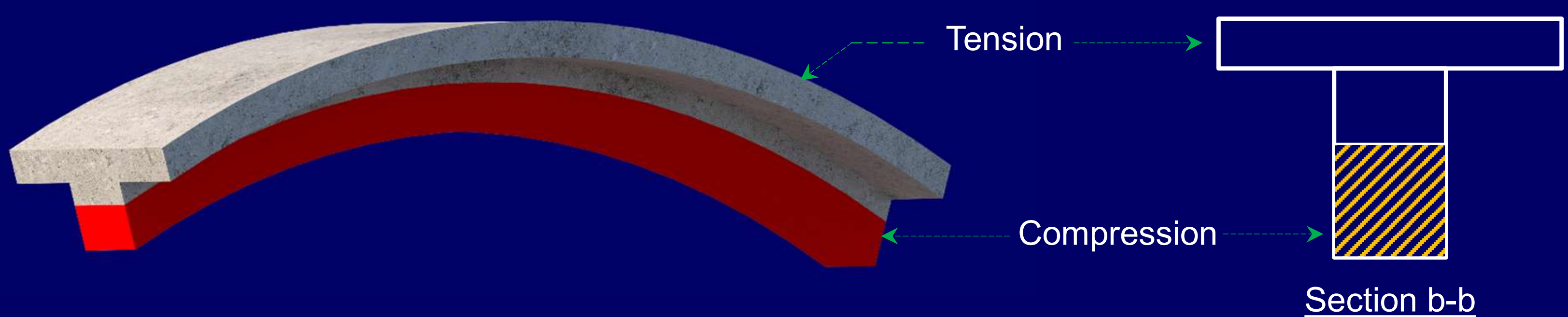


Design of Solid T and L Sections

❑ Behavior of T and L section Beams under gravity Loading

❖ Negative Bending Moment

- In the case of negative bending moment, the slab at the top of the stem (web) will be in tension, while the bottom of the stem will be in compression.
- This usually occurs at interior support of continuous beam.



Design of Solid T and L Sections

Calculation of Effective Flange Width

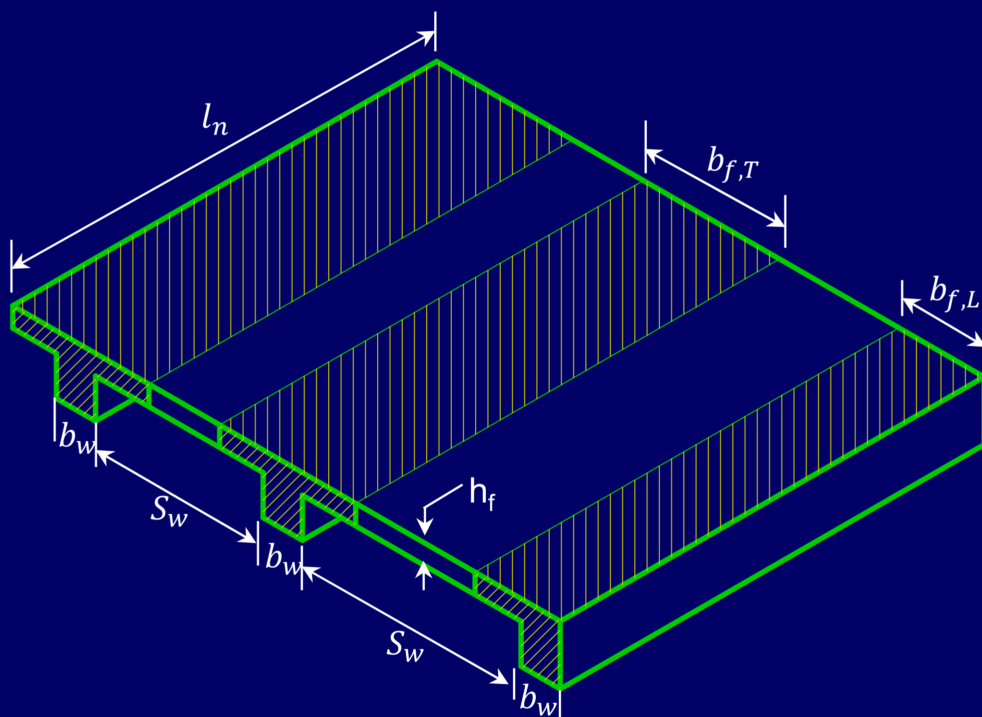
- As per ACI 318-19, the effective flange width b_f for T and L beams shall be calculated as per Table 6.3.2.1

- **For T beam**

$$b_{f,T} = least\ of \begin{cases} b_w + 16h_f \\ b_w + S_w \\ b_w + l_n/4 \end{cases}$$

- **For L beam**

$$b_{f,L} = least\ of \begin{cases} b_w + 6h_f \\ b_w + S_w/2 \\ b_w + l_n/12 \end{cases}$$



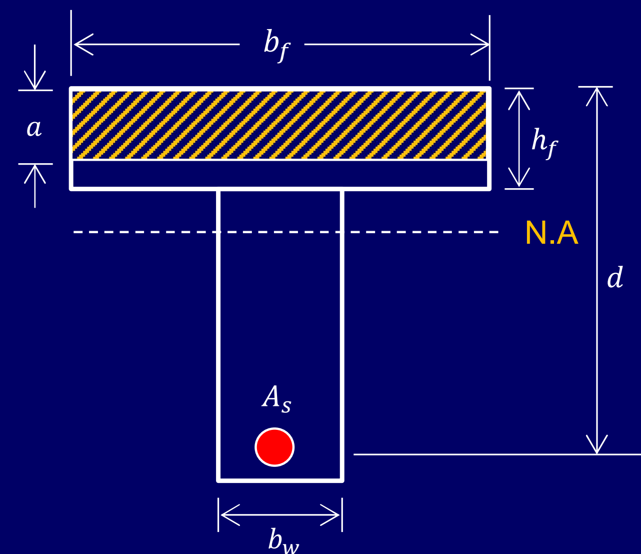
Design of Solid T and L Sections

❑ Design Cases

- In designing T or L beams for positive bending moment, there exist two conditions:

Case 1: Rectangular Compression Block

- When the value of compression block depth is less than or equal to flange thickness ($a \leq h_f$), it assumes a rectangular shape.
- In such a case, T-beam should be designed as a rectangular beam with a compression block width of b_f.
- Same is the case for L-Section



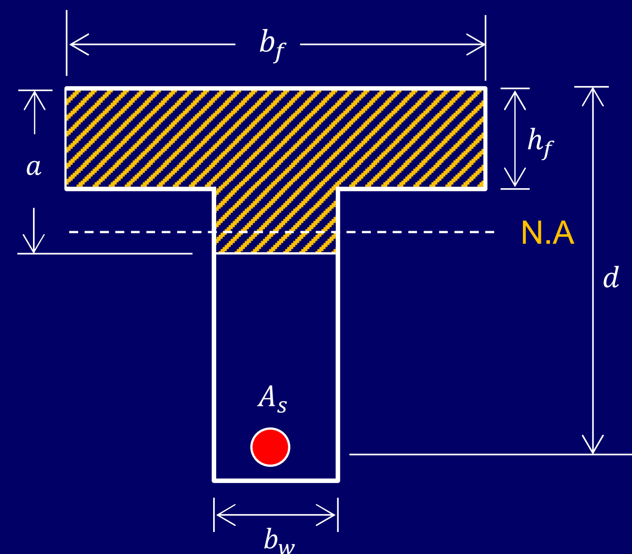
Design of Solid T and L Sections

Design Cases

- In designing T or L beams for positive bending moment, there exist two conditions:

Case 2: T-shaped Compression Block

- When the compression block covers the whole flange and extends into the web portion i.e. ($a > h_f$) , the compression block becomes a T-shaped.
- In such a condition, the T-Beam is designed as True T-beam.
- Same is the case for L-Section.



Design of Solid T and L Sections

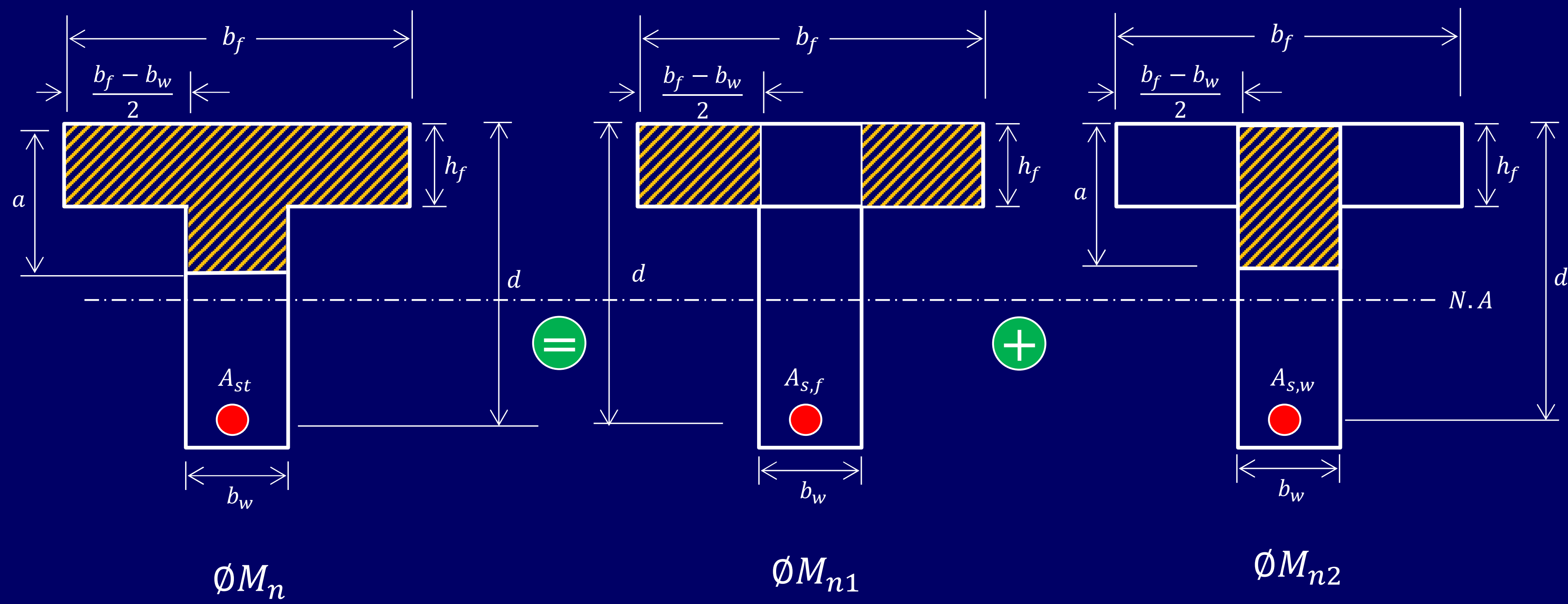
❑ Design Cases

- Lecture 04 of RCD – I provides comprehensive coverage of both Case – I and Case – II.
- However, in this lecture, we will briefly revisit Case – II, which involves the design of True-T or True-L sections.

Design of Solid T and L Sections

❑ Flexural Capacity of True-T beam

Method 1



$$\emptyset M_n = \emptyset M_{n1} + \emptyset M_{n2}$$

Design of Solid T and L Sections

Flexural Capacity of True-T beam

Method 1

- Calculation of $\emptyset M_{n1}$

From Stress diagram;

$C_1 = T_1$

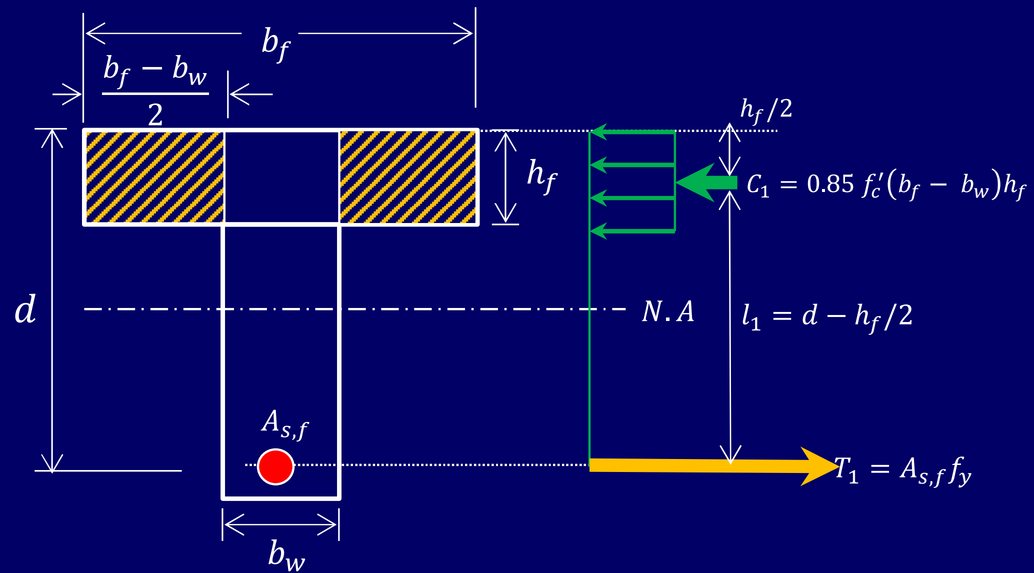
$C_1 = 0.85\, f_c'(b_f - b_w)h_f$

$T_1 = A_{s,f} f_y$

$0.85\, f_c'(b_f - b_w)h_f = A_{s,f} f_y$

$$A_{s,f} = \frac{0.85 f_c'(b_f - b_w)h_f}{f_y}$$

$$\emptyset M_{n1} = T_1 \times l_1 = \emptyset A_{s,f} f_y (d - h_f/2)$$



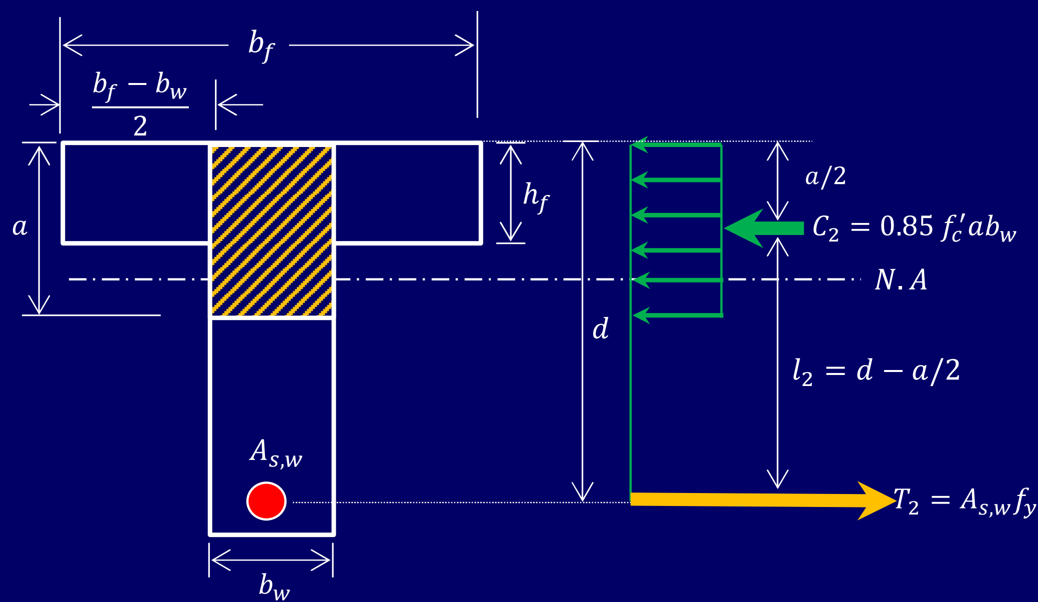
$A_{s,f}$ is the amount of steel to be resisted by flange part of the beam.

Design of Solid T and L Sections

Flexural Capacity of True-T beam

Method 1

- **Calculation of $\emptyset M_{n2}$**



From Stress diagram;

$$C_2 = T_2$$

$$C_2 = 0.85\ f_c' a b_w$$

$$T_2 = A_{s,w} f_y$$

$$0.85\ f_c' a b_w = A_{s,w} f_y$$

$$a = \frac{A_{s,w} f_y}{0.85 f_c' b_w}$$

$$\emptyset M_{n2} = T_2 \times l_2 = \emptyset A_{s,w} f_y (d - a/2)$$

$A_{s,w}$ is the amount of steel to be resisted by web part of the beam.

Design of Solid T and L Sections

Flexural Capacity of True-T beam

Method 1

- **Determination of Steel Area**

Steel area to be resisted by web part, $A_{s,w}$ is given by

$$\emptyset M_{n1} + \emptyset M_{n2} = M_u$$

$$\emptyset M_{n1} + \emptyset A_{s,w} f_y \left(d - \frac{a}{2}\right) = M_u$$

$$A_{s,w} = \frac{M_u - \emptyset M_{n1}}{\emptyset f_y (d - a/2)}$$

$$a = \frac{A_{s,w} f_y}{0.85 f_c' b_w}$$

$$\emptyset M_{n1} = \emptyset A_{s,f} f_y (d - h_f/2)$$

$$A_{s,f} = \frac{0.85 f_c' (b_f - b_w) h_f}{f_y}$$

- **Determination of Total Steel Area**

$$A_{st} = A_{sf} + A_{s,w}$$

Design of Solid T and L Sections

❑ Flexural Capacity of True-T beam

❖ Method 2

$$\emptyset M_n = M_u = \emptyset A_{st} f_y (d - x)$$

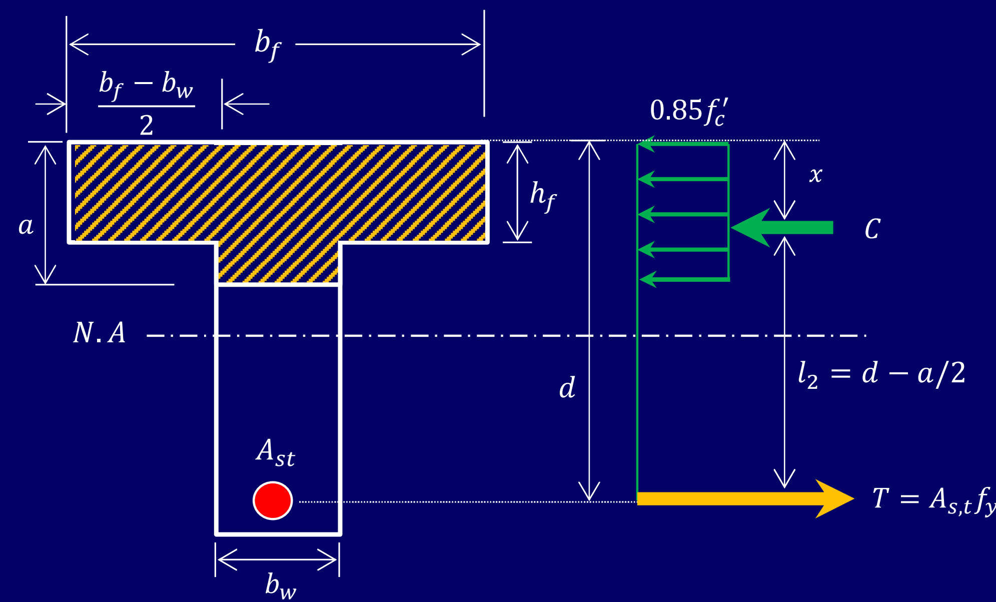
$$A_{st} = \frac{M_u}{\emptyset f_y (d - x)}$$

where;

$$x = \frac{b_w a^2 + (b_f - b_w) h_f^{\,2}}{2 b_w a + 2(b_f - b_w) h_f}$$

and

$$a = \frac{A_{st} f_y - 0.85 f_c' (b_f - b_w) h_f}{0.85 f_c' b_w}$$



Trial and Success Procedure

- Calculate x by assuming a
- Compute A_{st} by substituting x
- Recalculate a.
- Repeat the process until the values match.

Design of Solid T and L Sections

Maximum Reinforcement Limit

$$A_{st,max} = \frac{0.85 f_c' \beta_1 c b_w}{f_y} + \frac{A_{s,f} f_y}{f_y}$$

(refer to Lecture 04 of RCD – I for complete derivation)

Substituting $\beta_1 = 0.85$, and relevant c values, we get

$$A_{st,max(TT),40} = \frac{f_c' b_w d}{136} + A_{s,f}$$

and

$$A_{st,max(TT),60} = \frac{f_c' b_w d}{223} + A_{s,f}$$

$$c_{40} = 0.41d$$

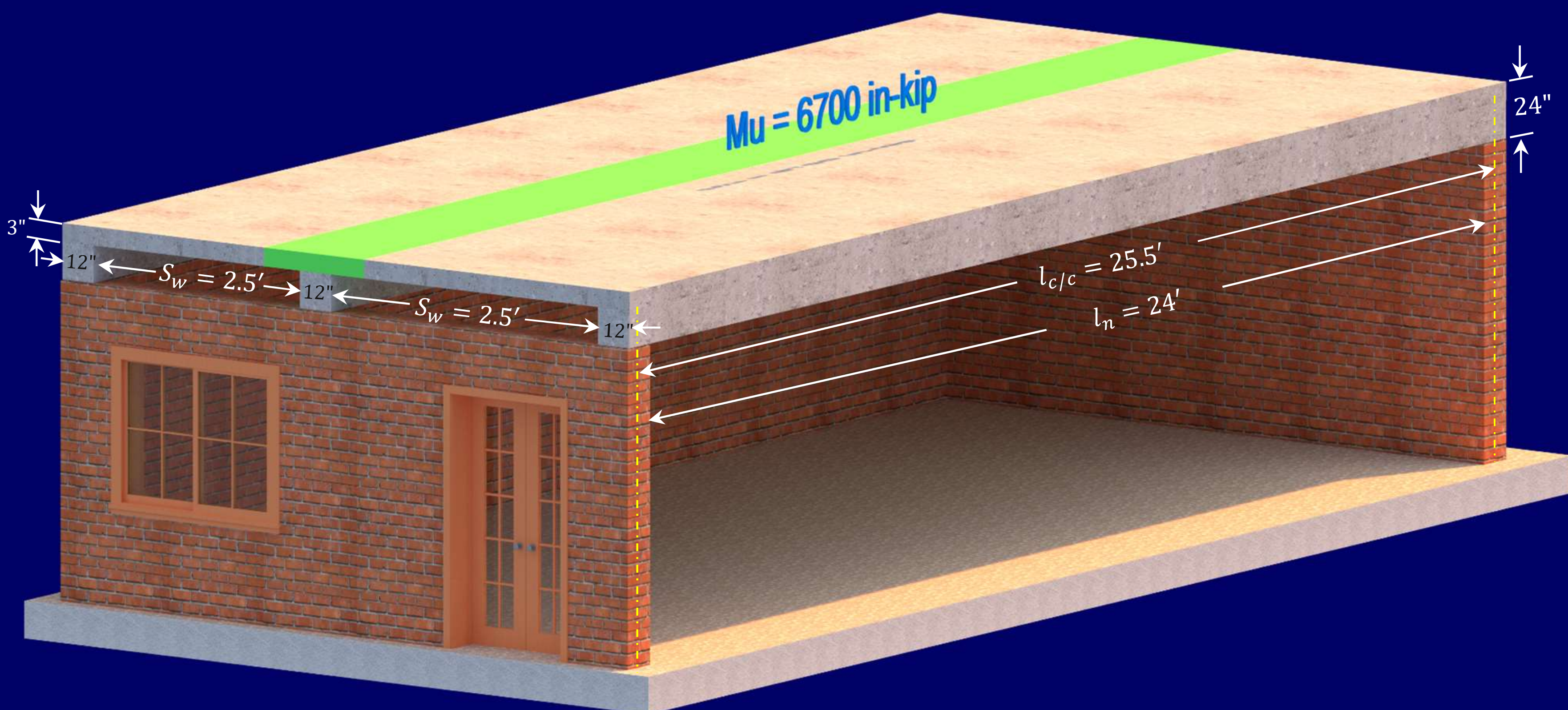
$$c_{60} = 0.38d$$

- The same formulae are applicable for L sections.

Design of Solid T and L Sections

Example 3.5

- ***Design*** the highlighted beam for the data provided in figure using $f_c{}' = 3ksi$ and $f_y = 60ksi$.



Design of Solid T and L Sections

❑ Solution

- **Given Data**

$b_w = 12", h = 24"$ *and* $h_f = 3"$

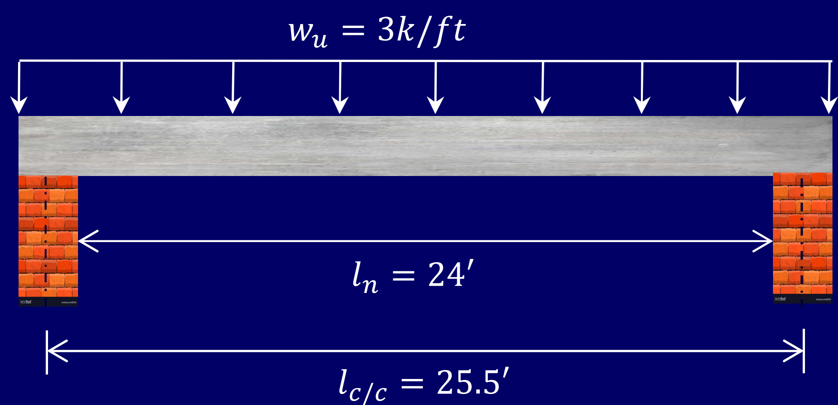
$l_{c/c} = 25.5'$, *and* $l_n = 24'$

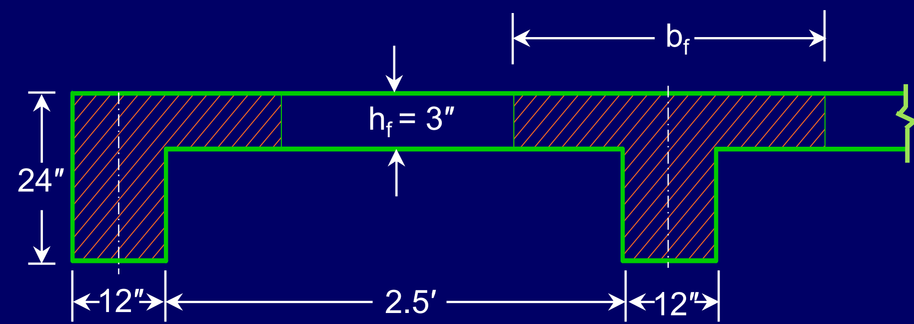
$S_w = 2.5'$

$M_u = 6700\ in.kip$

$f_c{}' = 3ksi$

$f_y = 60ksi$





- **Required Data**
  - Design the beam as per ACI 318 – 19

Design of Solid T and L Sections

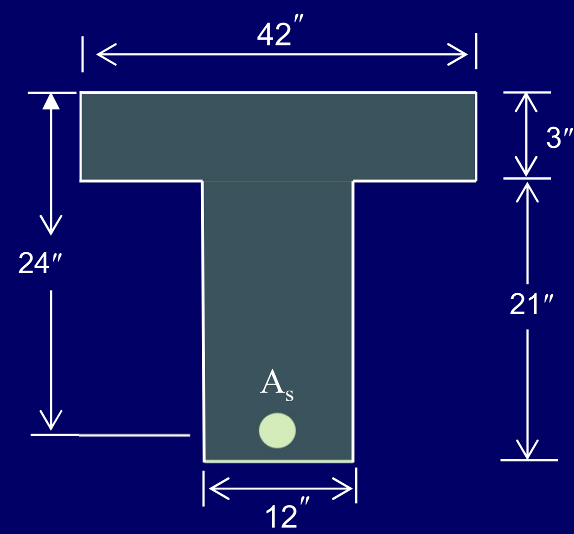
❑ Solution

➢ Step 1: Selection of Sizes

$b_w = 12"$, $h = 24"$, $h_f = 3"$ and assume $d = 24 - 2.5 = 21.5"$

Effective width of T- beam b_f is minimum of:

- $b_w + 16h_f = 12 + 16(3) = 60"$
- $b_w + s_w = 12 + 2.5 \times 12 = 42"$
- $b_w + \frac{l_n}{4} = 12 + \frac{24}{4} \times 12 = 84"$



Therefore, $b_f = 42"$

Design of Solid T and L Sections

Solution

Step 2: Calculation of Loads

We have directly given the ultimate moment

Step 3: Analysis

$$M_u = 6700\ in.kip$$

Step 4: Checking the behavior of Section

$$a = 21.5 - \sqrt{21.5^2 - \frac{2.614 \times 6700}{3 \times 42}} = 3.52''$$

Since $a > h_f$, the section True T- beam

- Now area of steel can be determined using Method 1 or Method 2.

Design of Solid T and L Sections

❑ Solution

- **Method 1**

- **Step 5: Determination of $A_{s,f}$, $\emptyset M_{n,1}$ and $A_{s,w}$**

$$A_{s,f} = \frac{0.85 f_c' (b_f - b_w) h_f}{f_y}$$

$$= \frac{0.85 \times 3(42 - 12)3}{60} = 3.83\ in^2$$

And

$$\emptyset M_{n1} = \emptyset A_{s,f} f_y \left(d - \frac{h_f}{2} \right)$$

$$= 0.9 \times 3.83 \times 60 \left(21.5 - \frac{3}{2} \right) = 4136.4\ in.kip$$

Design of Solid T and L Sections

Solution

Method 1

- **Step 5: Determination of $A_{s,f}$, $\emptyset M_{n,1}$ and $A_{s,w}$**

Now,

$$M_{u,w} = M_u - \emptyset M_{n1} = 6700 - 4136.4 = 2563.6\ in.kip$$

$$a = d - \sqrt{d^2 - \frac{2.614 M_{u,w}}{f_c' b_w}} = 21.5 - \sqrt{21.5^2 - \frac{2.614(2563.6)}{3 \times 12}} = 4.88\ in.$$

Putting value of a in equation (4.2) gives;

$$A_{s,w} = \frac{M_{u,w}}{\emptyset f_y (d - a/2)} = \frac{2563.6}{0.9 \times 60(21.5 - 4.88/2)} = 2.49 in^2$$

Design of Solid T and L Sections

❑ Solution

❖ Method 1

➢ Step 6: Determination of Total Steel Area

Total area of steel can be calculated as;

$A_{st} = A_{s,f} + A_{s,w}$

By Substituting values, we get

$A_{st} = 3.83 + 2.49 = 6.32\ in^2$

Using #8 bar with area of bar $A_b = 0.79\ in^2$

Number of bars $= 6.32/0.79 = 8$ bars

So, Provide 8- #8 bars in two layers (4+4)

Design of Solid T and L Sections

Solution

Method 2

Step 6: Determination of Total Steel Area

Alternatively, we can find total area of steel directly as follows:

Trial a, (in.) For first trial, assume $a = 0.2d$	x (in.) $x = \frac{b_w a^2 + (b_f - b_w)h_f^2}{2b_w a + 2(b_f - b_w)h_f}$	Calculated A_{st} (in^2) $A_{st} = \frac{M_u}{0.9f_y(d-x)}$	Calculated a (in.) $\frac{A_{st}f_y - 0.85f_c'(b_f - b_w)h_f}{0.85f_c' b_w}$
4.8	1.85	6.31	4.87
4.87	1.87	6.32	4.89
4.89	1.87	6.32	4.89 (converged!)

- Hence, we get $A_{st} = 6.32\ \text{in}^2$, (same as obtained using method 1).

Design of Solid T and L Sections

❑ Solution

➢ Step 7: Reinforcement Check

$$A_{s,min} = \frac{200}{f_y} b_w d \qquad \text{(for } f_c{}' \leq 4500psi\text{)}$$

$$A_{s,min} = \frac{200}{60000} \times 12 \times 21.5 = 0.86\ in^2$$

And

$$A_{st,max(TT),60} = \frac{f_c' b_w d}{223} + A_{sf} = \frac{3 \times 12 \times 21.5}{223} + 3.83$$

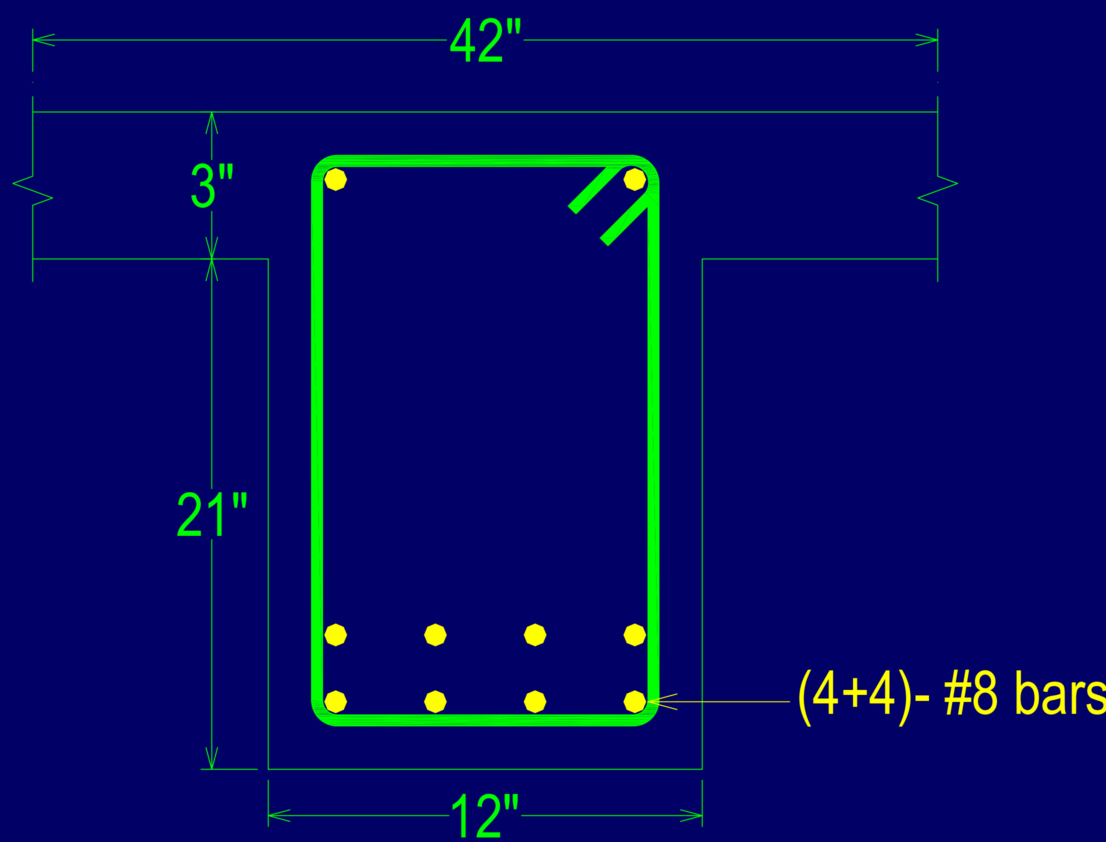
$$A_{st,max(TT),60} = 7.30 in^2$$

$$A_{s,min} < A_s < A_{st,max(TT),60} \rightarrow \text{OK!}$$

Design of Solid T and L Sections

❑ Solution

➢ Step 8: Drafting



Design of Solid T and L Sections

Design Procedure

- The design procedure for a True L-beam is identical to that of a True T-beam, with only one exception given below:
  - Calculate b_f of L section using the following equations

$$b_{f,L} = least\ of \begin{cases} b_w + 6h_f \\ b_w + S_w/2 \\ b_w + l_n/12 \end{cases}$$

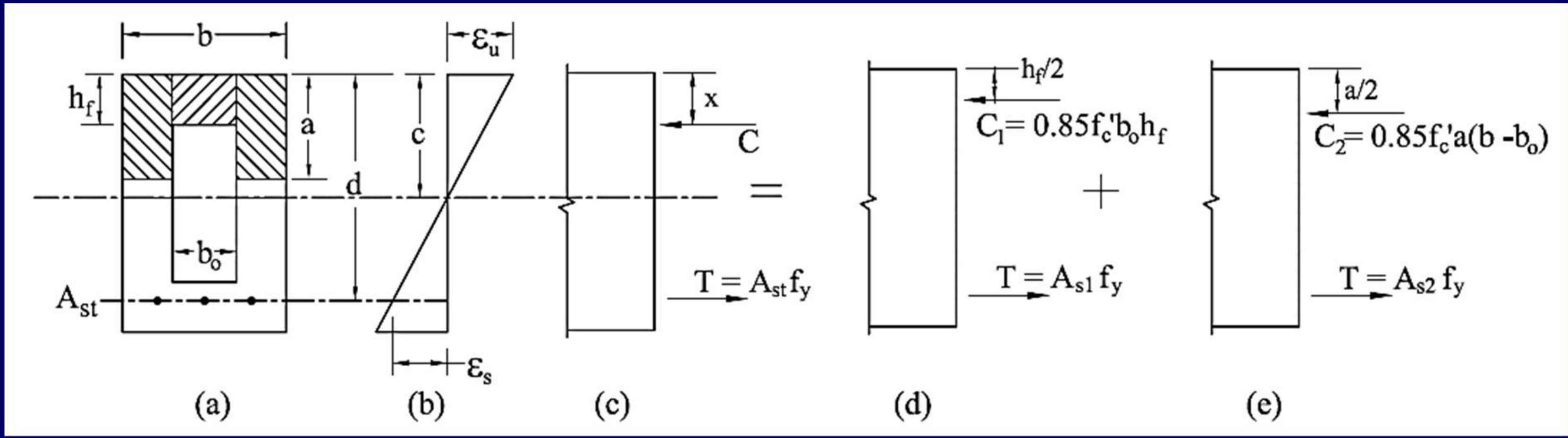
Design of Hollow Rectangular Sections

❑ Flexural Capacity

- The design flexural capacity of hollow sections is determined in similar manner as that for T section.

Method 1

$$\emptyset M_n = \emptyset M_{n1} + \emptyset M_{n2} = A_{s1} f_y \left(d - \frac{h_f}{2}\right) + A_{s2} f_y \left(d - \frac{a}{2}\right)$$



Design of Hollow Rectangular Sections

Flexural Capacity

Method 1

$$A_{s1} = \frac{0.85 f_c' b_o h_f}{f_y}$$

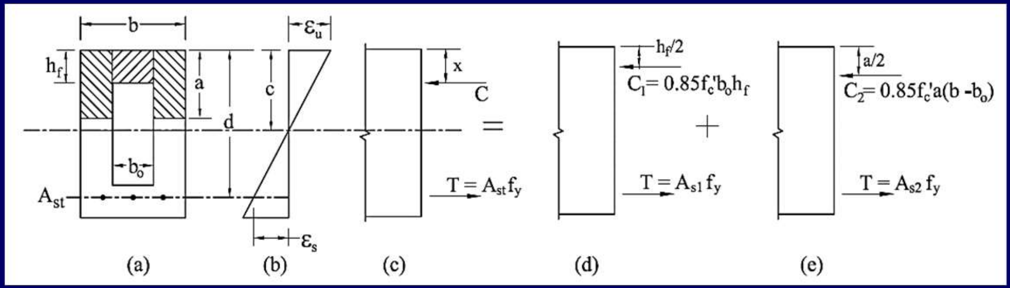
$$A_{s2} = \frac{M_u - \emptyset M_{n1}}{\emptyset f_y \left(d - \frac{a}{2}\right)}$$

and

$$a = \frac{A_{s2} f_y}{0.85 f_c' (b_w - b_o)}$$

Total steel area is given by:

$$A_{st} = A_{s1} + A_{s2}$$



- A_{s1} represents the steel area which when stressed to f_y, is required to balance the longitudinal compressive force in the rectangular portion of the area $b_o h_f$ that is stressed uniformly at $0.85f_c'$.
- A_{s2} is the steel area which when stressed to fy, is required to balance the longitudinal compressive force in the remaining portion of the section that is stressed uniformly at 0.85fc′.

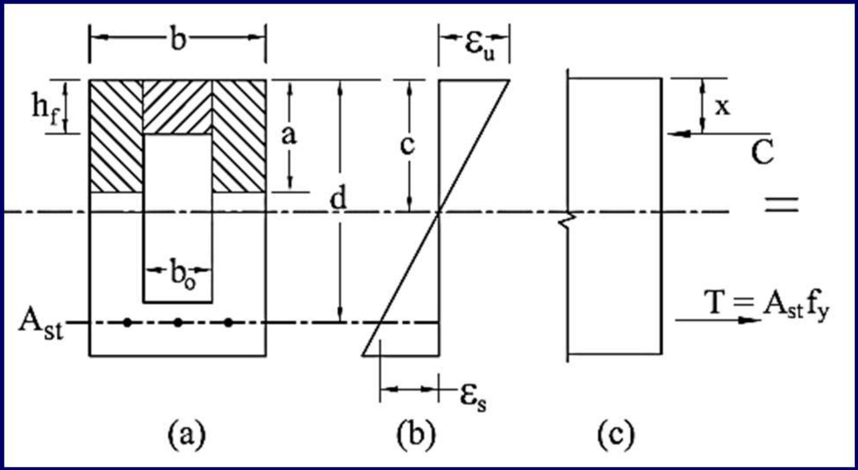
Design of Hollow Rectangular Sections

❑ Flexural Capacity

❖ Method 2

$$\emptyset M_n = M_u = \emptyset A_{st} f_y (d - x)$$

$$A_{st} = \frac{M_u}{\emptyset f_y (d - x)}$$



where;

$$x = \frac{(b_w - b_o)a^2 + b_o {h_f}^2}{2(b_w - b_o)a + 2b_o h_f}$$

and

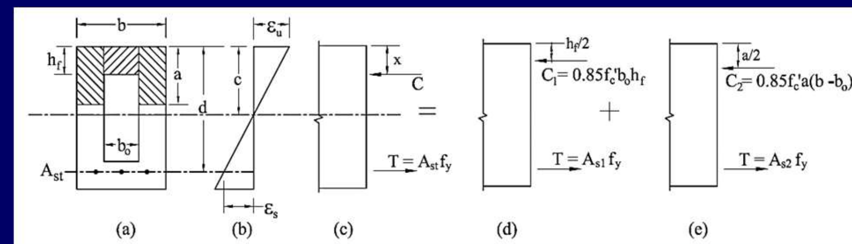
$$a = \frac{A_{st} f_y - 0.85 f_c' b_o h_f}{0.85 f_c' (b_w - b_o)}$$

Design of Hollow Rectangular Sections

❑ Maximum Reinforcement Limit

From summation of internal forces;

$$A_{st}f_y = 0.85f_c'ba - 0.85f_c'b_o(a - h_f)$$



For a = β_1c, we have

$$A_{st} = 0.85f_c'b\beta_1 c - 0.85f_c'b_o(\beta_1 c - h_f)$$

Setting $c = c_{max}$, we have $A_{st} = A_{st,max,h}$

$$A_{st,max,h} = \frac{0.85f_c'b\beta_1 c_{max}}{f_y} - 0.85f_c'b_o(\beta_1 c_{max} - h_f)/f_y$$

$\dfrac{0.85f_c'b\beta_1 c_{max}}{f_y} \rightarrow A_{s,max\ (SR)}$

$c_{max} \rightarrow c_{max} = \left(\dfrac{0.003}{0.006 + \epsilon_{ty}}\right)d$

Design of Hollow Rectangular Sections

Maximum Reinforcement Limit

- **Grade 40 Steel**

Taking $\beta_1 = 0.85$ and $c_{max,40} = 0.41d$

$$A_{st,max,h} = \frac{f_c'}{136} b_w d - \frac{0.85 f_c' b_o (0.35d - h_f)}{f_y}$$

- **Grade 60 Steel**

Taking $\beta_1 = 0.85$ and $c_{max,60} = 0.37d$

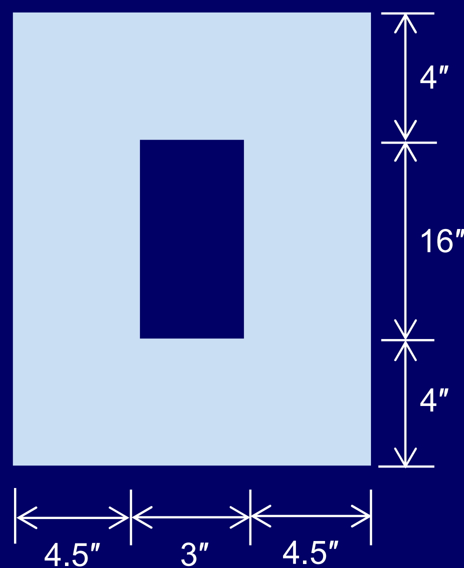
$$A_{st,max,h} = \frac{f_c'}{223} b_w d - \frac{0.85 f_c' b_o (0.31d - h_f)}{f_y}$$

Design of Hollow Rectangular Sections

Example 3.6

- A beam with a hollow rectangular section, as shown in the figure below, is subjected to a factored moment M_u of 2500 in-kip. Material strengths are, $f_c' = 3\ \text{ksi}$ and $f_y = 60\ \text{ksi}$.

Determine the required flexural steel for the section.



Design of Hollow Rectangular Sections

❑ Solution

- **Given Data**

$b_w = 12''$

$b_o = 3''$

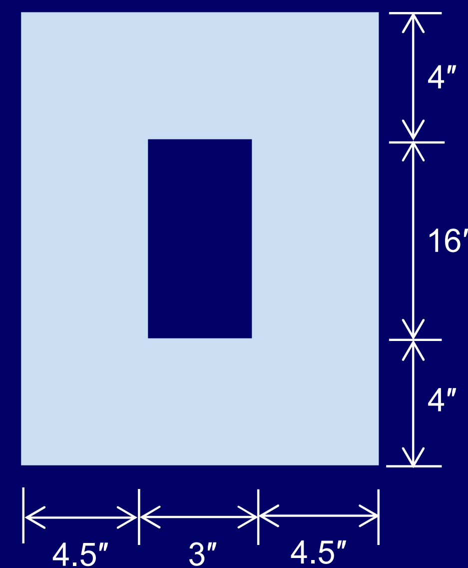
$h = 24''$

$h_f = 4''$

$M_u = 2500 \text{ in. kip}$

$f_c' = 3 \text{ ksi}$

$f_y = 60 \text{ ksi}$



- **Required Data**
  - Design the section for given demand

Design of Hollow Rectangular Sections

❑ Solution

➢ Step 1: Check the Behavior of Section

$$a = d - \sqrt{d^2 - \frac{2.614M_u}{f_c' b_w}} = 21.5 - \sqrt{21.5^2 - \frac{2.614 \times 2500}{3 \times 12}} = 4.75\ in.$$

$a > h_f = 4" \rightarrow$ the section should be designed as hollow section.

➢ Step 2: Determination of Steel Area

$$A_{st} = A_{s1} + A_{s2}$$

$$A_{s1} = \frac{0.85 f_c' b_o h_f}{f_y} = \frac{0.85 \times 3 \times 3 \times 4}{60} = 0.51\ \text{in}^2$$

Design of Hollow Rectangular Sections

Solution

Step 2: Determination of Steel Area

$$\emptyset M_{n1} = \emptyset A_{s1} f_y \left(d - \frac{h_f}{2}\right) = 0.9 \times 0.51 \times 60 \left(21.5 - \frac{4}{2}\right) = 537.03 \text{ in. kip}$$

$$\emptyset M_{n2} = M_u - \emptyset M_{n1} = 2500 - 537.03 = 1962.97 \text{ in. kip}$$

Now calculate A_{s2}

$$a = d - \sqrt{d^2 - \frac{2.614 \times \emptyset M_{n2}}{f_c'(b_w - b_o)}} = 21.5 - \sqrt{21.5^2 - \frac{2.614(1962.97)}{3 \times (12 - 4)}} = 5.74 \text{ in.}$$

$$A_{s2} = \frac{\emptyset M_{n2}}{0.9 f_y \left(d - \frac{a}{2}\right)} = \frac{1962.97}{0.9 \times 60 \left(21.5 - \frac{5.74}{2}\right)} = 1.95 \text{ in}^2$$

Design of Hollow Rectangular Sections

Solution

Step 2: Determination of Steel Area

$A_{st} = A_{s1} + A_{s2} = 0.51 + 1.95 = 2.46 \text{ in}^2$ (6-#6 bars)

Step 3: Reinforcement Check

$$A_{st,max,h} = \frac{f_c'}{223} b_w d - \frac{0.85 f_c' b_o (0.31d - h_f)}{f_y}$$

$$A_{st,max,h} = \frac{3}{223} \times 12 \times 21.5 - \frac{0.85 \times 3 \times 4(0.31 \times 21.5 - 4)}{60}$$

$$A_{st,max,h} = 3.01 \text{ in}^2 > \mathrm{A_{st,pvd}} = 6 \times 0.44 = 2.64 \text{ in}^2 \rightarrow \text{OK!}$$

- Area of steel can also be determined using method 2.

The End of Section – I

Discussion on **Section – II** will be continued in Part 2 of the lecture.