Lecture 04

Design of RC Members for Shear and Torsion

By:
Prof. Dr. Qaisar Ali
Civil Engineering Department
UET Peshawar

drqaisarali@uetpeshawar.edu.pk
https://drqaisarali.com

Lecture Contents

- **Section – I : Design of RC Members for Shear**
  - General
    - Shear Stresses in Rectangular Beams
    - Diagonal Tension in RC Beams Subjected to Flexure and Shear
    - Types of Cracks in RC Beams
  - Shear Strength of Concrete
  - Web Reinforcement Requirement
  - ACI Code Provisions for Shear Design

Lecture Contents

- **Section – II : Design of RC Members for Torsion**
  - Torsional Stresses in Solid Concrete Members
  - Torsional Strength of Concrete
  - Reinforcement Requirement
  - ACI Requirements for Torsion Design
  - Steps for Design of RC Member Subjected to Torsion
- Design Example
- References

Learning Outcomes

- **At the end of this lecture, students will be able to;**
  - ***Understand*** the behavior and mechanics of RC Members under Shear and Torsion.
  - ***Design*** RC Members for Combined Shear and Torsion

Section – I
Design of RC Members for Shear

General

❑ Introduction

- Unlike Flexural failure, shear failure is difficult to predict accurately.
- Despite many decades of experimental research and the use of highly sophisticated analytical tools it is not yet fully understood.
- Furthermore, if a beam without properly designed shear reinforcement is overloaded to failure, shear collapse is likely to occur suddenly, with no advance warning of distress.
- Therefore, reinforced concrete beams are generally provided with special shear reinforcement to ensure that flexural failure would occur before shear failure.

Shear Failure of RC Beam

General

❑ Shear stresses in Homogeneous Elastic Rectangular Beam

- The shear stress (v) at any point in the cross section is given by

$$v = \frac{VQ}{Ib}$$

Where;

V = Total shear at section.

I = Moment of inertia of cross section about neutral axis.

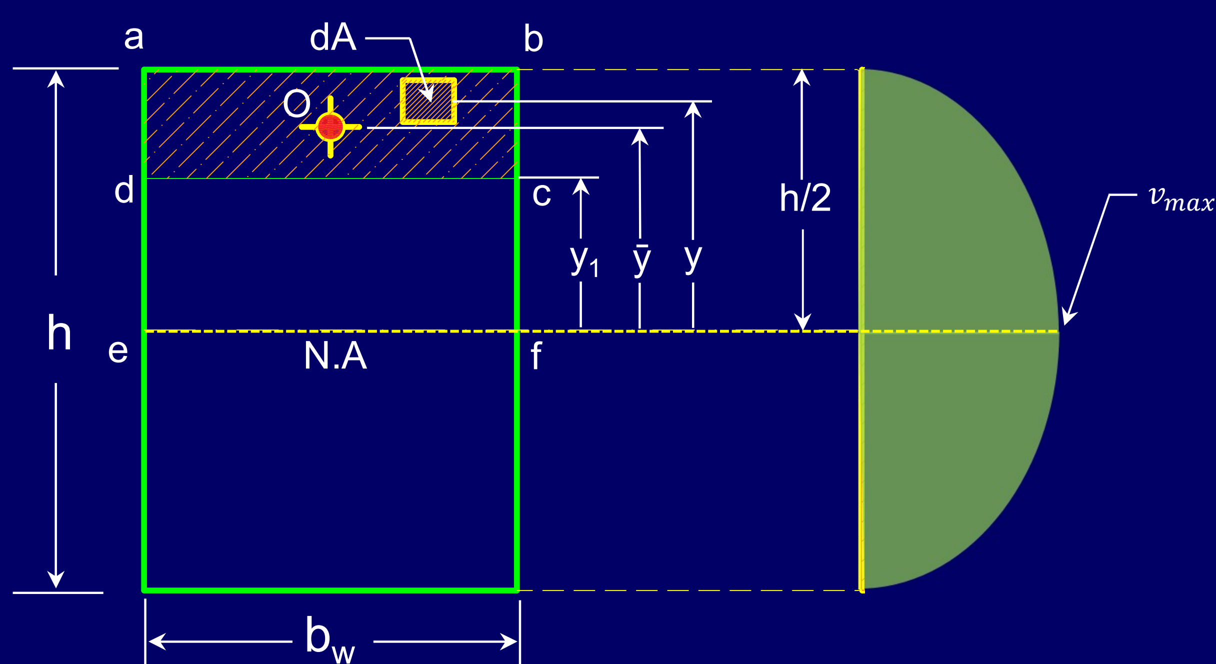
b = Width of beam at a given point.

Q = Statical moment about neutral axis of that portion of cross section lying between a line through point in question parallel to neutral axis and nearest face (upper or lower) of beam.

General

❑ Shear stresses in Homogeneous Elastic Rectangular Beam

- For the calculation of shear stress at level d-c in the given figure, Q will be equal to $A_{abcd}\ \bar{y}$, where A_{abcd} is area abcd and $\bar{y}$ is the centroidal distance of area abcd from N.A



General

❑ Shear stresses in Homogeneous Elastic Rectangular Beam

- For shear at neutral axis, we have

$$Q = A_{abef}\bar{y} = \frac{b_w h}{2} \times \frac{h}{4}$$

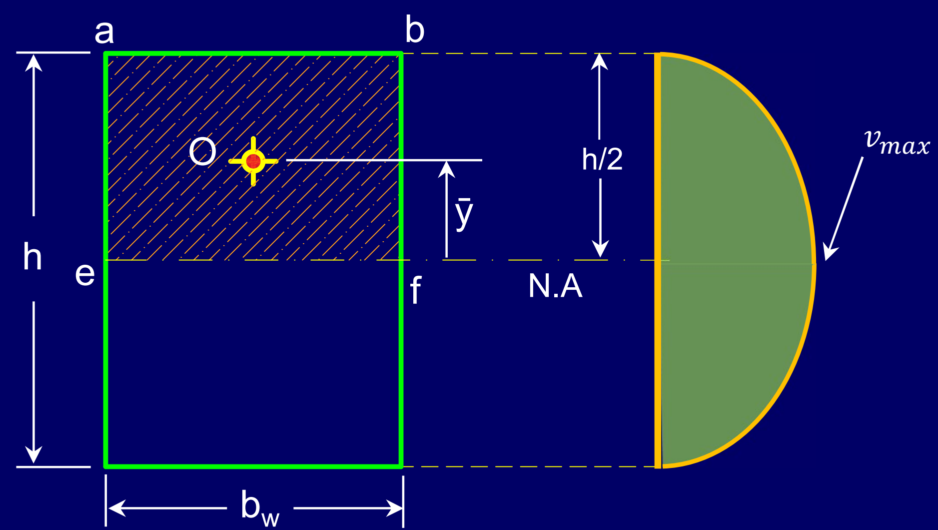
$$I = \frac{b_w h^3}{12}$$

Therefore,

$$v = \frac{VQ}{Ib} = \frac{V(b_w h^2/8)}{(b_w h^3/12)\times b_w}$$

which on simplification gives,

$$v_{max} = \frac{1.5V}{b_w h}$$



General

Shear Stresses in Reinforced Concrete Beam

- When load on the beam is such that stresses are no longer proportional to strain, then equation v = VQ/Ib for shear stress calculation does not govern.
- The exact distribution of shear stresses over the depth of reinforced concrete member in such a case is not fully known.

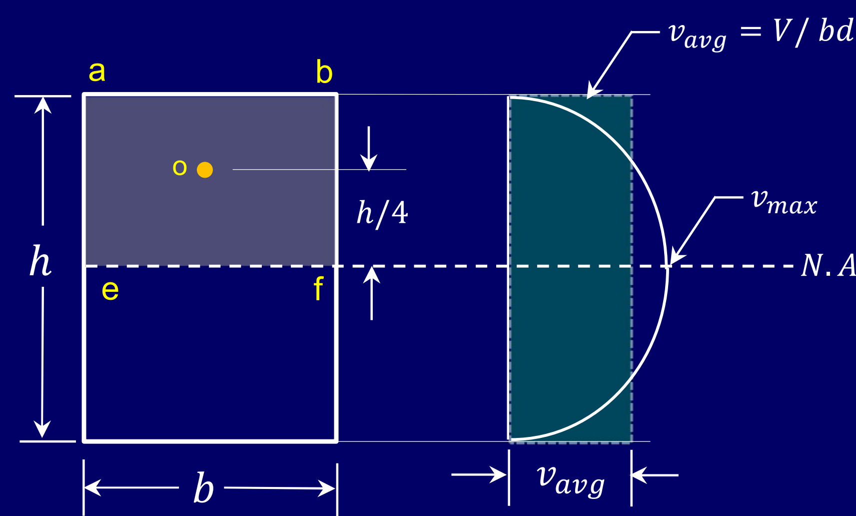
General

❑ Shear Stresses in Reinforced Concrete Beam

- Tests have shown that the average shear stress in a RC beam can be expressed by :

$$v_{avg} = V / bd$$

- The maximum value, which occurs at the neutral axis, will exceed this average by an unknown but moderate amount.



General

❑ Shear Strength in Presence of Cracks

- Many tests on beams have shown that in regions where small moment and large shear exist (web shear crack location) the nominal or average shear strength is taken as:

$$V_{cr} = 3.5\sqrt{f_c'}$$

- However, in the presence of large moments (for which adequate longitudinal reinforcement has been provided), the nominal shear strength corresponding to formation of diagonal tension cracks can be taken as:

$$V_{cr} = 2\sqrt{f_c'}$$

General

❑ Shear Strength in Presence of Cracks

- The same has been adopted by the ACI code (refer to ACI 22.5.5.1).
- This reduction of shear strength of concrete is due to the pre-existence of flexural cracks.
- It is important to mention here that this value of shear strength of concrete exists at the ultimate i.e., just prior to the failure condition.

General

❑ Diagonal Tension in RC Beams Under Flexure and Shear

❖ Conclusions

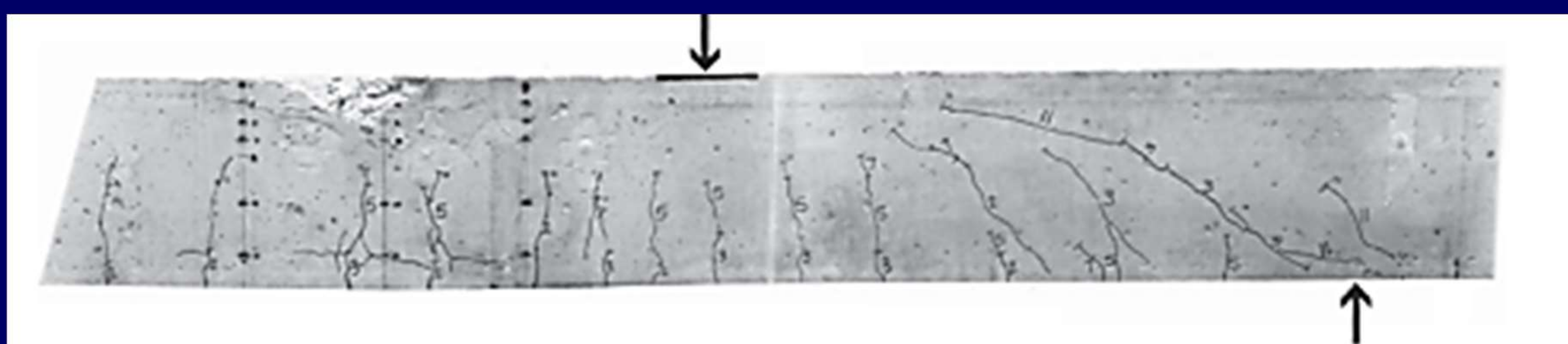
- The tensile stresses are not confined to horizontal bending stresses that are caused by bending alone.
- Tensile stresses of various inclinations and magnitudes resulting from shear alone (at the neutral axis) or from the combined action of shear and bending, exist in all parts of a beam and can impair its integrity if not adequately provided for.
- It is for this reason that the inclined tensile stresses, known as diagonal tension stress must be carefully considered in reinforced concrete design.

General

❑ Diagonal Tension in RC Beams Under Flexure and Shear

❖ Conclusions

- The cracking pattern in a test beam with longitudinal flexural reinforcement, but no shear reinforcement, is shown in Figure.
- Two types of cracks can be seen. The vertical cracks occurred first, due to flexural stresses.
- The inclined cracks near the ends of the beam are due to combined shear and flexure.



General

- Types of Cracks in Reinforced Concrete Beam

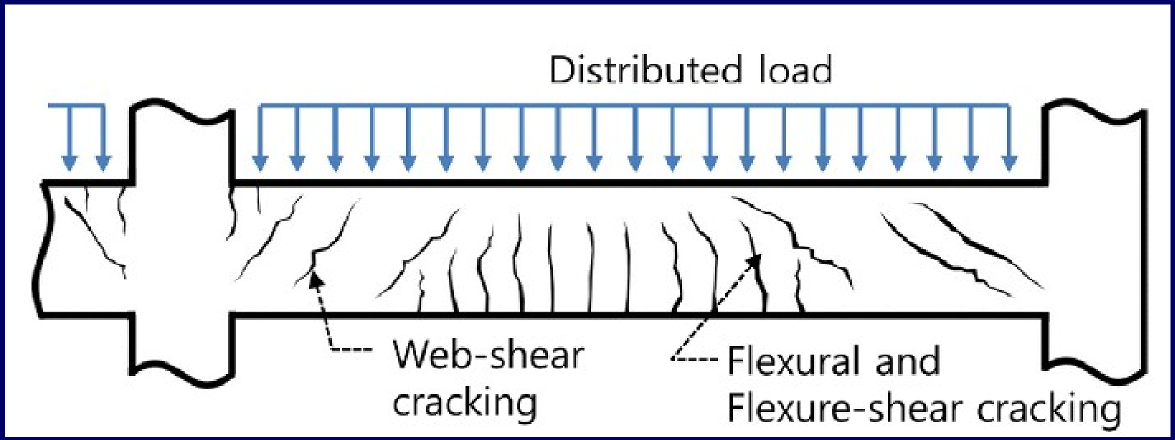
1. Flexural Cracks
2. Diagonal Tension Cracks

   i. Web-shear cracks

   - Formed at locations where flexural stresses are negligibly small.

   ii. Flexure shear cracks

   - Formed where shear force and bending moment have large values.



Web Reinforcement in RC Beams

❑ Nominal Shear Capacity V_n

- The general expression for shear capacity of reinforced concrete beam is given as:

$V_n = V_c + V_s$ (ACI 22.5.1.1)

Where;

V_c =Nominal shear capacity of concrete,

V_s = Nominal shear capacity of shear reinforcement.

- Note that in case of flexural capacity, $M_n = M_c + M_s$ where $M_c = 0$, at ultimate load. However, in case of shear capacity, the term $V_c \neq 0$.

Web Reinforcement in RC Beams

❑ Nominal Shear Capacity V_n

❖ Calculation of V_c

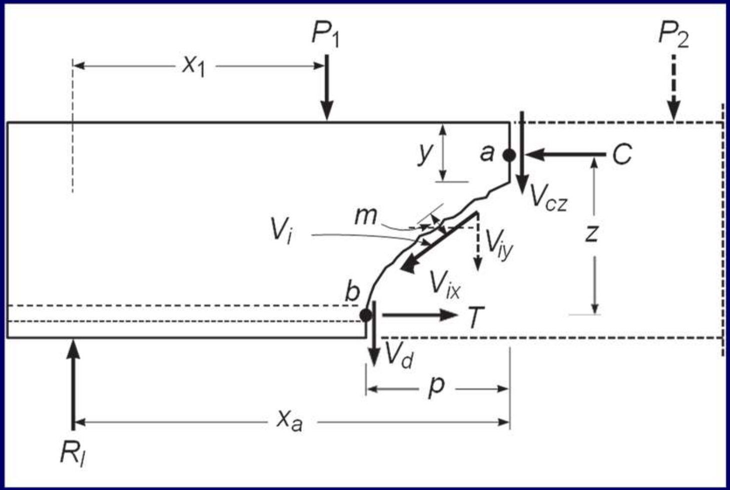
- Test evidence have led to the conservative assumption that just prior to failure of a web-reinforced beam, three internal shear components contributing to the total shear, the sum of which is referred as shear capacity of concrete V_c.

$$V_c = V_{cz} + V_d + V_{iy}$$

V_{cz} = Internal vertical forces in the uncracked portion of the concrete.

V_d = Internal vertical forces across the longitudinal steel, acting as a dowel.

V_{iy} = Vertical component of sizable interlock forces.



Web Reinforcement in RC Beams

Nominal Shear Capacity V_n

Calculation of V_c

- Nominal Shear capacity of concrete shall be calculated in accordance with ACI Table 22.5.5.1.

Table 22.5.5.1—V_c for nonprestressed members

Criteria	V_c		
$A_v \geq A_{v,min}$	Either of:	$\left[2\lambda\sqrt{f'_c} + \frac{N_u}{6A_g}\right]b_w d$	(a)
		$\left[8\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right]b_w d$	(b)
$A_v < A_{v,min}$	$\left[8\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right]b_w d$		(c)

Notes:
1. Axial load, N_u, is positive for compression and negative for tension.
2. V_c shall not be taken less than zero.

Web Reinforcement in RC Beams

❑ Nominal Shear Capacity V_n

❖ Calculation of V_c

- The equation which is applicable in most of the cases is given as follows;

$$V_c = \left[2\lambda\sqrt{f_c'} + \frac{N_u}{6A_g} \right] b_w d$$

- In case of beams, Axial force N_u is very small and can be ignored. Taking $N_u = 0$ and $\lambda = 1\ (\text{normalweight concrete})$, the above equation reduces to;

$$V_c = 2\sqrt{f_c'}\,b_w d$$

Web Reinforcement in RC Beams

❑ Nominal Shear Capacity V_n

❖ Calculation of V_s

- Shear capacity of reinforcement V_s can be determined in accordance with ACI 22.5.8.5.3 as:

$$V_s = nA_v f_y$$

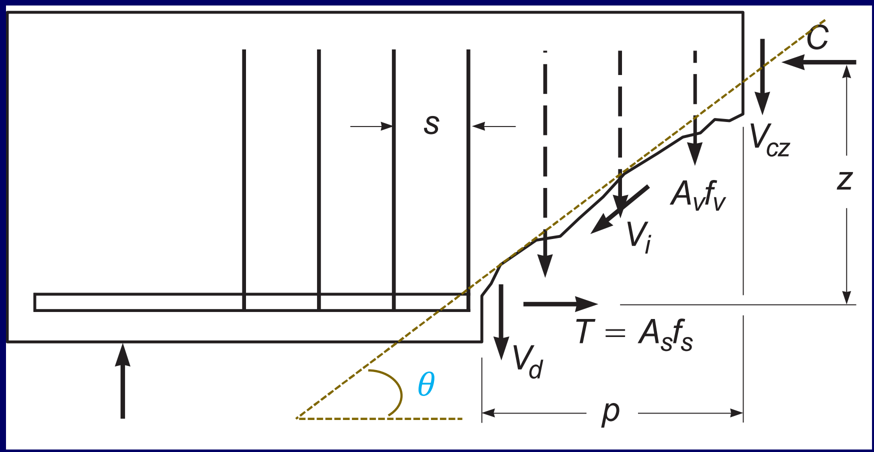
n = number of stirrups

A_v = shear reinforcement

f_y = yield tensile strength of steel

Setting n = d/s

$$V_s = A_v f_y d/s$$



$P \approx d$ and $\theta \approx 45^o$

Web Reinforcement in RC Beams

❑ Nominal Shear Capacity V_n

- Now the total shear capacity is determined as;

$$V_n = V_c + V_s \Rightarrow \text{ or } \quad \emptyset V_n = \emptyset V_c + \emptyset V_s$$

For no failure, $\emptyset V_n \geq V_u$. Setting $\emptyset V_n = V_u$, we get

$$V_u = \emptyset V_c + \emptyset V_s$$

Substituting value of V_s

$$V_u = \emptyset V_c + \frac{\emptyset A_v f_y d}{s}$$

$$s = \frac{\emptyset A_v f_y d}{V_u - \emptyset V_c}$$

Where;

$V_u - \emptyset V_c = \emptyset V_s$

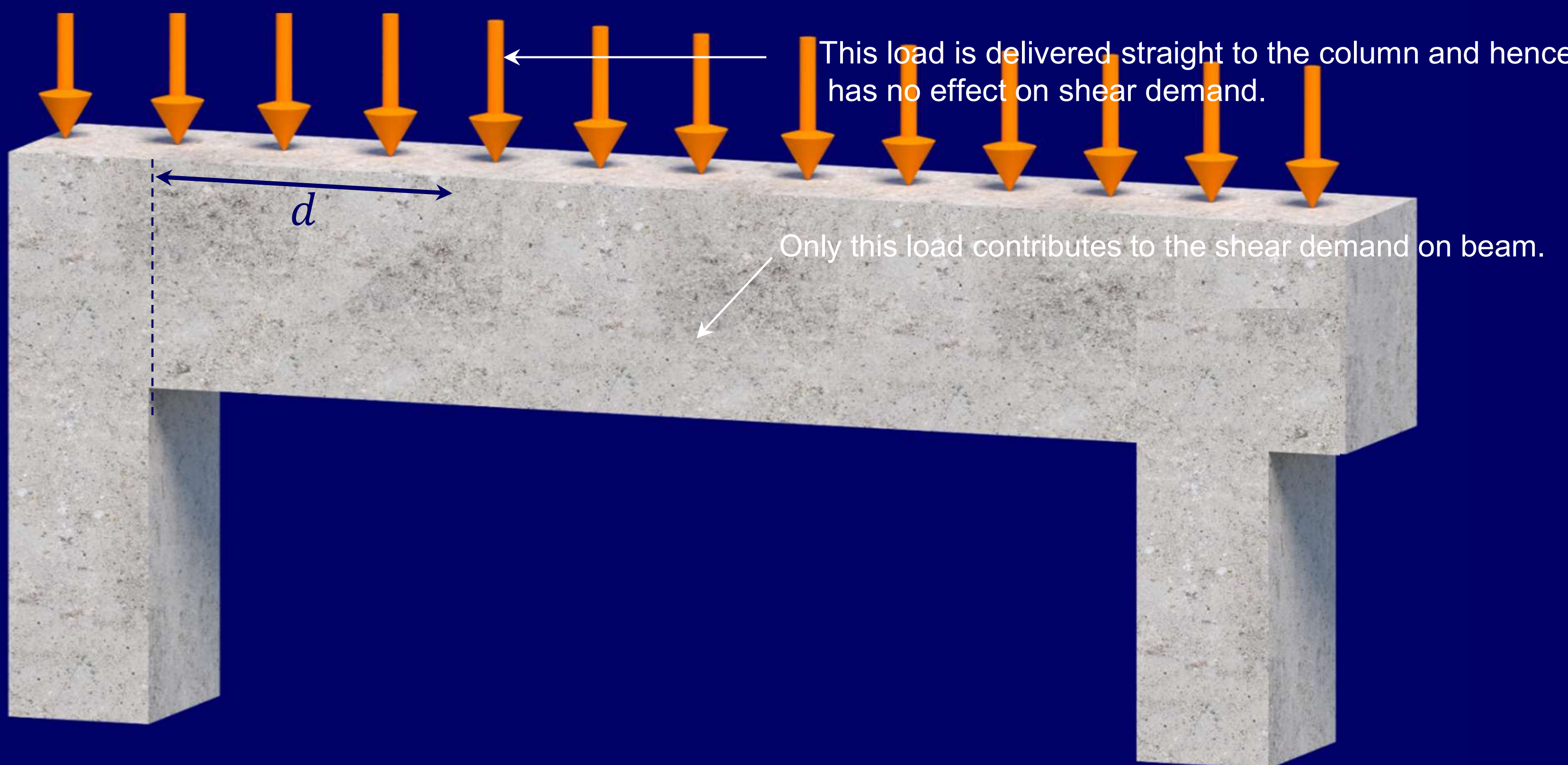
$A_v = nA_b$. For 2-legged stirrups

$A_v = 2A_b$

Web Reinforcement in RC Beams

❑ Location of Critical Section for ultimate shear V_u

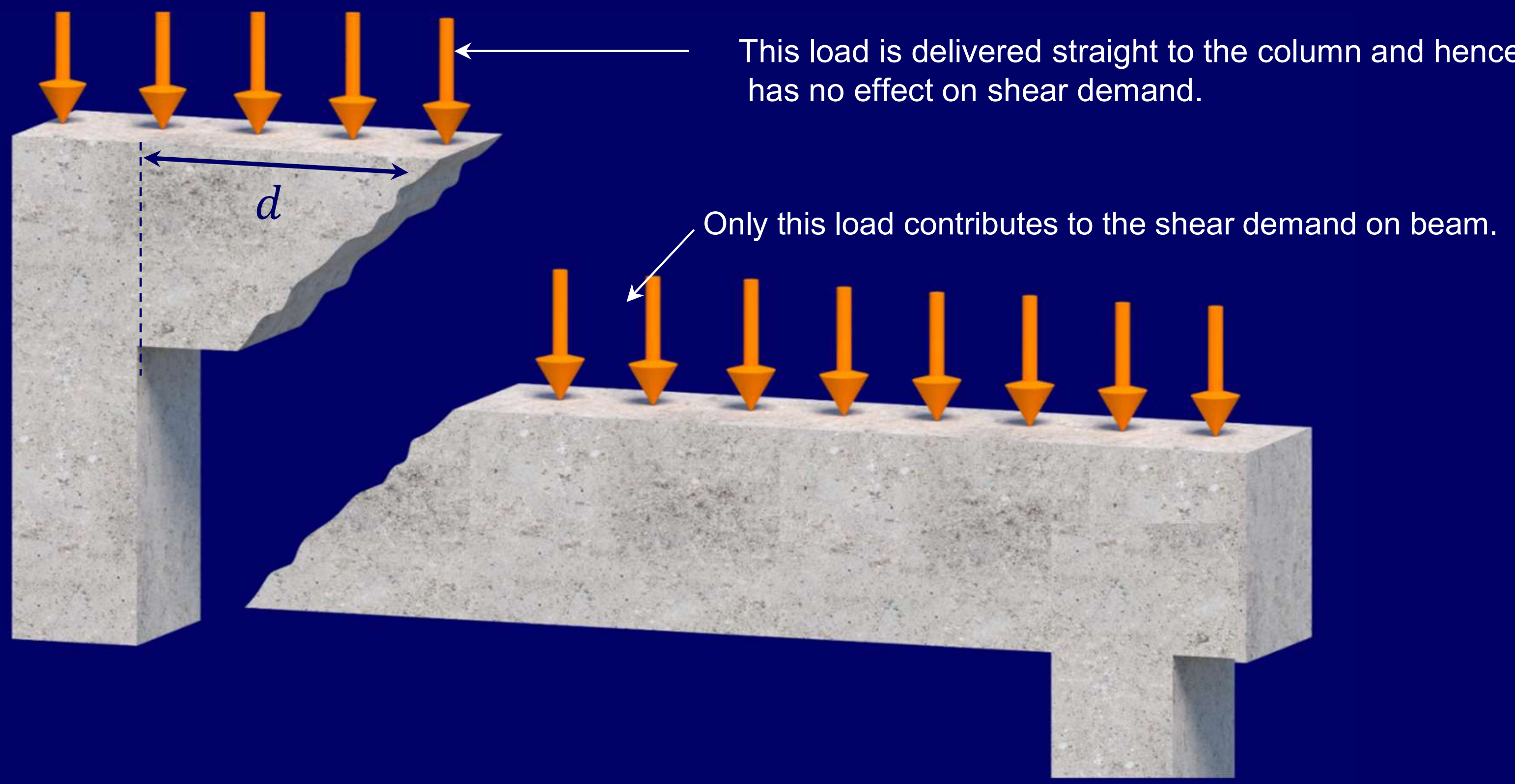
- In most of the cases, for the design of shear, critical shear is taken at a distance "d" from the support instead of maximum shear at the face of the support. This is due to the following reason.



Web Reinforcement in RC Beams

❑ Location of Critical Section for ultimate shear V_u

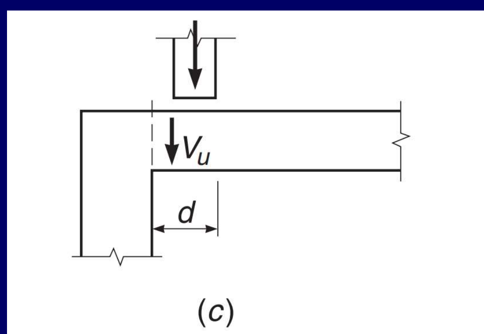
- In most of the cases, for the design of shear, critical shear is taken at a distance "d" from the support instead of maximum shear at the face of the support. This is due to the following reason.



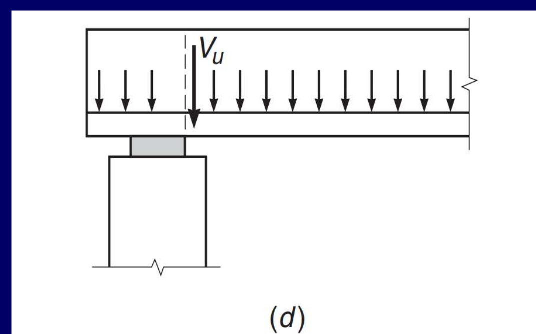
Web Reinforcement in RC Beams

❑ Location of Critical Section for ultimate shear V_u

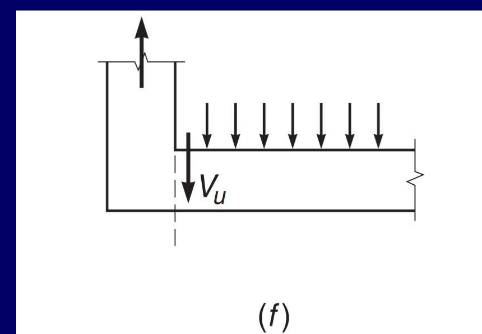
- The critical section for shear is taken at different positions in different situations, as shown in the diagrams below.



Concentrated load within distance “d”



Inverted T beam

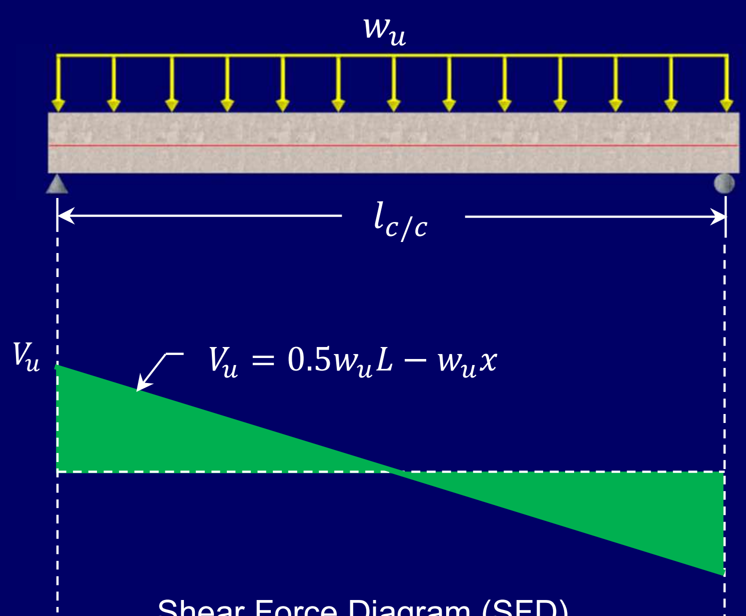


Suspended slab held by tie column

Web Reinforcement in RC Beams

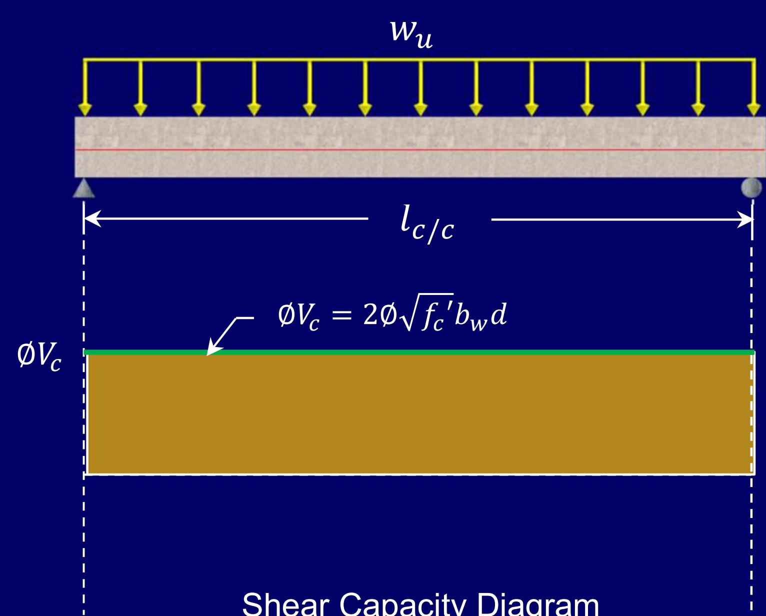
❑ Design of RC Beams for Shear

- Consider the following typical Shear force diagram, and shear capacity diagram of beam.



Shear Force Diagram (SFD)

(Varies along the length of beam)



Shear Capacity Diagram

(Remains constant throughout the length of beam)

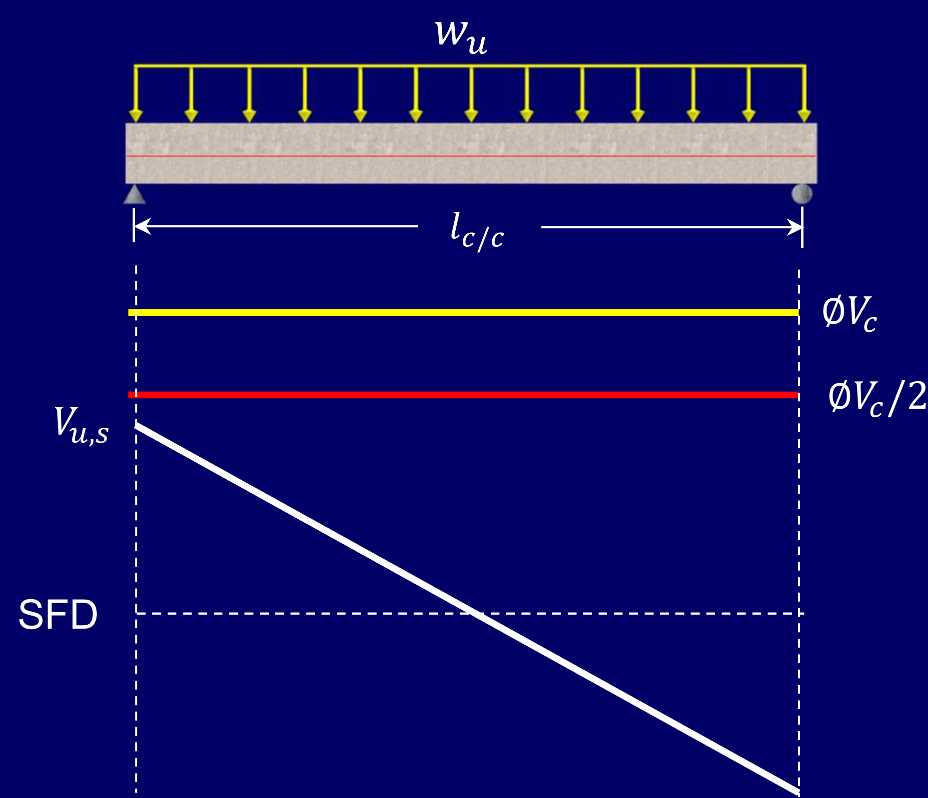
Web Reinforcement in RC Beams

Design of RC Beams for Shear

- If shear capacity is plotted on shear force diagram, then depending on the value of V_u, the following three cases are possible.

Case I

$$\frac{\emptyset V_c}{2} \geq V_u$$



Web Reinforcement in RC Beams

Design of RC Beams for Shear

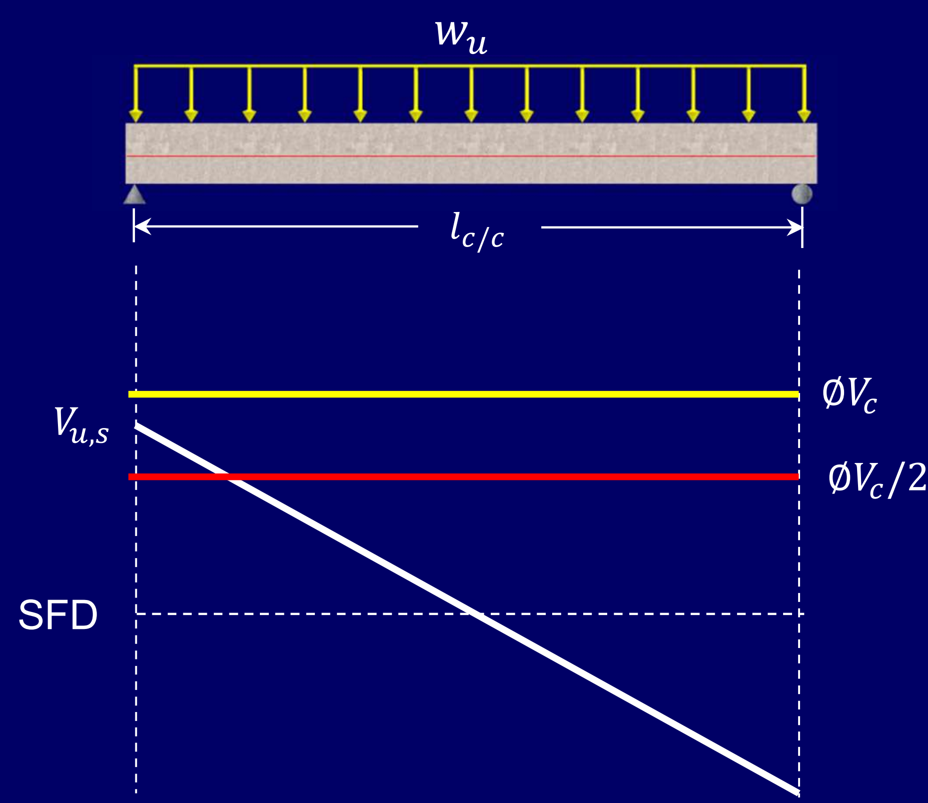
- If shear capacity is plotted on shear force diagram, then depending on the value of V_u, the following three cases are possible.

  - **Case I**

    $$\frac{\emptyset V_c}{2} \geq V_u$$

  - **Case II**

    $$\emptyset V_c \geq V_u \text{ but } \frac{\emptyset V_c}{2} < V_u$$



Web Reinforcement in RC Beams

Design of RC Beams for Shear

- If shear capacity is plotted on shear force diagram, then depending on the value of V_u, the following three cases are possible.

  - **Case I**

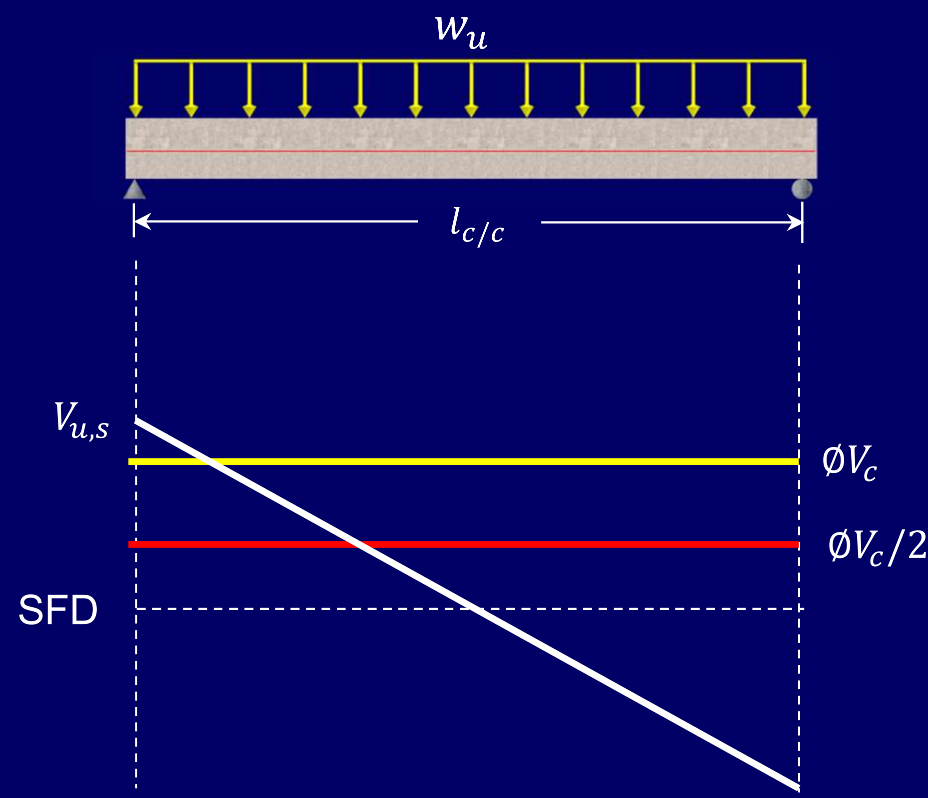
    $$\frac{\emptyset V_c}{2} \geq V_u$$

  - **Case II**

    $$\emptyset V_c \geq V_u \text{ but } \frac{\emptyset V_c}{2} < V_u$$

  - **Case III**

    $$\emptyset V_c < V_u$$



Web Reinforcement in RC Beams

Design of RC Beams for Shear

Case I: $\emptyset V_c /2 > V_u$

- No web reinforcement is required.

Case II: $\emptyset V_c \geq V_u$ but $\emptyset V_c /2 < V_u$

- Theoretically no web reinforcement is required. However, minimum web reinforcement in the form of maximum spacing S_{max} shall be provided (ACI 9.7.6.2.2 and 10.6.2.2).

$$s_{max} = min\left\{\frac{A_v f_y}{50 b_w}, \frac{A_v f_y}{0.75\sqrt{f_c{}'} b_w}, \frac{d}{2}, 24"\right\}$$

Web Reinforcement in RC Beams

Design of RC Beams for Shear

Case III: $\emptyset V_c < V_u$

- Web reinforcement is required. The required spacing s can be calculated using:

$$s = \frac{\emptyset A_v f_y d}{V_u - \emptyset V_c}$$

Shear Checks

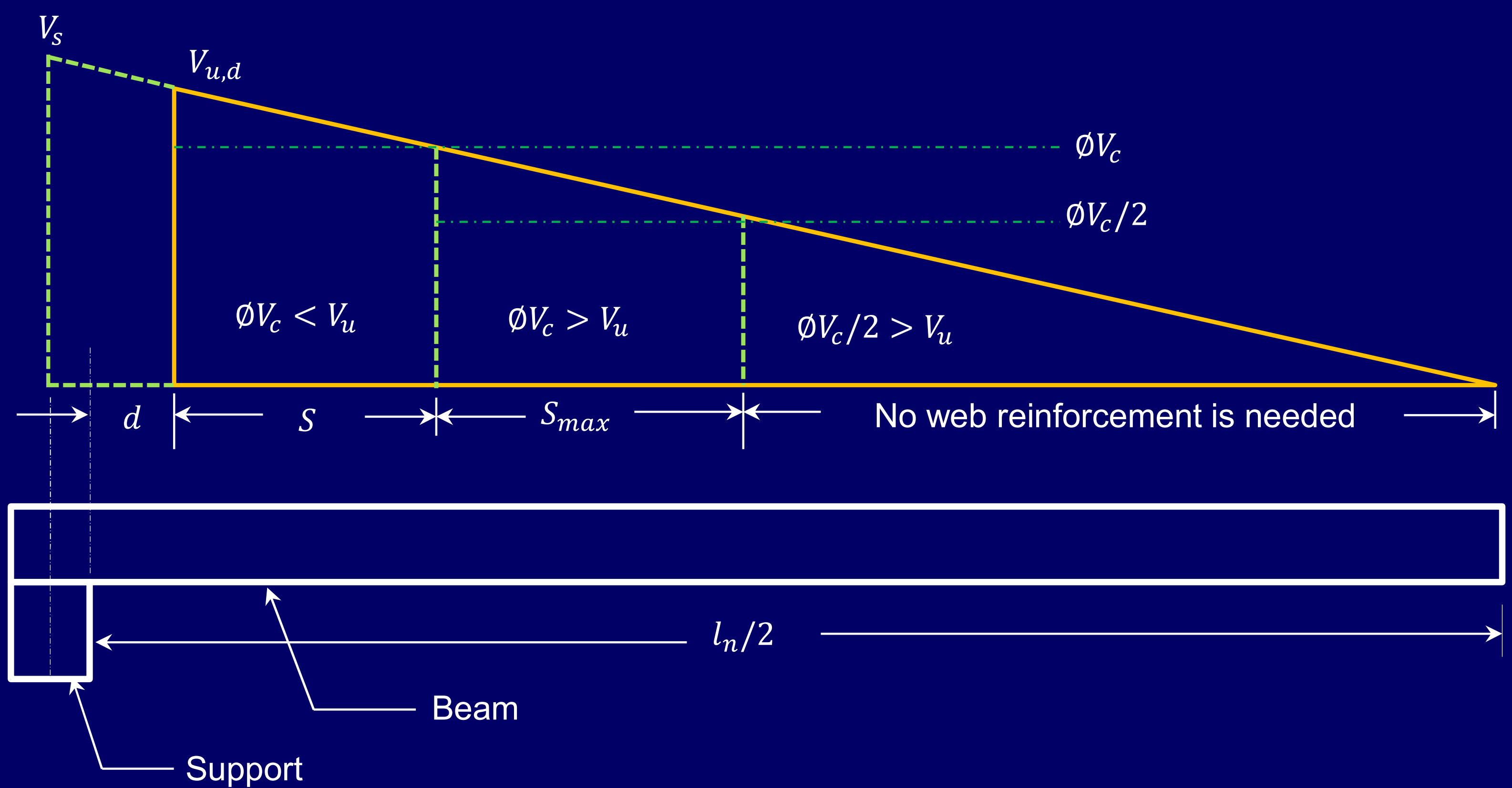
$\emptyset V_s \leq \emptyset 8\sqrt{f_c{}'} b_w d$ → depth of beam is OK!, otherwise increase depth

$\emptyset V_s \leq \emptyset 4\sqrt{f_c{}'} b_w d$ → S_{max} is OK!, otherwise divide S_{max} by 2.

Web Reinforcement in RC Beams

❑ Design of RC Beams for Shear

❖ Placement of Reinforcement

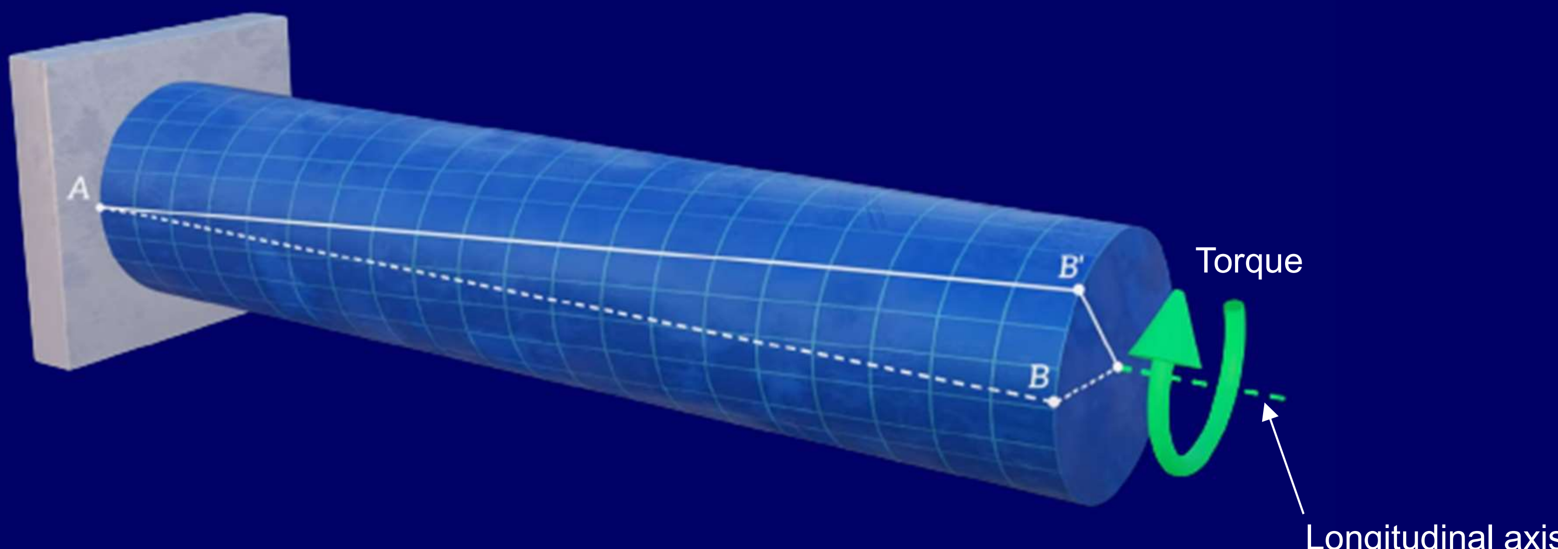


Section – II
Design of RC Members for Torsion

Design of RC Members for Torsion

❑ Torsional Stresses

- A moment acting about the longitudinal axis of a member is called a twisting moment, a torque, or a torsional moment, T.
- The shear stress induced due to applied torque on a member is called as torsional shear stress or torsional stress.



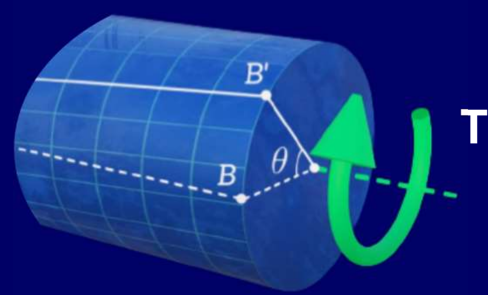
Design of RC Members for Torsion

❑ Torsional Stresses

Circular Solid Members

- Torsional stresses in solid circular members can be computed as:

$$\tau = \frac{T\rho}{J}$$

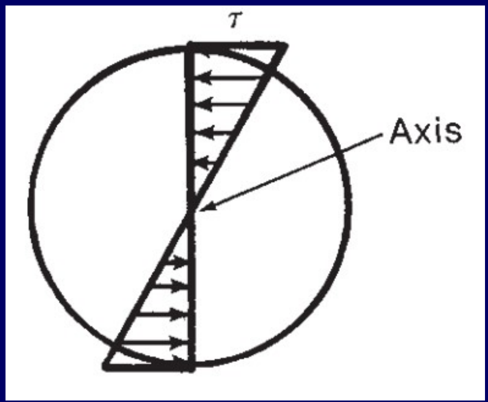


Where;

T = applied torque,

ρ = radial distance,

J = polar moment of inertia.



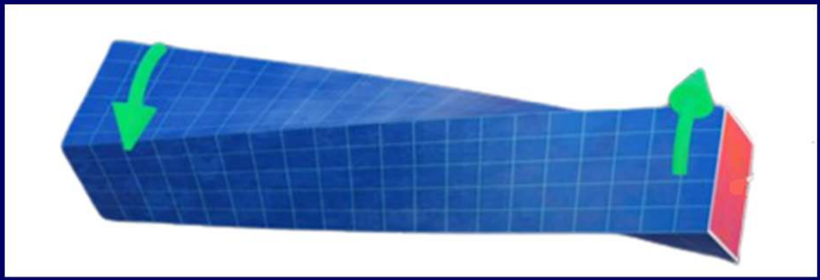
Shear stress distribution

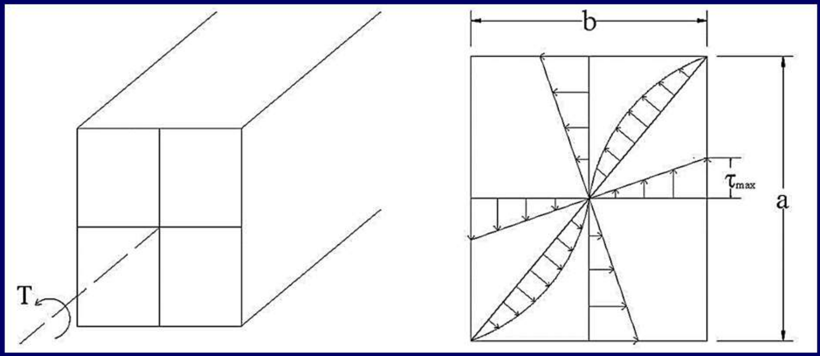
Design of RC Members for Torsion

Torsional Stresses

Rectangular Members

- Torsional stress variation in rectangular members is relatively complicated.
- Torsional stress close to the faces of the rectangular member is much greater than that of interior section.





Design of RC Members for Torsion

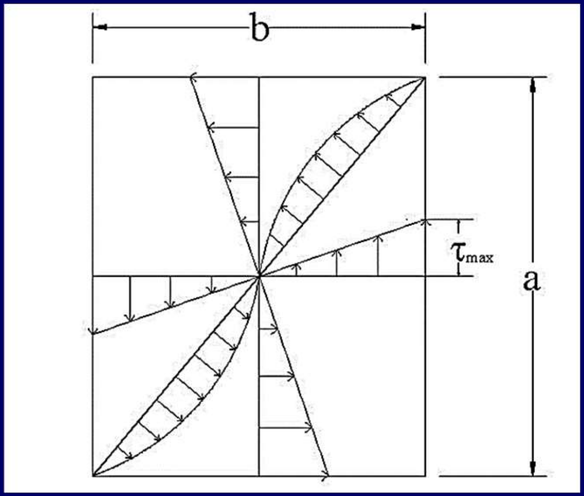
❑ Torsional Stresses

❖ Rectangular Members

- The largest stress occurs at the middle of the wide face . The stress at the corners is zero.
- Stress distribution at any other location is less than that at the middle and greater than zero.

$$\tau_{max} = \frac{T}{\alpha b^2 a}$$

Variation of α with ratio a/b.				
a/b	1	1.5	2	3
α	0.2	0.23	0.24	0.267

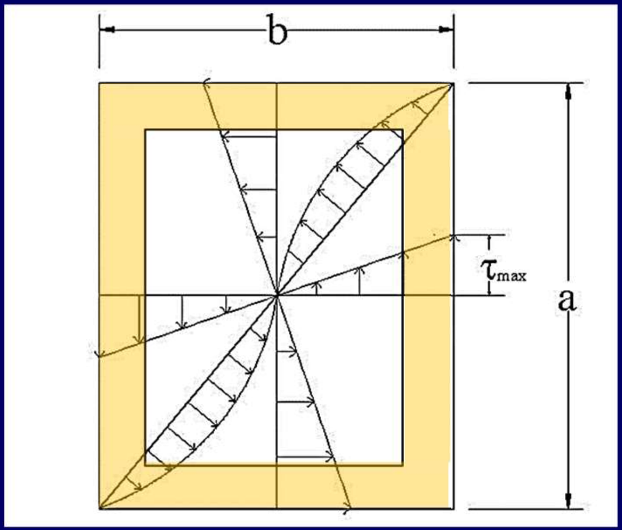


Design of RC Members for Torsion

Torsional Stresses

Rectangular Members

- As can be observed in the figure, Torsional stresses are concentrated in a thin outer skin of the solid cross section.
- This leads to the concept of thin-walled tube analogy.

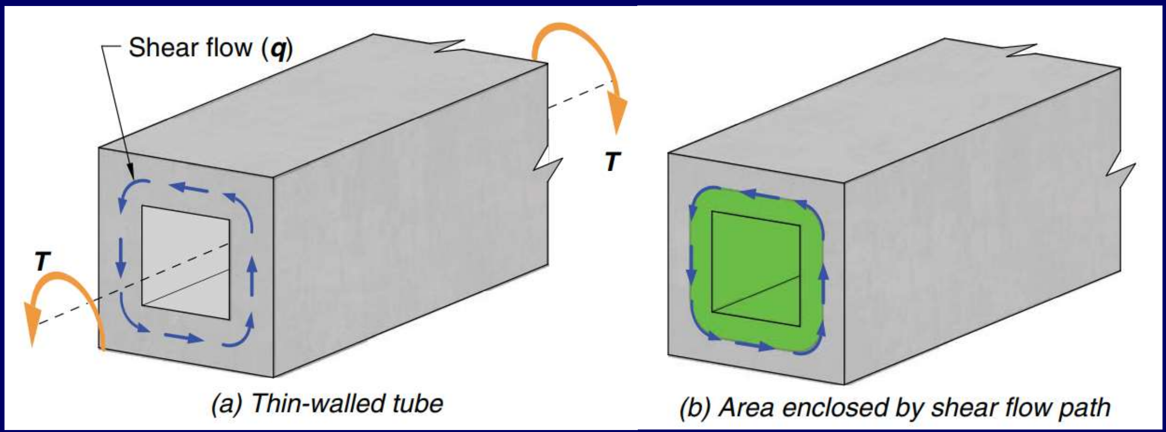


Design of RC Members for Torsion

❑ Thin-Walled Tube Analogy (ACI R22.7)

- A beam subjected to torsion is idealized as a thin-walled tube with the core concrete cross section in a solid beam neglected. The strength is assumed to be provided by the outer skin of the cross section roughly centered on the closed stirrups.

- The product of the shear stress τ and the wall thickness t at any point in the perimeter is known as the shear flow (q), which remains constant within the thin walls of the tube.



(a) Thin-walled tube

(b) Area enclosed by shear flow path

Design of RC Members for Torsion

Torsional Stress Formula

Shear stress τ = Force / Area

$$\tau_1 = \frac{V_1}{x_o t}$$

Shear flow = Shear stress x thickness

$$q_1 = \tau_1 \times t = V_1/x_o$$

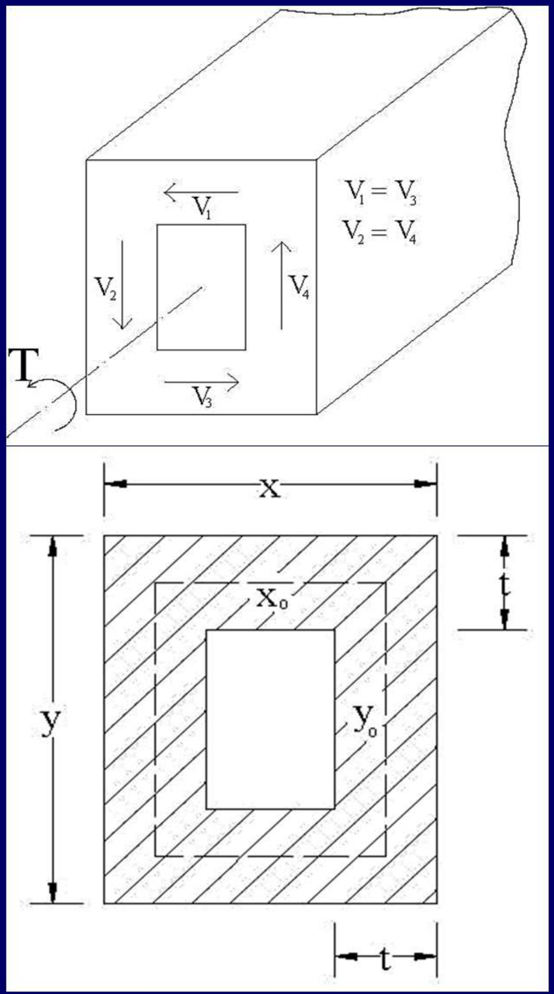
$$q_2 = \tau_2 \times t = V_2/y_o$$

According to thin-walled tube theory,

$$q_1 = q_2 = q_3 = q_4 = q = constant$$

$$\frac{V_1}{x_o} = \frac{V_2}{y_o} = \frac{V_3}{x_o} = \frac{V_4}{y_o} = q$$

$q = \tau t$ (for same wall thickness)



Design of RC Members for Torsion

❑ Torsional Stress Formula

As the terms V_1 through V_4 are induced shear and cannot be easily determined, therefore, they can be expressed in terms of torque. Taking moments about centerline of thin-walled tube.

$$T = V_4\left(\frac{x_o}{2}\right) + V_2\left(\frac{x_o}{2}\right) + V_1\left(\frac{y_o}{2}\right) + V_3\left(\frac{y_o}{2}\right) = (V_2 + V_4)\frac{x_o}{2} + (V_1 + V_3)\frac{y_o}{2}$$

Since $V_1 = V_3 \;\; and \;\; V_2 = V_4$;

$$T = (2V_2)\left(\frac{x_o}{2}\right) + (2V_1)\left(\frac{y_o}{2}\right) = V_2 x_o + V_1 y_o$$

Substituting the values of V_1 and V_2

$$T = (q_2 y_o)x_o + (q_1 x_o)y_o = 2q\, x_o y_o = 2(\tau t)A_o$$

$$\tau = \frac{T}{2A_o t}$$

$$q_1 = \frac{V_1}{x_o} \;\; and \;\; q_2 = \frac{V_2}{y_o}$$

$$V_1 = q_1 x_o \;\; and \;\; V_2 = q_2 y_o$$

$$q_1 = q_2 = q = \tau t$$

$$x_o y_o = A_o$$

Design of RC Members for Torsion

❑ Torsional Strength of Concrete

In case of shear, shear strength of concrete is given as:

$$v_c = 2\sqrt{f_c'}$$

Since average shear stress $v_{ave} = V/bd$ therefore, $V/bd = 2\sqrt{f_c'}$

$$V = 2\sqrt{f_c'}bd$$

In case of torsion induced shear stresses (torsional stresses), ACI 22.7.5 states that "cracking is assumed when tensile stresses reach $4\sqrt{f_c'}$". Therefore;

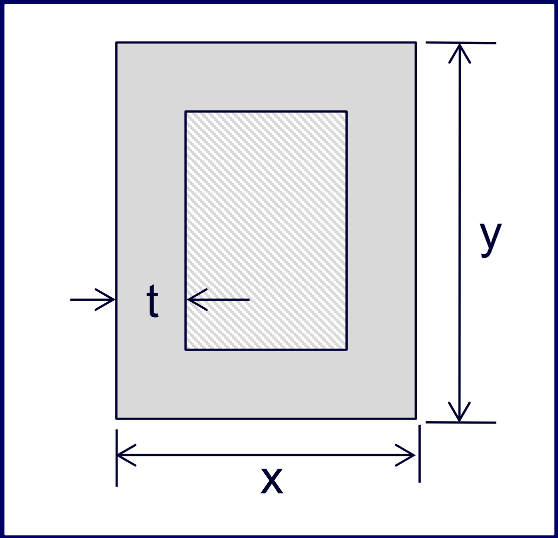
$$\tau_c = 4\sqrt{f_c'} \quad --- \text{(a)}$$

Design of RC Members for Torsion

❑ Torsional Capacity of Concrete

From the previous discussion on torsional stresses in thin-walled tube

$$\tau = \frac{T}{2A_o t} ---- (b)$$



Equating eq. (a) and (b), we get

$$T_c = 4\sqrt{f_c'} \times 2A_o t$$

According to ACI R22.7.5;

$$A_o = (2/3)A_{cp} \; , t = (3/4)A_{cp}/p_{cp}$$

Where; $A_{cp} = xy, \;\; P_{cp} = 2(x+y)$ (full section of the member)

Substituting values of A_o and t equation (b) becomes;

$$T_c = 4\sqrt{f_c'}\, A_{cp}{}^2/p_{cp}$$

Reinforcement Requirement for Torsion

Review of Reinforcement Requirement for Flexure

- To understand the reinforcement requirement for torsion, recall the concept of flexural design of RC beam.

Elastic Range Flexural Capacity

For an uncracked concrete beam, the flexural stresses are given by:

$$f = \frac{My}{I}$$

Taking $f = f_r = 7.5\sqrt{f_c'}$

$$M = M_{cr} = \frac{f_r I}{y}$$

$$M_{cr} = 7.5\sqrt{f_c'}\ \frac{I}{y}$$

Reinforcement Requirement for Torsion

❑ Review of Reinforcement Requirement for Flexure

❖ Ultimate Flexural Capacity

For a cracked RC beam at ultimate stage, the flexural capacity is given as:

$$M_n = M_c + M_s$$

As concrete is weak in tension (Refer ACI 9.5.2.1 for concrete tensile strength), $M_c \approx 0$, therefore,

$$M_n = M_s = A_s f_y \left(d - \frac{a}{2}\right)$$

Hence, the tension reinforcement along with the concrete in compression acts as a couple to resist the flexural demand on the member.

Reinforcement Requirement for Torsion

Review of Reinforcement Requirement for Shear

- Similarly, recall the concept of shear design of RC beam.

Elastic Range Shear Capacity

For an uncracked concrete beam, the shear stress is given by:

$$v = \frac{VQ}{Ib}$$

According to ACI Code, $v = v_{cr} = 2\sqrt{f_c'}$ is the nominal shear strength corresponding to formation of diagonal tension cracks. Therefore, shear capacity of section at that stage is:

$$V_{cr} = \frac{v_{cr}Ib}{Q} = 2\sqrt{f_c'}\,Ib/Q$$

Reinforcement Requirement for Torsion

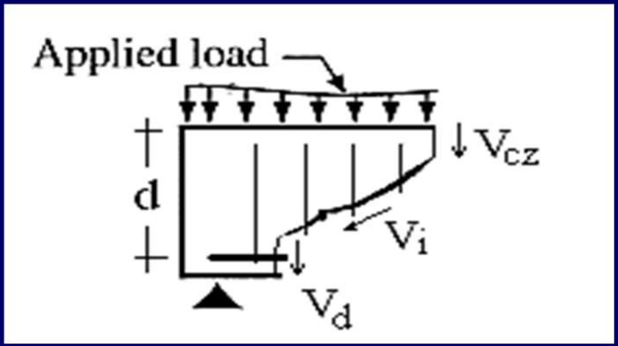
Review of Reinforcement Requirement for Shear

Ultimate Shear Capacity

For a cracked RC beam at ultimate stage, the shear capacity is given as:

$$V_n = V_c + V_s$$

Unlike flexure, the term $V_c \neq 0$ because from test evidence:



$$V_c = V_{cz} + V_d + V_{iy} = 2\sqrt{f_c'}\, b_w d \quad \text{[ACI 22.5.5.1]}$$

Therefore, shear steel (stirrups) along with the contribution of concrete (V_c) acts together to resists the shear demand due to applied load on the member.

Reinforcement Requirement for Torsion

❑ Torsional Capacity of RC Beams

- The total design torsional capacity of an RC member is given by

$$\emptyset T_n = \emptyset T_c + \emptyset T_s$$

Where, $T_c = 4\sqrt{f_c'}\, A_{cp}{}^2/p_{cp}$

- ACI Code (R22.7) requires that the concrete contribution to torsional strength shall be ignored. Therefore,

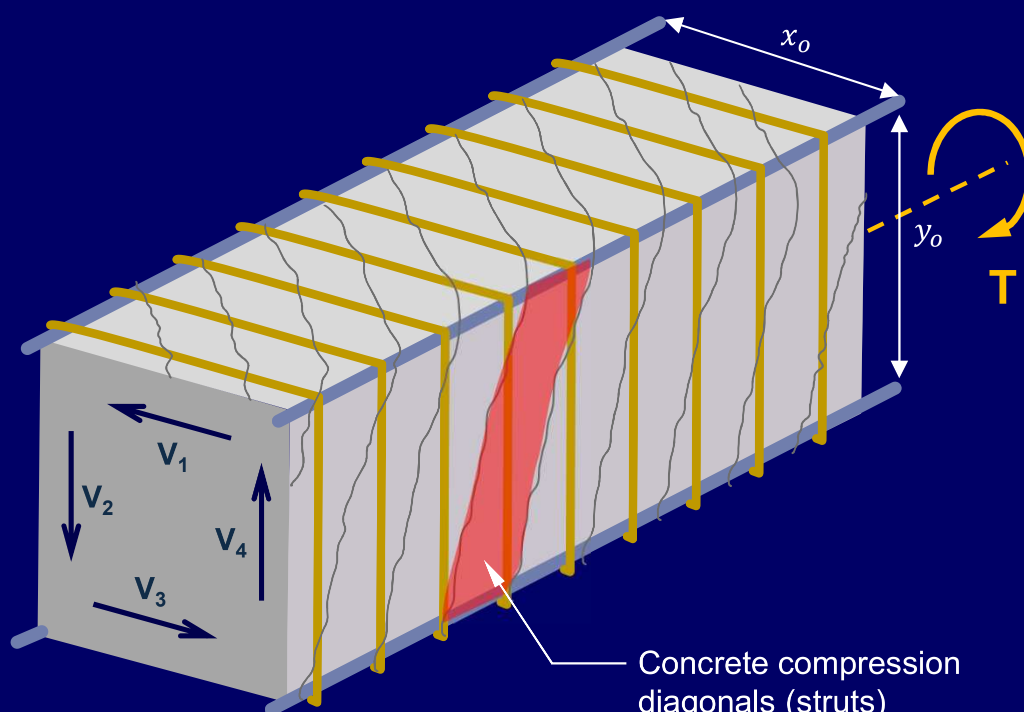
$$\emptyset T_n = \emptyset T_s$$

- ΦT_s can be determined from space truss analogy discussed next.

Reinforcement Requirement for Torsion

Space Truss Analogy

- From thin-walled tube analogy, internal effects in the form of induced shear forces (V_1 to V_4) will be generated due to applied torque T.
- Such internally induced shear forces will crack the member.
- Due to cracks, the member splits up into diagonal compressive portions or struts.



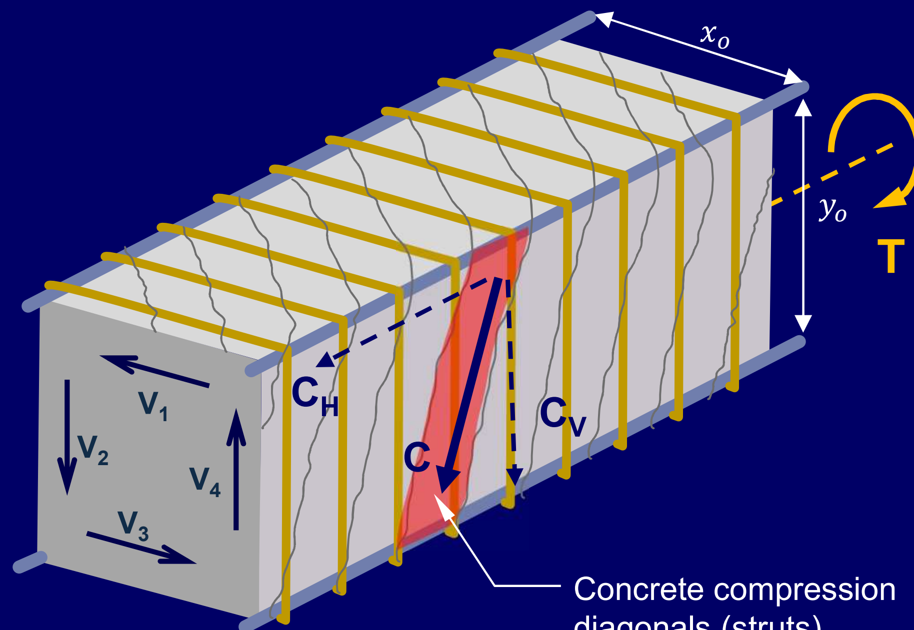
Reinforcement Requirement for Torsion

❑ Space Truss Analogy

- If the compressive force in the strut is C, then it can be resolved into two components.

C_H = horizontal component

C_V = vertical component



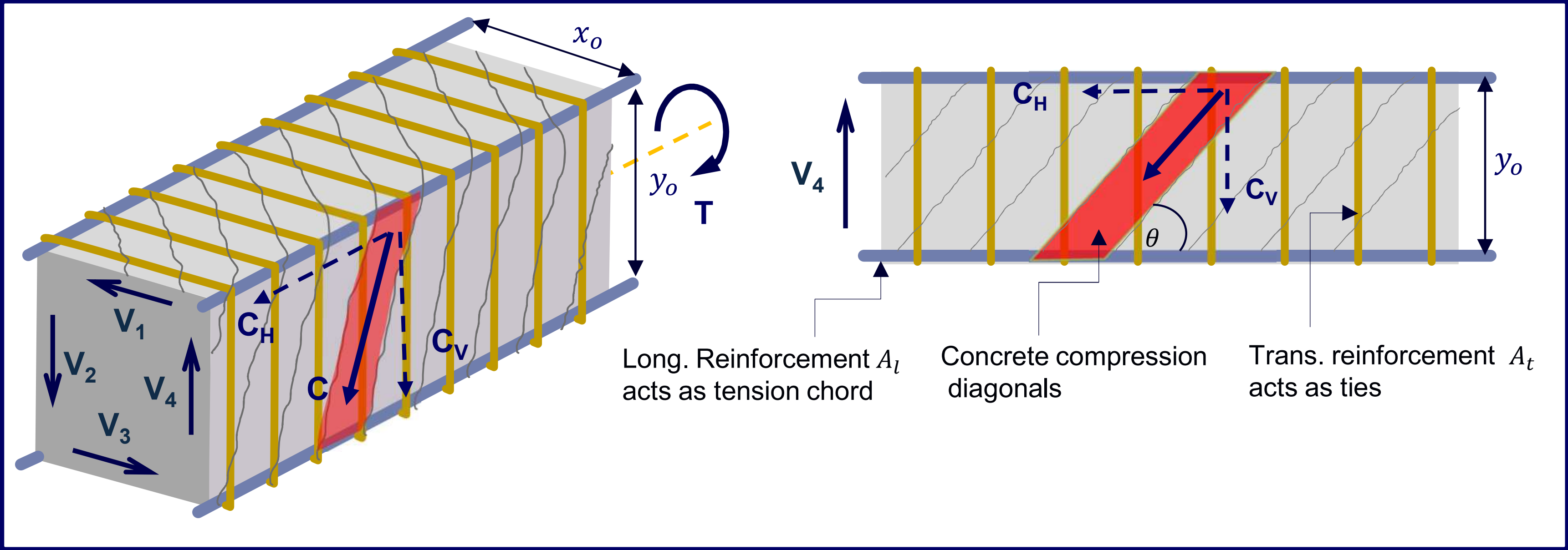
- Longitudinal reinforcement shall be provided to resist C_H and vertical stirrups shall be provided to resist C_V.

- This leads to space truss analogy.

Reinforcement Requirement for Torsion

❑ Space Truss Analogy

- In space truss analogy, the concrete compression diagonals (struts), vertical/transverse reinforcement in tension (ties), and longitudinal reinforcement (tension chords) act together.
- The analogy derives that torsional stress will be resisted by the vertical stirrups as well as by the longitudinal steel.



Reinforcement Requirement for Torsion

❑ Transverse Reinforcement A_t

Refer to figure (a), we have

$$C_V = V_4$$

From figure (b)

$$V_4 = n \times A_t f_{yt} = \frac{y_o \cot\theta}{s} \times A_t f_{yt}$$

Since $V_4 = V_2$ so,

$$V_2 = V_4 = \frac{y_o \cot\theta A_t f_{yt}}{s} \quad \text{--- (i)}$$

Similarly,

$$V_1 = V_3 = \frac{x_o \cot\theta A_t f_{yt}}{s} \quad \text{--- (ii)}$$

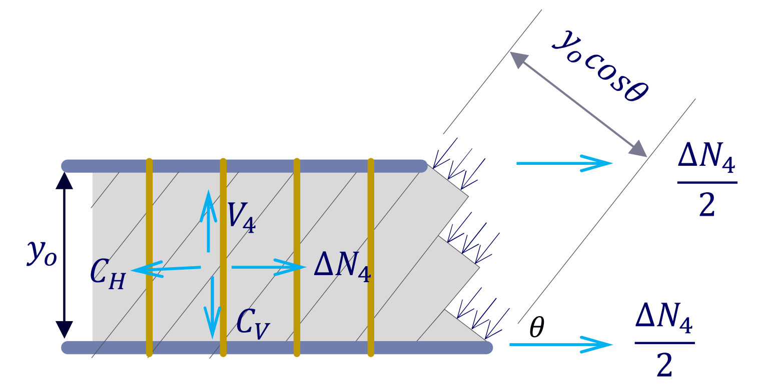


Figure a) Face 4

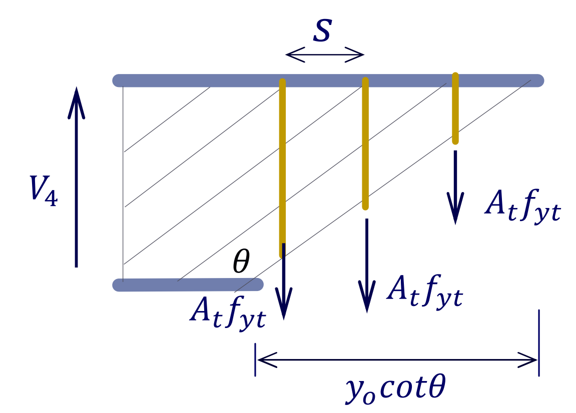


Figure b)

Reinforcement Requirement for Torsion

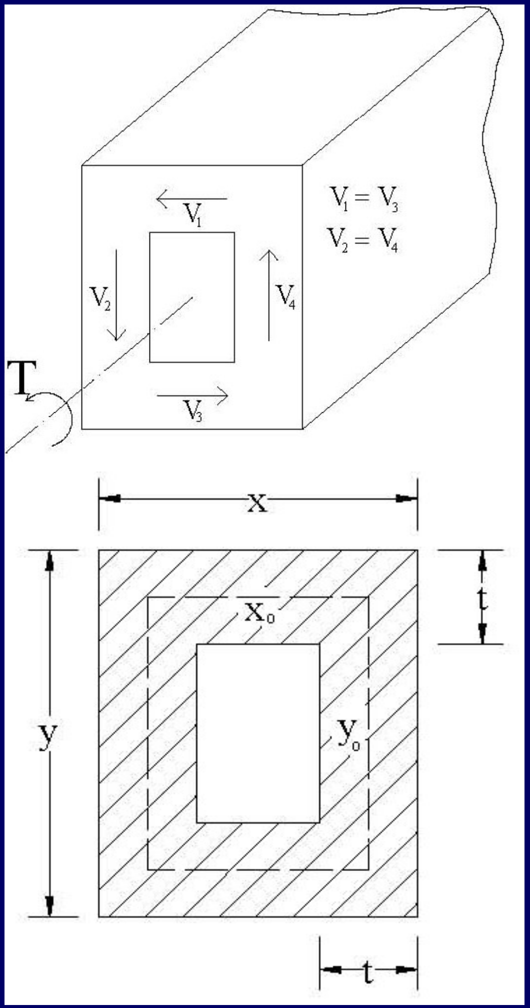
❑ Transverse Reinforcement A_t

- If V_1 to V_4 are known, A_t can be determined from previous equations.
- However as discussed earlier, it is convenient to express V_1 to V_4 in terms of T by taking moments about centerline of thin-walled tube.

$$T_n = \frac{V_4 x_o}{2} + \frac{V_2 x_o}{2} + \frac{V_1 y_o}{2} + \frac{V_3 y_o}{2}$$

With $V_4 = V_2$ and $V_1 = V_3$, we have

$$T_n = V_2 x_o + V_1 y_o$$



Reinforcement Requirement for Torsion

❑ Transverse Reinforcement A_t

Substituting values of V_1 V_2 from eq (i) and (ii), the equation becomes

$$T_n = V_2 x_o + V_1 y_o = \left(\frac{y_o cot\theta A_t f_{yt}}{s}\right) x_o + \left(\frac{x_o cot\theta A_t f_{yt}}{s}\right) y_o$$

Which on simplifying gives

$$T_n = \frac{2A_t}{s} f_{yt} x_o y_o cot\theta$$

Setting $x_o y_o = A_o$ and taking $\theta = 45^o$

$$T_n = \frac{2A_t}{s} f_{yt}\, A_o$$

The value of θ shall not be taken less than 30° and greater than 60°. It is permitted to take $\theta =$ 45° for non-prestressed members (ACI 22.7.6).

Reinforcement Requirement for Torsion

❑ Transverse Reinforcement A_t

For no failure, torsional capacity of the member shall be greater than or equal to torsional demand i.e. $\emptyset T_n \geq T_u$ (where $\emptyset = 0.75$)

For $\emptyset T_n = T_u$ equation (iii) becomes

$$\emptyset \frac{2A_t}{s} f_{yt}\ A_o = T_u \quad \Rightarrow \quad A_t = \frac{T_u s}{2\emptyset f_{yt}\ A_o}$$

Note that A_t is the steel area of single leg of stirrup. For 2-legged stirrups, we have

$$A_{t(2\ leg)} = \frac{T_u s}{\emptyset f_{yt}\ A_o} \quad \text{--- (iii)}$$

Reinforcement Requirement for Torsion

❑ Transverse Reinforcement A_t

- The total shear reinforcement requirement is therefore the sum of the shear reinforcement requirements due to direct shear and torsion both.

$$A_{v+t} = A_{v(2\text{ leg})} + A_{t(2\text{ leg})} = \frac{(V_u - \emptyset V_c)s}{\emptyset f_{yt} d} + \frac{T_u s}{\emptyset f_{yt} A_o}$$

Reinforcement Requirement for Torsion

❑ Longitudinal Reinforcement A_l

Refer to figure b and c for face 4

$\Delta N_4 = V_4 cot\theta = V_4 \ \ (\theta = 45^o)$

Similarly,

$\Delta N_1 = V_1$, $\Delta N_2 = V_2$ & $\Delta N_3 = V_3$

Total longitudinal reinforcement for torsion is:

$$\Delta N = \Delta N_1 + \Delta N_2 + \Delta N_3 + \Delta N_4$$

$$\Delta N = V_1 + V_2 + V_3 + V_4$$

As $V_1 = V_3$ and $V_2 = V_4$ therefore,

$$\Delta N = 2V_1 + 2V_4$$

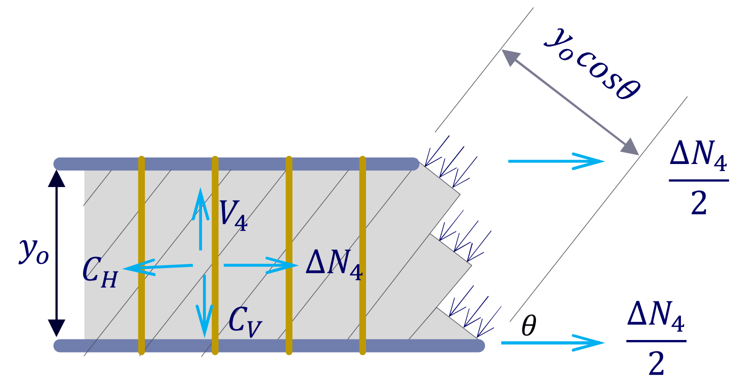


Figure a) Face 4

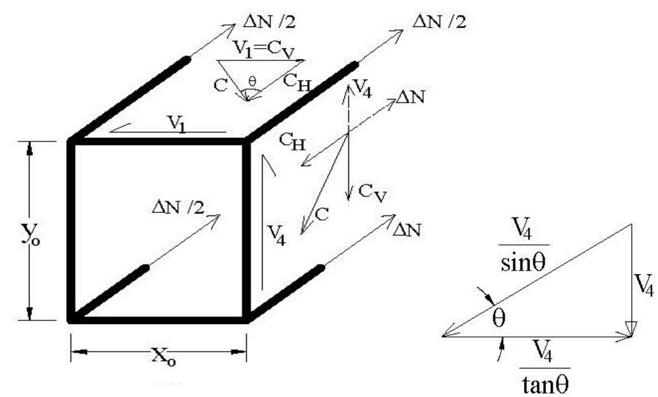


Figure c)

Reinforcement Requirement for Torsion

❑ Longitudinal Reinforcement A_l

Substituting values of V_1 and V_4

$$\Delta N = 2V_1 + 2V_4$$

$$\Delta N = 2\frac{x_o cot\theta A_t f_{yt}}{s} + 2\frac{y_o cot\theta A_t f_{yt}}{s}$$

$$\Delta N = 2(x_o + y_o)\frac{A_t f_{yt}}{s}$$

Setting $\Delta N = A_l f_{yl}$ & $2(x_o + y_o) = P_h$

$$A_l f_{yl} = P_h A_t f_{yt}/s$$

$$A_l = \frac{A_t P_h f_{yt}}{s f_{yl}}$$

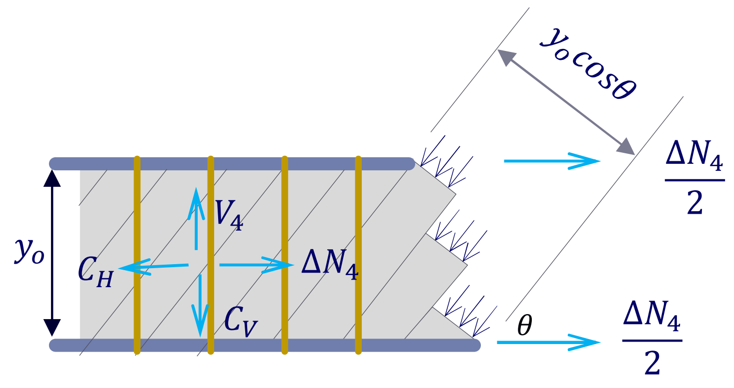


Figure a) Face 4

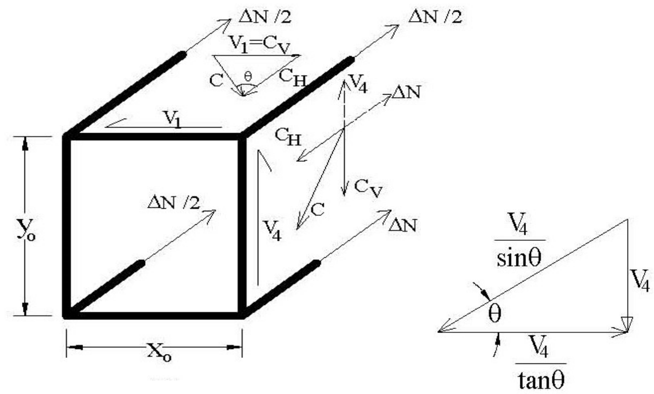


Figure c)

Reinforcement Requirement for Torsion

❑ Longitudinal Reinforcement A_l

Now, as derived earlier

$$A_{t(1\ leg)} = \frac{T_u s}{\emptyset 2 f_{yt} A_o}$$

Therefore, the preceding equation becomes

$$A_l = \frac{A_t P_h f_{yt}}{s f_{yl}} = \frac{T_u s}{\emptyset 2 f_{yt} A_o} \times \frac{P_h f_{yt}}{s f_{yl}} = \frac{T_u}{\emptyset 2 A_o} \times \frac{P_h}{f_{yl}}$$

$$A_l = \frac{T_u P_h}{\emptyset 2 A_o f_{yl}}$$

This expression can be used to find longitudinal reinforcement due to torsion.

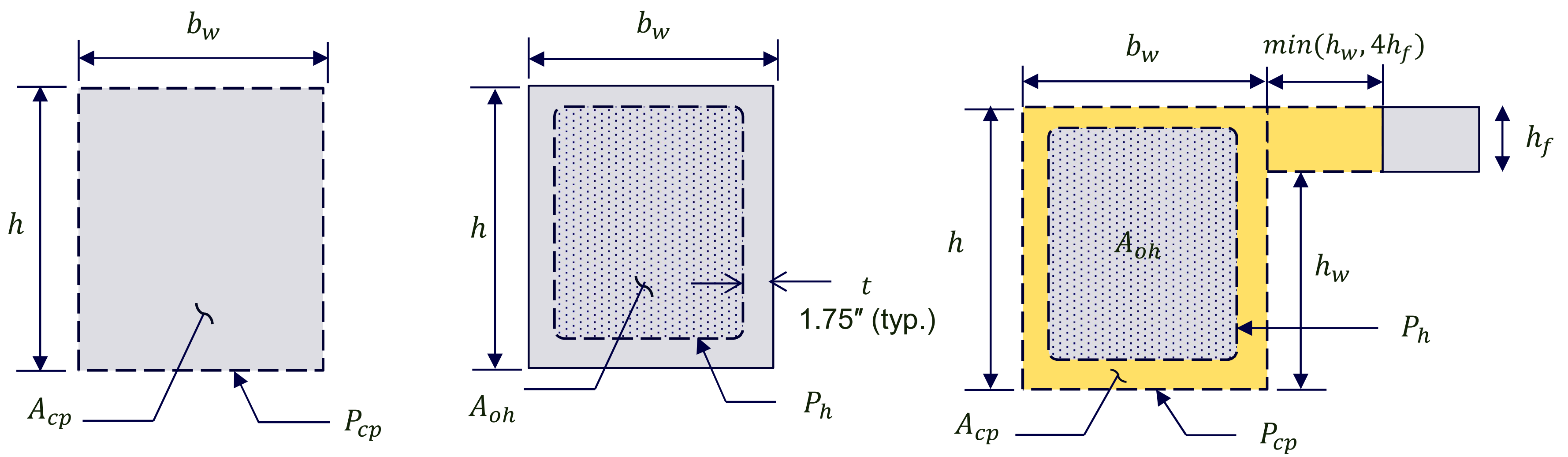
ACI Code Provisions for Torsion

Concrete Contribution in Torsional Capacity (ACI R22.7.6)

- In the calculation of T_n, all the torque is assumed to be resisted by stirrups and longitudinal steel with $T_c = 0$.
- At the same time, the shear resisted by concrete V_c is assumed to be unchanged by the presence of torsion.

ACI Code Provisions for Torsion

❑ Definition of Various Terms related to Torsion



$\boldsymbol{A_{cp}}$ = area enclosed by outside perimeter of concrete cross section

$\boldsymbol{P_{cp}}$ = outside perimeter of the concrete cross section

$\boldsymbol{A_{oh}}$ = area enclosed by centerline of the outermost closed transverse torsional reinforcement

$\boldsymbol{A_o} = 0.85A_{oh}$[1]

$\boldsymbol{P_h}$ = perimeter of centerline of outermost closed transverse torsional reinforcement

Note:

[1] It should be noted that $A_o = 2/3A_{cp}$ serves only as part of the derivation for calculating the cracking torsional capacity. However, in the design context, the value used for A_o is taken as $0.85A_{oh}$ (ACI 22.7.6.1.1).

ACI Code Provisions for Torsion

- **Definition of A_{oh}**

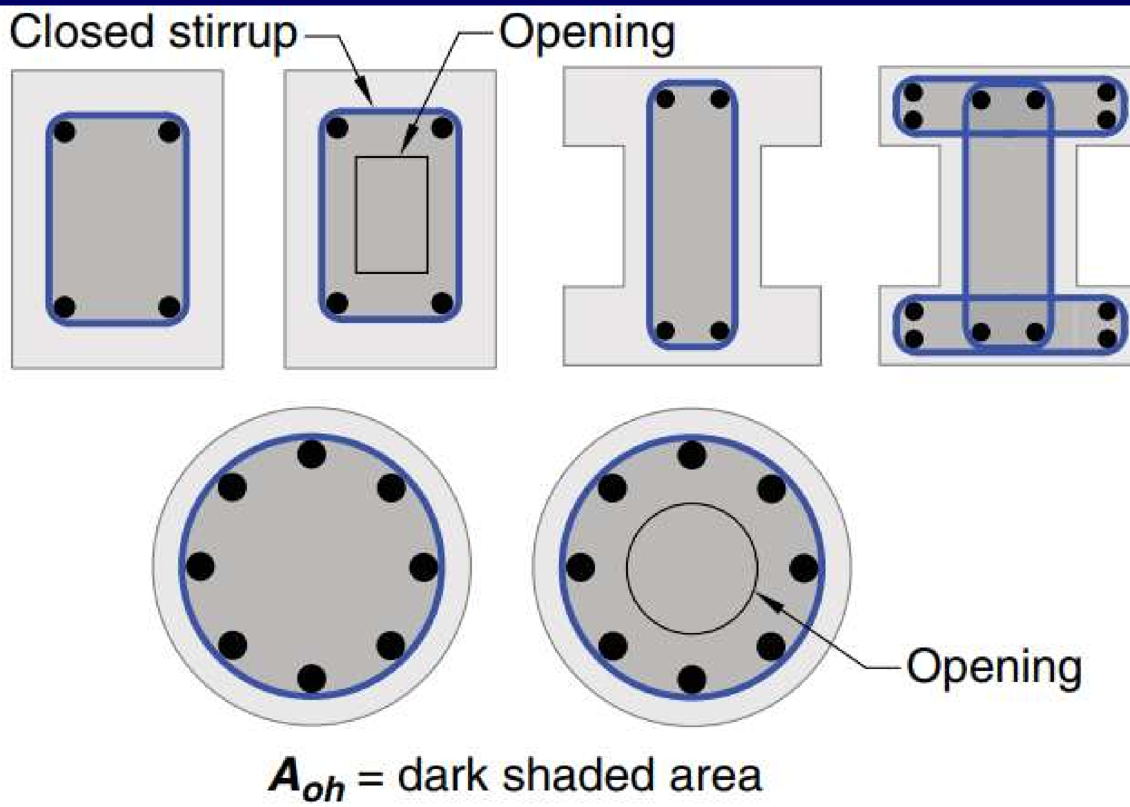


Fig. R22.7.6.1.1—Definition of A_{oh}.

ACI Code Provisions for Torsion

❑ Cracking Torsion T_{cr}

- Cracking torsion T_{cr} shall be calculated in accordance with ACI Table 22.7.5.1. For nonprestressed members, we have

$$\emptyset T_{cr} = \emptyset \lambda 4\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{N_u}{4A_g\lambda\sqrt{f_c'}}}$$

- $\emptyset = \mathbf{0.75}$
- $\boldsymbol{N_u}$ is positive for compression and negative for tension

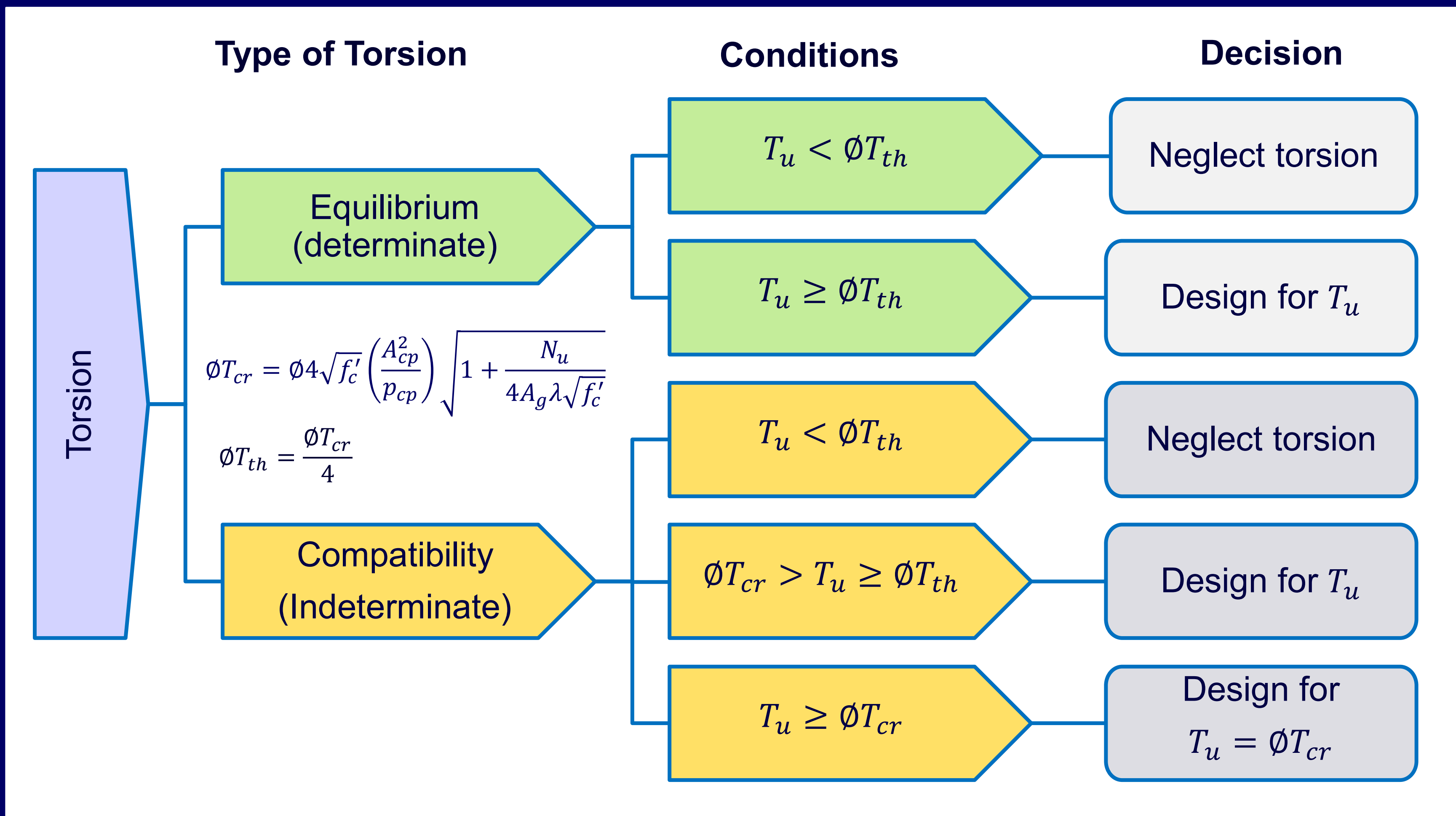
❑ Threshold Torsion T_{th}

- The threshold torsion is defined as one-fourth the cracking torsional moment T_{cr} (ACI R22.7.4).

$$\emptyset T_{th} = \frac{T_{cr}}{4}$$

ACI Code Provisions for Torsion

❑ Consideration of Torsional Effects



ACI Code Provisions for Torsion

❑ Cross sectional Limits (ACI 22.7.7)

Cross section shall be selected such that (a) or (b) is satisfied

a) Solid Sections

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \leq \emptyset\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right)$$

Direct shear stress | Torsional shear stress | Shear capacity | ACI restriction

b) Hollow Sections

$$\frac{V_u}{b_w d} + \frac{T_u p_h}{1.7A_{oh}^2} \leq \emptyset\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right)$$

Direct shear stress | Torsional shear stress | Shear capacity | ACI restriction

Why Size Limits?

As per ACI R22.7.7.1 The size of a cross section is limited for two reasons:

1. To reduce unsightly cracking
2. To prevent crushing of the surface concrete due to inclined compressive stresses due to shear and torsion.

ACI Code Provisions for Torsion

❑ Reinforcement Limits (ACI 22.7.7)

a) Minimum Transverse Reinforcement (ACI 9.6.4.2,9.7.6.3.3)

$$\frac{A_{(v+t),min}}{s} = max\left(0.75\sqrt{f_c'}, 50\right)\frac{b_w}{f_{yt}} \quad ; \quad A_{v+t} = A_v + A_{t(2\text{ leg})}$$

Calculated spacing shall not exceed S_{max}

$$S_{max} = min\left(\frac{p_h}{8}, 12\right)$$

b) Minimum Longitudinal Reinforcement (ACI 9.6.4.3)

$$A_{l,min} = \frac{5\sqrt{f_c'}\,A_{cp}}{f_y} - max\left(\frac{A_{t(2\text{ leg})}}{2s}, \frac{25b_w}{f_{yt}}\right)\frac{p_h f_{yt}}{f_y}$$

ACI Code Provisions for Torsion

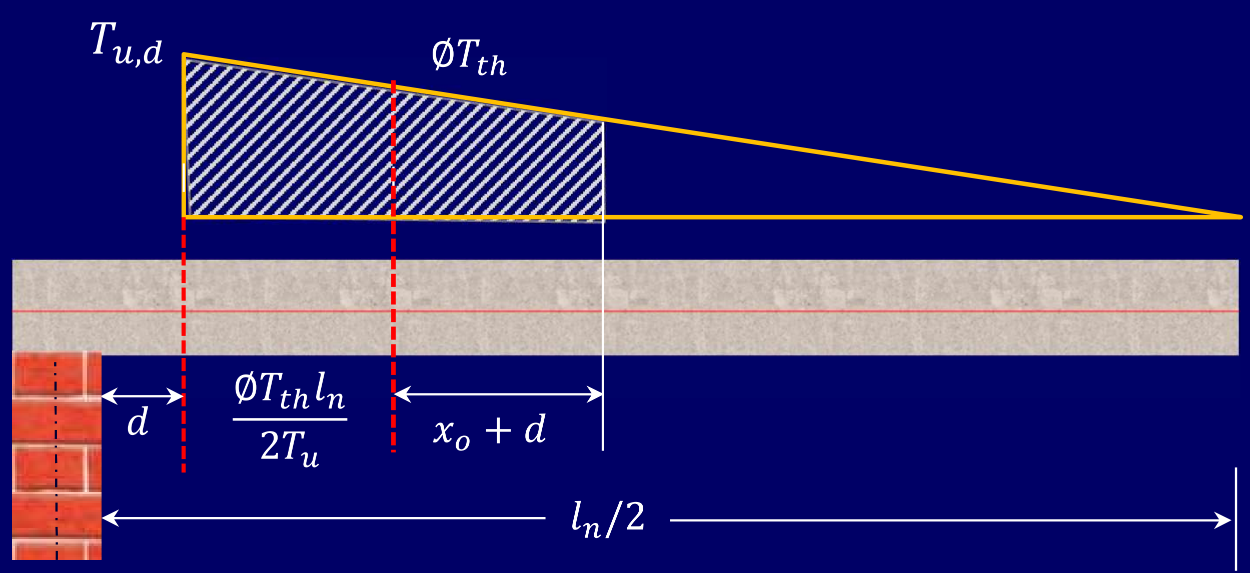
❑ Reinforcement Detailing (ACI 9.7.5)

- Transverse torsional reinforcement shall be detailed in the same manner as shear reinforcement.
- Longitudinal torsional bars should be evenly distributed around the perimeter of the cross-section, with spacing along the depth not exceeding 12 inches.
- The longitudinal reinforcement shall be inside the stirrup or hoop, and at least one longitudinal bar shall be placed in each corner.
- The diameter of longitudinal bars should be greater of 3/8″ and 0.042s where s is spacing of stirrups.

ACI Code Provisions for Torsion

❑ Reinforcement Detailing (ACI 9.7.5)

- The torsional moment varies from maximum at the face of the support to zero at span mid-length.
- Bars can be discontinued per the following criteria. however, in practice, bars are extended over the full length of the beam.



ACI Code Provisions for Torsion

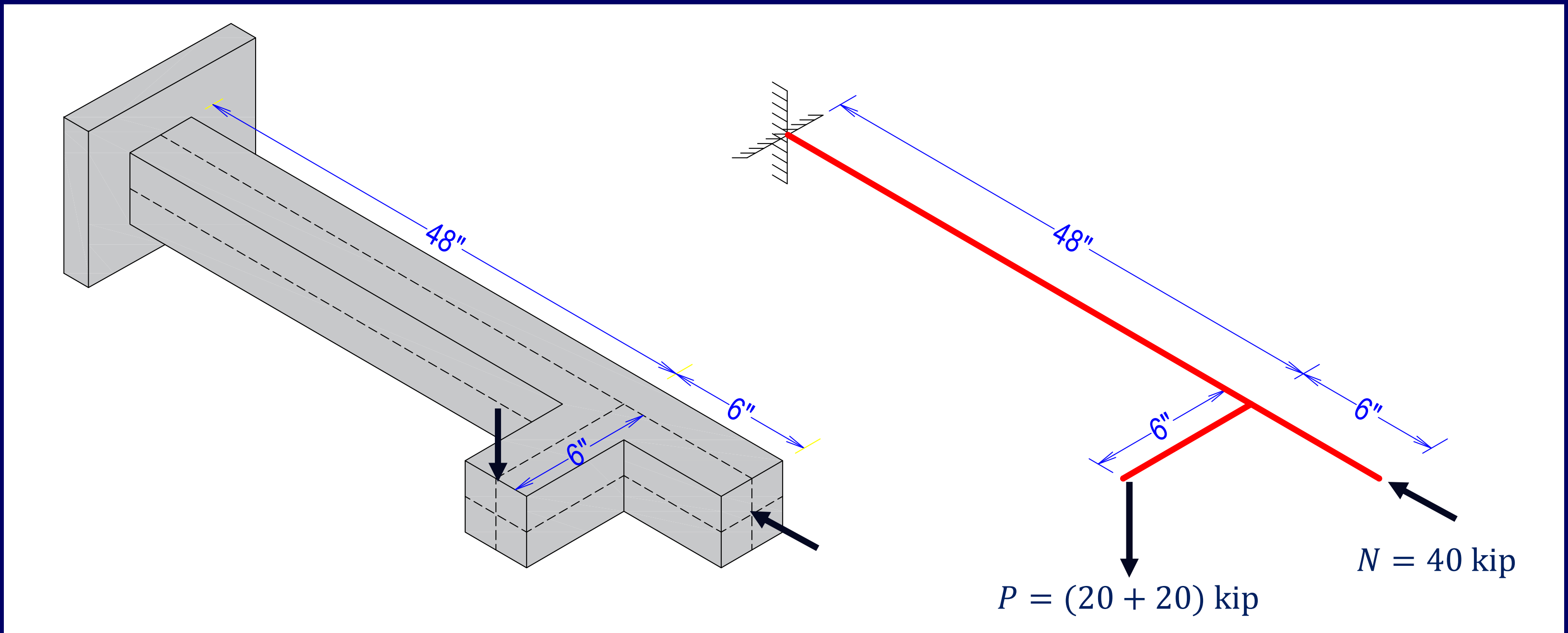
Summary of Steps for Torsion Design

- **Step 1:** Determine factored torsion T_u
- **Step 2:** Determine special section properties
- **Step 3:** Check need for torsional reinforcement
- **Step 4:** Check adequacy of cross section for torsion
- **Step 5:** Determine torsional reinforcement
- **Step 6:** Apply minimum torsional reinforcement and spacing checks
- **Step 7:** Perform detailing of reinforcement

Design Example

❑ Problem Statement

- A 54-in long RC cantilever beam supports its own dead load plus a concentrated load $\boldsymbol{P}$ which consists of 20-kip dead and 20-kip live load. The beam also supports an unfactored axial compressive dead load $\boldsymbol{N}$ of 40 kip. Using f_c' = 3 ksi and $f_y = f_{yt}$ = 60 ksi, ***Design*** the beam for Flexure, Shear and Torsion.



Design Example

Solution

Step 1: Selection of Sizes

Minimum depth for cantilever beam as per ACI 9.3.1.1 is given by

$$h_{min} = \frac{l}{8} = \frac{54}{8} = 6.75''$$

Though any depth of beam greater than 6.75″ can be taken as per ACI minimum requirement, we will use a depth equal to 24″.

Assume width of 14″ and effective depth of 24 - 2.5 = 21.5″.

So finally selected sizes are:

$$b_w = 14" , \text{ h} = 24" \text{ and d} = 21.5"$$

Design Example

Solution

Step 2: Calculation of Loads

- **Factored Self-weight**

$$W_u = 1.2(b_w h \times \gamma_c) = 1.2\left(\frac{14 \times 24}{144} \times 0.150\right) = \mathbf{0.42\ kip/ft}$$

- **Factored Concentrated Load**

$$P_u = 1.2D + 1.6L = 1.2(20) + 1.6(20) = \mathbf{56\ kip}$$

- **Factored Axial Compressive Load**

$$N_u = 1.2D = 1.2(40) = \mathbf{48\ kip}$$

Design Example

❑ Solution

➢ Step 3: Analysis

❖ **Factored Moment**

$$M_u = \frac{W_u l^2}{2} + P_u(l - 0.5)$$

$$M_u = \frac{0.42(4.5)^2}{2} + 56(4) = 228.3 \text{ ft.kip}$$

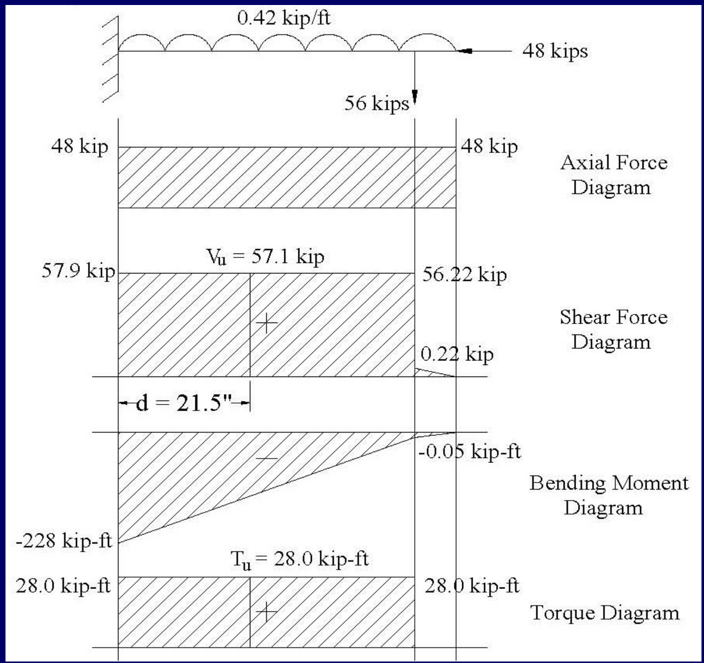
❖ **Factored Shear**

$$V_{u,max} = W_u l + P_u = 0.42(4.5) + 56 = 57.9 \text{ kip}$$

$$V_{u,d} = 57.1 \text{ kip}$$

❖ **Factored Torsion**

$$T_u = P_u \times 0.5 = 56 \times 0.5 = 28 \text{ ft.kip}$$



Design Example

Solution

Step 4: Determination of Reinforcement

Flexural Reinforcement

Axial load can be ignored in the flexural design if:

$$N_u \leq 0.1 A_g f_c'$$

$$0.1 A_g f_c' = 0.1 \times 14 \times 24 \times 3 = 100.8 \text{ kip}$$

$N_u = 48 \text{ kip} < 0.1 A_g f_c' = 100.8 \text{ kip} \rightarrow$ axial load can be neglected

In case $N_u > 0.1 A_g f_c'$, the member shall be designed for bending and axial load both.

Design Example

❑ Solution

➢ Step 4: Determination of Reinforcement

❖ Flexural Reinforcement

Determine required reinforcement A_s

$$a = d - \sqrt{d^2 - \frac{2.614 M_u}{f_c' b}} = 21.5 - \sqrt{21.5^2 - \frac{2.614(228.3 \times 12)}{3 \times 14}} = 4.42"$$

$$A_s = \frac{M_u}{\emptyset f_y \left(d - \frac{a}{2}\right)} = \frac{228.25 \times 12}{0.9 \times 60 \left(21.5 - \frac{4.42}{2}\right)} = 2.63\ in^2$$

Using #6 bar, with $A_b = 0.44$ in^2

$$n = \frac{2.63}{0.44} = 5.9 \approx 6$$

Design Example

Solution

Step 4: Determination of Reinforcement

Flexural Reinforcement

Apply minimum and maximum checks on flexural reinforcement

$$A_{s,min} = max\left(3\sqrt{f_c'}, 200\right)\frac{b_w d}{f_y} = max\left(3\sqrt{3000}, 200\right)\frac{14 \times 21.5}{60{,}000} = 1.0\ in^2$$

and

$$A_{s,max[60]} = \frac{f_c'}{223}bd = \frac{3 \times 14 \times 21.5}{223} = 4.01\ in^2$$

$$A_{s,min} = 1.0 < A_{s,pvd} = 2.64 < A_{s,max[60]} = 4.01 \rightarrow OK$$

Design Example

❑ Solution

➢ Step 4: Determination of Reinforcement

❖ Shear Reinforcement

The shear reinforcement due to direct shear is required if:

$\emptyset V_c < V_{u,d}$

The design shear capacity of normal-weight concrete neglecting size effect factor is given by;

$$\emptyset V_c = \emptyset\left[2\sqrt{f_c'} + \frac{N_u}{6A_g}\right]b_w d = 0.75\left[2\sqrt{3000} + \frac{48 \times 1000}{6(14 \times 24)}\right]14 \times 21.5 = 30104\ lb$$

$\emptyset V_c = 30.1 \text{ kip} < V_{u,d} = 57.1 \text{ kip} \rightarrow$ Shear reinforcement is required.

Design Example

❑ Solution

➢ Step 4: Determination of Reinforcement

❖ Shear Reinforcement

The required spacing of shear reinforcement due to direct shear is given by:

$$S = \frac{\emptyset A_v f_{yt} d}{V_{u,d} - \emptyset V_c}$$

From which we get

$$\frac{A_v}{s} = \frac{V_{u,d} - \emptyset V_c}{\emptyset f_{yt} d} = \frac{57.1 - 30.1}{0.75 \times 60 \times 21.5}$$

$$\frac{A_v}{s} = 0.0279 \text{ in}^2/\text{in}$$ ---- (a)

Design Example

Solution

Step 4: Determination of Reinforcement

Torsion Reinforcement

1) Special Section Properties

$$A_{cp} = b_w h = 14 \times 24 = 336\ in^2$$

$$P_{cp} = 2b_w + 2h = 2 \times 14 + 2 \times 24 = 76\ in$$

With

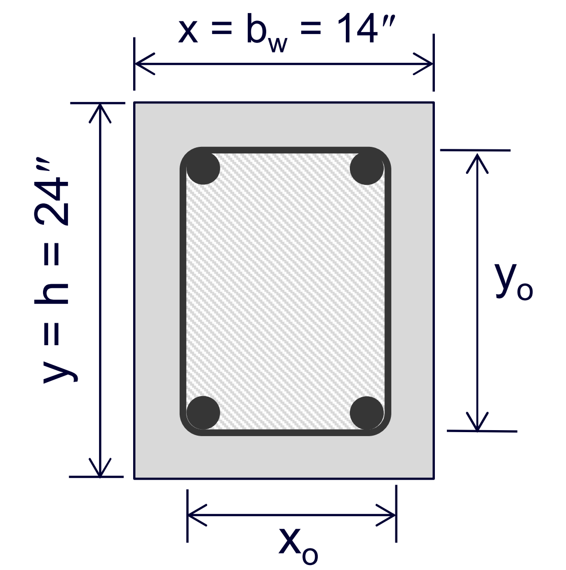
$$x_o = 14 - 2(1.5) - 4/8 = 10.5''$$

$$y_o = 20 - 2(1.5) - 4/8 = 20.5''$$

$$A_{oh} = x_o y_o = 10.5 \times 20.5 = 215.25\ in^2$$

$$A_o = 0.85 A_{oh} = 0.85(215.25) = 182.96\ in^2$$

$$P_h = 2x_o + 2y_o = 2(10.5) + 2(20.5) = 62\ in$$



$$x_o = x - 2c - d_h$$

$$y_o = y - 2c - d_h$$

Design Example

❑ Solution

➢ Step 4: Determination of Reinforcement

❖ Torsion Reinforcement

2) Check Need for Torsional reinforcement

The given system is determinate, so this is equilibrium torsion case.

$$\emptyset T_{cr} = \emptyset 4\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{N_u}{4A_g\sqrt{f_c'}}} = 0.75 \times 4\sqrt{3000}\left(\frac{336^2}{76}\right)\sqrt{1+\frac{48 \times 1000}{4(336)\sqrt{3000}}}$$

$$\emptyset T_{cr} = 313731.79 \text{ in.lb or } 26.14 \text{ ft.kip}$$

$$\emptyset T_{th} = \frac{\emptyset T_{cr}}{4} = \frac{26.14}{4} = 6.54 \text{ ft.kip}$$

Since, $\emptyset T_{th} = 6.54 \text{ ft.kip} < T_u = 28 \text{ ft.kip} \rightarrow$ Torsional reinforcement is required

Design Example

Solution

Step 4: Determination of Reinforcement

Torsion Reinforcement

3) Check Adequacy of Cross section

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \emptyset\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right)$$

$$\sqrt{\left(\frac{57.1 \times 1000}{14 \times 21.5}\right)^2 + \left(\frac{(28 \times 12 \times 1000) \times 62}{1.7 \times 215.25^2}\right)^2} \le \left(\frac{30.1 \times 1000}{14 \times 21.5} + 0.75 \times 8\sqrt{3000}\right)$$

$325.48 \text{ psi} < 428.63 \text{ psi} \rightarrow$ the section is adequate for torsion.

Design Example

Solution

Step 4: Determination of Reinforcement

Torsion Reinforcement

4) Transverse Reinforcement A_t

$$\frac{A_{t(2\text{ leg})}}{s} = \frac{T_u}{\emptyset f_{yt}\ A_o} = \frac{28 \times 12}{0.75 \times 60 \times 182.96} = 0.0408\ in^2/in$$

The total shear reinforcement requirement is the sum of the shear reinforcement requirements due to direct shear and torsion both.

$$\frac{A_{v+t}}{s} = \frac{A_v}{s} + \frac{A_{t(2\text{ leg})}}{s} = 0.0279 + 0.0408 = 0.0687\ in^2/in$$

Design Example

Solution

Step 4: Determination of Reinforcement

Torsion Reinforcement

4) Transverse Reinforcement A_t

Check for minimum transverse reinforcement

$$\frac{A_{(v+t),min}}{s} = max\left(0.75\sqrt{f_c'}, 50\right)\frac{b_w}{f_{yt}} = max\left(0.75\sqrt{3000}, 50\right)\frac{14}{60} = 0.0117 \text{ in}^2/\text{in} \rightarrow OK$$

Using 2-legged #4 closed ties;

$$S = \frac{0.40}{0.0687} = 5.8"$$

$$S_{max} = min\left(\frac{p_h}{8}, 12, \frac{d}{2}\right) = min\left(\frac{62}{8}, 12, \frac{21.5}{2}\right) = 10.8" > 5.8'' \rightarrow OK$$

Design Example

❑ Solution

➢ Step 4: Determination of Reinforcement

❖ Torsion Reinforcement

5) Longitudinal Reinforcement A_l

$$A_l = \frac{T_u P_h}{\emptyset 2 A_o f_{yl}} = \frac{(28 \times 12)62}{0.75 \times 2 \times 182.96 \times 60} = 1.265\ in^2$$

Check for minimum longitudinal torsional reinforcement

$$A_{l,min} = \frac{5\sqrt{f_c'}\,A_{cp}}{f_y} - max\left(\frac{A_{t(2\ \mathrm{leg})}}{2s}, \frac{25b_w}{f_{yt}}\right)\frac{p_h f_{yt}}{f_y}$$

$$A_{l,min} = \frac{5\sqrt{3000} \times 336}{60{,}000} - max\left(\frac{0.0408}{2}, \frac{25 \times 14}{60{,}000}\right)62\left(\frac{60{,}000}{60{,}000}\right) = 0.269\ in^2$$

$$A_l > A_{l,min} \rightarrow OK$$

Design Example

❑ Solution

➢ Step 5: Detailing of Reinforcement

❖ Total Transverse Reinforcement

- 2-legged #4 closed ties will be provided throughout the span @ 5″ c/c .
- The first stirrup will be placed at a distance S/2 = 2″ from the face of support.

❖ Total Longitudinal Reinforcement

- The bar diameter for longitudinal torsional reinforcement shall be at least greater of (3/8, 0.042s) = 0.21″ and the spacing must not exceed 12 in.
- Using #6 bars with A_b = 0.44 in^2;

Number of bars = 1.265/0.44 = 2.8 ≈ 3.

Design Example

Solution

Step 5: Detailing of Reinforcement

Total Longitudinal Reinforcement

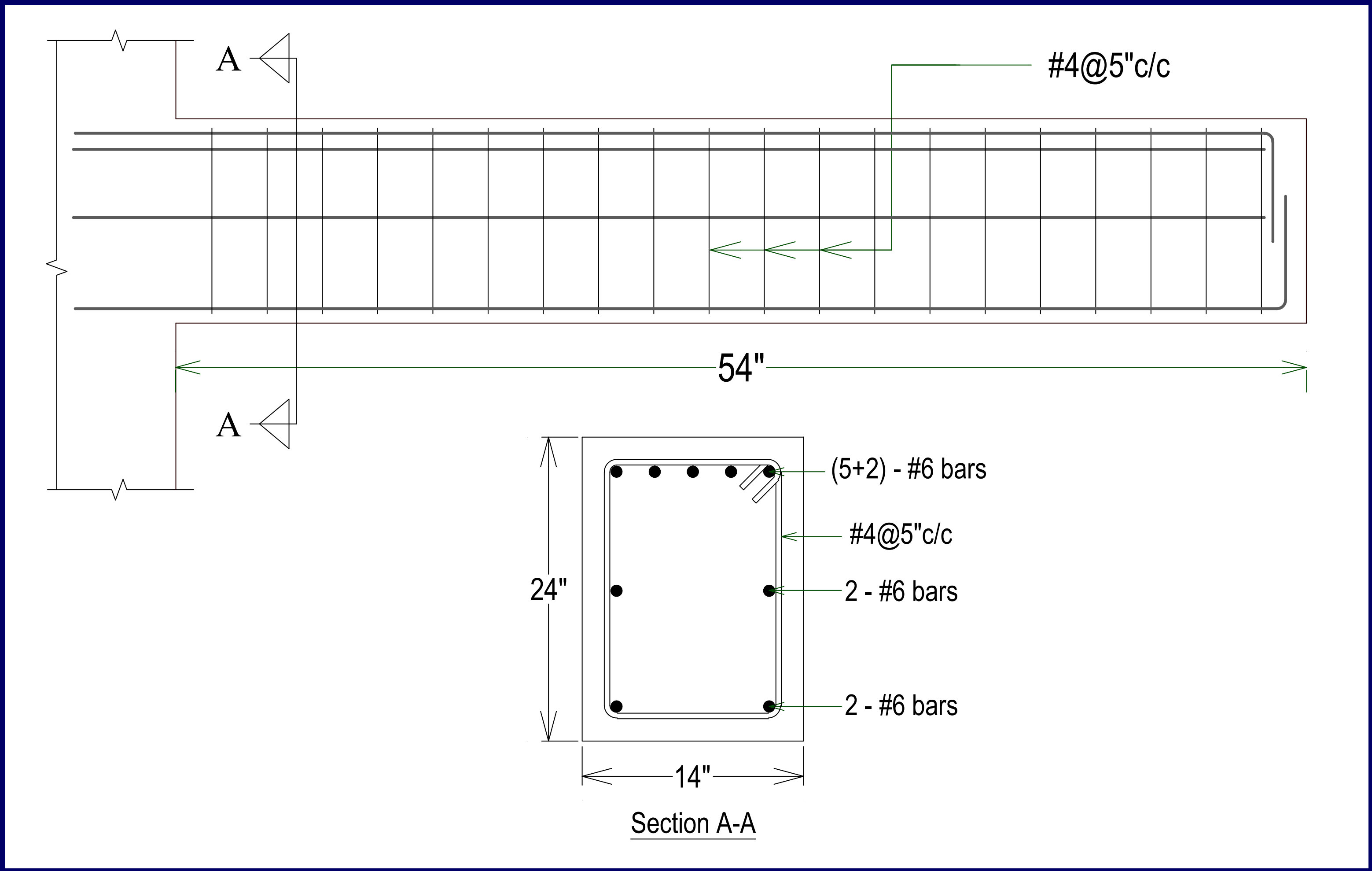
- Reinforcement will be placed at the top, mid depth, and bottom of the member, each level to provide not less than 1.265/3 = 0.422 in^2.
- 2 - #6 bars will be used at mid depth, and reinforcement to be placed for flexure will be increased by 0.422 in2 at the top and bottom of member.
- Final flexural reinforcement is given by

A_s = 2.62 + 0.422 = 3.042 in^2 (7 #6 bars)

Design Example

❑ Solution

➢ Step 6: Drafting



References

- Reinforced Concrete - Mechanics and Design (7th Ed.) by James MacGregor.
- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-19)
- Portland Cement Association (PCA 2002)

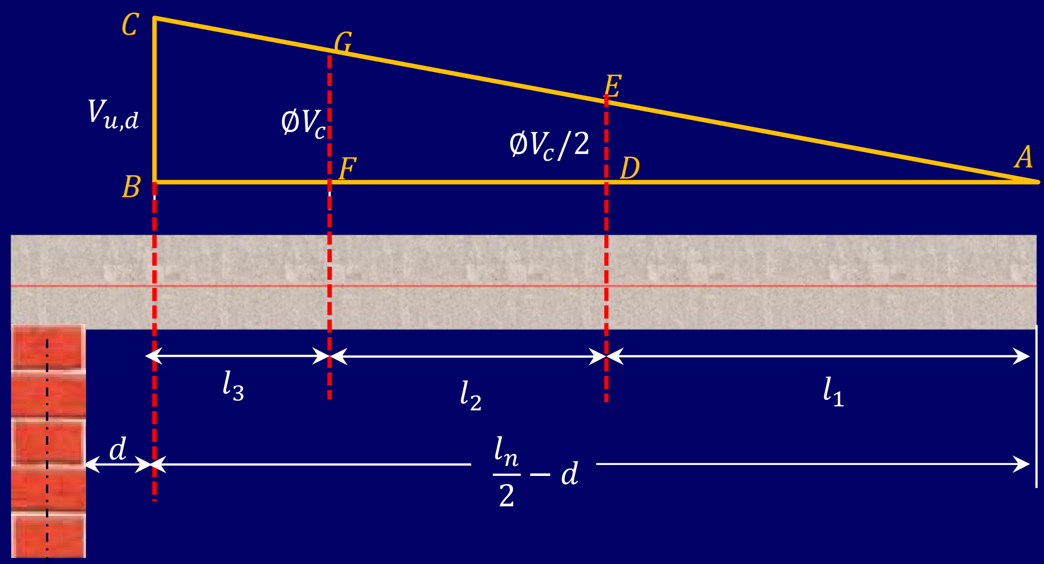
Appendix

Calculation of Distances of Shear Reinforcement Regions

Comparing $\Delta ADE \leftrightarrow \Delta ABC$

$$\frac{AD}{AB} = \frac{DE}{BC} \Rightarrow \frac{l_1}{l_n/2 - d} = \frac{\emptyset V_c/2}{V_{u,d}}$$

$$l_1 = \frac{\emptyset V_c}{2V_{u,d}}\left(\frac{l_n}{2} - d\right)$$



Similarly, comparing $\Delta ADE \leftrightarrow \Delta AFG$

$$\frac{AD}{AF} = \frac{DE}{FG} \Rightarrow \frac{l_1}{l_1 + l_2} = \frac{\emptyset V_c/2}{\emptyset V_c}$$

$$\frac{l_1}{l_1 + l_2} = \frac{1}{2} \Rightarrow \frac{l_1 + l_2}{l_1} = 2 \Rightarrow 1 + \frac{l_2}{l_1} = 2 \;\; which\ gives \;\; l_2 = l_1$$

$$l_3 = \frac{l_n}{2} - d - 2l_1$$

Appendix

Special section properties for Solid Sections

The formulae for calculating geometric parameters related to Torsion Design for solid rectangular and T or L sections are tabulated below.

Parameters	Solid Rectangular Section	Solid T or L Section
A_{cp}	$b_w h$	$b_w h + (b_f - b_w)h_f$
P_{cp}	$2(b_w + h)$	$2(b_f + h)$
A_{oh}	$(b_w - 3.5)(h - 3.5)$	$(b_w - 3.5)(h - 3.5)$
P_h	$2(b_w - 3.5) + 2(h - 3.5)$	$2(b_w - 3.5) + 2(h - 3.5)$