Lecture 03

Design of RC Members for Flexural and Axial Loads (Part – II)

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Section – II

RC Members under Axial and Combined Loads (Columns)

General

Introduction

- A structural member (usually vertical) , used primarily to support axial compressive load is called column.
- However, columns would generally carry bending moments as well, about one or both axes of the cross section.

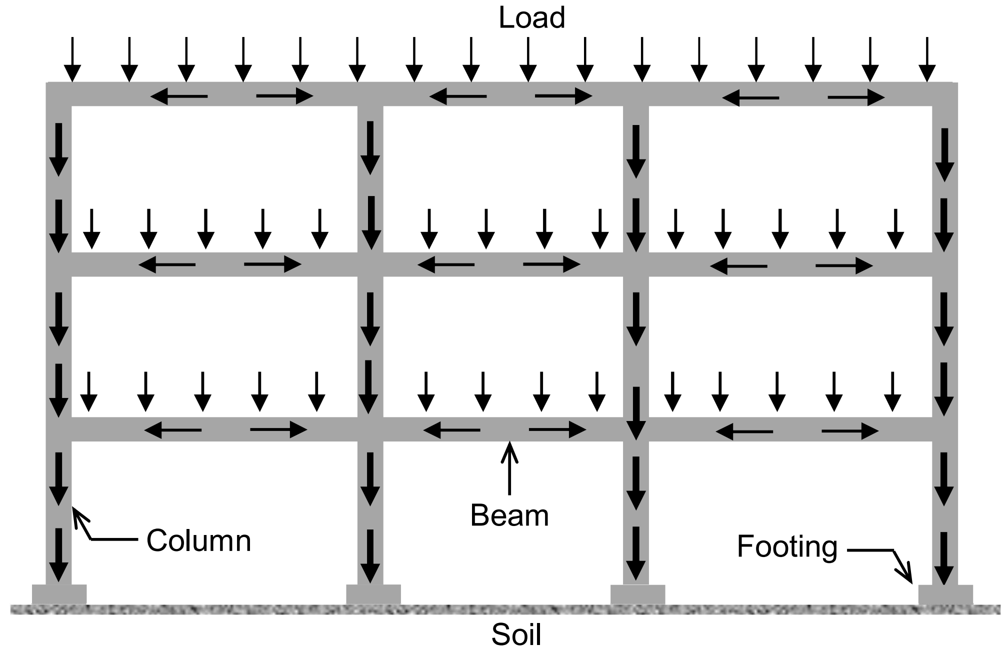
Column for 60-story Bank of America Corporate Center, North Carolina

Column for 20-story Dominion Mall & Apartments, Pakistan (Designed by DQA).

General

❑ Introduction

- Columns transmit loads from upper floor levels to the lower floor levels and ultimately to the ground through the foundations.
- Unlike beams and slabs that carry the load of a single floor, columns bear the load of multiple floors above them, resulting in an accumulation of load.

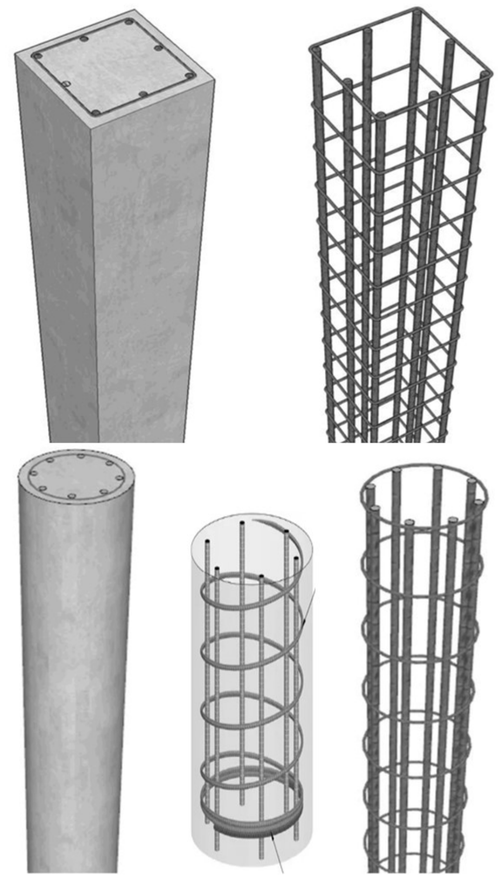


General

❑ Reinforcement in RC Columns

- **Longitudinal Reinforcement**
  - They are provided parallel to the direction of the load to resist the Bending moment as well as the Compression.

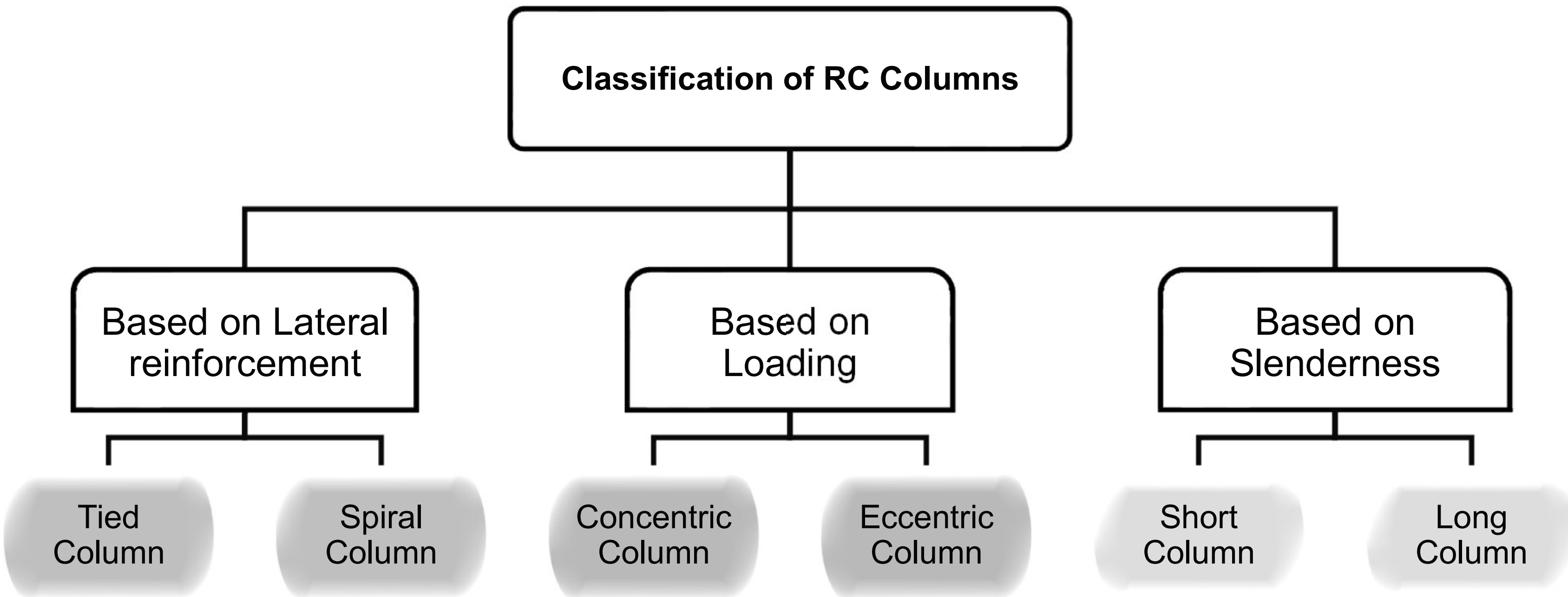
- **Lateral Reinforcement**
  - The lateral reinforcement is provided in the form of ties or continuous spiral to resist Shear and to hold the longitudinal bars.



General

❑ Classification of RC Columns

- RC columns can be classified on various bases as shown below.

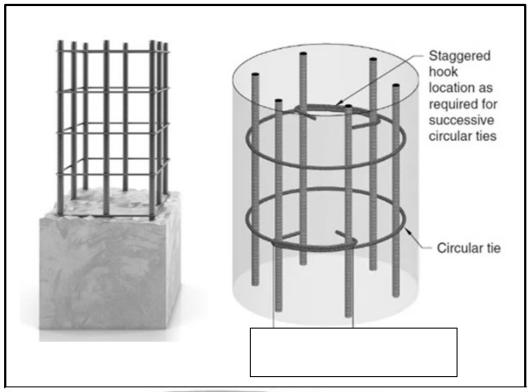


General

❑ Types of RC Columns (based on lateral reinforcement)

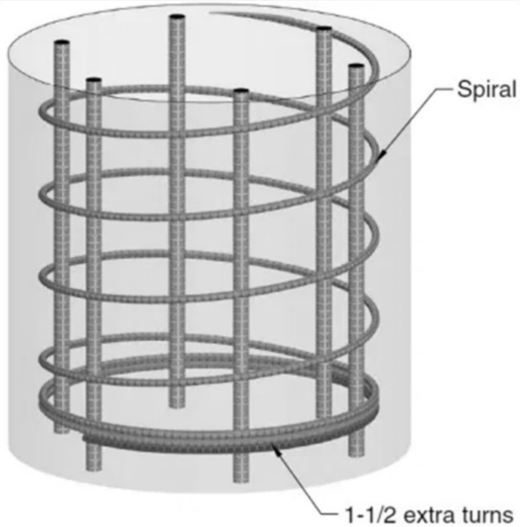
1. Tied Columns

- Columns (of any shape) with closely spaced lateral ties/hoops.



2. Spiral Columns

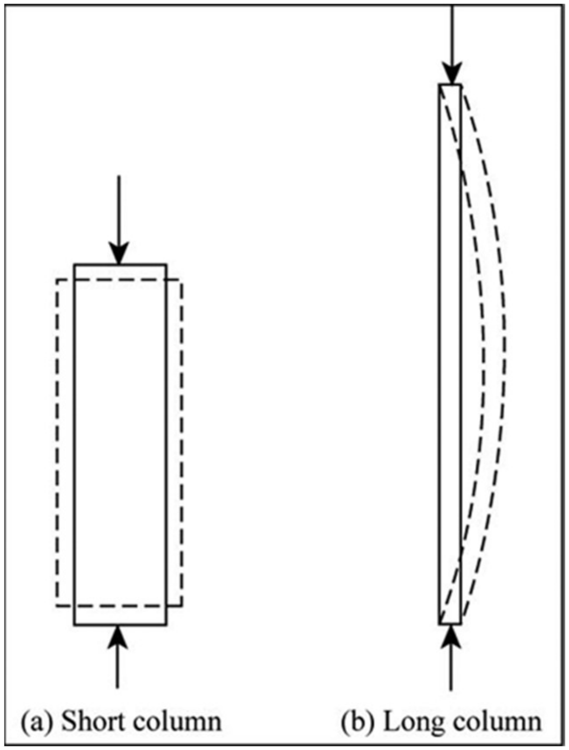
- Columns (of any shape) with continuous spiral reinforcement wound in a helical pattern.
- They are generally more efficient than tied columns.



General

Types of RC Columns (based on slenderness)

1. **Short Columns**
   - Columns that fail due to the failure of materials are called short columns.
   - Most of the concrete columns fall in this category.
2. **Long /Slender columns**
   - Columns in which failure occurs due to geometric instability (buckling) are called long columns.



General

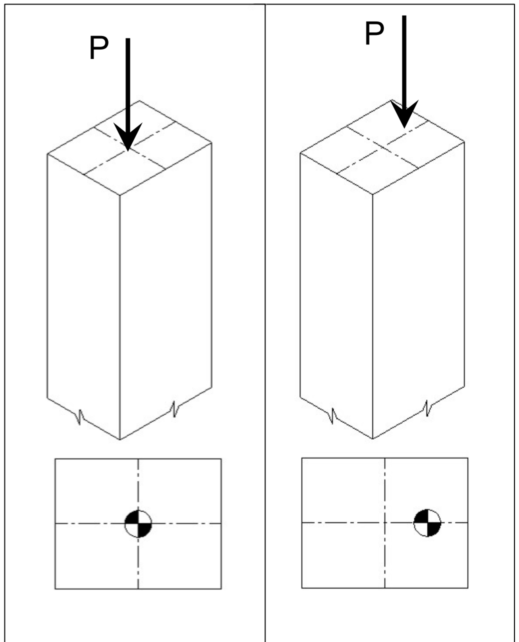
❑ Types of RC Columns (based on loading)

1. Concentric Columns

- Columns in which applied load is aligned with its central axis, resulting in uniform compression throughout the column's cross-section.

2. Eccentric Columns

- Columns in which applied load does not coincide with its central axis, causing an uneven distribution of compression forces across the column's cross-section. They can be
  1. Uniaxially eccentric
  2. Biaxially eccentric



a) Concentric column b) Eccentric column

General

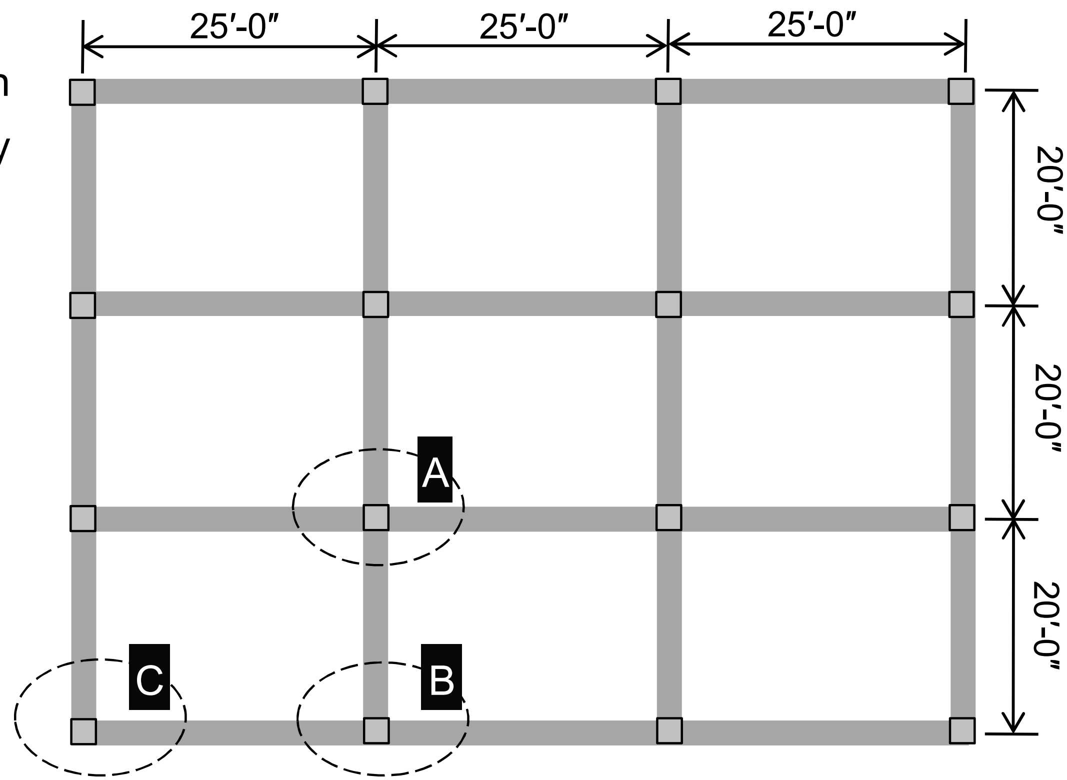
❑ Types of RC Columns (based on loading)

When the spans are equal in both directions and the loading is uniformly distributed then

A) **Interior columns ⇒ Concentric**

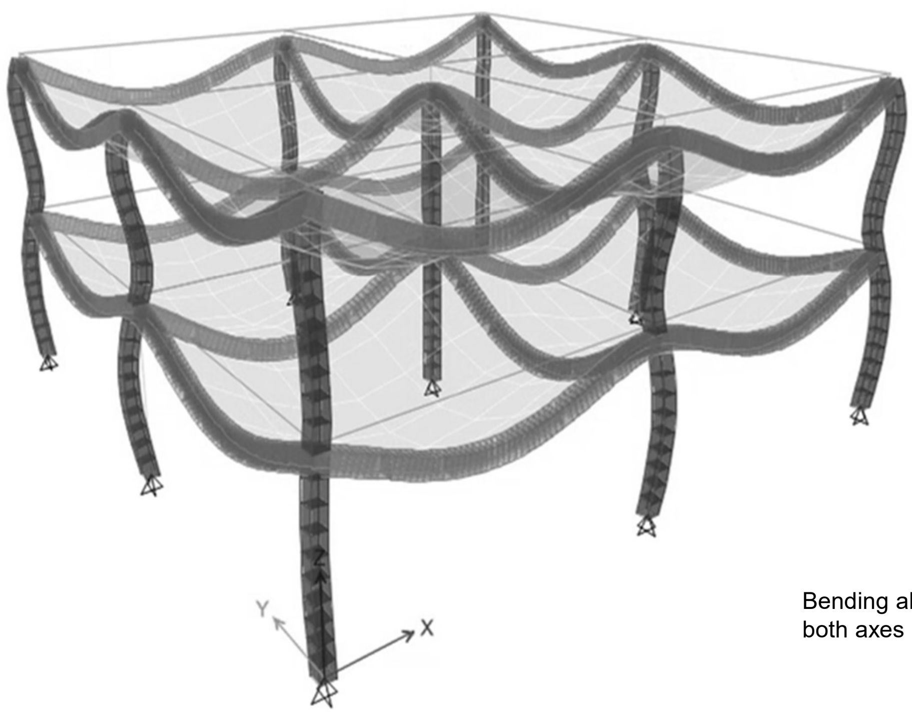
B) **Edge columns ⇒ Uniaxially eccentric**

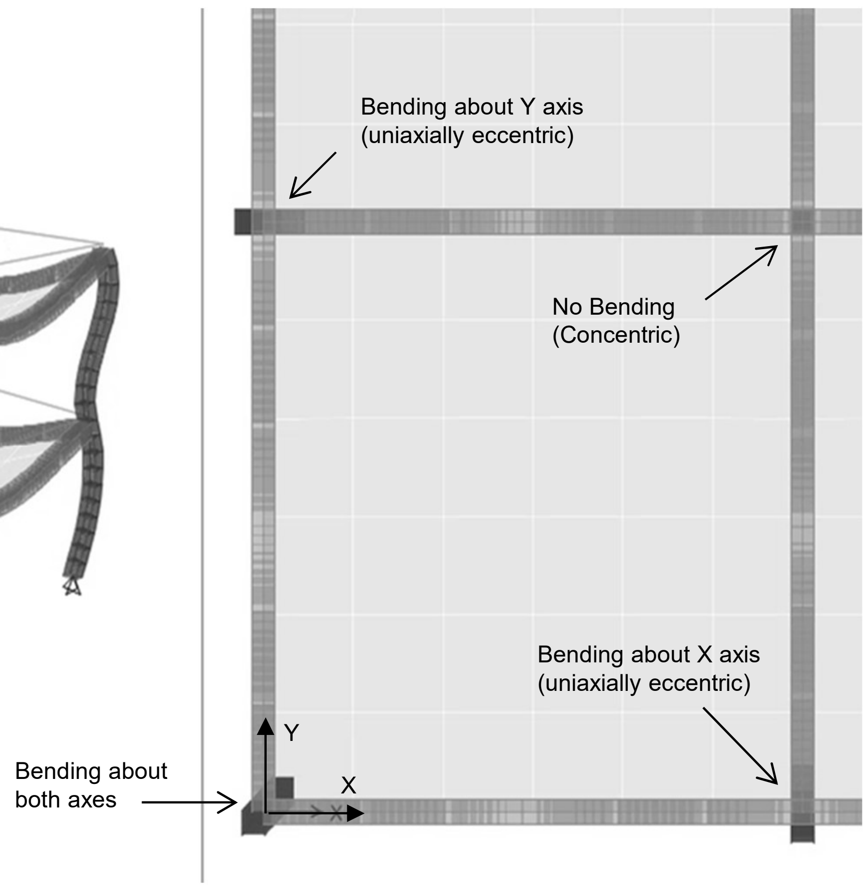
C) **Corner Columns ⇒ Biaxially eccentric**



General

❑ Types of RC Columns (based on loading)





General

❑ Dimensional Limits

- The ACI Code does not specify minimum column sizes for columns that are not part of the seismic-force-resisting system.

❑ Reinforcement Limits

a) Longitudinal reinforcement (ACI 10.6.1.1)

- Area of longitudinal reinforcement shall be at least $0.01A_g$ but shall not exceed $0.08A_g$.
- Minimum Reinforcement is necessary to provide resistance to bending, and to reduce the effects of creep and shrinkage of the concrete under sustained compressive stresses.

General

❑ Reinforcement Limits

a) Longitudinal Reinforcement

- Maximum amount of longitudinal reinforcement is limited to ensure that concrete can be effectively consolidated around the bars.

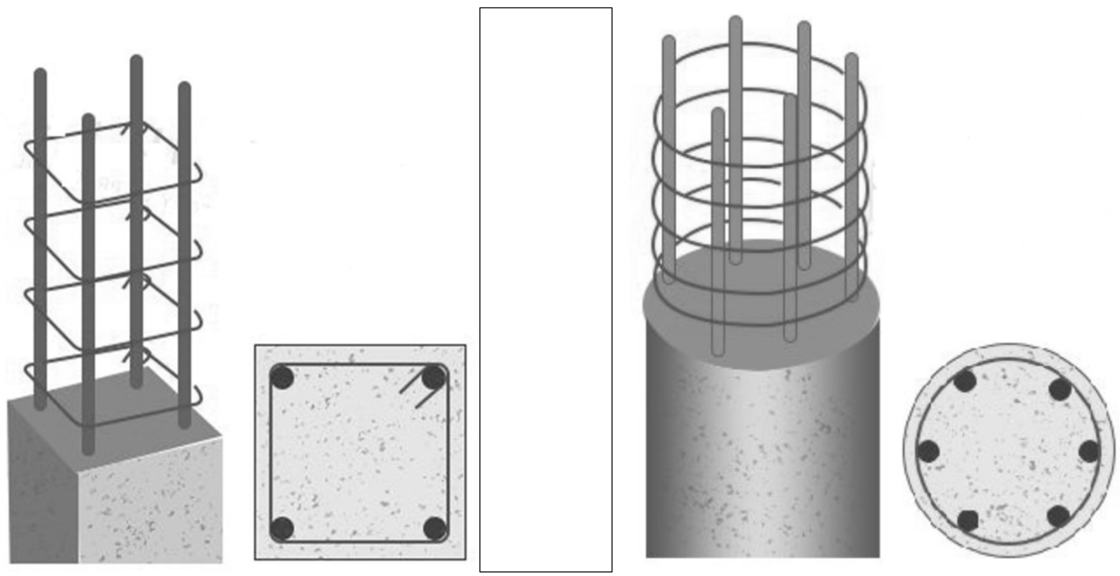
- Longitudinal reinforcement in columns usually does not exceed 4 percent as the lap splice zone will have twice as much reinforcement, if all lap splice occur at the same location.

General

❑ Reinforcement Limits

a) Longitudinal Reinforcement

- Minimum diameter ⇒ #4 (ACI 10.7.3)
- Minimum number of bars ⇒ 4 for rectangular columns
  6 for circular columns.

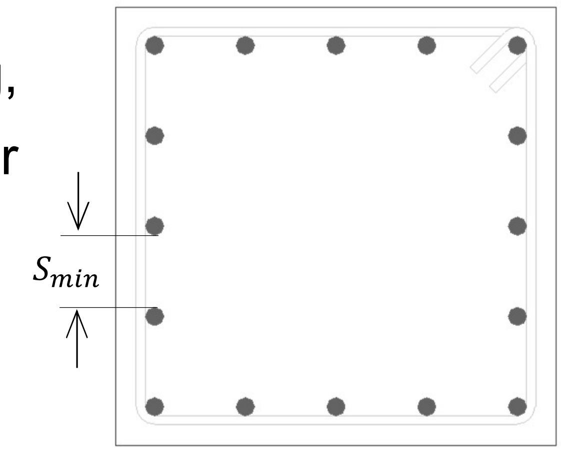


General

❑ Reinforcement Limits

a) Longitudinal Reinforcement

- **Minimum Spacing Between Longitudinal bars (ACI 25.2.3)**
  - Clear spacing between longitudinal bars shall be at least the greatest of; 1.5 in. and $1.5d_b$ (where d_b is the diameter of longitudinal bar).
  - However, to ensure proper concreting, it is better to maintain a minimum clear spacing of 3 inches.



General

❑ Reinforcement Limits

b) Shear Reinforcement

- **Maximum Spacing of Lateral ties (ACI 25.7.2.1)**

- Maximum spacing S_{max} shall not exceed the least of;

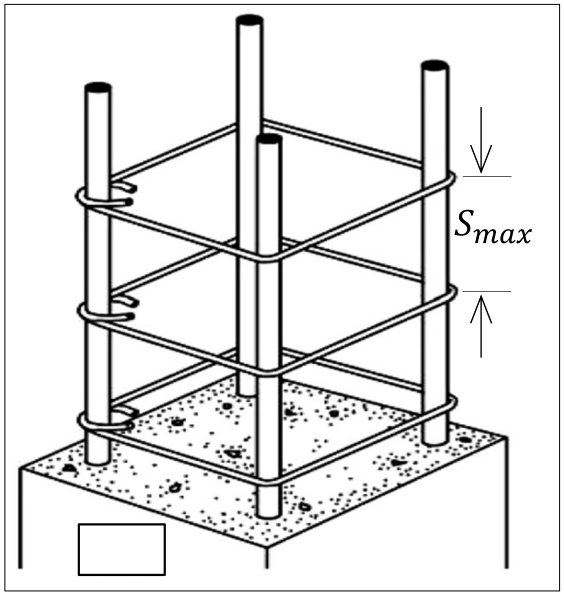
i. $\dfrac{A_v f_y}{50b}$

ii. $\dfrac{A_v f_y}{0.75\sqrt{f_c{}'}b}$

iii. $16d_b$ of longitudinal bar

iv. $48d_h$ of hoop/tie bar

v. Smallest dimension of member



Note: These spacing requirements are for gravity loads only.

General

Reinforcement Limits

b) Shear Reinforcement

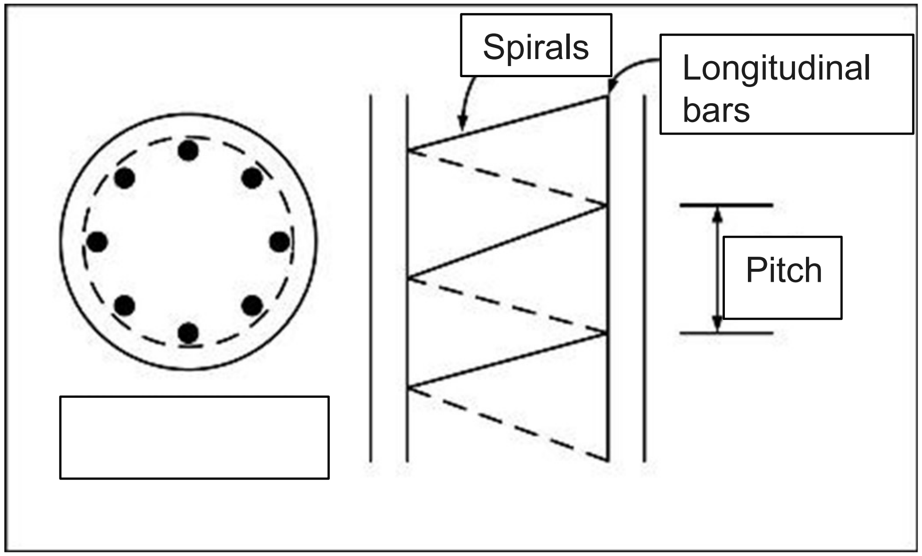
- **Minimum Diameter of Lateral Ties (ACI 25.7.2.2)**
  - Diameter of tie bar shall be at least:
    - i. #3 for longitudinal bars having size up to #10.
    - ii. #4 for longitudinal bars having size larger than #10.

General

❑ Reinforcement Limits

b) Shear Reinforcement

- **Diameter and Spacing of Spiral Reinforcement (ACI 25.7.3)**
  - The minimum spiral reinforcement size is 3/8 in.
  - Spacing/pitch of spiral must not be less than 1 in. and greater than 3 in.

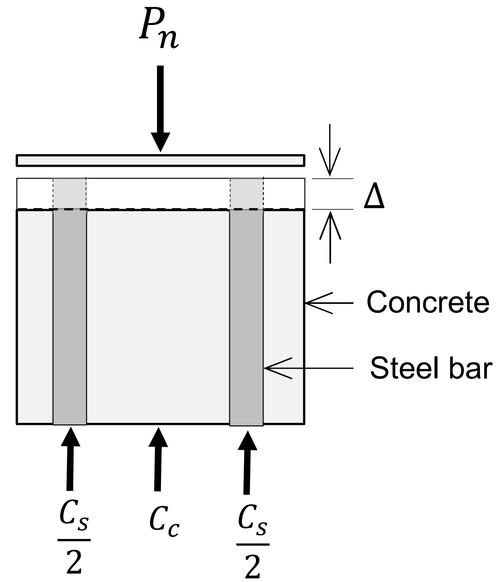


Design of RC Members Under Axial Loads

❑ Axial Capacity

From the figure shown below, we have

$$P_n = C_c + C_s = f_c A_c + f_s A_s$$



Because of the perfect bonding between concrete and steel bars, the strain in both materials will be identical. As a result, steel bars with a grade of 80 or lower will yield at the ultimate stage ($\epsilon_u = 0.003$).

$$f_c = 0.85 f_c' \quad \text{and} \quad f_s = f_y \ \ (for\ f_y \leq 80ksi)$$

$$P_n = 0.85 f_c' A_c + f_y A_s$$

$$\epsilon_{y,40} = \frac{f_y}{E_s} = \frac{40}{29000} = 0.0014 < \epsilon_u = 0.003$$

$$\epsilon_{y,60} = \frac{60}{29000} = 0.0021 < \epsilon_u$$

$$\epsilon_{y,80} = \frac{80}{29000} = 0.0028 < \epsilon_u$$

Design of RC Members Under Axial Loads

❑ Axial Capacity

Taking $A_c = A_g - A_{st}$ the preceding equation becomes

$$P_n = 0.85f_c'(A_g - A_{st}) + f_y A_{st}$$

From which Design Axial capacity can be determined as;

$$\alpha\emptyset P_n = \alpha\emptyset\left[0.85f_c'(A_g - A_{st}) + f_y A_{st}\right]$$ (for tied column)

$$\alpha\emptyset P_n = \alpha\emptyset\left[0.85f_c'(A_g - A_{st}) + f_y A_{st}\right]$$ (for spiral column)

where;

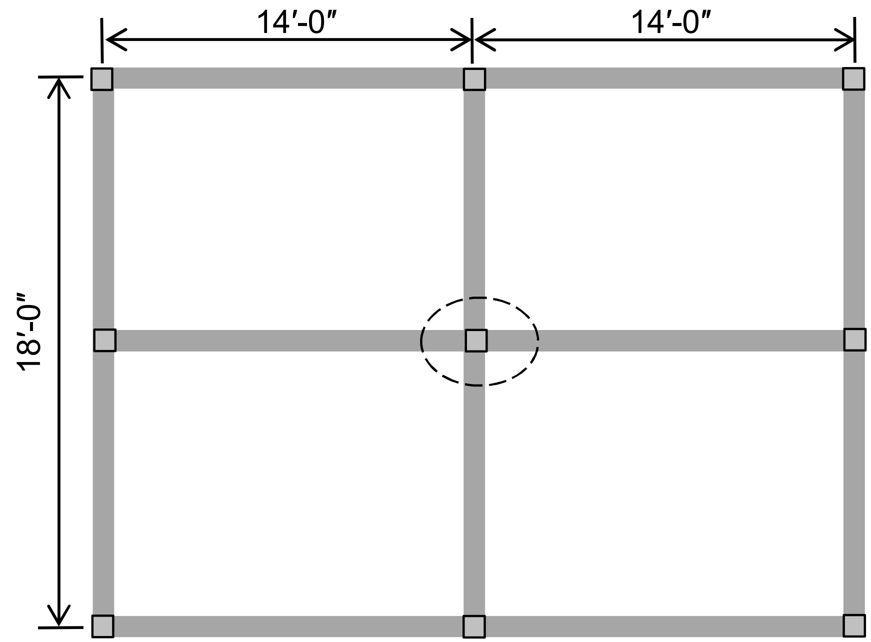
$\emptyset = 0.65$ for tied columns and 0.75 for spiral columns (ACI Table 21.2.2)

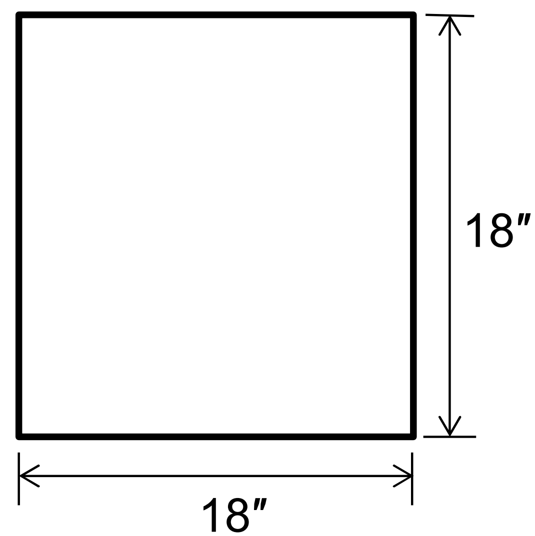
$\alpha = 0.8$ for tied columns and 0.85 for spiral columns.

Design of RC Members Under Axial Loads

❑ Example 3.7

- ***Design*** the interior column shown in figure to support a factored axial compressive load of 500 kips. The specified material strengths are; $f_c' = 3$ ksi and $f_y = 60$ ksi.





Design of RC Members Under Axial Loads

❑ Solution

- **Given Data**

$b = 18''$

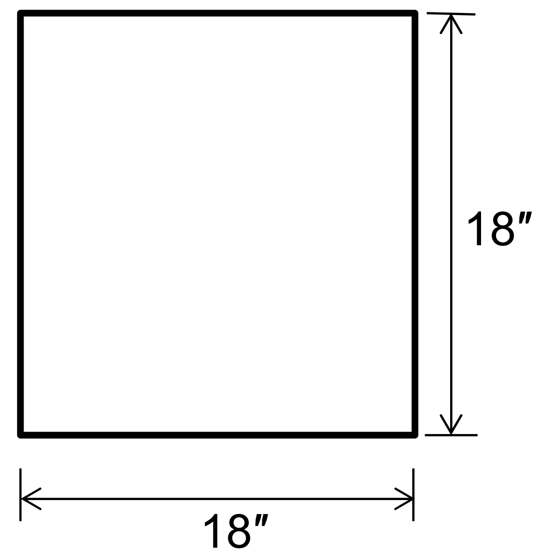
$h = 18''$

$A_g = 18'' \times 18'' = 324\ in^2$

$P_u = 500\ kip$

$f_c' = 3\ ksi$

$f_y = 60\ ksi$



- **Required Data**

Design the column for the given axial load

Design of RC Members Under Axial Loads

❑ Solution

➢ **Step 1: Determination of Longitudinal Reinforcement**

Calculate design axial capacity of column by assuming 1% steel area and compare the calculated capacity with demand axial load.

$A_{st} = 0.01A_g$

$\alpha\emptyset P_n = 0.80 \times 0.65\left[0.85 \times 3\,(A_g - 0.01A_g) + (60)0.01A_g\right] = 1.625A_g$

$\alpha\emptyset P_n = 1.625(324) = 526.5 \text{ kip} > P_u \rightarrow$ OK!

Therefore, $A_{st} = 0.01A_g = 0.01(324) = 3.24\ in^2$

Design of RC Members Under Axial Loads

❑ Solution

➢ Step 2: Determination of Longitudinal Reinforcement

Using #6 bar with $A_b = 0.44 in^2$

Number of bars $= \frac{A_s}{A_b} = \frac{3.24}{0.44} = 7.36 \approx 8$

Hence use 8,#6 bars.

Note:

- To maintain the symmetrical distribution along the perimeter of the cross-section, the number of bars in a square column should be a multiple of 4.
- The configuration may alter for a rectangular or circular column.

Design of RC Members Under Axial Loads

❑ Solution

➢ Step 2: Detailing of Lateral / shear Reinforcement

Using #3 bar with $A_b = 0.11in^2$, maximum spacing S_{max} is the least of:

i. $\frac{A_v f_y}{50b}$ = 0.22 x 60,000/ (50x18) = 14.6″

ii. $\frac{A_v f_y}{0.75\sqrt{f_c{}'}b}$ = 0.22 x 60,000 / (0.75$\sqrt{3000}$ x18) = 17.9″

iii. $16d_b$ of longitudinal bar = 16 x 0.75 = 12″

iv. $48d_h$ of hoop/tie bar = 48 x 3/8 = 18″

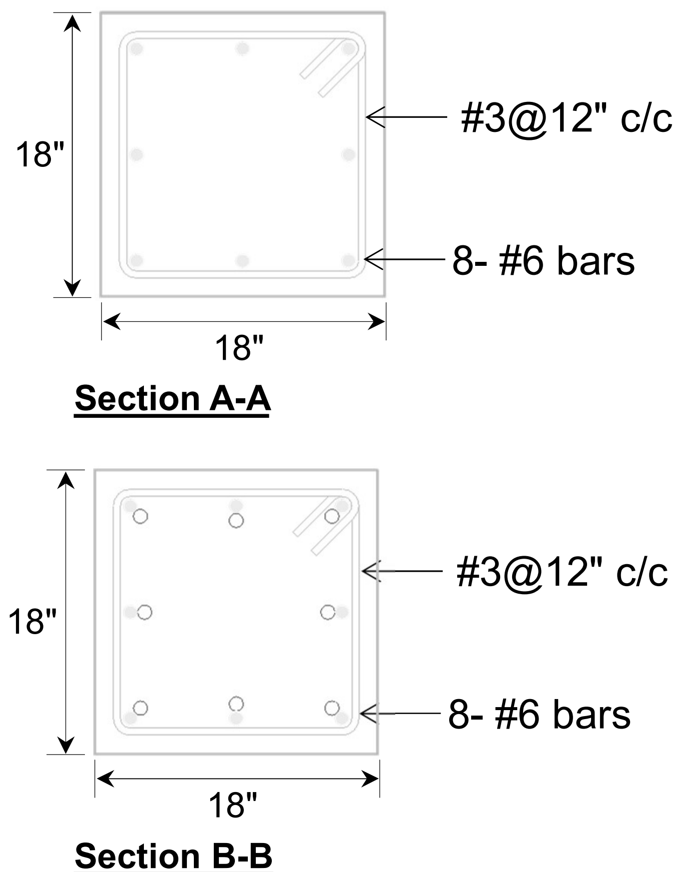
v. Smallest dimension of member = 18″

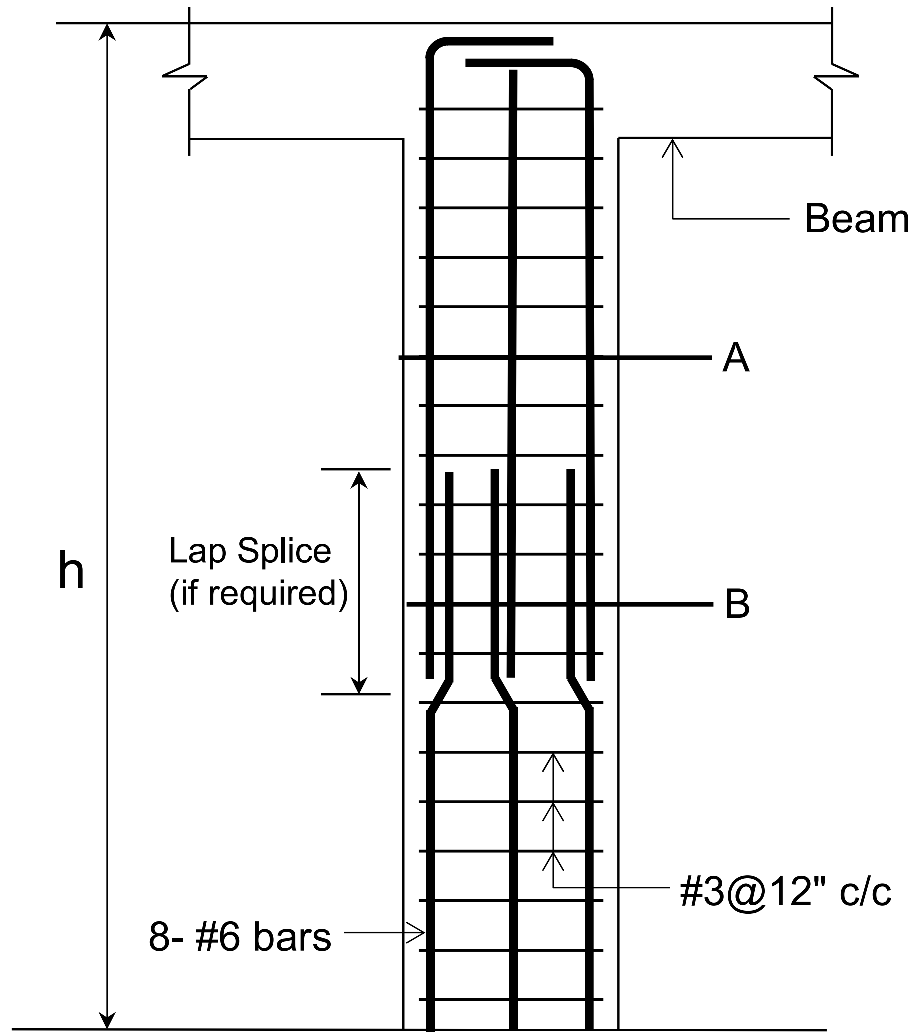
Therefore, $S_{max} = 14.6″$. Finally provide #3 ties @ 12″ c/c

Design of RC Members Under Axial Loads

- **Solution**

  - **Step No.3: Drafting**

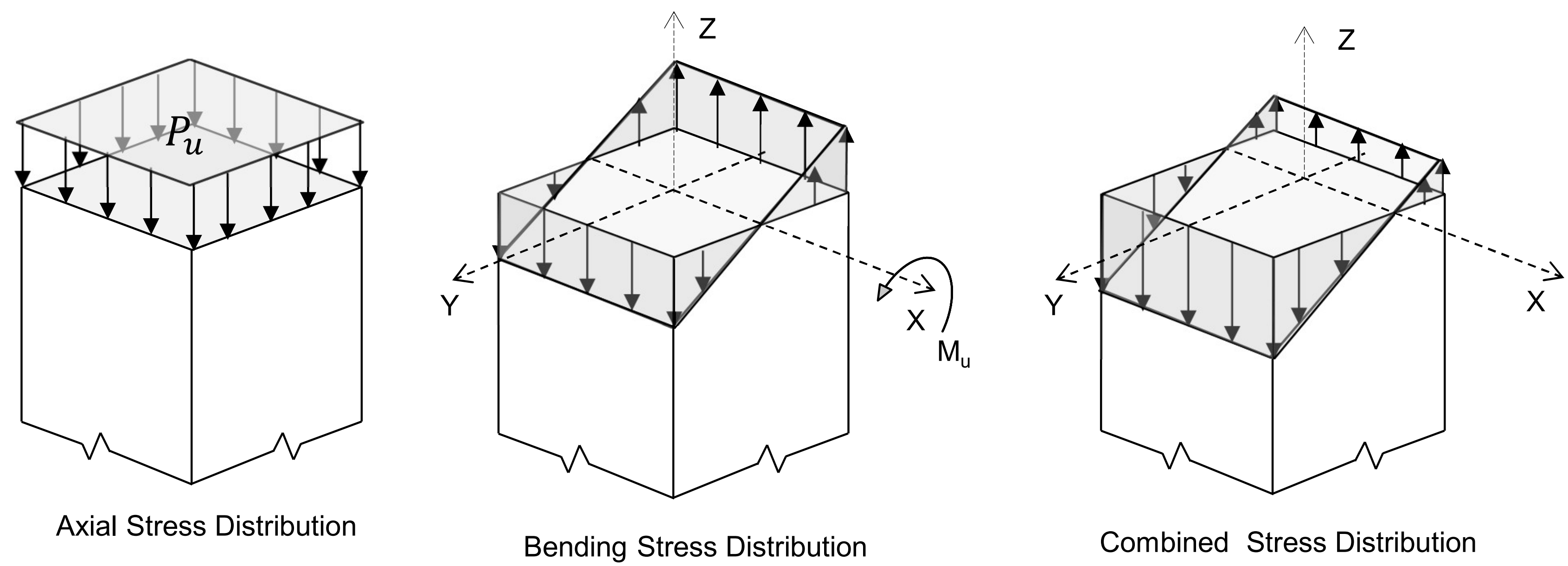




Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Introduction

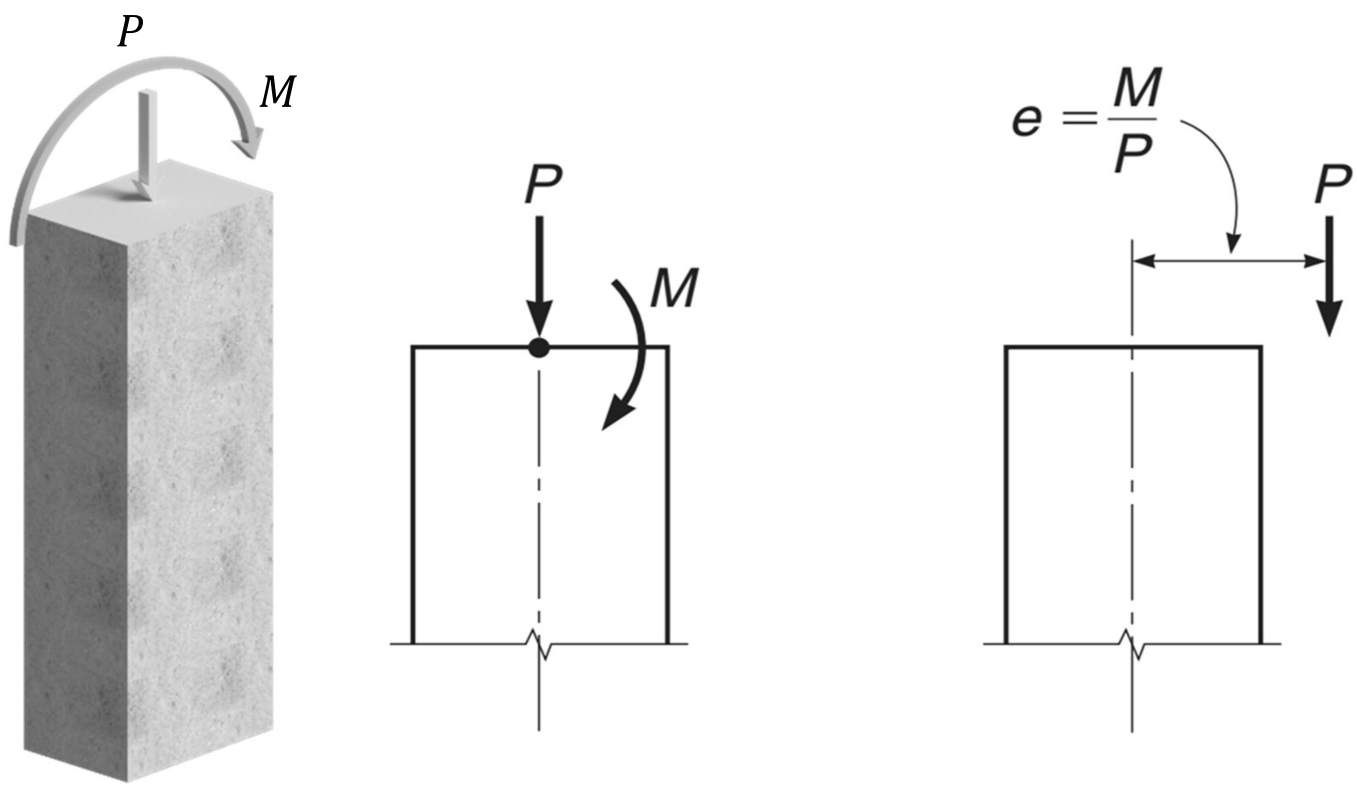
- An eccentrically loaded column is one that is subjected to both axial load and bending moment simultaneously.
- As a result, combined stresses are induced in the section as shown below.



Design of RC Members Under Axial Loads with Uniaxial Bending

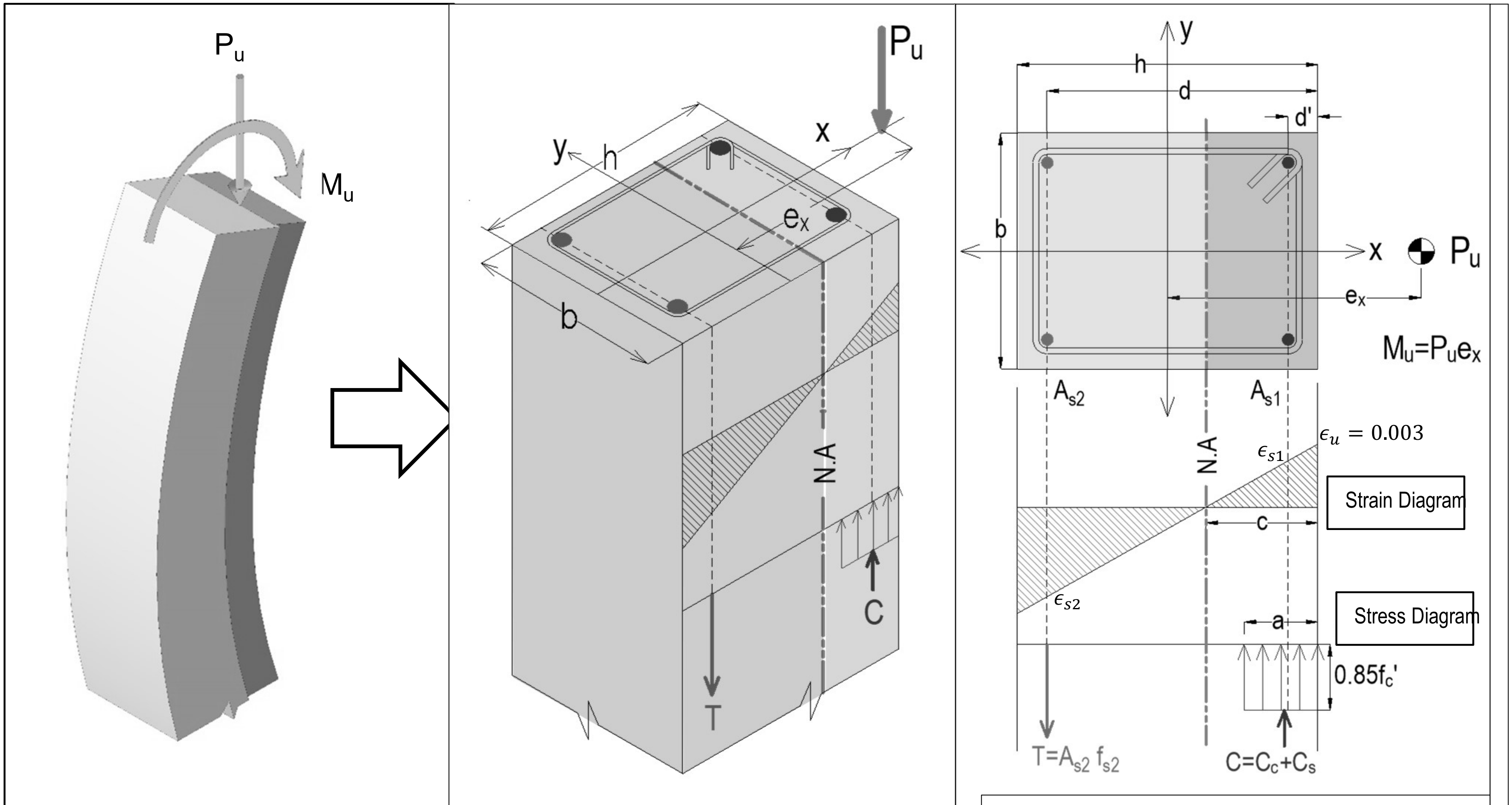
❑ Introduction

- To simplify the computations, this coupled action can be transformed into P and the equivalent eccentricity e.



Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Introduction



Bending moment can be transformed into eccentricity using $M_u = P_u\, e_x$

Red Region: Compression
Blue Region : Tension

Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Calculation of Capacity

a. Axial Capacity

From the Figure;

$$P_n = C_c + C_s - T_s$$

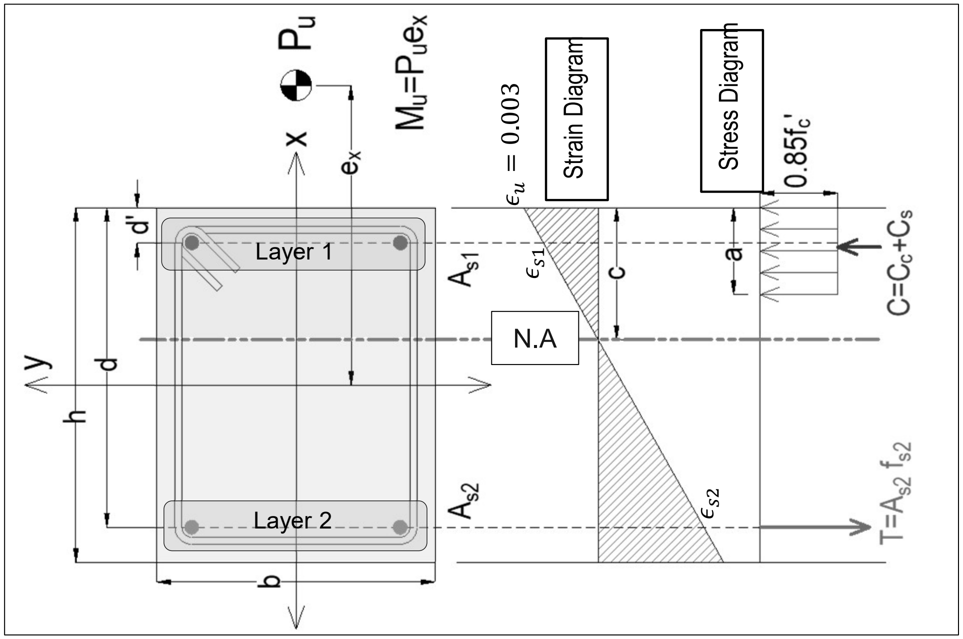
$$P_n = 0.85 f_c' ab + f_{s1} A_{s1} - f_{s2} A_{s2}$$

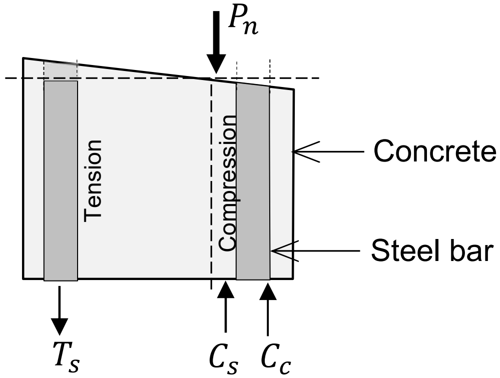
$$P_n = 0.85 f_c' \beta_1 cb + A_s (f_{s1} - f_{s2})$$

Taking $\beta_1 = 0.85$ gives

$$\emptyset P_n = \emptyset [0.72 f_c' bc + A_s (f_{s1} - f_{s2})] \quad \text{---- (3.3)}$$

(Note that A_s is steel area of a SINGLE layer, not the total steel area)



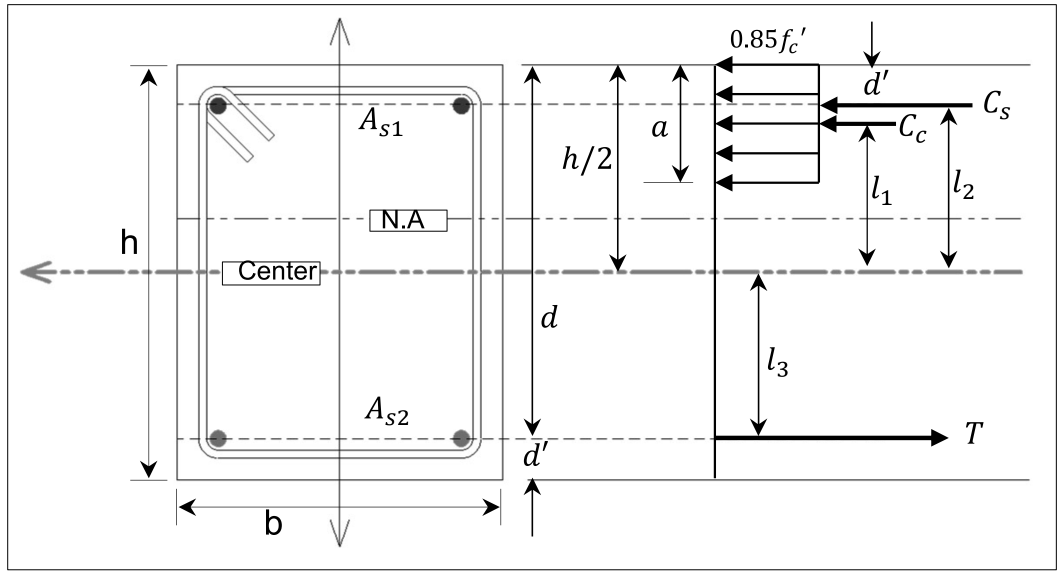


Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Calculation of Capacity

b. Flexural Capacity

$$M_n = C_c l_1 + C_s l_2 + T_s l_3$$



From figure;

$$l_1 = \frac{h}{2} - \frac{a}{2}$$

$$l_2 = \frac{h}{2} - d'$$

$$l_3 = \frac{h}{2} - d'$$

Now, taking moment about the center of section,

$$M_n = C_c\left(\frac{h}{2} - \frac{a}{2}\right) + C_s\left(\frac{h}{2} - d'\right) + T_s\left(\frac{h}{2} - d'\right)$$

Where;

$$C_c = 0.85f_c'ab = 0.85f_c'\beta_1 bc$$

$$C_s = A_{s1}f_{s1}$$

$$T_s = A_{s2}f_{s2}$$

Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Calculation of Capacity

b. Flexural Capacity

$$M_n = 0.85 f_c' \beta_1 bc\left(\frac{h}{2} - \frac{a}{2}\right) + A_{s1} f_{s1}\left(\frac{h}{2} - d'\right) + A_{s2} f_{s2}\left(\frac{h}{2} - d'\right)$$

Since $A_{s1} = A_{s2} = A_s$, therefore

$$M_n = \frac{0.85^2}{2} f_c' bc(h - a) + A_{s1}(h/2 - d')(f_{s1} + f_{s2})$$

$$M_n = 0.36 f_c' bc(h - 0.85c) + A_s(h/2 - d')(f_{s1} + f_{s2})$$ [taking $\beta = 0.85$]

From which the design flexural capacity is determined as,

$$\emptyset M_n = \emptyset[0.36 f_c' bc(h - 0.85c) + A_s(h/2 - d')(f_{s1} + f_{s2})]$$ ---- (3.4)

Design of RC Members Under Axial Loads with Uniaxial Bending

Calculation of Capacity

- **Calculation of Normal Stresses in Steel (f_{s1} and f_{s2})**

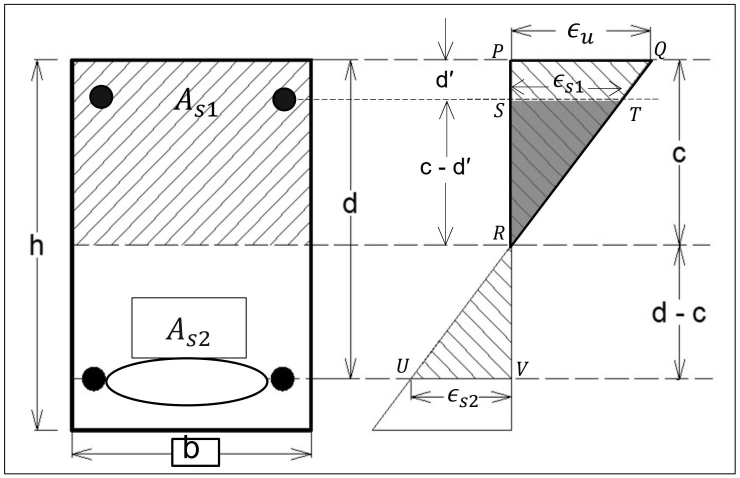
  - Compressive stress f_{s1}

$$f_{s1} = E_s \epsilon_{s1}$$

From $\Delta PQR \leftrightarrow \Delta STR$, we have

$$\frac{\epsilon_{s1}}{c - d'} = \frac{\epsilon_u}{c} \Rightarrow \epsilon_{s1} = \frac{\epsilon_u (c - d')}{c}$$

$$f_{s1} = E_s \frac{\epsilon_u (c - d')}{c}$$



Substituting $E_s = 29000$ ksi and $\epsilon_u = 0.003$, we get

$$f_{s1} = 87\left(1 - \frac{d'}{c}\right)$$

Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Calculation of Capacity

- **Calculation of Normal Stresses in Steel (f_{s1} and f_{s2})**

  - Tensile stress f_{s2}

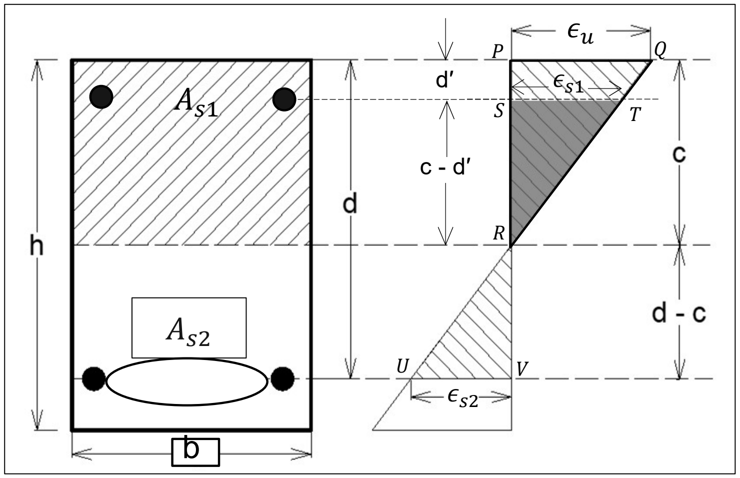
$$f_{s2} = E_s \epsilon_{s2}$$

From $\Delta PQR \leftrightarrow \Delta VUR$, we have

$$\frac{\epsilon_{s2}}{\mathrm{d-c}} = \frac{\epsilon_u}{c} \Rightarrow \epsilon_{s2} = \frac{\epsilon_u(d-c)}{c}$$

$$f_{s2} = E_s \frac{\epsilon_u(d-c)}{c}$$

$$f_{s2} = 87\left(\frac{d}{c} - 1\right)$$



Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Limitations of Equations 3.3 and 3.4

- It is important to note that equations 3.3 and 3.4 are valid for

  1. Two layers of reinforcement.
  2. $f'_c \leq 4000$ psi (since $\beta_1 = 0.85$ was used)

- For intermediate layers of reinforcement, the corresponding terms with "A_s" shall be added in the equations.

Design of RC Members Under Axial Loads with Uniaxial Bending

Design Approaches

- In case of flexural members (with no or negligible axial load), the flexural capacity is expressed as:

$$\emptyset M_n = \emptyset A_s f_y \left(d - \frac{a}{2}\right) \; ; \; a = \frac{A_s f_y}{0.85 f_c' b}$$

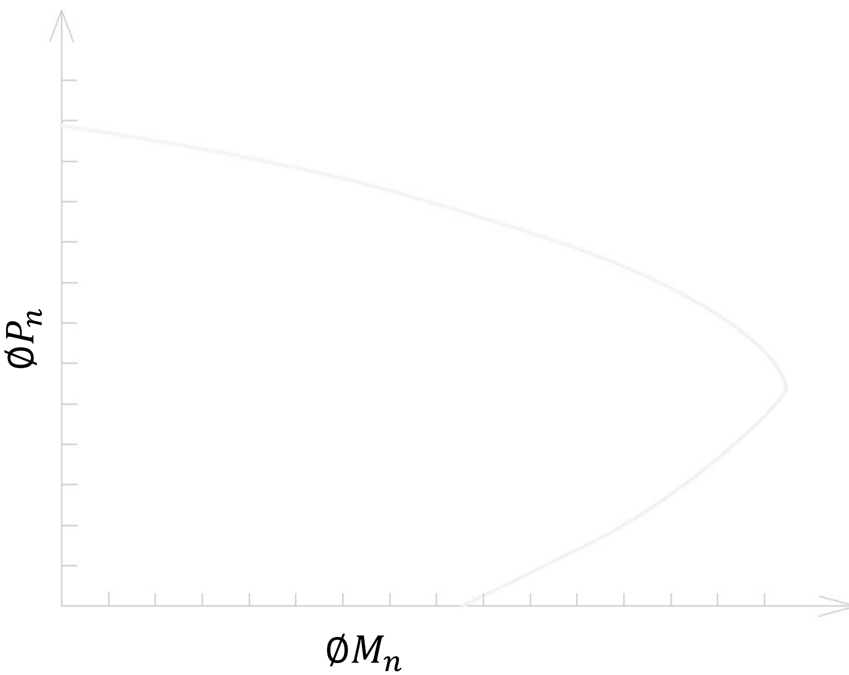
- However, such straightforward equations cannot be derived when members are subjected to combined loading.
- This is because the flexural and axial capacities are inherently coupled (dependent on each other) and cannot be separately dealt with. Consequently, for such members, two commonly used approaches are:

1. Interaction Diagram 2. Design Aids

Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Interaction Diagram

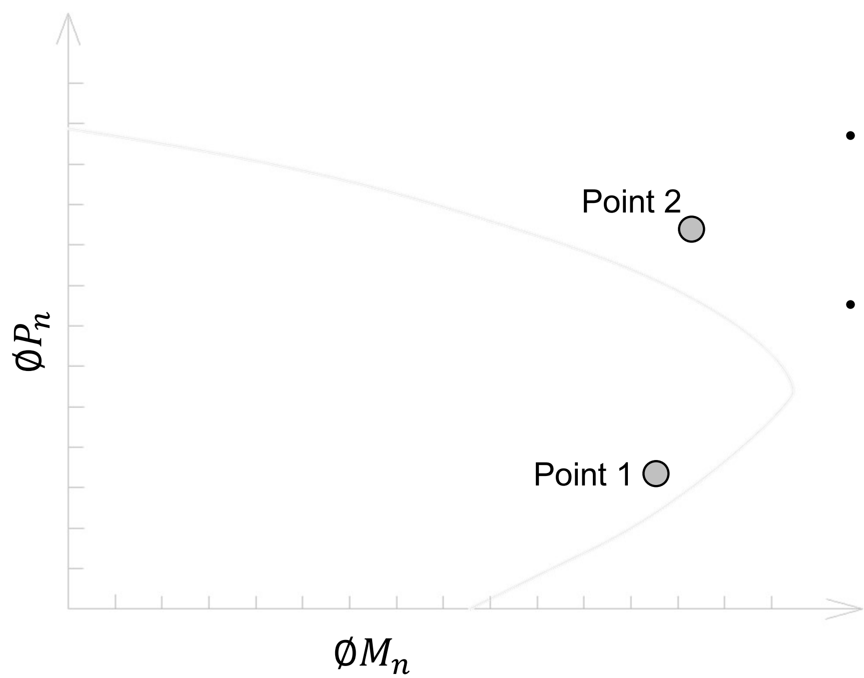
- A graphical representation that shows the interaction/relationship between axial capacity and flexural capacity of a structural member having known material properties, dimensions and reinforcement is called Interaction diagram or Capacity curve.



Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Interaction Diagram

- If the factored demand in the form of P_u and M_u lies inside or at the border line of the design interaction diagram, the column will be deemed safe against the given demand, otherwise it is failed.

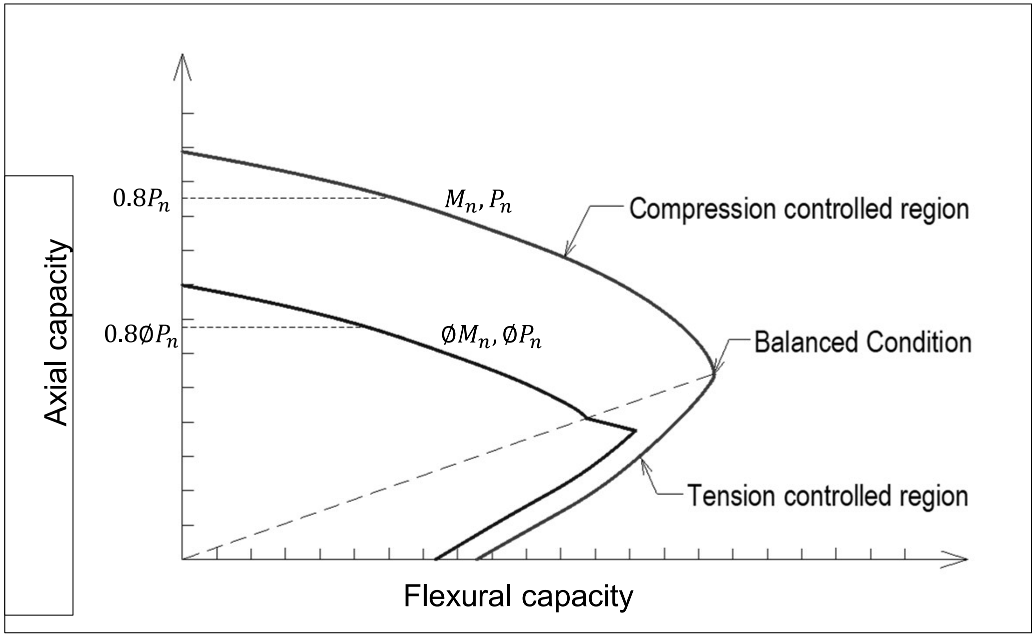


- **Point 1** lies within the curve, indicating that the column is safe against the demand.
- **Point 2** falls outside the curve, showing that the column's capacity is insufficient to carry the given demand.

Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Interaction Diagram

- The horizontal cutoff at upper end of the curve at a value of $\alpha \emptyset P_n$ represents the maximum design load specified in the ACI 318-19 10.4.2.1 for small eccentricities i.e., large axial loads.



Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Interaction Diagram

- Linear Variation of Strength Reduction Factor ∅

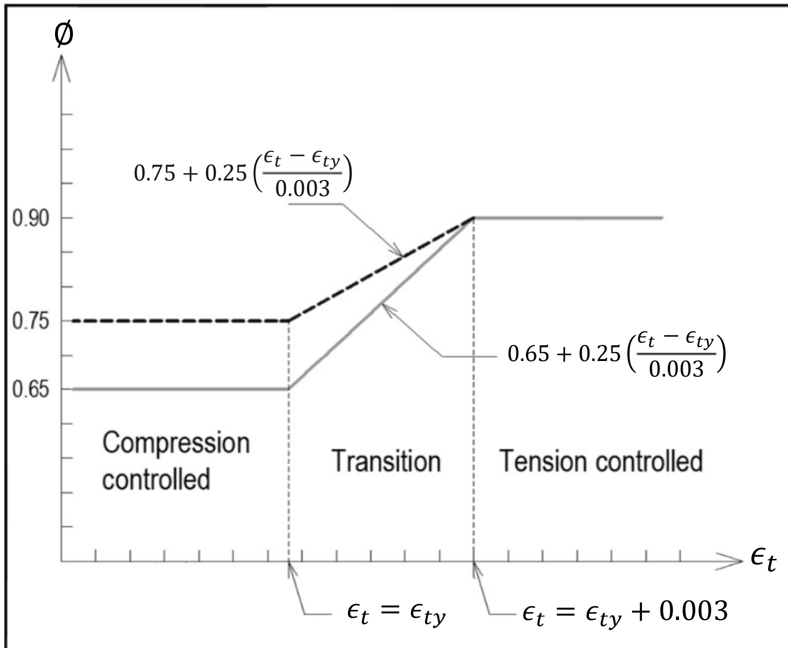
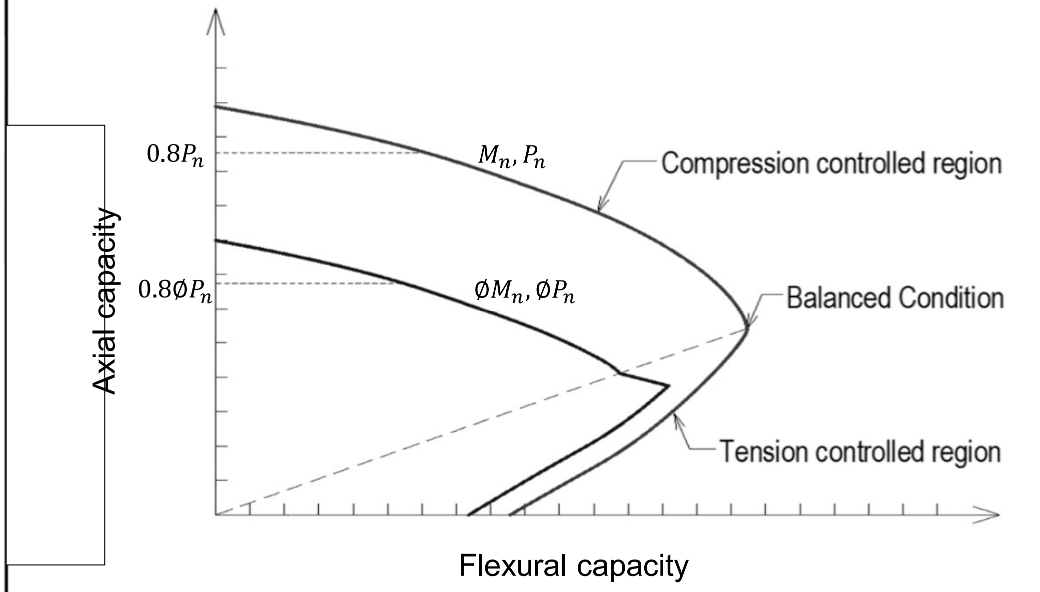


Fig. R21.2.2b



Interaction Diagram

Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Development of Interaction Diagram

- The interaction diagram can be developed by calculating certain points at key locations, using different values of *c*. These points are obtained from equations 3.3 and 3.4 as described below.

$$\emptyset P_n = \emptyset[0.72 f_c' bc + A_s(f_{s1} - f_{s2})]$$

$$\emptyset M_n = \emptyset[0.36 f_c' bc(h - 0.85c) + A_s(h/2 - d')(f_{s1} + f_{s2})]$$

$$f_{s1} = 87\left(1 - \frac{d'}{c}\right) \le f_y$$

$$f_{s2} = 87\left(\frac{d}{c} - 1\right) \le f_y$$

For a given set of material properties (f_c', f_y), dimensions (b, h, d, d') and area of reinforcement (A_s), the only variable that remains unknown is the depth of the neutral axis, c.

Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Development of Interaction Diagram

- Point 1 is determined using equation of concentrically loaded column ignoring α factor. $\emptyset P_n = \emptyset\left[0.85 f_c{}'(A_g - A_{st}) + f_y A_{st}\right]$
- All other control points can be obtained using the following 3 steps.

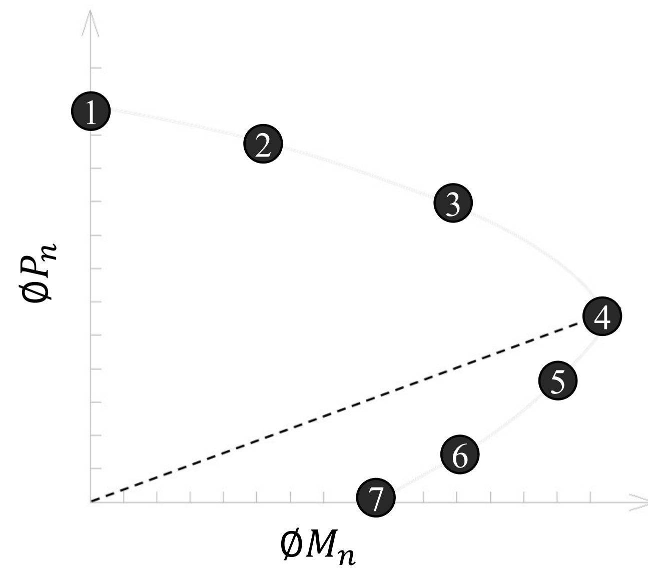
1. Assume reasonable value of c.
2. Compute f_{s1} and f_{s2}

$$f_{s1} = 87\left(1 - \frac{d'}{c}\right) \leq f_y \text{ and } f_{s2} = 87\left(\frac{d}{c} - 1\right) \leq f_y$$

3. Calculate $\emptyset P_n$ and $\emptyset M_n$

$$\emptyset P_n = \emptyset[0.72 f_c' bc + A_s(f_{s1} - f_{s2})]$$

$$\emptyset M_n = \emptyset[0.36 f_c' bc(h - 0.85c) + A_s(h/2 - d')(f_{s1} + f_{s2})]$$

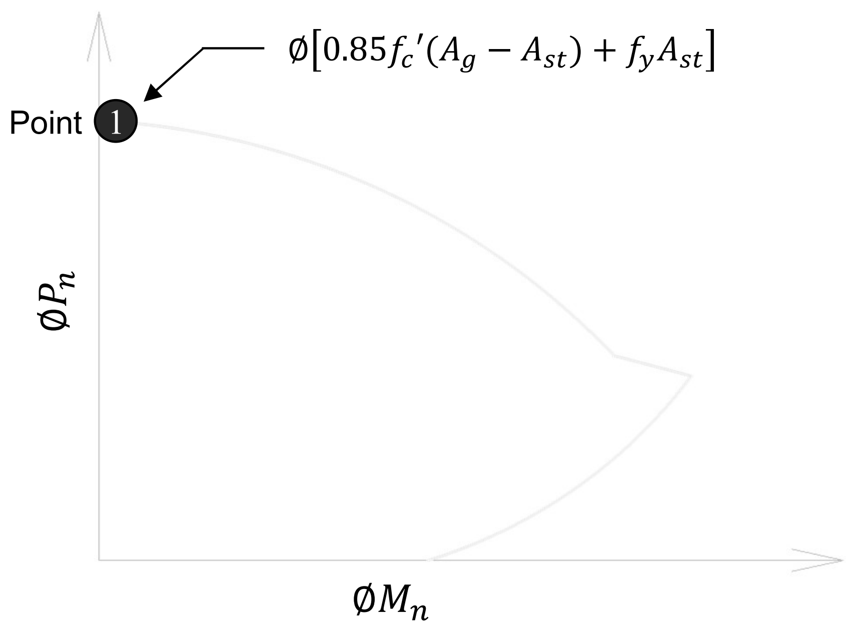


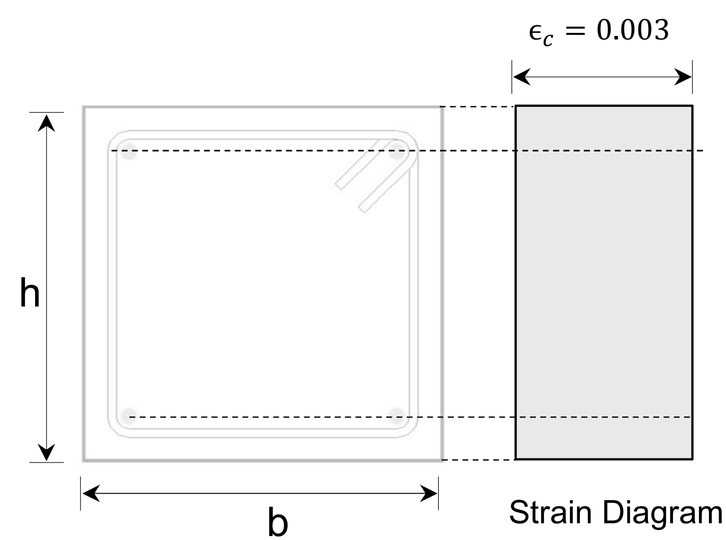
Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Development of Interaction Diagram

❖ Point 1

- Point representing capacity of column when concentrically loaded.
- This is the point at which M_n = 0.
- Design axial capacity equation of concentric column will be used.



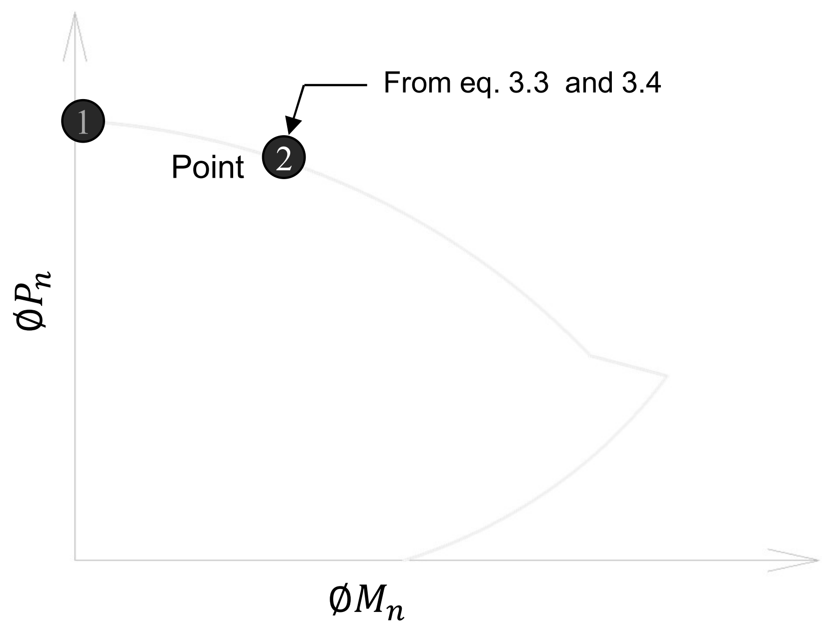


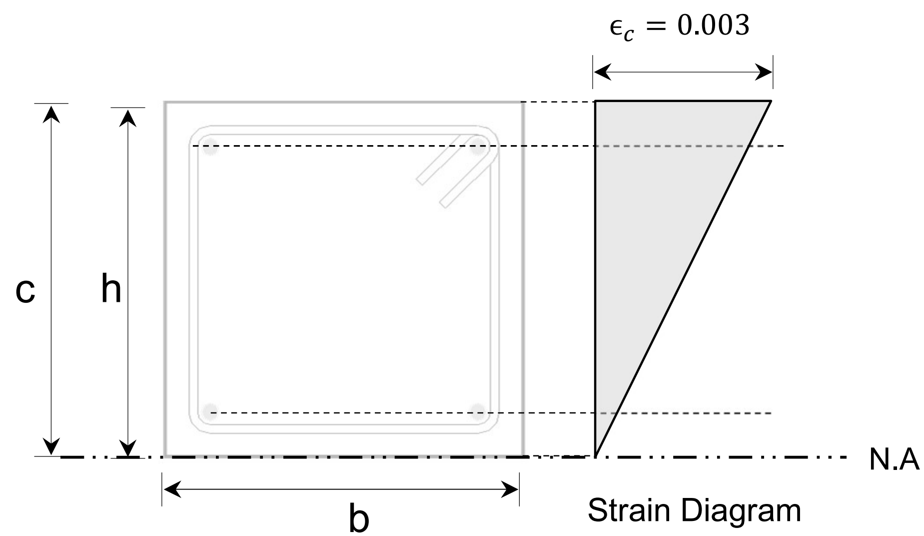
Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Development of Interaction Diagram

❖ Point 2

- This point corresponds to crushing of the concrete at the compression face of the section and zero stress at the other face.
- $c = h$ and $\emptyset = 0.65$



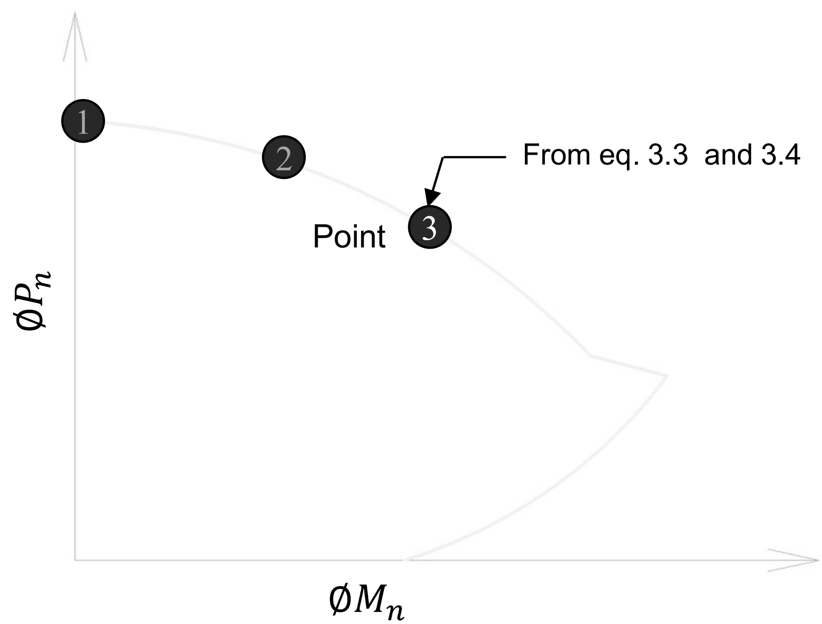


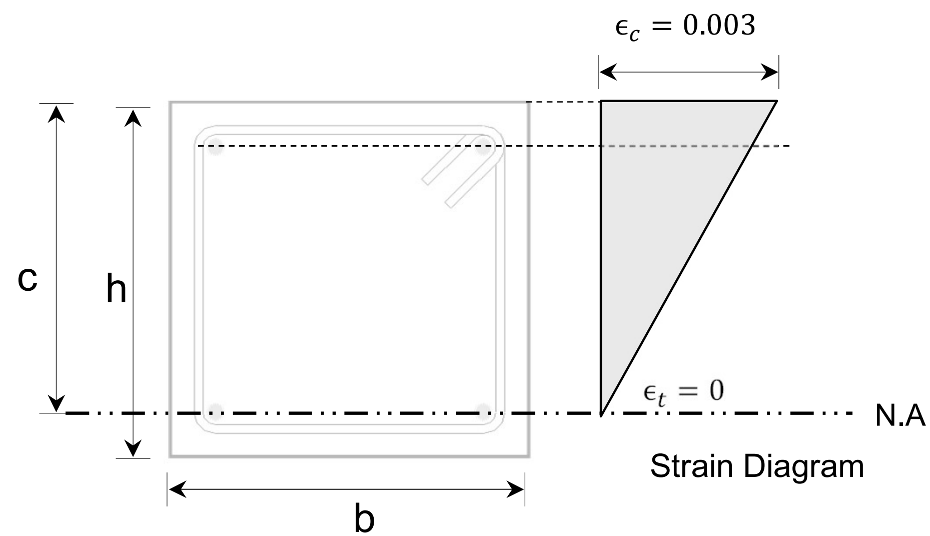
Design of RC Members Under Axial Loads with Uniaxial Bending

Development of Interaction Diagram

Point 3

- At Point 3, the strain in the reinforcing bars farthest from the compression face is equal to zero.
- $c = h - d'$ and $\emptyset = 0.65$





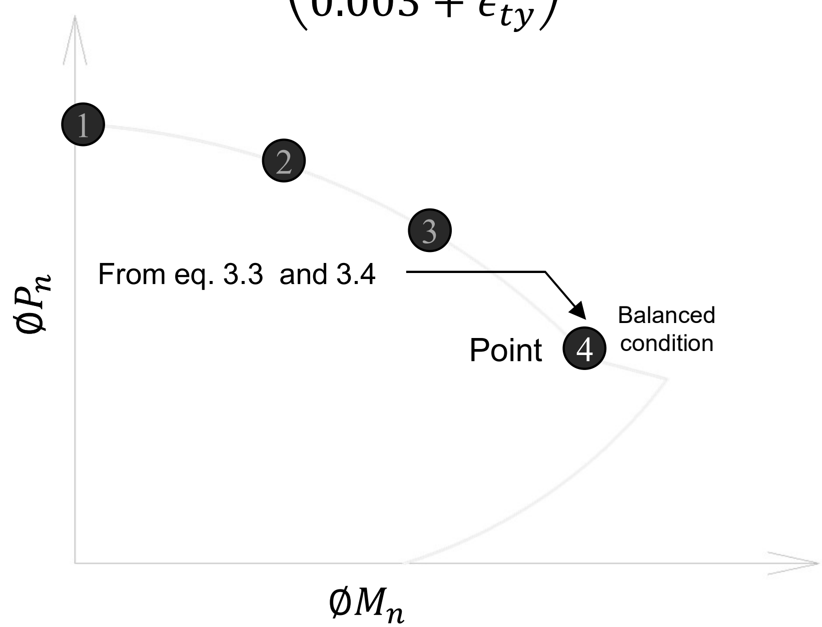
Design of RC Members Under Axial Loads with Uniaxial Bending

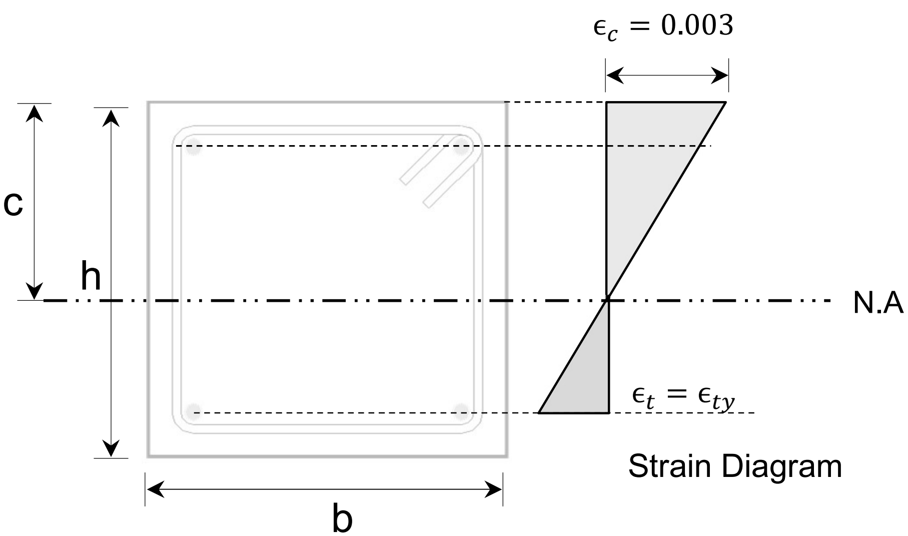
❑ Development of Interaction Diagram

❖ Point 4

- Point representing capacity of column for balance failure condition $\epsilon_t = \epsilon_{ty}$, $\epsilon_c = 0.003$ and $\emptyset = 0.65$

$$c = \left(\frac{0.003}{0.003 + \epsilon_{ty}}\right) d \quad \Rightarrow c_{40} = 0.69d \quad and\ c_{60} = 0.59d$$





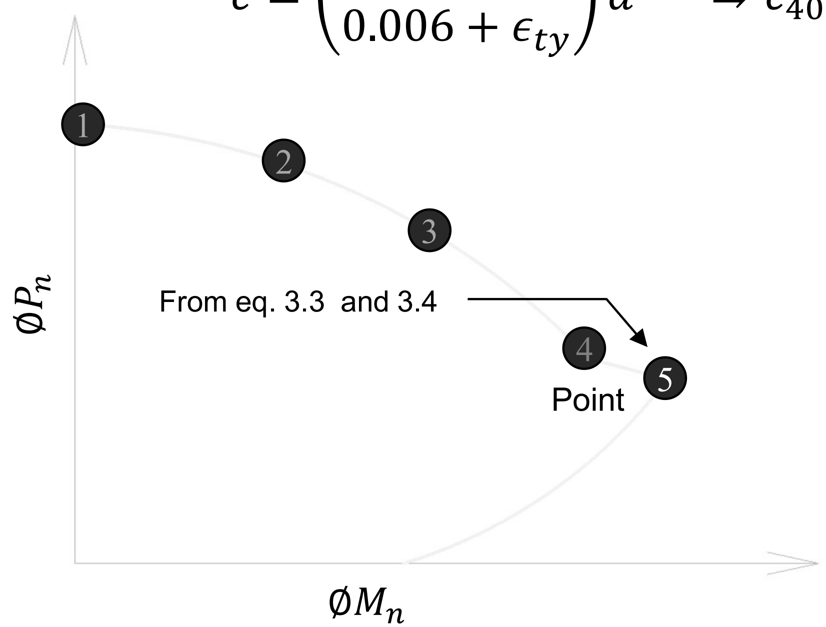
Design of RC Members Under Axial Loads with Uniaxial Bending

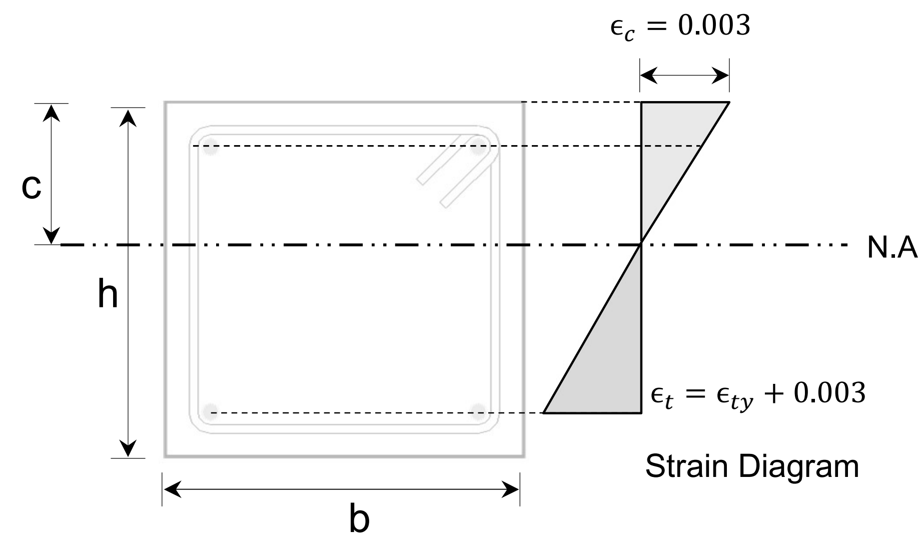
❑ Development of Interaction Diagram

❖ Point 5

- Point on capacity curve for which $\epsilon_t = \epsilon_{ty} + 0.003$, $\epsilon_c = 0.003$
- $\emptyset = 0.90$ or 0.65 (designer's preference)

$$c = \left(\frac{0.003}{0.006 + \epsilon_{ty}}\right) d \quad \Rightarrow c_{40} = 0.41d \;\; and \; c_{60} = 0.37d$$





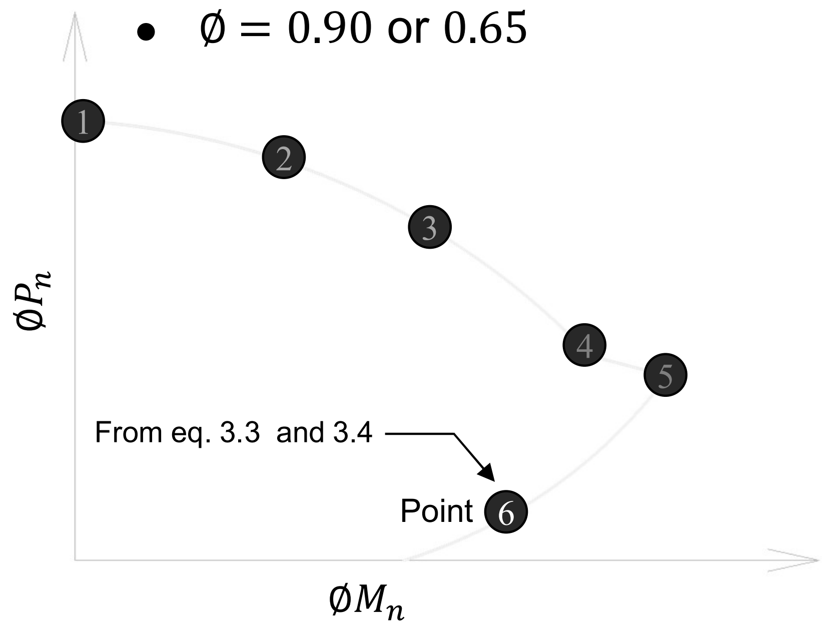
Strain Diagram

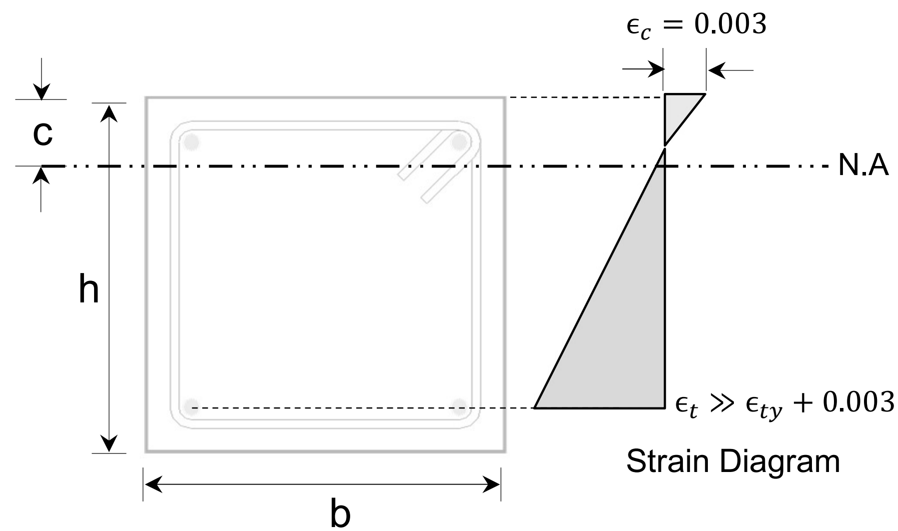
Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Development of Interaction Diagram

❖ Point 6

- Point on capacity curve at which the strain in tension steel is sufficiently greater than yield. Let consider ϵ_t two times that of point 5, then
- $c_{40} = 0.25d$, $c_{60} = 0.23d$ (for simplicity, assume c = 0.25d for both grades)
- $\emptyset = 0.90$ or 0.65





Design of RC Members Under Axial Loads with Uniaxial Bending

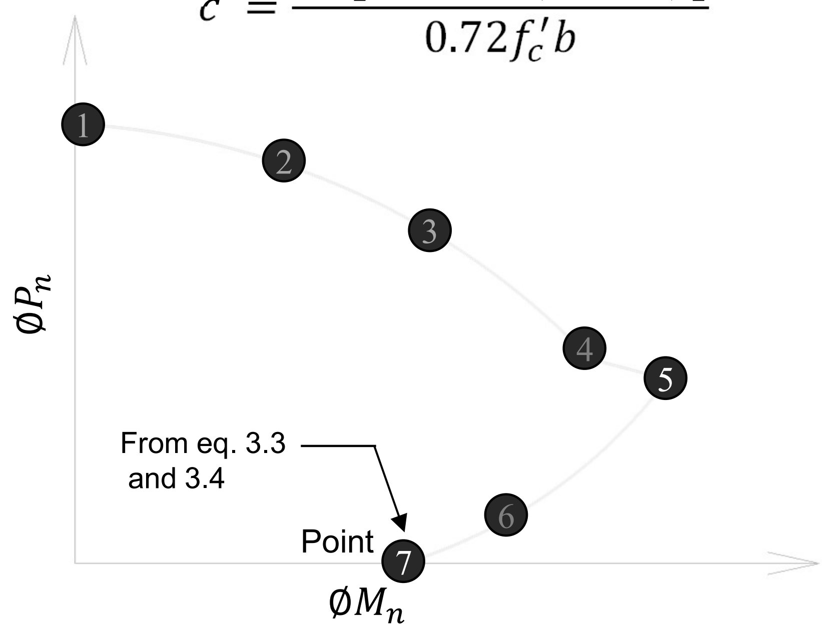
❑ Development of Interaction Diagram

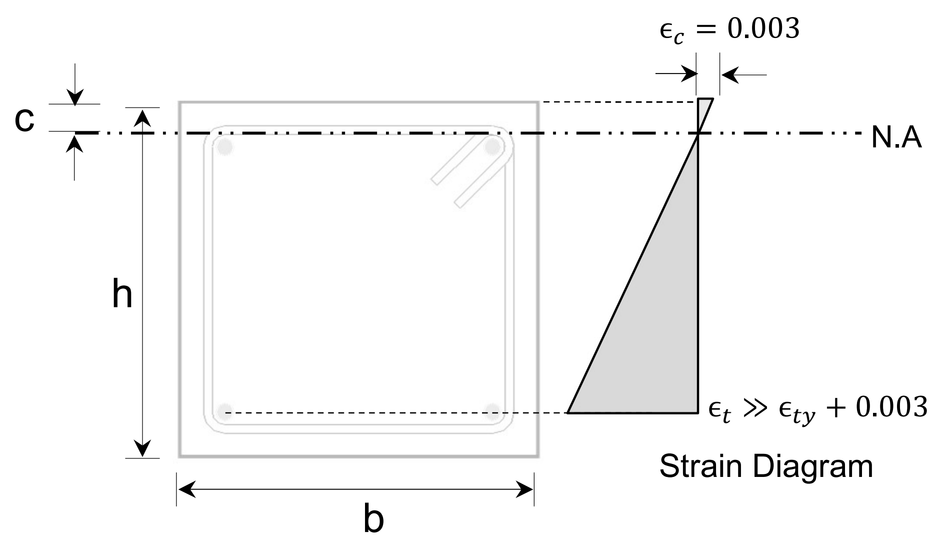
❖ Point 7

- This is the pure bending case on capacity curve at which the axial load is zero and and $\emptyset = 0.90$ or 0.65 and c can be taken as;

$$c = \frac{A_s\left[f_y - 87\left(1 - \frac{d'}{c}\right)\right]}{0.72 f_c' b}$$

(Please refer to the Appendix for the derivation of this equation.)

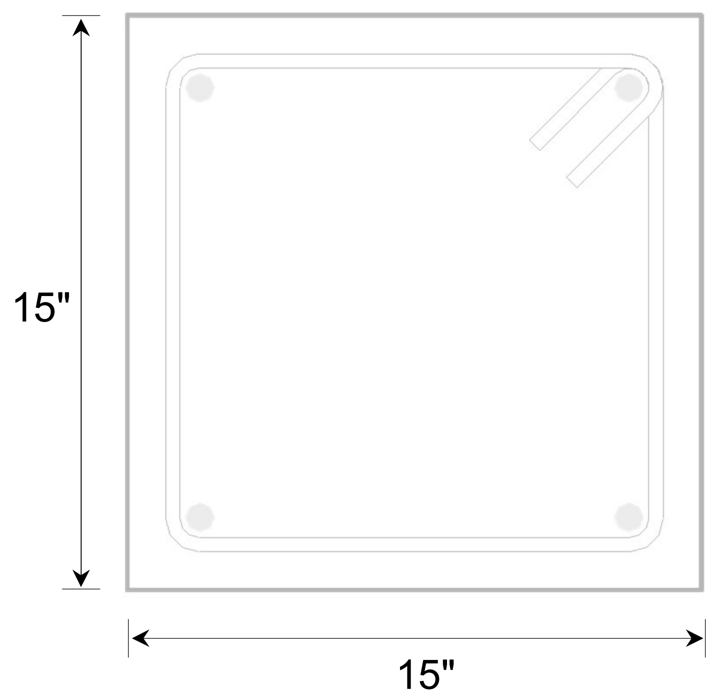




Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Example 3.8

- *Develop* interaction diagram for the given column. The material strengths are $f_c' = 3\text{ ksi}$ and $f_y = 60\text{ ksi}$ with 4 - #8 bars.



Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Solution

- **Given Data**

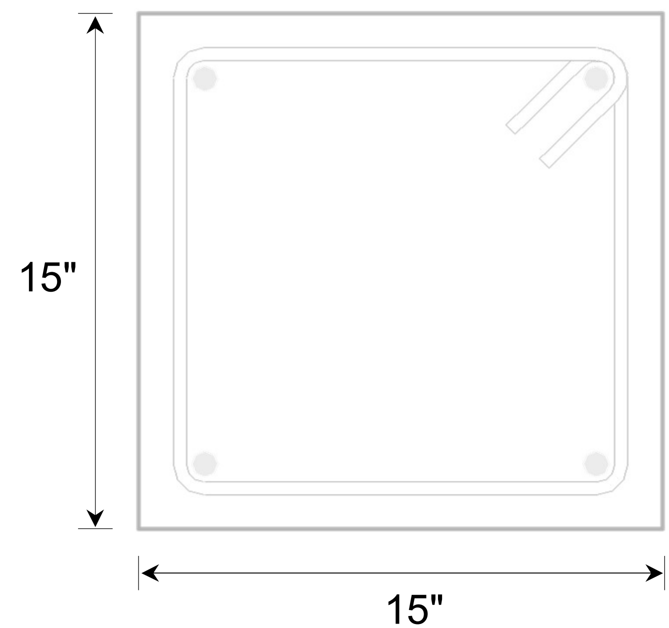
  $b = 15''$

  $h = 15''$

  $A_s = 4 \times 0.79 = 3.16 \text{ in}^2$

  $f_c' = 3 \text{ ksi}$

  $f_y = 60 \text{ ksi}$



- **Required Data**

  Develop Interaction diagram

Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Solution

❖ Point 1: Pure Axial Condition

The pure axial capacity of column (ignoring α) is given by

$$\emptyset P_n = 0.65\left[0.85 f_c{}'(A_g - A_s) + f_y A_s\right]$$

On substituting values;

$$\emptyset P_n = 0.65[0.85 \times 3\ (225 - 3.16) + 60 \times 3.16]$$

$$\emptyset P_n = 490.9 \text{ kip}$$

And

$$\emptyset M_n = 0$$

Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Solution

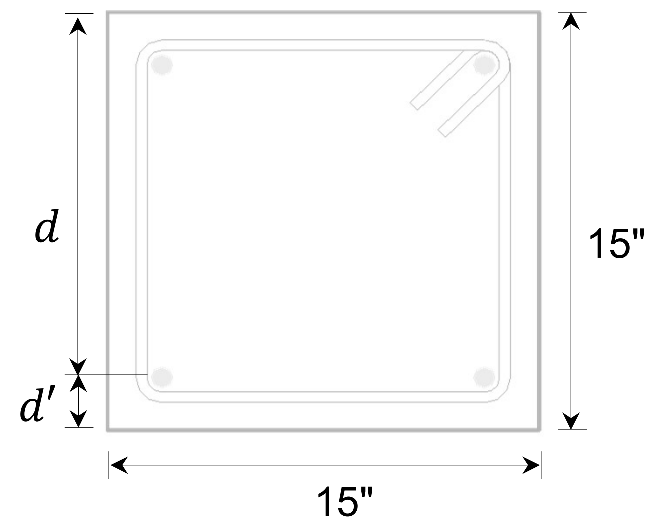
❖ Point 2

d' and d can be calculated as;

$$d' = 1.5 + \frac{3}{8} + \frac{8}{16} = 2.375"$$

and

$$d = 15 - d' = 12.625"$$



Now, with c = h = 15″

$$f_{s1} = 87(1 - d'/c) = 87(1 - 2.375/15) = 73.2 \text{ ksi} > f_y \rightarrow use\ f_{s1} = 60 \text{ ksi}$$

and

$$f_{s2} = 87(d/c - 1) = 87(12.625/15 - 1) = -13.8 \text{ ksi} < f_y \rightarrow use\ f_{s2} = -13.8 \text{ ksi}$$

Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Solution

❖ Point 2

Now, from eq.(3.3) and (3.4) we have

$\emptyset P_n = \emptyset[0.72 f_c' bc + A_s(f_{s1} - f_{s2})]$ ⇐ Note that A_s is steel area of single layer.

$= 0.65[0.72 \times 3 \times 15 \times 15 + 1.58(60 + 13.8)] = \mathbf{391.7\ kip}$

Similarly,

$\emptyset M_n = \emptyset[0.36 f_c' bc(h - 0.85c) + A_s(h/2 - d')(f_{s1} + f_{s2})]$

$= 0.65[0.36 \times 3 \times 15 \times 15(15 - 0.85 \times 15) + 1.58(7.5 - 2.375)(60 - 13.8)]$

$= 598.56\ \text{in.kip}$ or $\mathbf{49.9\ ft.kip}$

Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Solution

❖ Point 3

with $c = h - d' = 15 - 2.375 = 12.625''$

$f_{s1} = 87(1 - 2.375/12.625) = 70.6 \text{ ksi} > f_y \rightarrow use\ f_{s1} = 60 \text{ ksi}$

$f_{s2} = 87(12.625/12.625 - 1) = 0$

Now,

$\emptyset P_n = 0.65[0.72 \times 3 \times 15 \times 12.625 + 1.58(60 - 0)] = \mathbf{327.5\ kip}$

$\emptyset M_n = 0.65[0.36 \times 3 \times 15 \times 12.625(15 - 0.85 \times 12.625) + 1.58(5.125)(60 + 0)]$

$= 883.29 \text{ in. kip} \quad \text{or} \quad \mathbf{73.6\ ft.kip}$

Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Solution

❖ Point 4: Balanced Condition

with $c_{60} = 0.59d = 0.59 \text{ x } 12.625 = 7.45''$

$f_{s1} = 87(1 - 2.375/7.45) = 59.3\ ksi < f_y \rightarrow use\ f_{s1} = 59.3 \text{ ksi}$

$f_{s2} = 87(12.625/7.45 - 1) = 60.43 > f_y \rightarrow use\ f_{s2} = 60 \text{ ksi}$

Now,

$\emptyset P_n = 0.65[0.72 \times 3 \times 15 \times 7.45 + 1.58(59.3 - 60)] = \mathbf{156.2\ kip}$

$\emptyset M_n = 0.65[0.36 \times 3 \times 15 \times 7.45(15 - 0.85 \times 7.45) + 1.58(5.125)(119.30)]$

$= 1307.87 \text{ in.kip} \quad \text{or} \quad \mathbf{109.0\ ft.kip}$

Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Solution

❖ Point 5

with $c_{60} = 0.37d = 0.37 \text{ x } 12.625 = 4.67''$

$f_{s1} = 87(1 - 2.375/4.67) = 42.8\ ksi < f_y \rightarrow use\ f_{s1} = 42.8 \text{ ksi}$

$f_{s2} = 87(12.625/4.67 - 1) = 148.3 > f_y \rightarrow use\ f_{s2} = 60 \text{ ksi}$

Now,

$\emptyset P_n = 0.90[0.72 \times 3 \times 15 \times 4.67 + 1.56(42.8 - 60)] = \mathbf{111.8\ kip}$

$\emptyset M_n = 0.90[0.36 \times 3 \times 15 \times 4.67(15 - 0.85 \times 4.67) + 1.58(5.125)(42.8 + 60)]$

$= 1500.23 \text{ in.kip } or\ \mathbf{125.0\ ft.kip}$

Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Solution

❖ Point 6

with $c = 0.25d = 0.25 \text{ x } 12.625 = 3.16''$

$f_{s1} = 87(1 - 2.375/3.16) = 21.6 \text{ ksi} < f_y \rightarrow use\ f_{s1} = 21.6 \text{ ksi}$

$f_{s2} = 87(12.625/3.16 - 1) = 260.6 > f_y \rightarrow use\ f_{s2} = 60 \text{ ksi}$

Now,

$\emptyset P_n = 0.90[0.72 \times 3 \times 15 \times 3.16 + 1.58(21.6 - 60)] = \mathbf{37.5\ kip}$

$\emptyset M_n = 0.90[0.36 \times 3 \times 15 \times 3.16(15 - 0.85 \times 3.16) + 1.58(5.125)(21.6 + 60)]$

$= 1162.02 \text{ in.kip} \quad \text{or} \quad \mathbf{96.8\ ft.kip}$

Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Solution

❖ Point 7: Pure Bending Condition

$$c = \frac{A_s\left[f_y - 87\left(1 - \frac{d'}{c}\right)\right]}{0.72 f_c' b} \Rightarrow \text{on solving and neglecting negative root}, \text{c} = 2.58''$$

$$f_{s1} = 87(1 - 2.375/2.58) = 6.9 \text{ ksi} < f_y \rightarrow use\ f_{s1} = 6.9 \text{ ksi}$$

$$f_{s2} = 87(12.625/2.58 - 1) = 338.7 \text{ ksi} > f_y \rightarrow use\ f_{s2} = 60\ ksi$$

Now,

$$\emptyset P_n = 0$$

$$\emptyset M_n = 0.90[0.36 \times 3 \times 15 \times 2.58(15 - 0.85 \times 2.58) + 1.58(5.125)(66.9)]$$

$$= 969.30 \text{ in. kip} \quad \text{or} \quad \mathbf{80.8\ ft.kip}$$

Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Solution

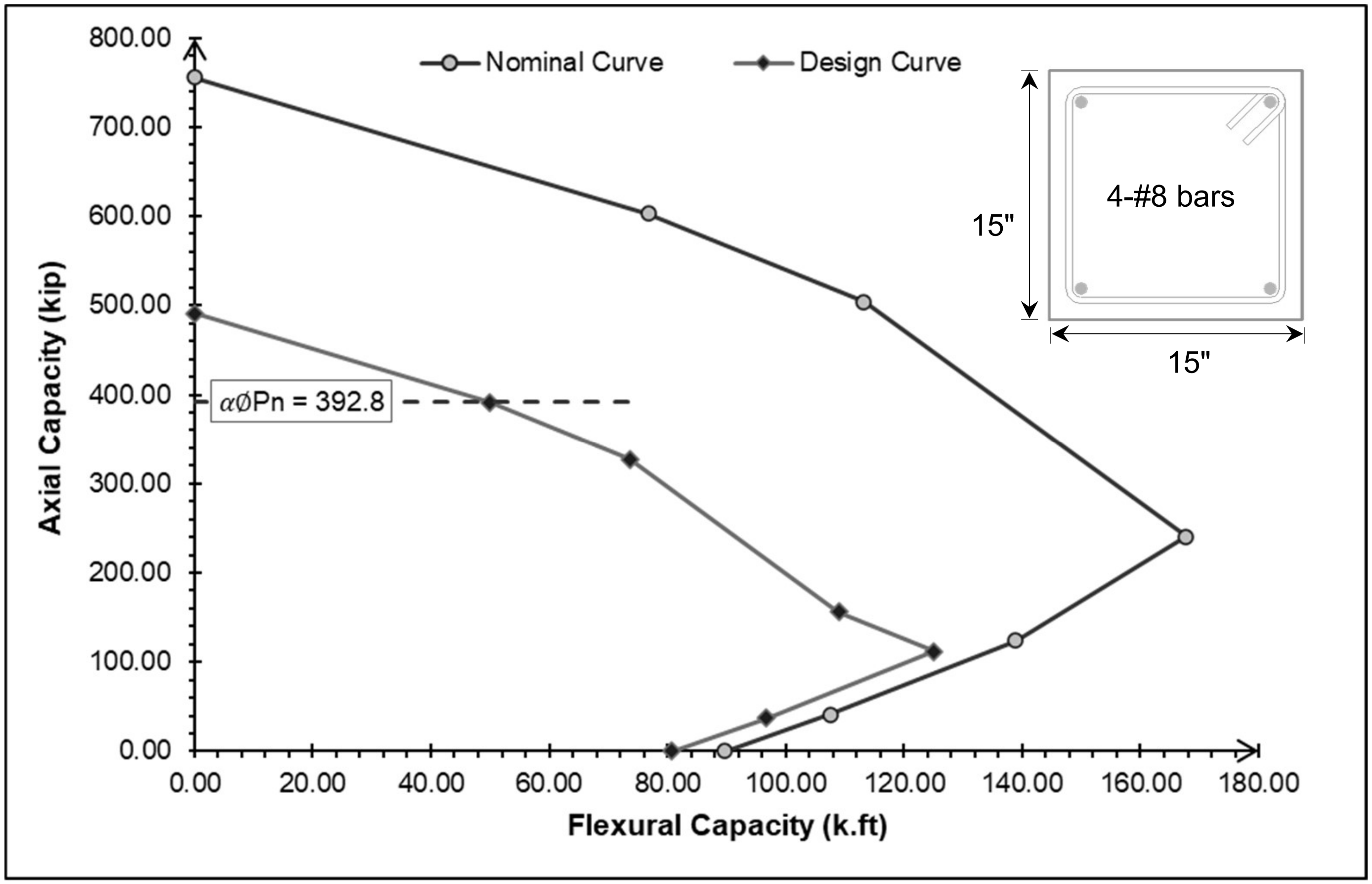
❖ Summary of Calculations

Point	c $(in.)$	f_{s1} (ksi)	f_{s2} (ksi)	$\emptyset P_n$ (kip)	$\emptyset M_n$ $(kip.ft)$	Remarks
1	---	---	---	281.5	0	Compression controlled region
2	15.00	60.0	-13.8	391.7	49.9	
3	12.625	60.0	0.0	327.5	73.6	
4	7.45	59.3	60.0	156.2	109.0	Balanced condition
5	4.67	42.8	60.0	111.8	125.0	Tension controlled region
6	3.16	21.6	60.0	37.5	96.8	
7	2.58	6.9	60.0	0.0	80.8	

Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Solution

❖ Plot of Interaction Curve



Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Design Aids

- In practice, Design Aids are used for the design of eccentrically loaded RC columns.
- They can be found in handbooks and special volumes published by the American Concrete Institute (ACI).
- They cover the most frequent practical cases, such as symmetrically reinforced rectangular and square columns and circular spirally reinforced columns.
- Design Aids for different ranges of f_c' and f_y are provided in Appendix. (at the end of this lecture).

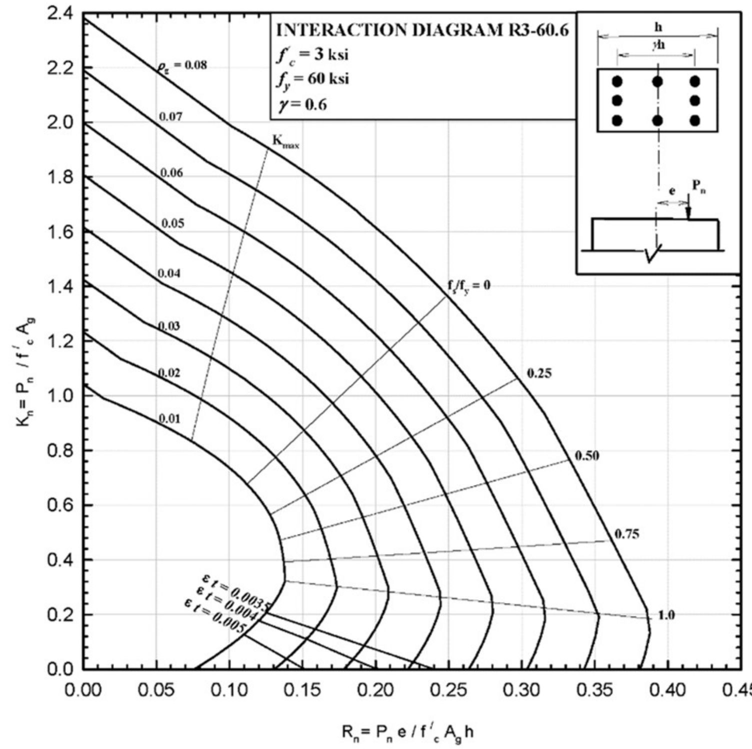
Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Procedure of using Design Aids

1. Select a trial cross-sectional dimensions b and h

2. Calculate the ratio γ based on required cover distances to the bar centroids and select the corresponding column design chart.

$$\gamma = \frac{h - 2d'}{h}$$



Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Procedure of using Design Aids

4. Calculate K_n and R_n factor

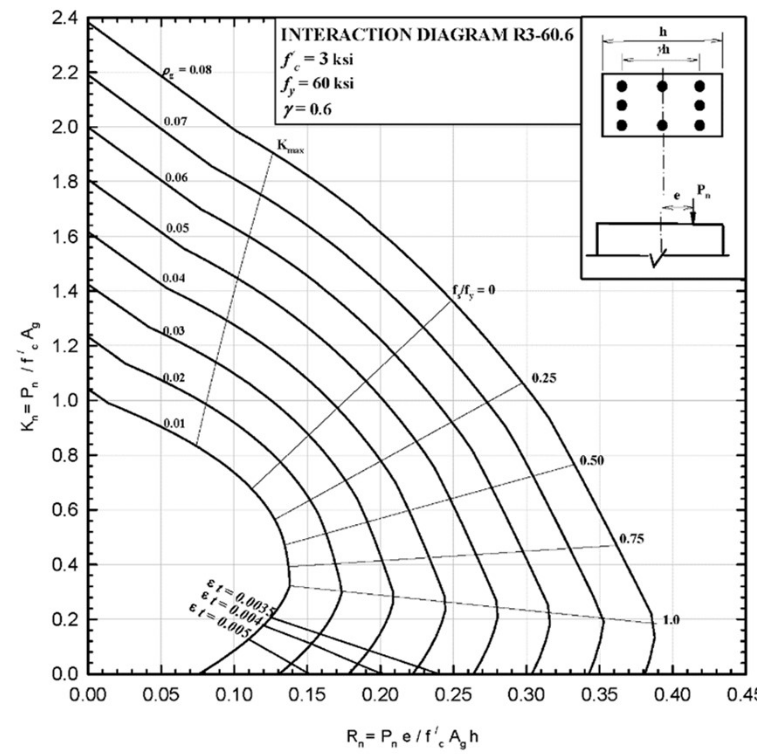
$$K_n = \frac{P_u}{\emptyset f_c' bh}$$

$$R_n = \frac{M_u}{\emptyset f_c' bh^2}$$

5. Using values of K_n and R_n, read the required reinforcement ratio ρ_g from the graph.

6. Calculate the total steel area

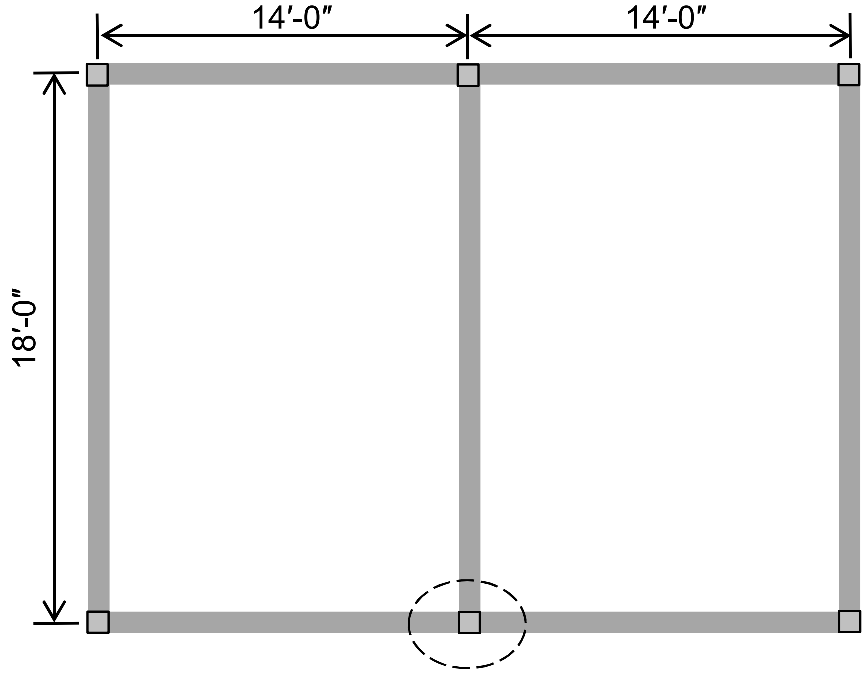
$$A_{st} = \rho_g bh$$

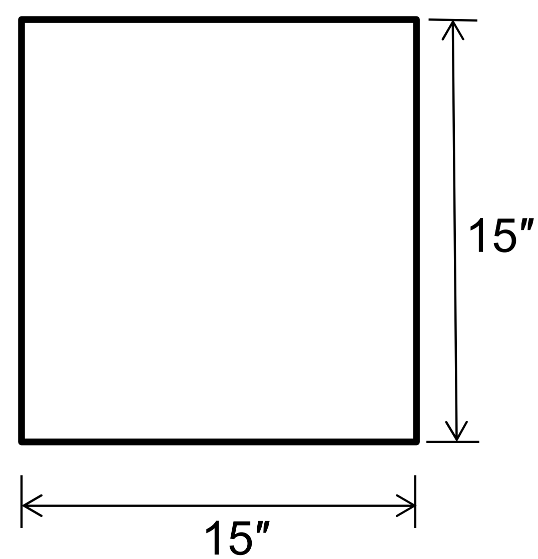


Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Example 3.9

- ***Design*** the highlighted edge column to support a factored load of 450 kip and a factored moment of 80 ft.kip. The material strengths are $f_c{}' = 4$ ksi and $f_y = 60$ ksi.





Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Solution

1. Dimensions are already given to us

   $b = h = 15"$

2. Calculate ratio γ

   $$\gamma = \frac{h - 2d'}{h}$$

   Assuming $d' = 2.5\ in$

   $$\gamma = \frac{15 - 2(2.5)}{15} = 0.67$$

   $$\gamma \approx 0.70$$

Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Solution

3. Calculate K_n and R_n factor

$$K_n = \frac{P_u}{\emptyset f_c' bh} = \frac{450}{0.65 \times 4 \times 15 \times 15}$$

$$K_n = 0.77$$

$$R_n = \frac{M_u}{\emptyset f_c' bh^2} = \frac{80 \times 12}{0.65 \times 4 \times 15 \times 15^2}$$

$$R_n = 0.11$$

For $\gamma = 0.70$, $f_c' = 4\ \mathrm{ksi}$ and $f_y = 60\ \mathrm{ksi}$, the relevant Design Aid is DA-6 (from Appendix).

Design of RC Members Under Axial Loads with Uniaxial Bending

❑ Solution

3. Read ρ_g from the graph

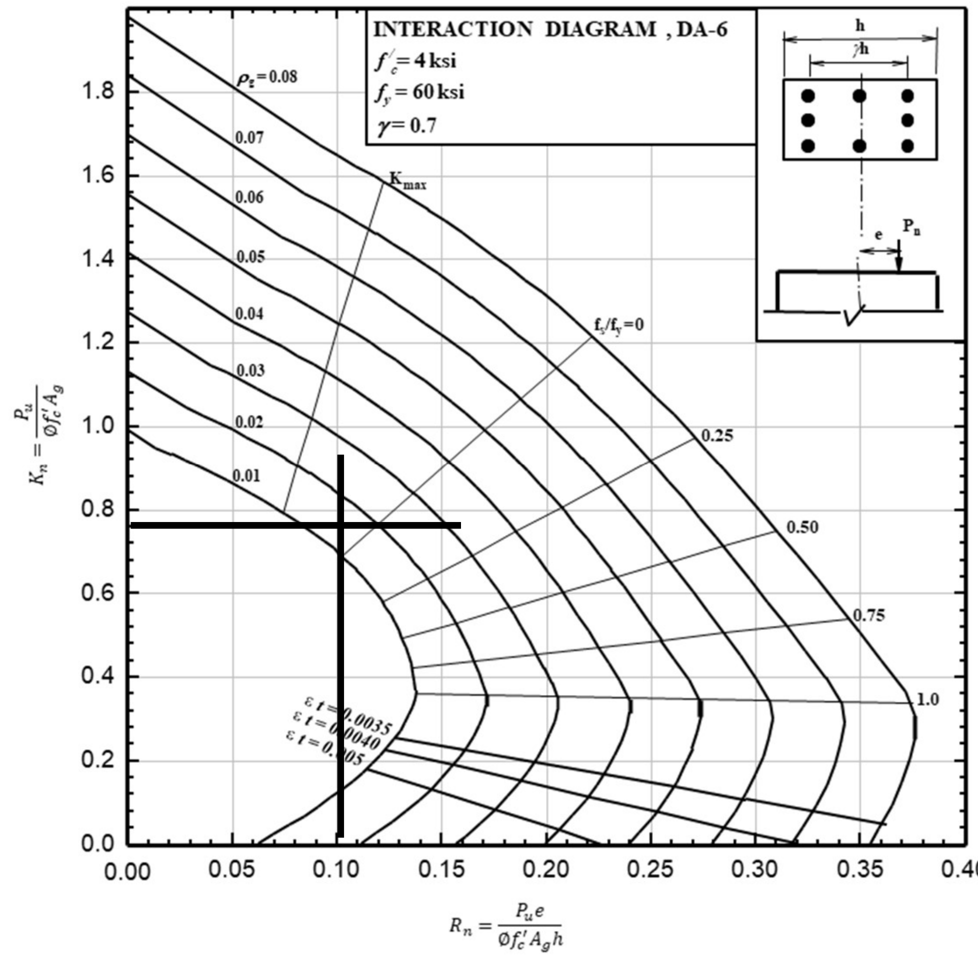
$$\rho_g = 0.015$$

Calculate Area of steel

$$A_{st} = 0.015A_g = 3.38\ in^2$$

Using #6 bar

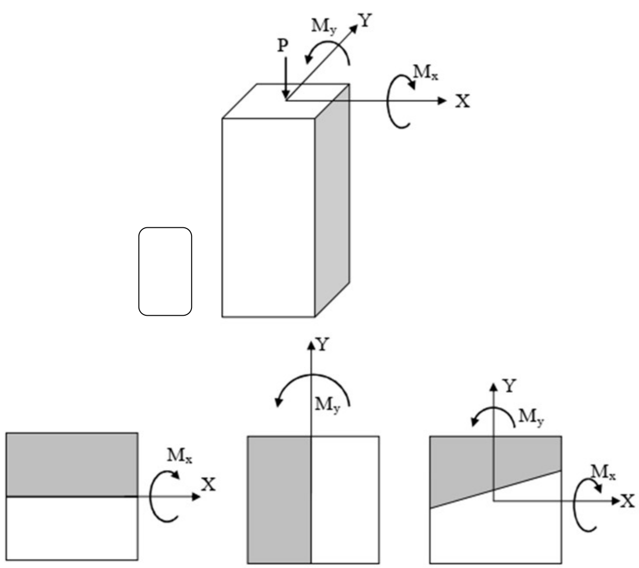
$$\text{No. of bars } = \frac{3.38}{0.44} \approx 8$$



Design of RC Members Under Axial Loads with Biaxial Bending

❑ Introduction

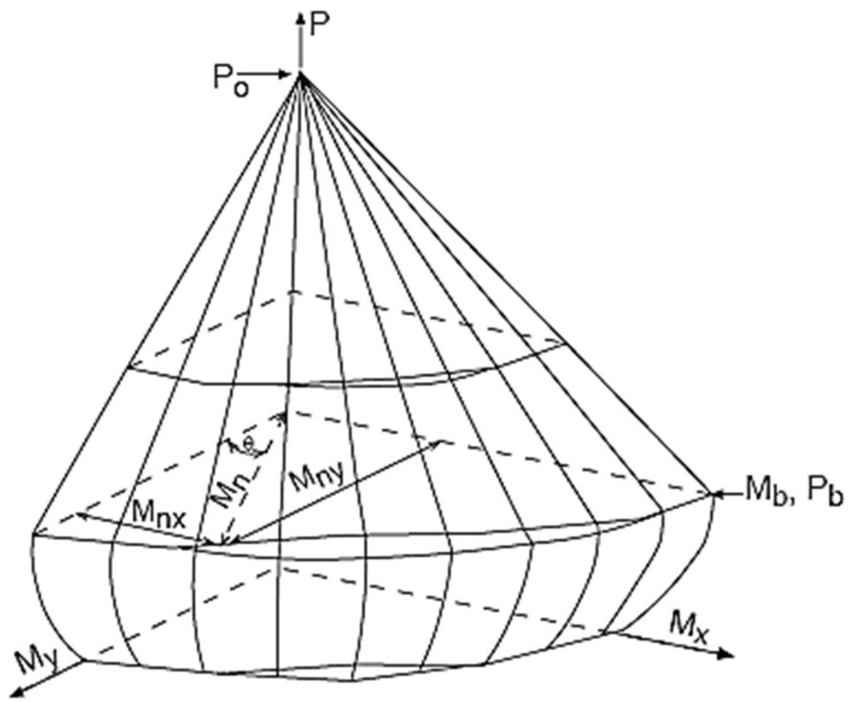
- Column section subjected to compressive load (P_u) at eccentricities e_x and e_y along x and y axes causing moments M_{uy} and M_{ux} respectively.



Design of RC Members Under Axial Loads with Biaxial Bending

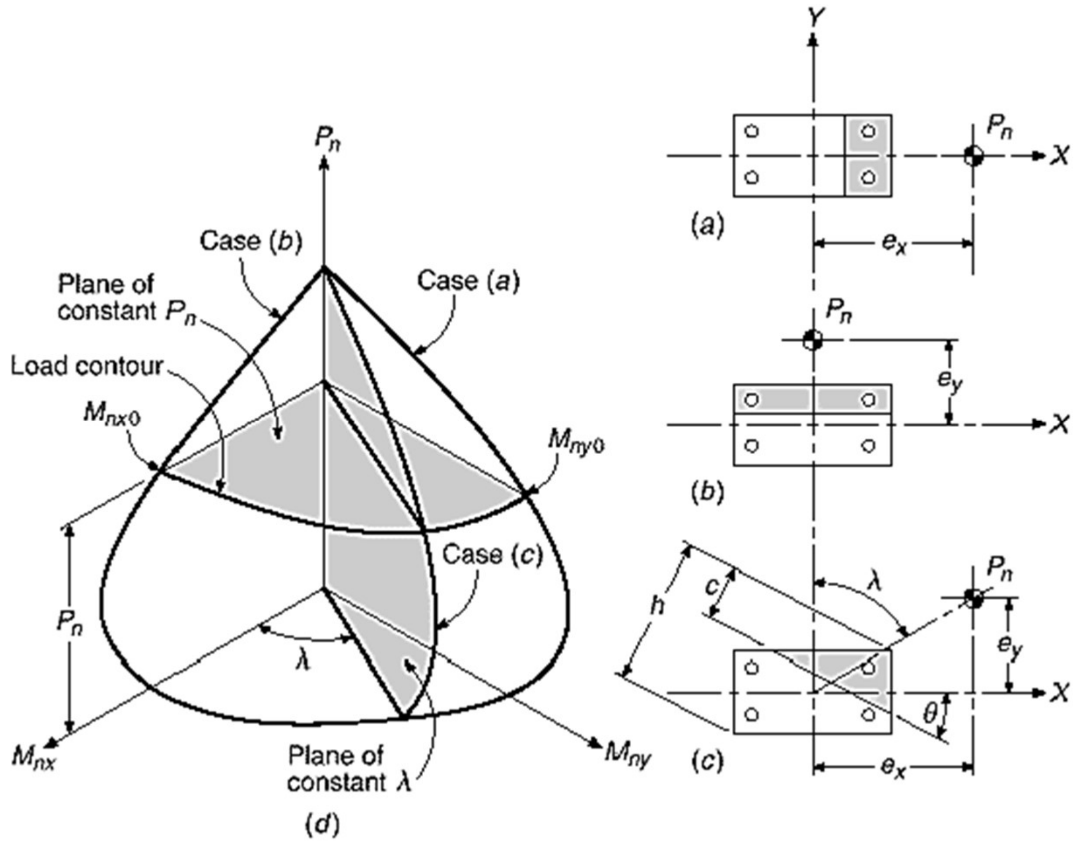
❑ Behavior of Columns Subjected to Biaxial Bending

- The biaxial bending resistance of an axially loaded column can be represented as a surface formed by a series of uniaxial interaction curves drawn radially from the P axis.



Design of RC Members Under Axial Loads with Biaxial Bending

❑ Behavior of Columns Subjected to Biaxial Bending

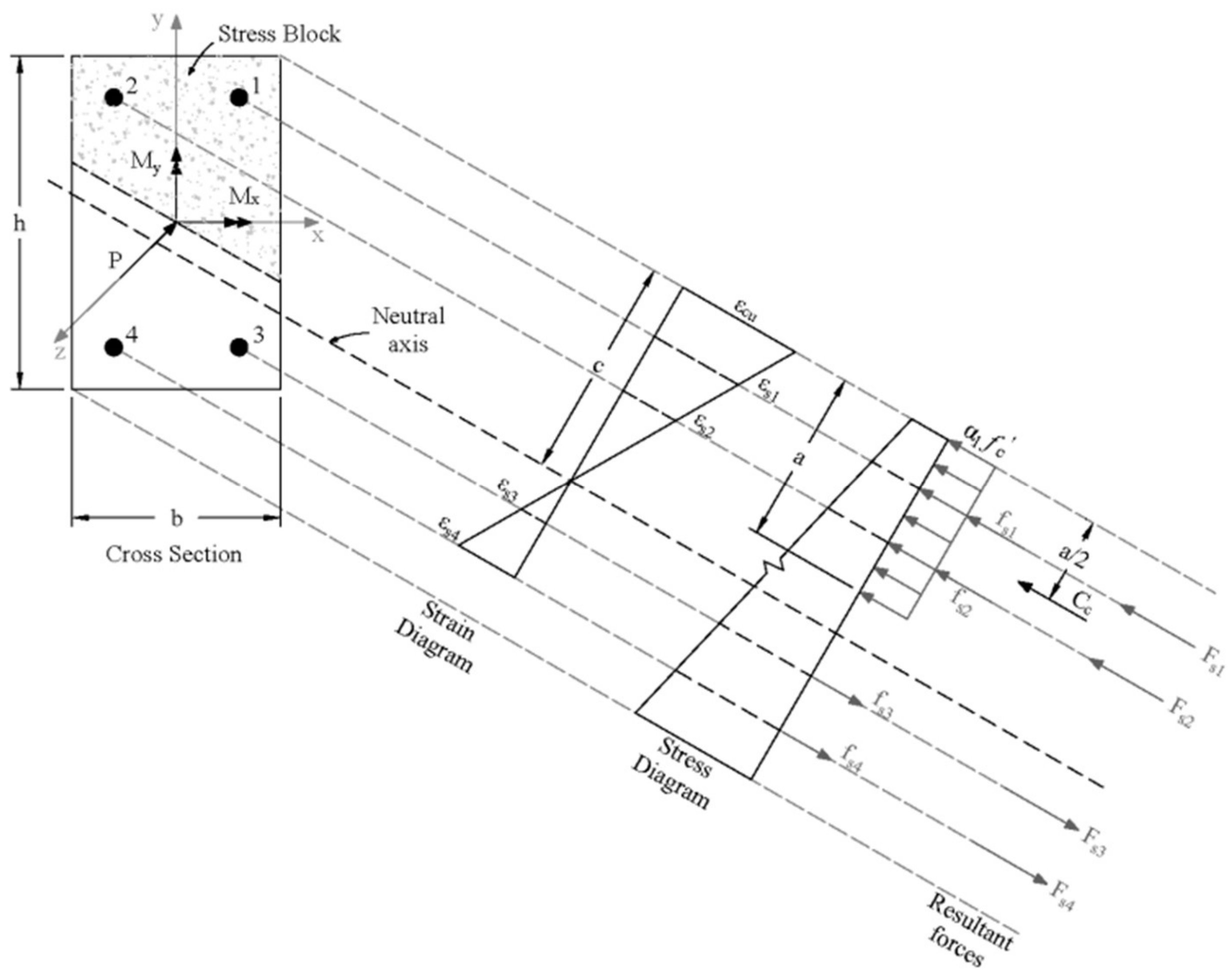


a) Uniaxial Bending about y Axis, b) Uniaxial Bending about x Axis,

c) Biaxial bending about Diagonal Axis.

Design of RC Members Under Axial Loads with Biaxial Bending

❑ Behavior of Columns Subjected to Biaxial Bending



Force, Strain and Stress Distribution Diagrams for Biaxial Bending

Design of RC Members Under Axial Loads with Biaxial Bending

Difficulties in Constructing Biaxial Interaction Surface

- The triangular or trapezoidal compression zone.
- Neutral axis, not in general, perpendicular to the resultant eccentricity.

Design of RC Members Under Axial Loads with Biaxial Bending

Analysis Methods

- Following are the Approximate methods for analyzing RC Members Under Axial Loads with Biaxial Bending:
  - PCA Approximate Method
  - Bressler's Reciprocal Load Method
  - Bresler Load Contour Method

Design of RC Members Under Axial Loads with Biaxial Bending

❑ PCA Approximate Method

- The Portland Cement Association (PCA) has developed equations to transform biaxial demands into equivalent uniaxial demands.
- The method is suitable for rectangular sections with reinforcement equally distributed on all faces.

$$M_{nox} = M_{nx} + \frac{h}{b}\left(\frac{1-\beta}{\beta}\right)M_{ny} \qquad -(Eq.20, Ch\#7, PCA)$$

$$M_{noy} = M_{ny} + \frac{b}{h}\left(\frac{1-\beta}{\beta}\right)M_{nx} \qquad -(Eq.17, Ch\#7, PCA)$$

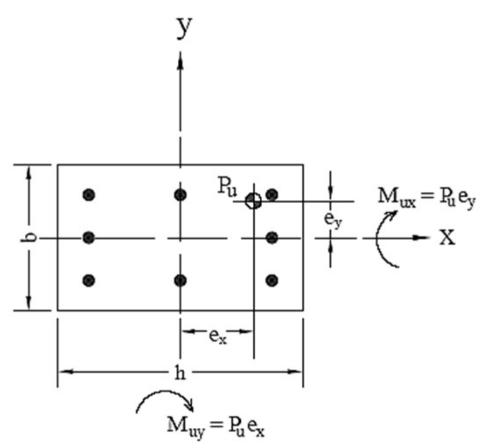
Design of RC Members Under Axial Loads with Biaxial Bending

❑ PCA Approximate Method

- In the above equations the factor β ranges from 0.65 to 0.7.
- A value of 0.65 for β is generally a good initial choice in a biaxial bending analysis.
- Taking value of β = 0.65, and converting nominal moments to factored moments, the equations can be simplified as below:

$$M_{uox} = M_{ux} + 0.54M_{uy}\left(\frac{h}{b}\right) \quad \text{---- (3.5)}$$

$$M_{uoy} = M_{uy} + 0.54M_{ux}\left(\frac{b}{h}\right) \quad \text{---- (3.6)}$$

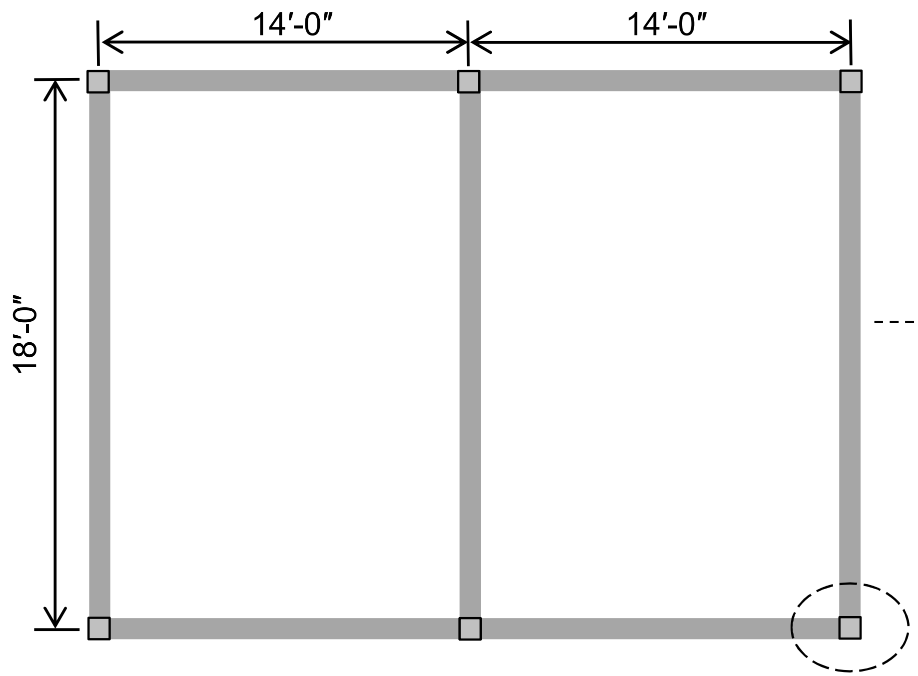


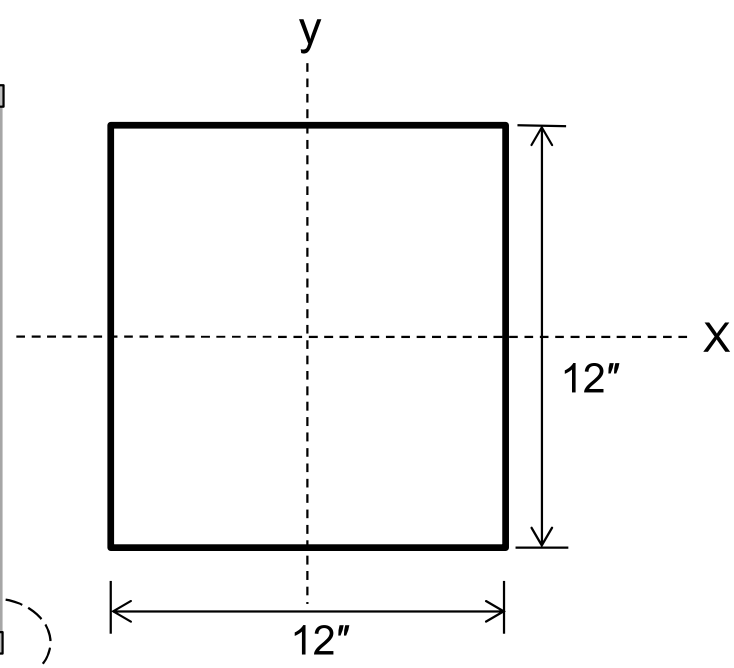
NOTE: Pick the larger moment for onward calculations.

Design of RC Members Under Axial Loads with Biaxial Bending

❑ Example 3.10

- Using PCA Approximate Method, ***Determine*** Area of longitudinal reinforcement for the highlighted corner column, to support factored axial load of 190 kip and factored moments of 35 ft.kip about x axis and 50 ft.kip about y axis. Take f_c' = 4 ksi and f_y = 60 ksi.





Design of RC Members Under Axial Loads with Biaxial Bending

❑ Solution

➢ **Step 1: Converting Biaxial Case to Uniaxial Case**

Determine the values of M_{uox} and M_{uoy} as follows:

$$M_{uox} = 35 + 0.54 \times 50(12/12) = 62 \text{ ft. kip}$$

$$M_{uoy} = 50 + 0.54 \times 35(12/12) = 68.9 \text{ ft. kip}$$

Take the larger value

The biaxial column can now be designed as an equivalent uniaxial column with moment $M_u = 68.9 \text{ ft. kip}$

Design of RC Members Under Axial Loads with Biaxial Bending

❑ Solution

➢ **Step 2: Calculate Reinforcement using Design Aids**

$$\gamma = \frac{12 - 2(2.5)}{12} = 0.583 \approx 0.60$$

$$K_n = \frac{P_u}{\emptyset f_c' bh} = \frac{190}{0.65 \times 4 \times 12 \times 12} = 0.51$$

$$R_n = \frac{M_u}{\emptyset f_c' bh^2} = \frac{68.9 \times 12}{0.65 \times 4 \times 12 \times 12^2} = 0.18$$

- For $\gamma = 0.60$, $f_c' = 4$ ksi and $f_y = 60$ ksi, the relevant Design Aid is DA – 2 (from Appendix)

Design of RC Members Under Axial Loads with Biaxial Bending

❑ Solution

➢ **Step 2: Calculate Reinforcement using Design Aids**

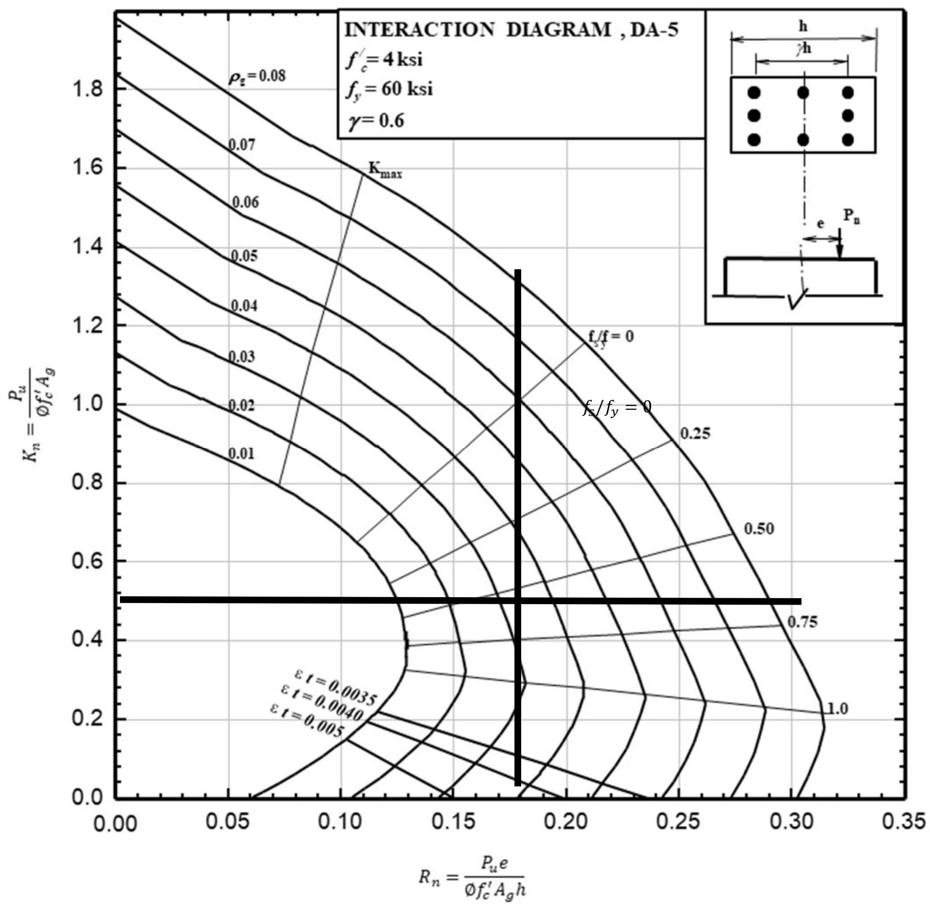
- From graph: $\rho_g = 0.033$
- Calculate Area of Steel

$A_{st} = 0.033A_g = 4.75\ in^2$

Using #6 bar:

No. of bars $= \dfrac{4.75}{0.44} = 10.8$

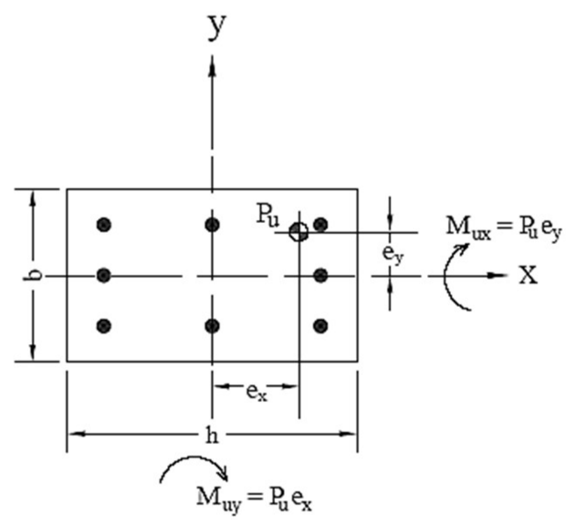
Provide 12-#6 bars



Design of RC Members Under Axial Loads with Biaxial Bending

❑ Bressler's Approximate Methods

1. **Reciprocal Load Method**
   - Suitable for columns having factored axial load $P_u \geq 0.1A_g f_c'$.
2. **Load Counter Method**
   - Appropriate for columns having factored axial load $P_u < 0.1A_g f_c'$.



Design of RC Members Under Axial Loads with Biaxial Bending

1. Reciprocal Load Method

- Bressler's reciprocal load equation can be derived from the geometry of the approximating plane. It can be shown that:

$$\frac{1}{P_n} = \frac{1}{P_{nx0}} + \frac{1}{P_{ny0}} + \frac{1}{P_0}$$

Where;

P_n = approximate value of nominal load in biaxial bending with eccentricities e_x and e_y.

$f_s/f_y = 0$

P_{nyo} = nominal axial capacity when only eccentricity e_x is present (e_y = 0),

P_{nxo} = nominal axial capacity when only eccentricity e_y is present (e_x = 0),

P_{no} = nominal axial capacity for concentrically loaded column

Design of RC Members Under Axial Loads with Biaxial Bending

1. Reciprocal Load Method

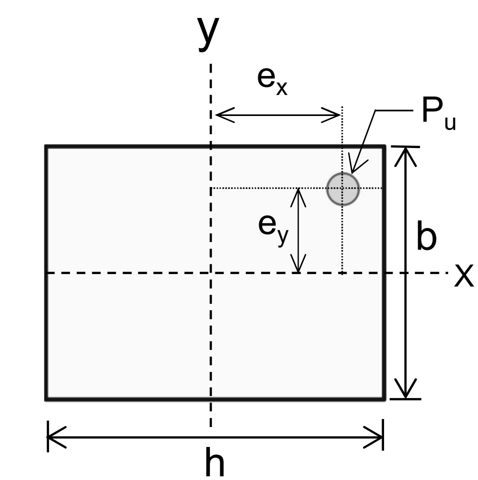
❖ Stepwise Procedure

- **Step 1: Check Applicability of Method**

  $P_n \geq 0.1 A_g f_c'$ → applies, otherwise not.

- **Step 2: Calculate Necessary Parameters**

Bending about X axis	Bending about Y axis
$\gamma = \frac{h - 2d'}{h}$	$\gamma = \frac{b - 2d'}{b}$
$\frac{e_y}{b} = \frac{M_{ux}}{P_u b}$	$\frac{e_x}{h} = \frac{M_{uy}}{P_u h}$
Assume $\rho = A_s/bh$	---
Select relevant graph based on given f_c', f_y and γ	Select relevant graph based on given f_c', f_y and γ



Design of RC Members Under Axial Loads with Biaxial Bending

1. Reciprocal Load Method

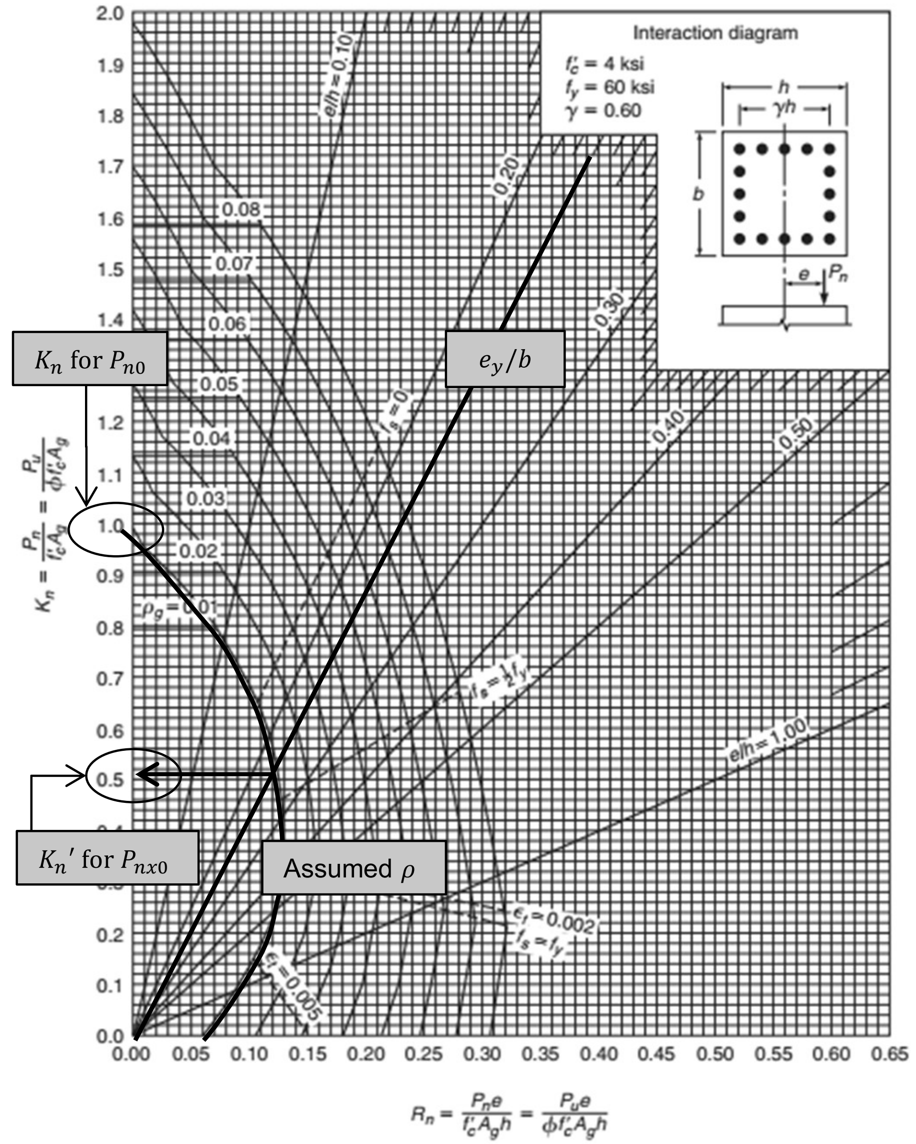
❖ Stepwise Procedure

➢ **Step 3: Calculate P_{n0}, P_{nx0} and P_{ny0}**

▪ Bending about X axis

$$P_{n0} = k_n A_g f_c'$$

$$P_{nx0} = k_n' A_g f_c'$$



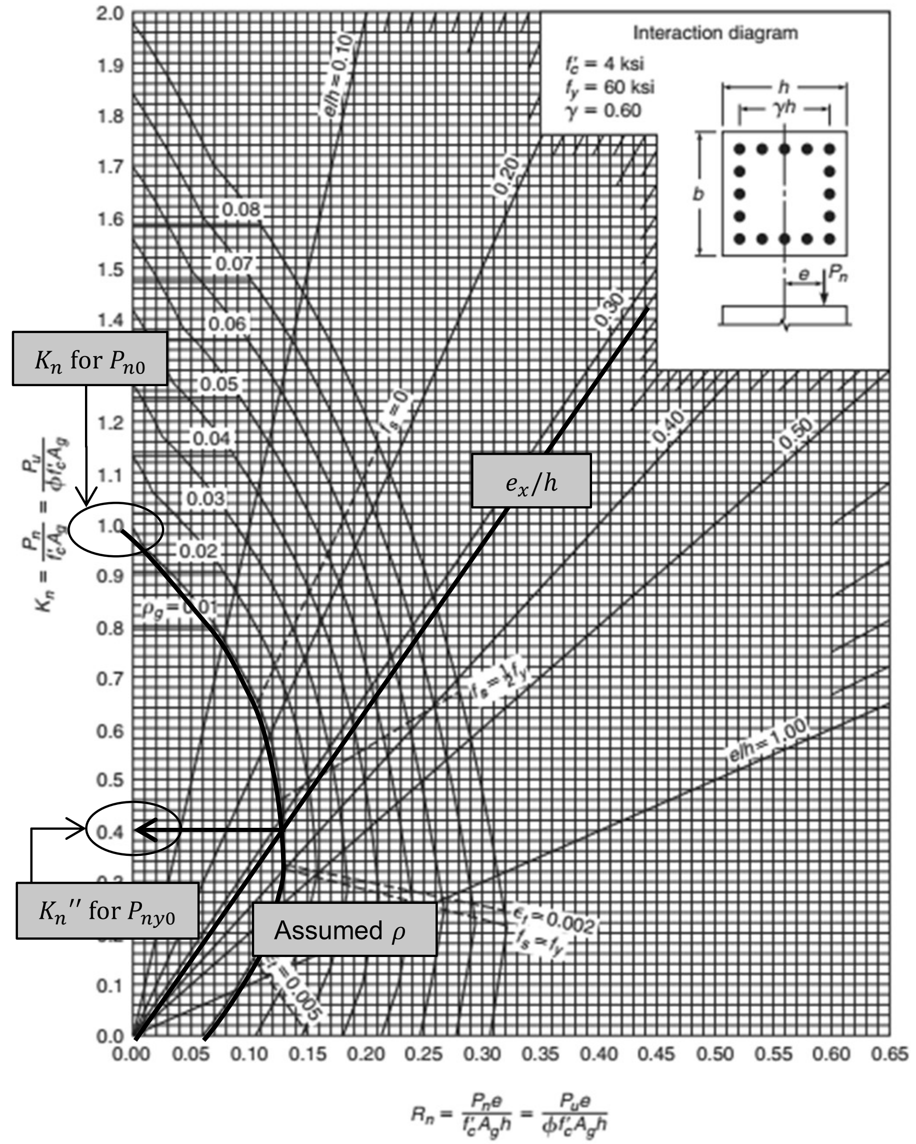
Design of RC Members Under Axial Loads with Biaxial Bending

1. Reciprocal Load Method

- **Stepwise Procedure**
  - **Step 3: Calculate P_{n0}, P_{nx0} and P_{ny0}**
    - Bending about X axis

$$P_{n0} = k_n A_g f_c'$$

$$P_{ny0} = k_n'' A_g f_c'$$



Design of RC Members Under Axial Loads with Biaxial Bending

1. Reciprocal Load Method

❖ Stepwise Procedure

➢ Step 4: Calculate Axial Capacity

Calculate P_n using the following equation

$$\frac{1}{P_n} = \frac{1}{P_{nx0}} + \frac{1}{P_{ny0}} + \frac{1}{P_0}$$

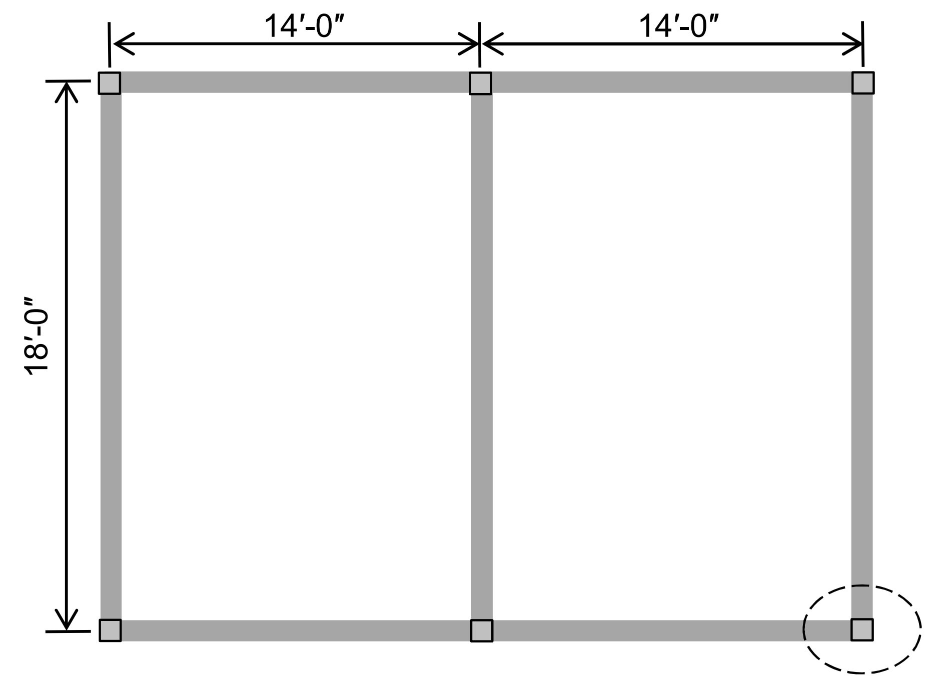
If $\emptyset P_n > P_u$ → Design is OK!

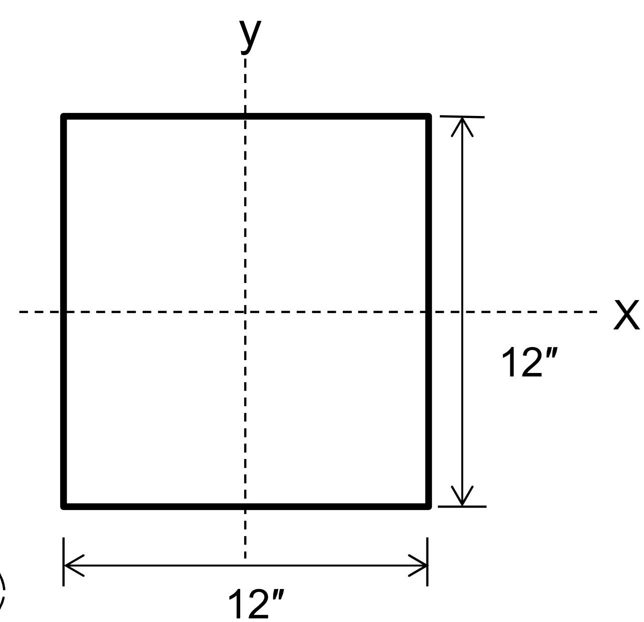
Otherwise, adjust material properties (f_c', f_y) or geometric properties (b, h), or the reinforcement area (A_s), and repeat the above steps.

Design of RC Members Under Axial Loads with Biaxial Bending

❑ Example 3.11

- Using Reciprocal Load Method, ***Determine*** area of longitudinal reinforcement for the corner column highlighted in figure, to support P_u = 185 kip, M_{ux} = 30 ft.kip and M_{uy} = 34 ft.kip. Take f_c' = 4 ksi and f_y = 60 ksi.





Design of RC Members Under Axial Loads with Biaxial Bending

❑ Solution

➢ **Step 1: Check Applicability of Method**

$$P_n = \frac{P_u}{\emptyset} = \frac{185}{0.65} = 284.62 \text{ kip}$$

$$0.1A_g f_c' = 0.1 \times (12 \times 12) \times 4 = 57.6 \text{ kip}$$

$P_n = 284.62 \text{ kip} > 0.1A_g f_c' = 57.6 \text{ kip}$ → Reciprocal Load Method applies

Design of RC Members Under Axial Loads with Biaxial Bending

❑ Solution

➢ Step 2: Calculate Necessary Parameters

Bending about X axis	Bending about Y axis
$\gamma = \frac{h - 2d'}{h} = \frac{12 - 2(2.5)}{12} \approx 0.60$	$\gamma = \frac{b - 2d'}{b} = \frac{12 - 2(2.5)}{12} \approx 0.60$
$\frac{e_y}{b} = \frac{M_{ux}}{P_u b} = \frac{30}{185(1)} = 0.16$	$\frac{e_x}{h} = \frac{M_{uy}}{P_u h} = \frac{34}{185(1)} = 0.18$
$\rho = \frac{A_s}{bh} = \frac{4(0.44)}{12 \times 12} = 0.012$	---
For f_c' = 4 ksi, f_y = 60 ksi and $\gamma = 0.60$, Graph A.5 of Nilson 14th Ed. applies	For f_c' = 4 ksi, f_y = 60 ksi and $\gamma = 0.60$, Graph A.5 of Nilson 14th Ed. applies

Design of RC Members Under Axial Loads with Biaxial Bending

❑ Solution

➢ **Step 3: Calculate P_{n0}, P_{nx0} and P_{ny0}**

- **Bending about X axis**

From Grapgh, the curve ρ interest Y axis at K_n = 1.09.

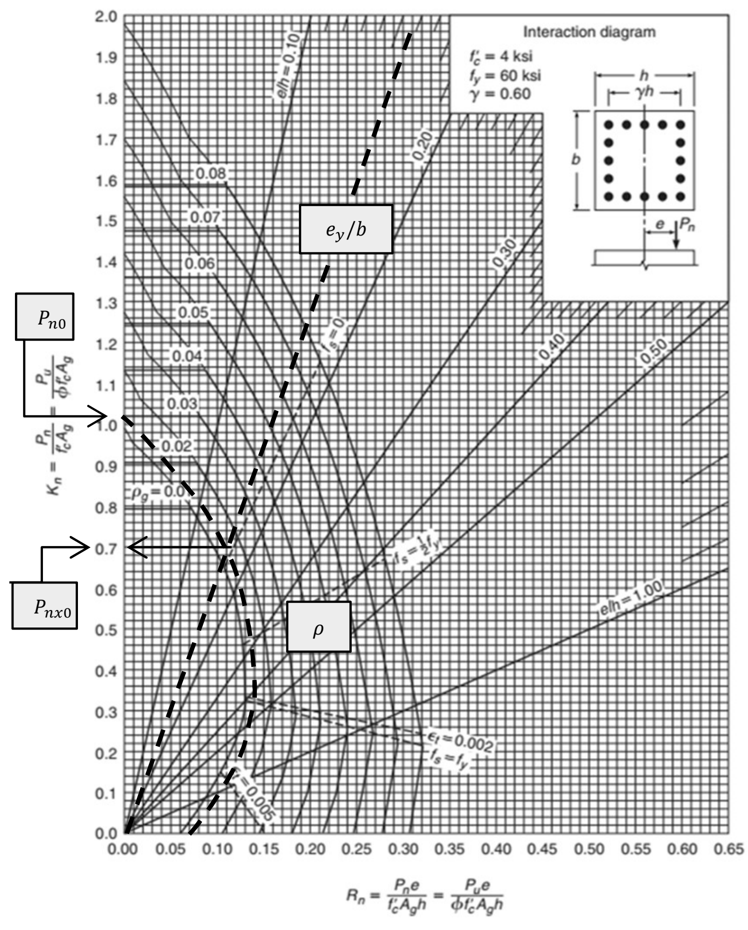
$$P_{n0} = K_n A_g f_c' = 1.09 \times 144 \times 4$$

$$P_{n0} = 627.84 \text{ kip}$$

Again, from Grapgh, the intersecting point of curve ρ and the line e_y/b is $K_n' = 0.7$.

$$P_{nx0} = 0.7 \times (144) \times 4$$

$$P_{nx0} = 403.2 \text{ kip}$$



Design of RC Members Under Axial Loads with Biaxial Bending

❑ Solution

- **Step 3: Calculate P_{n0}, P_{nx0} and P_{ny0}**

  - **Bending about X axis**

From Grapgh, the curve ρ interest Y axis at K_n = 1.09.

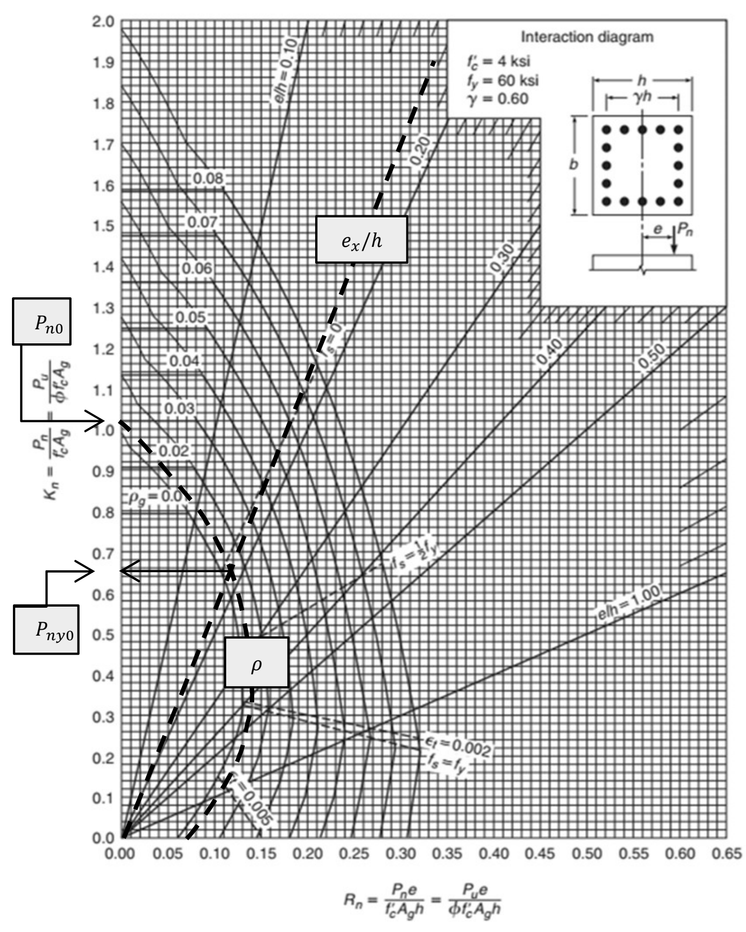
$P_{n0} = K_n A_g f_c' = 1.09 \times 144 \times 4$

$P_{n0} = 627.84 \text{ kip}$

Again, from Grapgh, the intersecting point of curve ρ and the line e_x/h is $K_n' = 0.67$.

$P_{ny} = 0.67 \times (144) \times 4$

$P_{ny} = 385.92 \text{ kip}$



Design of RC Members Under Axial Loads with Biaxial Bending

❑ Solution

➢ Step 4: Calculate Design Axial Capacity

Calculate P_n using the following equation

$$\frac{1}{P_n} = \frac{1}{P_{nx0}} + \frac{1}{P_{ny0}} - \frac{1}{P_0} = \frac{1}{403.2} + \frac{1}{385.92} - \frac{1}{627.84} = 0.003479$$

$$P_n = \frac{1}{0.003479} = 287.43\text{kip}$$

$$\emptyset P_n = 0.65 \times 287.43 = 186.82 \text{ kip} > \mathrm{P_u} = 185 \text{ kip} \rightarrow \text{OK!}$$

Design of RC Members Under Axial Loads with Biaxial Bending

2. Load Contour Method

- The load contour method is based on representing the failure surface of 3D interaction diagram by a family of curves corresponding to constant values of P_n. The equation is given below:

$$\left(\frac{M_{nx}}{M_{nx0}}\right)^{\alpha_1} = \left(\frac{M_{ny}}{M_{ny0}}\right)^{\alpha_2} \leq 1$$

Where;

$M_{nx} = P_n e_y$; $M_{nx0} = M_{nx}$ (when $M_{ny} = 0$)

$M_{ny} = P_n e_x$; $M_{ny0} = M_{ny}$ (when $M_{nx} = 0$)

α_1 & α_2 are exponents depending on column dimensions, amount and distribution of reinforcement, concrete cover and size of transverse ties or spiral.

Design of RC Members Under Axial Loads with Biaxial Bending

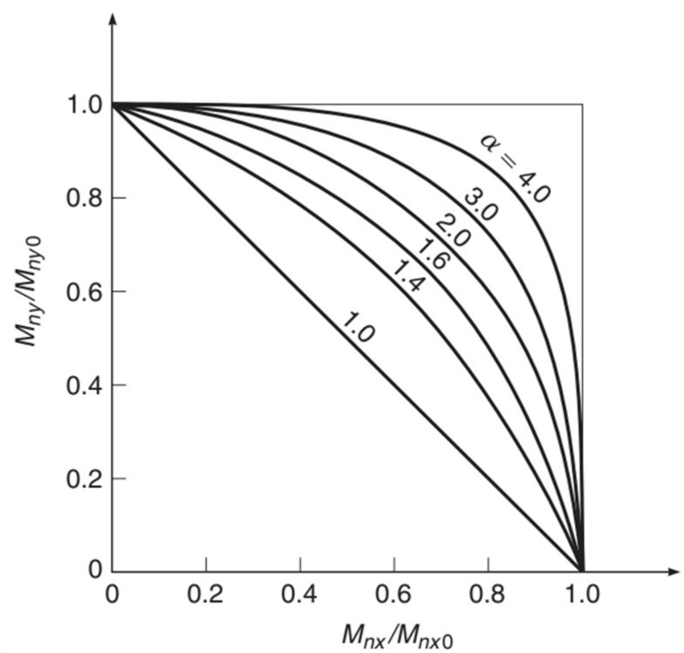
2. Load Contour Method

- Calculations reported by Bressler indicate that α falls in the range from 1.15 to 1.55 for square and rectangular columns.
- Values near the lower end of that range are the more conservative.

Design of RC Members Under Axial Loads with Biaxial Bending

2. Load Contour Method

- When $\alpha_1 = \alpha_2 = \alpha$, the shapes of such interaction contours are as shown for specific α values.
- For values of M_{nx}/M_{nxo} and M_{ny}/M_{nyo} , α can be determined from the given graph.



Design of RC Members Under Axial Loads with Biaxial Bending

2. Load Contour Method

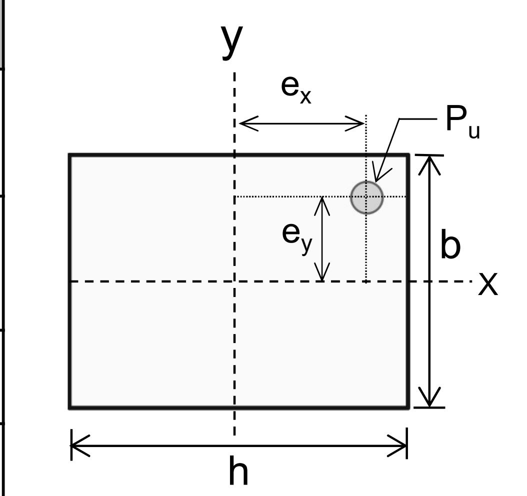
❖ Stepwise Procedure

➢ **Step 1: Check Applicability of Method**

$P_n < 0.1 A_g f_c'$ → applies, otherwise not.

➢ **Step 2: Calculate Necessary Parameters**

Bending about X axis	Bending about Y axis
$\gamma = \frac{h - 2d'}{h}$	$\gamma = \frac{b - 2d'}{b}$
$\frac{e_y}{b} = \frac{M_{ux}}{P_u b}$	$\frac{e_x}{h} = \frac{M_{uy}}{P_u h}$
Assume $\rho = A_s/bh$	---
Select relevant graph based on given f_c', f_y and γ	Select relevant graph based on given f_c', f_y and γ



Design of RC Members Under Axial Loads with Biaxial Bending

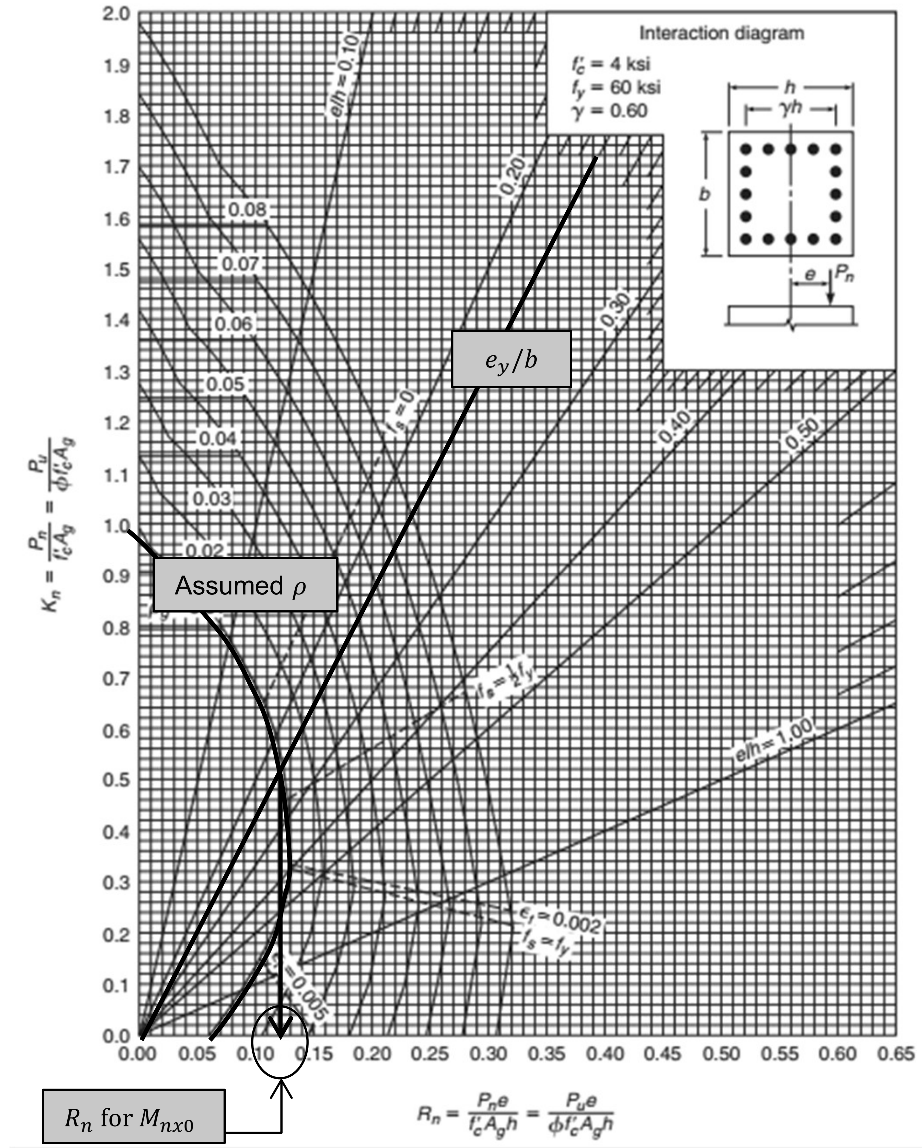
2. Load Contour Method

❖ Stepwise Procedure

➢ **Step 3: Calculate M_{nx0} and M_{ny0}**

▪ Bending about X axis

$$M_{nx} = R_n A_g f_c' b$$



Design of RC Members Under Axial Loads with Biaxial Bending

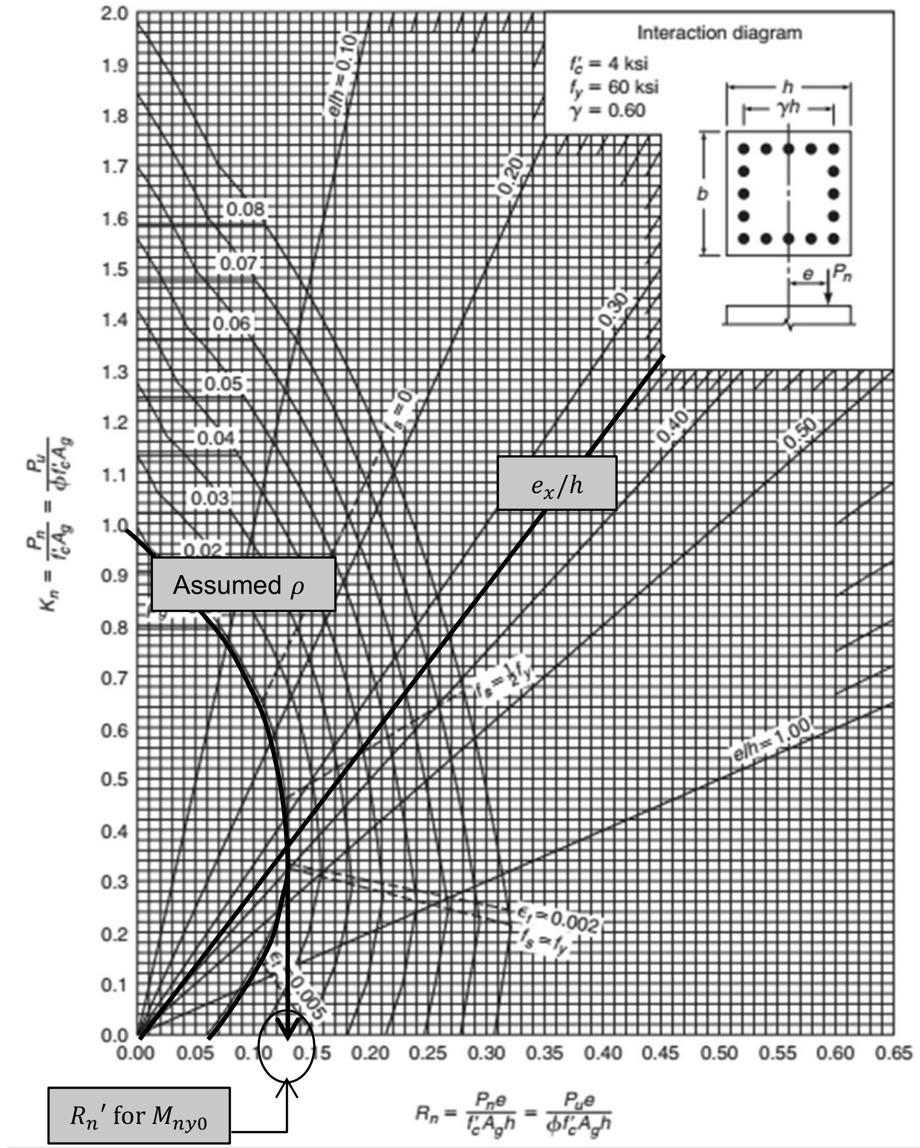
2. Load Contour Method

❖ Stepwise Procedure

➢ **Step 3: Calculate M_{nx0} and M_{ny0}**

▪ Bending about Y axis

$$M_{ny0} = R_n' A_g f_c' h$$



Design of RC Members Under Axial Loads with Biaxial Bending

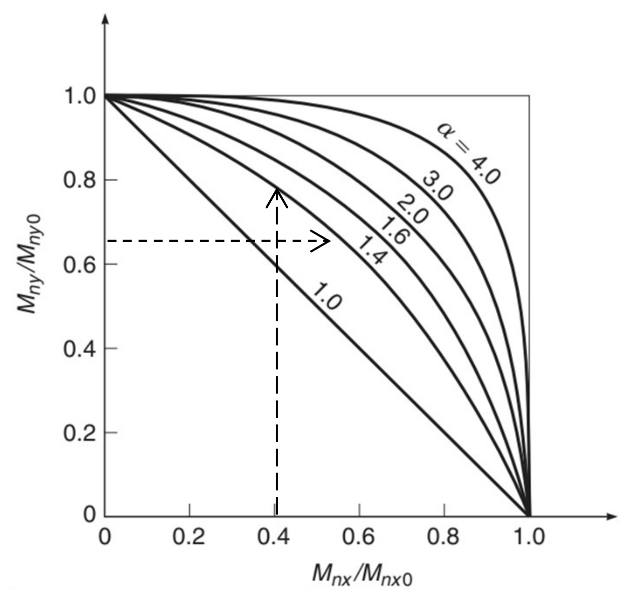
1. Reciprocal Load Method

❖ Stepwise Procedure

➢ Step 4: Check the Capacity

- Knowing the required values, select $\alpha_1 = \alpha_2 = \alpha$ from graph
- Substitute values in Load contour equation

$$\left(\frac{M_{nx}}{M_{nx0}}\right)^{\alpha_1} + \left(\frac{M_{ny}}{M_{ny0}}\right)^{\alpha_2} \leq 1$$



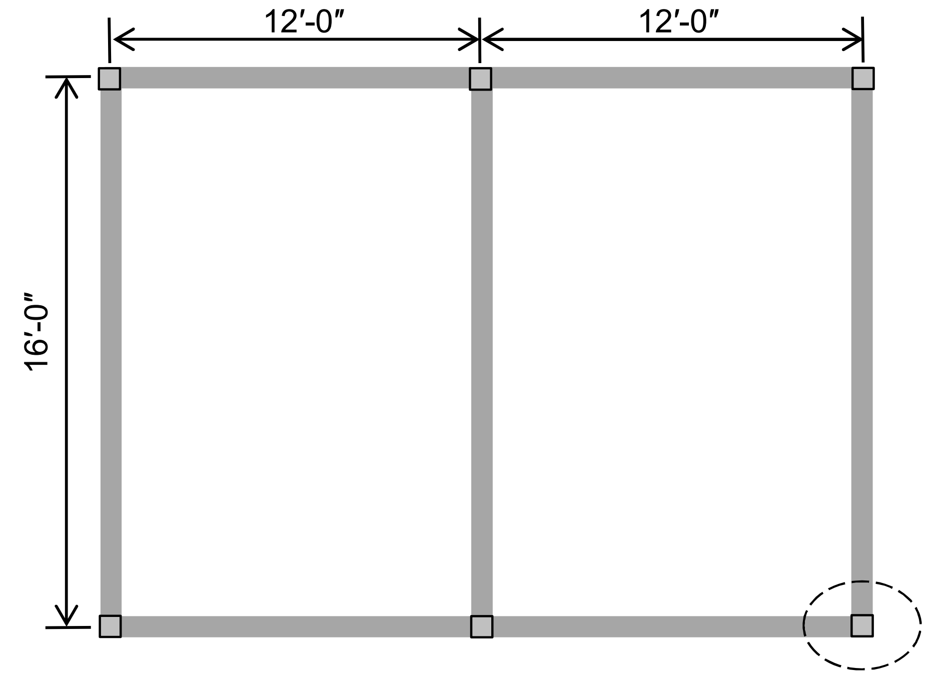
- If $\text{LHS} \leq 1 \rightarrow$ Design is OK!,

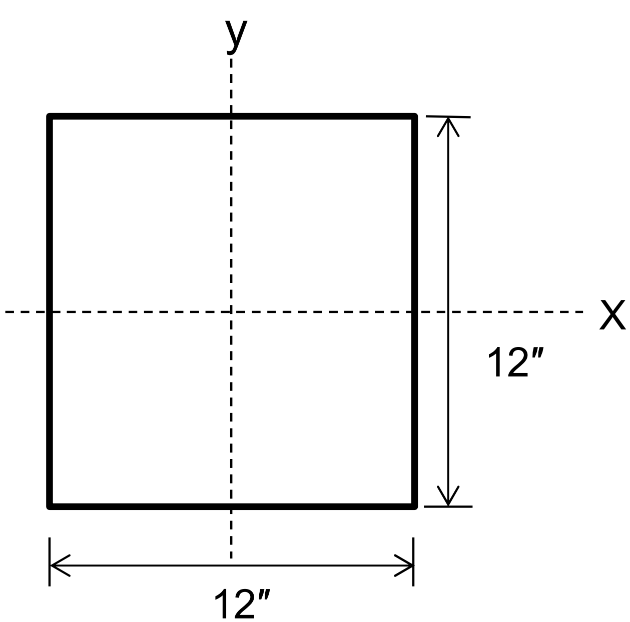
  otherwise repeat the process.

Design of RC Members Under Axial Loads with Biaxial Bending

❑ Example 3.11

- Using Load Contour Method, ***determine*** area of longitudinal reinforcement for the corner column highlighted in figure, to support factored load of $P_u = 30$ kip, $M_{ux} = 20$ ft.kip and $M_{uy} = 30$ ft.kip. Take f_c' = 4 ksi and f_y = 60 ksi.





Design of RC Members Under Axial Loads with Biaxial Bending

Solution

- **Step 1: Check Applicability of Method**

$$P_n = \frac{P_u}{\emptyset} = \frac{30}{0.65} = 46.15 \text{ kip}$$

$$0.1A_g f_c' = 0.1 \times (12 \times 12) \times 4 = 57.6 \text{ kip}$$

$P_n = 46.15 \text{ kip} < 0.1A_g f_c' = 57.6 \text{ kip} \rightarrow$ Load Contour Method applies

Design of RC Members Under Axial Loads with Biaxial Bending

❑ Solution

➢ **Step 2: Calculate Necessary Parameters**

Bending about X axis	Bending about Y axis
$\gamma = \frac{h - 2d'}{h} = \frac{12 - 2(2.5)}{12} \approx 0.60$	$\gamma = \frac{b - 2d'}{b} = \frac{12 - 2(2.5)}{12} \approx 0.60$
$\frac{e_y}{b} = \frac{M_{ux}}{P_u b} = \frac{20}{30(1)} = 0.67$	$\frac{e_x}{h} = \frac{M_{uy}}{P_u h} = \frac{30}{30(1)} = 1$
$\rho = \frac{A_s}{bh} = \frac{4(0.44)}{12 \times 12} = 0.012$	---
For f_c' = 4 ksi, f_y = 60 ksi and $\gamma = 0.60$, Graph A.5 of Nilson 14th Ed. applies	For f_c' = 4 ksi, f_y = 60 ksi and $\gamma = 0.60$, Graph A.5 of Nilson 14th Ed. applies

Design of RC Members Under Axial Loads with Biaxial Bending

❑ Solution

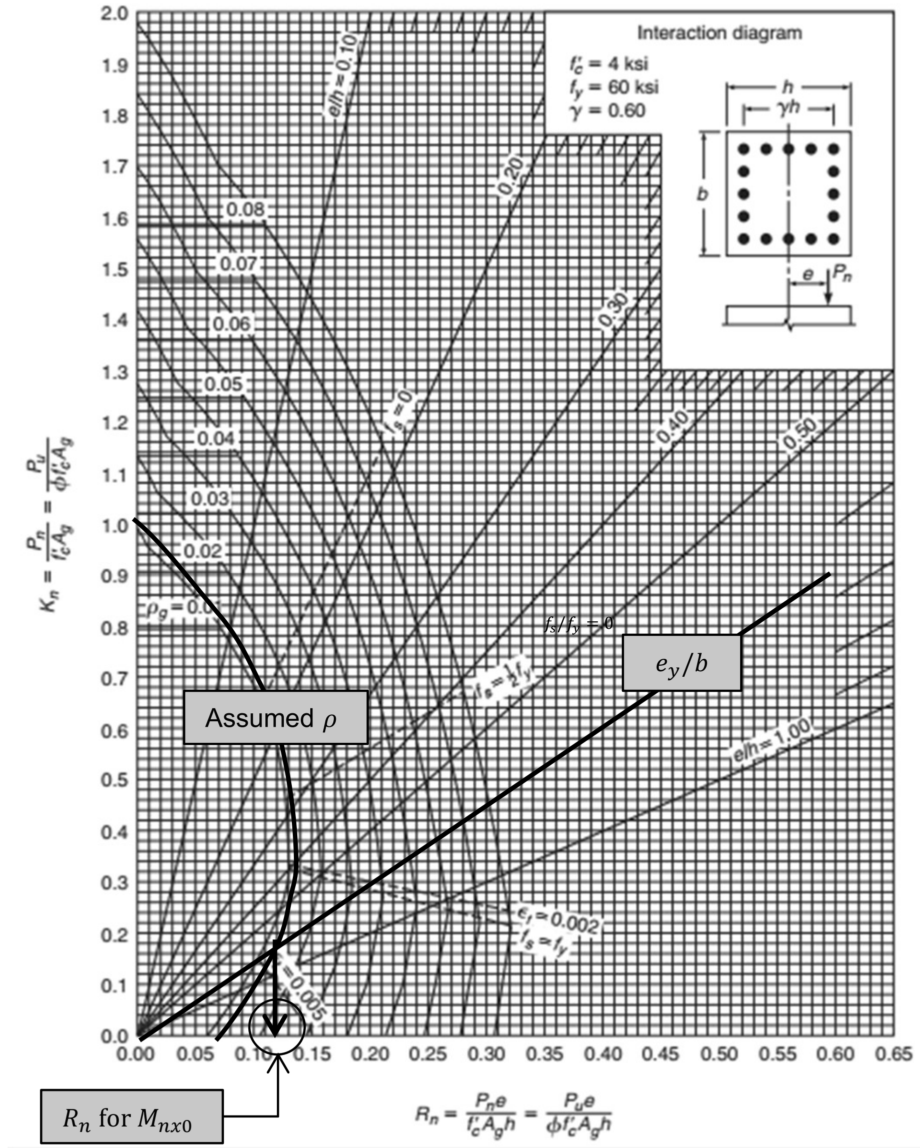
- **Step 3: Calculate M_{nx0} and M_{ny0}**

▪ **Bending about X axis**

$$M_{nx0} = R_n A_g f_c' b$$

$$M_{nx} = 0.12 \times 144 \times 4 \times 12$$

$$M_{nx} = 829.44 \text{ in. kip}$$



Design of RC Members Under Axial Loads with Biaxial Bending

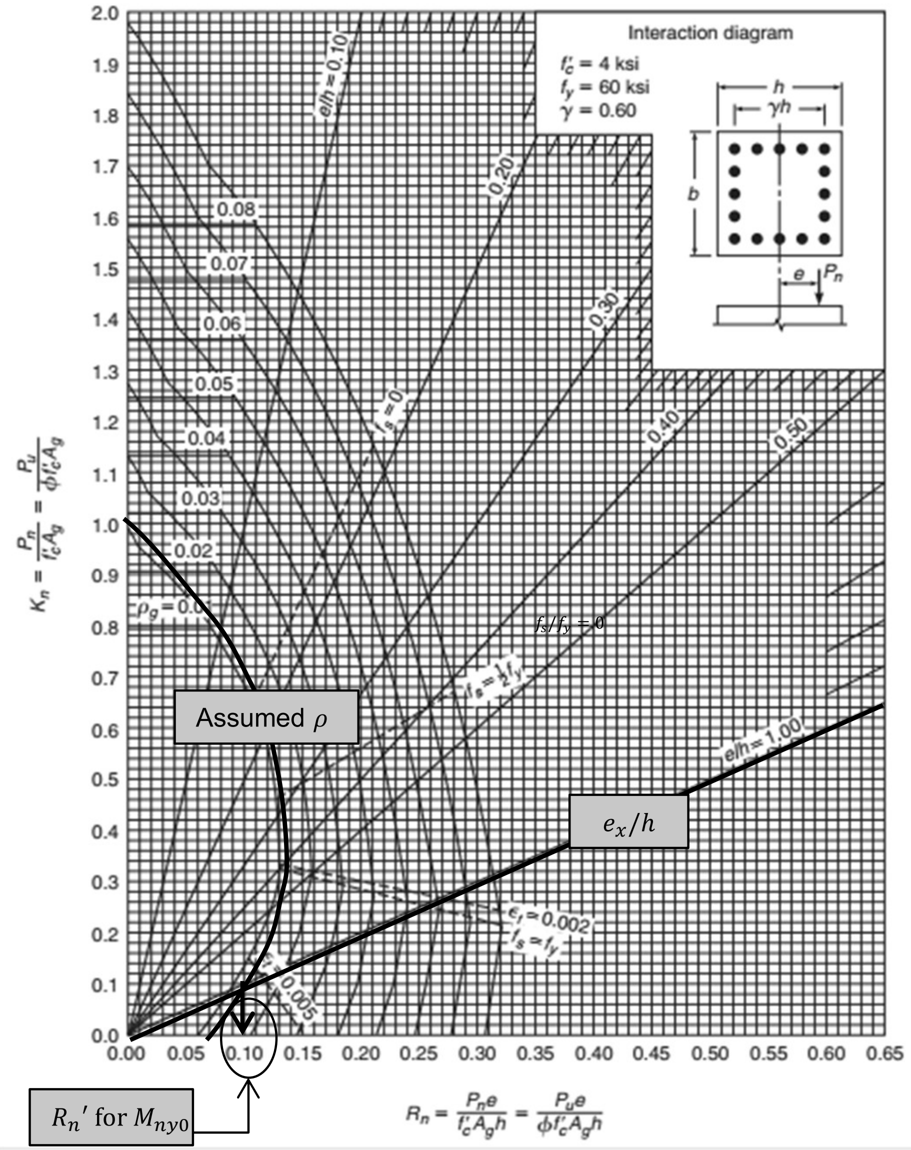
Solution

- **Step 3: Calculate M_{nx0} and M_{ny0}**
  - **Bending about Y axis**

$$M_{ny0} = R_n{}' A_g f_c' h$$

$$M_{ny} = 0.10 \times 144 \times 4 \times 12$$

$$M_{ny0} = 691.2 \text{ in. kip}$$



Design of RC Members Under Axial Loads with Biaxial Bending

❑ Solution

➢ Step 4: Check Capacity

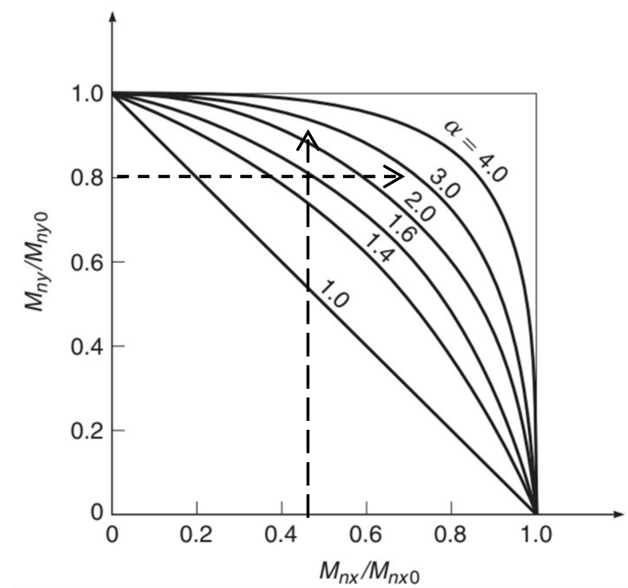
$$\frac{M_{nx}}{M_{nx0}} = \frac{(20/0.65) \times 12}{829.44} = 0.45 \quad \& \quad \frac{M_{ny}}{M_{ny0}} = \frac{(30/0.65) \times 12}{691.2} = 0.8$$

From grapgh, $\alpha_1 = \alpha_2 = \alpha = 1.6$

Substitute values in Load contour equation

$$\left(\frac{M_{nx}}{M_{nx0}}\right)^{\alpha_1} + \left(\frac{M_{ny}}{M_{ny0}}\right)^{\alpha_2} = (0.45)^{1.6} + (0.8)^{1.6}$$

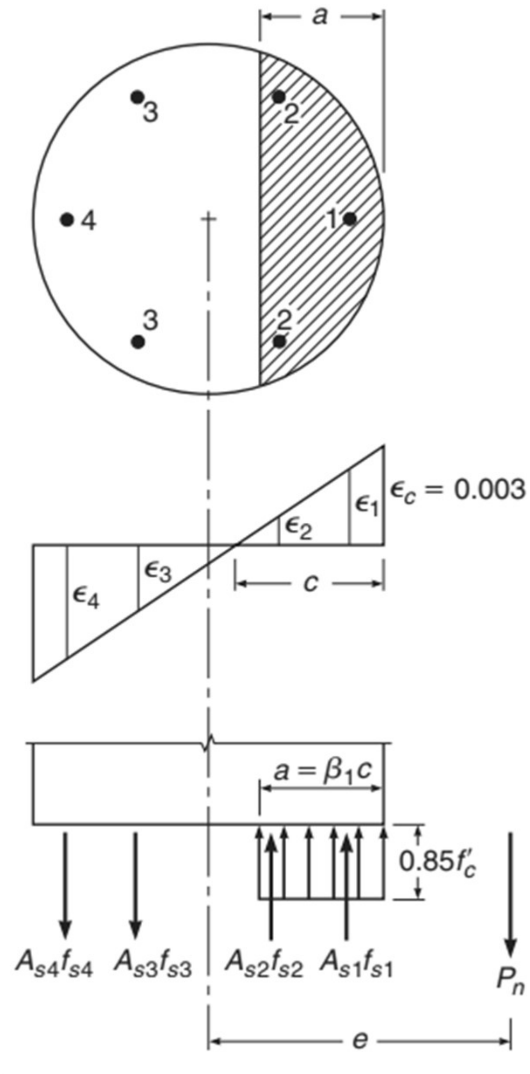
$0.978 < 1 \rightarrow \text{OK!}$



Design of RC Members Under Axial Loads with Biaxial Bending

❑ Behavior of Circular Columns

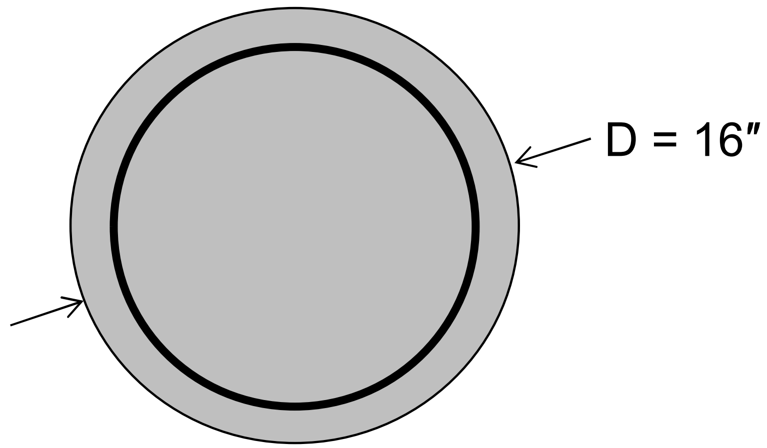
- The Strain distribution at ultimate load is shown in figure.
- The concrete compression zone subject to the equivalent rectangular stress distribution has the shape of a segment of a circle, shown shaded.



Design of RC Members Under Axial Loads with Biaxial Bending

❑ Example 3.12

- ***Design*** a circular column section shown in figure using approximate methods to support factored loads $P_u = 60$ kip, $M_{ux} = 20$ ft.kip and $M_{uy} = 30$ ft.kip. Take f_c' = 4 ksi and f_y = 60 ksi.



Design of RC Members Under Axial Loads with Biaxial Bending

❑ Solution

➢ **Step 1: Check Applicability of Method**

$$P_n = \frac{P_u}{\emptyset} = \frac{60}{0.65} = 92.3 \text{ kip}$$

$$0.1A_g f_c' = 0.1 \times \left(\frac{\pi \times 16^2}{4}\right) \times 4 = 80.42 \text{ kip}$$

$P_n = 92.3 \text{ kip} > 0.1A_g f_c' = 80.42 \text{ kip} \rightarrow$ Reciprocal Load Method applies

Design of RC Members Under Axial Loads with Biaxial Bending

❑ Solution

➢ **Step 2: Calculate Necessary Parameters**

Bending about X axis	Bending about Y axis
$\gamma = \frac{D-2d'}{D} = \frac{12-2(2.5)}{12} \approx 0.70$	$\gamma = \frac{D-2d'}{D} = \frac{16-2(2.5)}{16} \approx 0.70$
$\frac{e_y}{D} = \frac{M_{ux}}{P_u D} = \frac{20 \times 12}{60(16)} = 0.25$	$\frac{e_x}{D} = \frac{M_{uy}}{P_u D} = \frac{30 \times 12}{60(16)} = 0.36$
$\rho = \frac{A_s}{bh} = \frac{6(0.44)}{12 \times 12} = 0.018$	---
For f_c' = 4 ksi, f_y = 60 ksi and $\gamma = 0.70$, Graph A.14of Nilson 14th Ed. applies	For f_c' = 4 ksi, f_y = 60 ksi and $\gamma = 0.70$, Graph A.14 of Nilson 14th Ed. applies

Design of RC Members Under Axial Loads with Biaxial Bending

❑ Solution

➢ **Step 3: Calculate P_{n0}, P_{nx0} and P_{ny0}**

- **Bending about X axis**

From Grapgh, the curve ρ interest Y axis at K_n = 1.08.

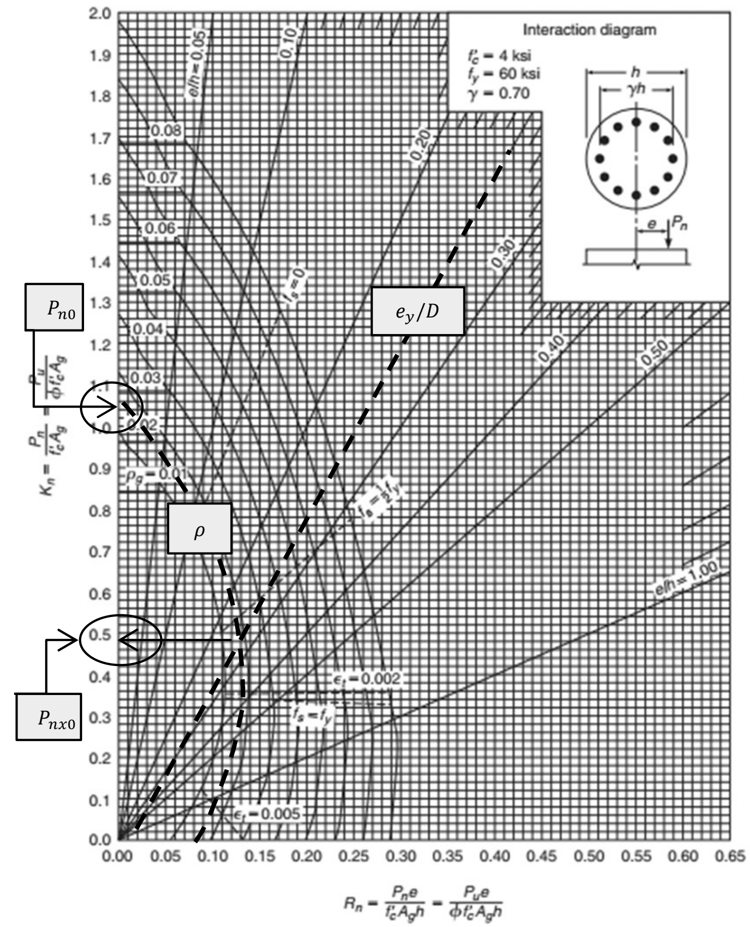
$P_{n0} = K_n A_g f_c' = 1.08 \times (201.06) \times 4$

$P_{n0} = 868.58 \text{ kip}$

Again, from Grapgh, the intersecting point of curve ρ and the line e_y/D is $K_n' = 0.48$.

$P_{nx0} = 0.48 \times (201.06) \times 4$

$P_{nx0} = 386.04 \text{ kip}$



Design of RC Members Under Axial Loads with Biaxial Bending

❑ Solution

- **Step 3: Calculate P_{n0}, P_{nx0} and P_{ny0}**

  - **Bending about Y axis**

From Grapgh, the curve ρ interest Y axis at K_n = 1.08.

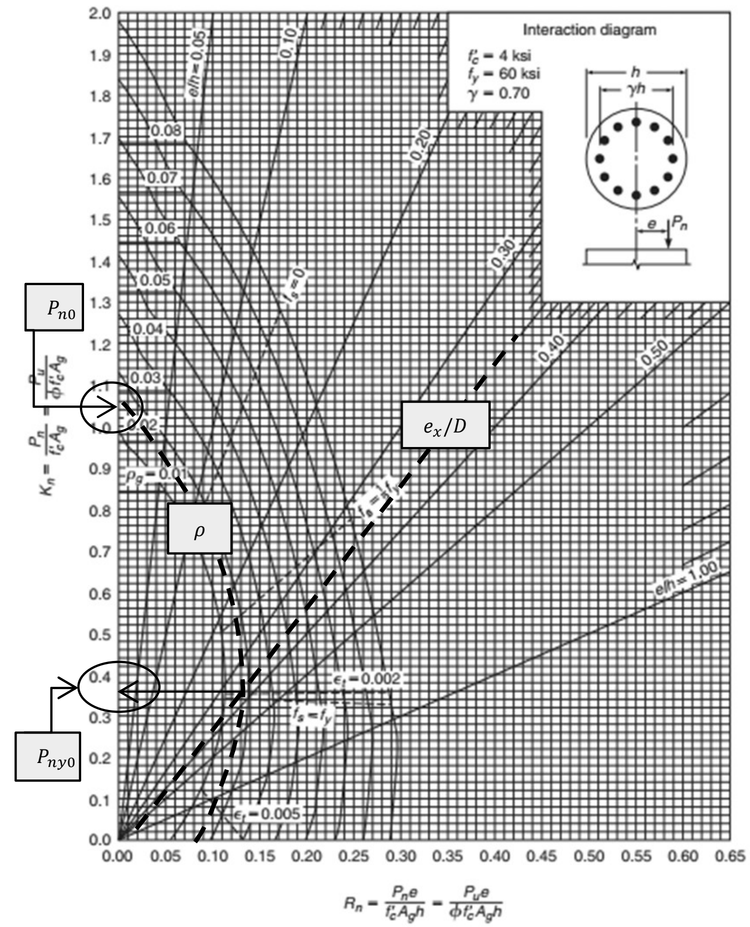
$$P_{n0} = K_n A_g f_c' = 1.08 \times (201.06) \times 4$$

$$P_{n0} = 868.58 \text{ kip}$$

Again, from Grapgh, the intersecting point of curve ρ and the line e_x/D is $K_n' = 0.36$.

$$P_{ny0} = 0.36 \times (201.06) \times 4$$

$$P_{ny0} = 289.52 \text{ kip}$$



Design of RC Members Under Axial Loads with Biaxial Bending

❑ Solution

➢ Step 4: Calculate Design Axial Capacity

Calculate P_n using the following equation

$$\frac{1}{P_n} = \frac{1}{P_{nx0}} + \frac{1}{P_{ny0}} - \frac{1}{P_0} = \frac{1}{386.04} + \frac{1}{289.52} - \frac{1}{868.58} = 0.00489$$

$$P_n = \frac{1}{0.00489} = 204.5 \text{ kip}$$

$$\emptyset P_n = 0.65 \times 204.5 = 132.93 \text{ kip} > \mathrm{P_u} = 60 \text{ kip} \rightarrow \text{OK!}$$

References

- Reinforced Concrete - Mechanics and Design (7th Ed.) by James MacGregor.
- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-19)
- Portland Cement Association (PCA 2002)

Appendix

❑ Derivation of c for Pure Bending Condition

As we know that;

$$P = C_c + C_s - T_s$$

For pure bending case, $P = 0$

$$T_s = C_c + C_s$$

$$A_{s2}f_2 = 0.85f_c'ab + A_{s1}f_{s1} \quad \Rightarrow a = \frac{A_{s2}f_{s2} - A_{s1}f_{s1}}{0.85f_c'b}$$

Here $A_{s1} = A_{s2} = A_s$, $f_{s1} = 87(1 - d'/c)$, $f_{s2} = f_y$ and $a = 0.85c$

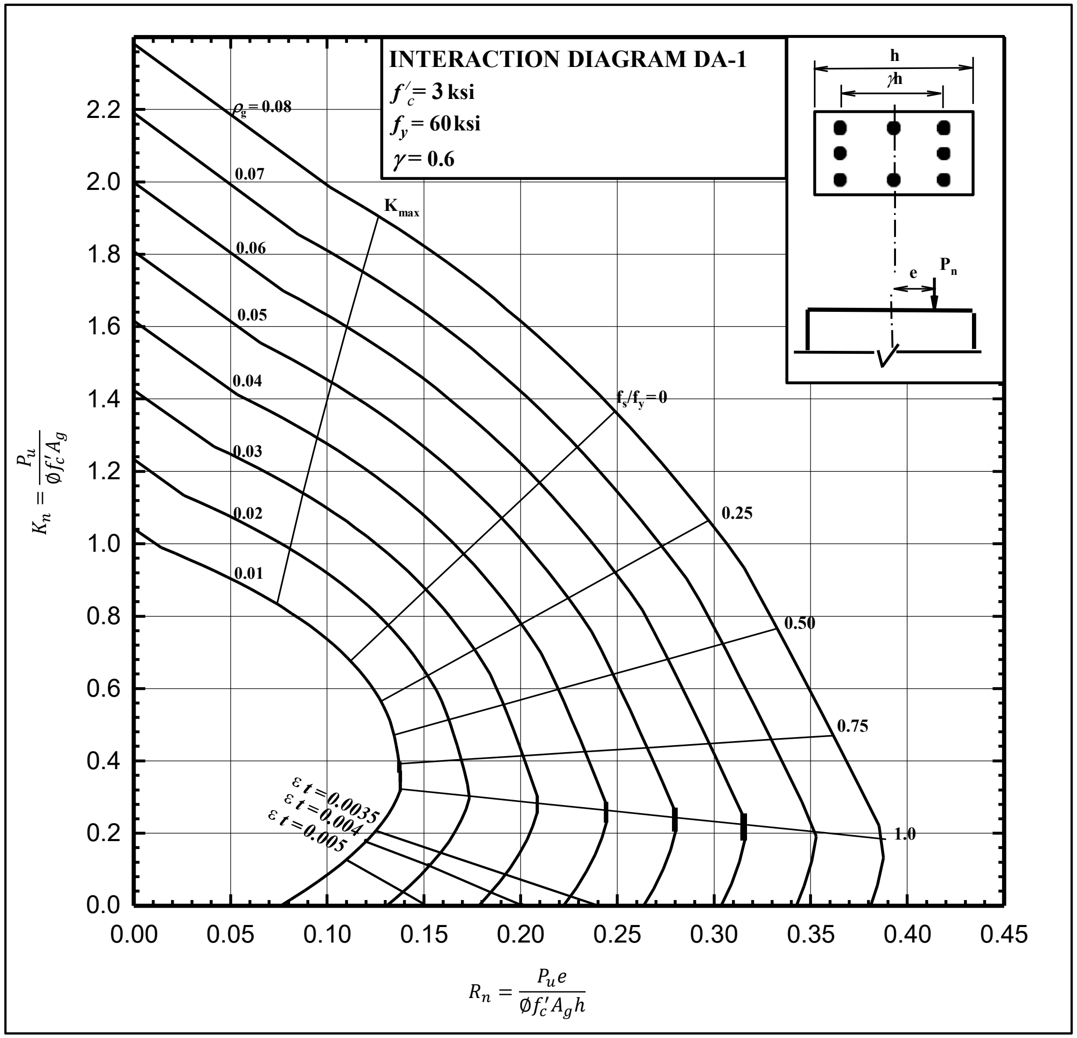
Substituting the above values, we get

$$c = \frac{A_s\left[f_y - 87\left(1 - \frac{d'}{c}\right)\right]}{0.72f_c'b}$$

(This is an implicit equation, hence shall be solved by Equation Solver)

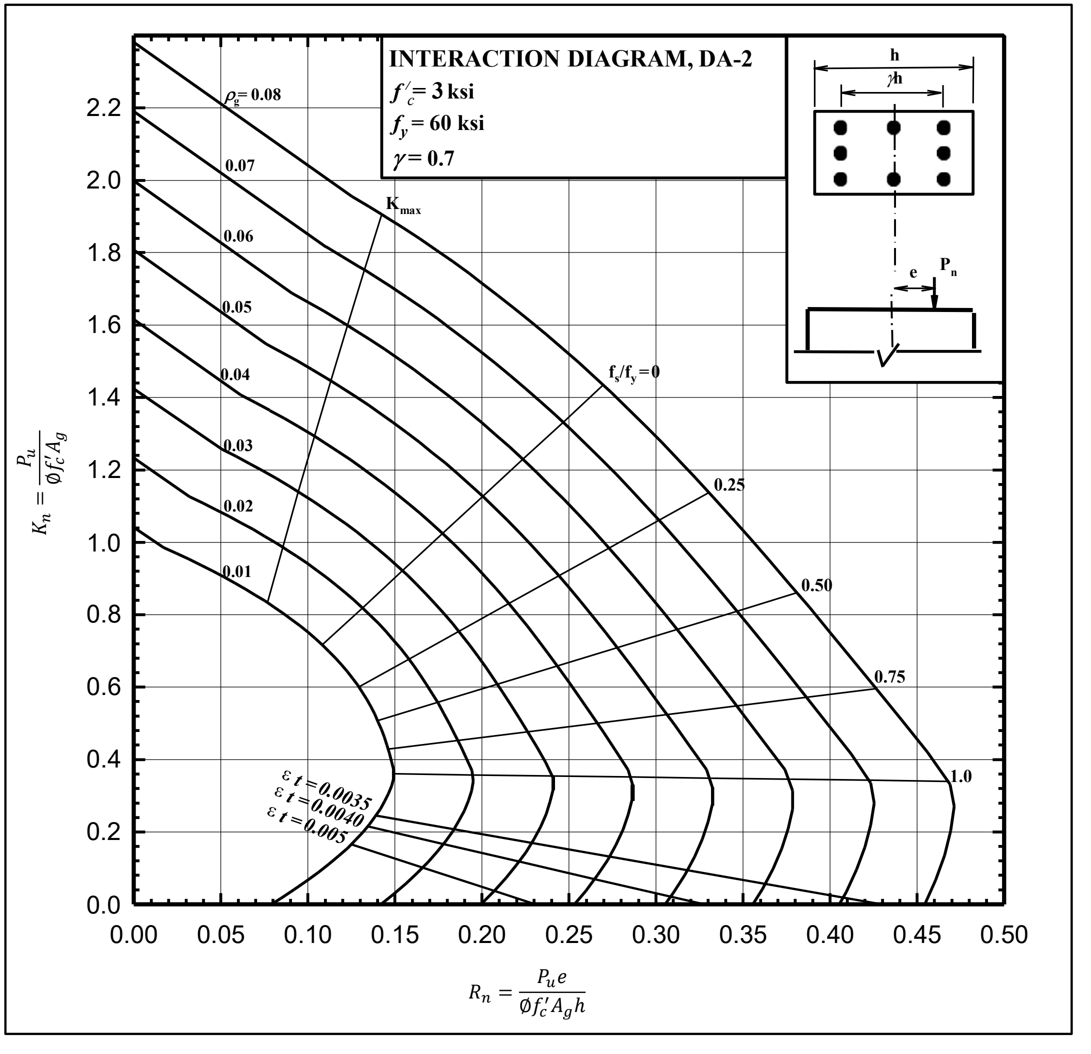
Appendix

❑ DESIGN AIDS (DA-1)



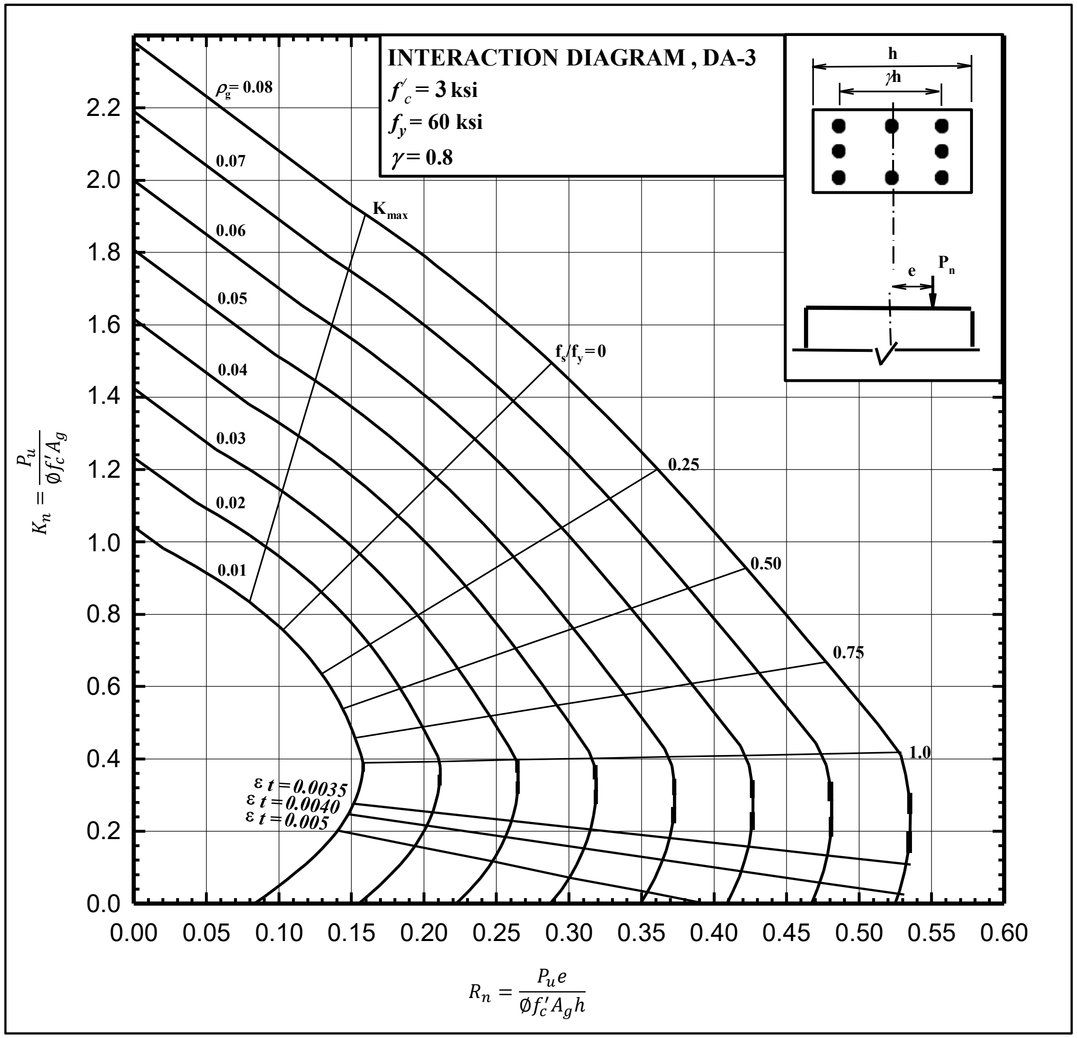
Appendix

❑ DESIGN AIDS (DA-2)



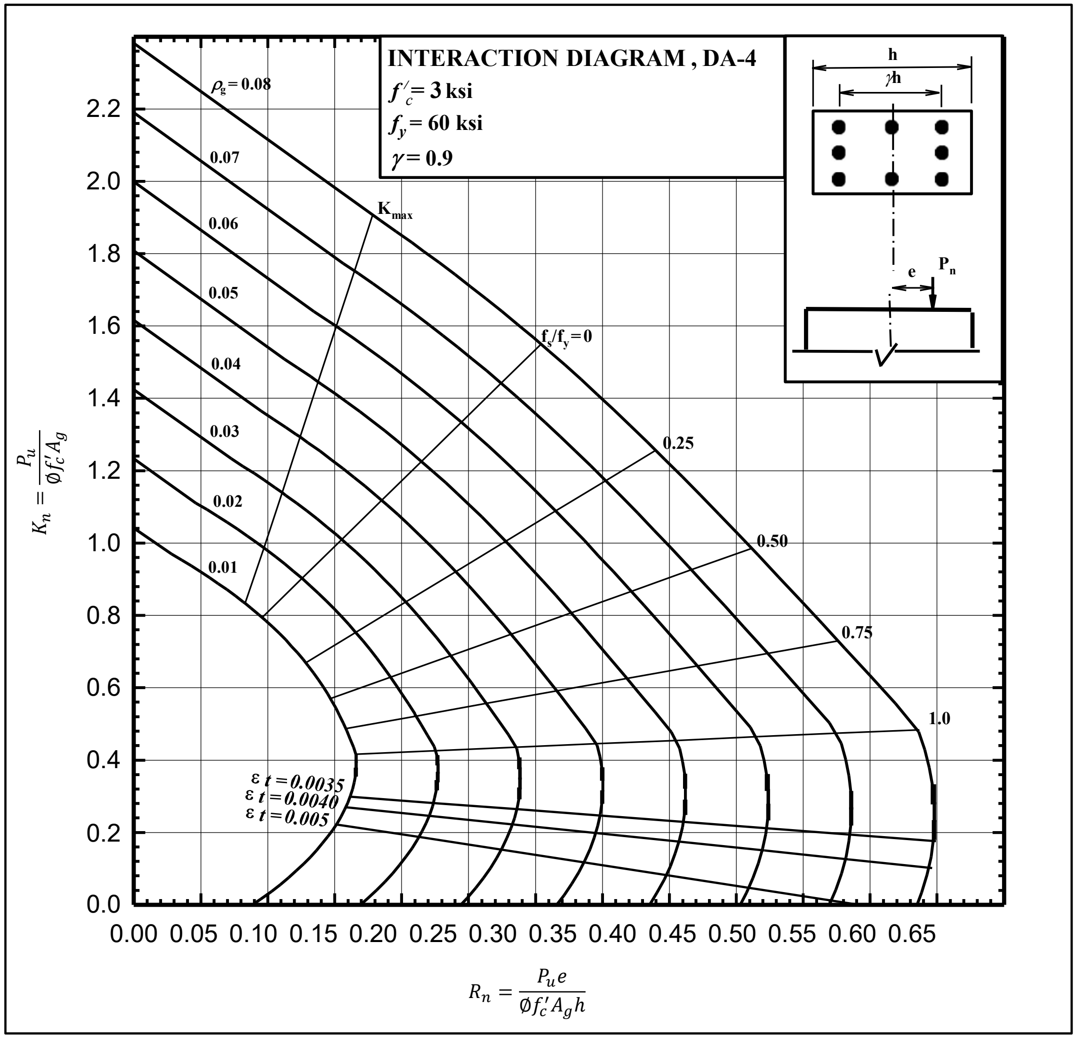
Appendix

❑ DESIGN AIDS (DA-3)



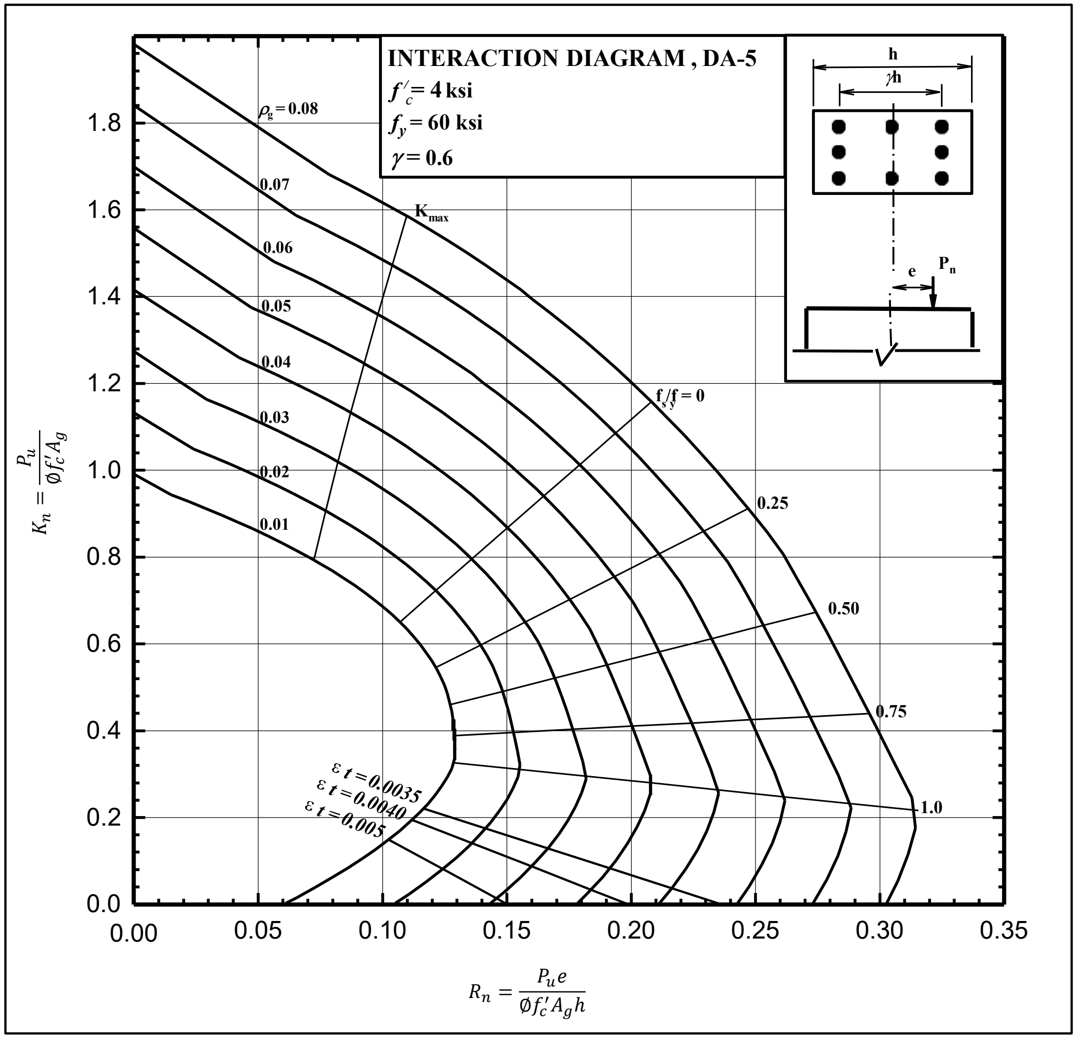
Appendix

❑ DESIGN AIDS (DA-4)



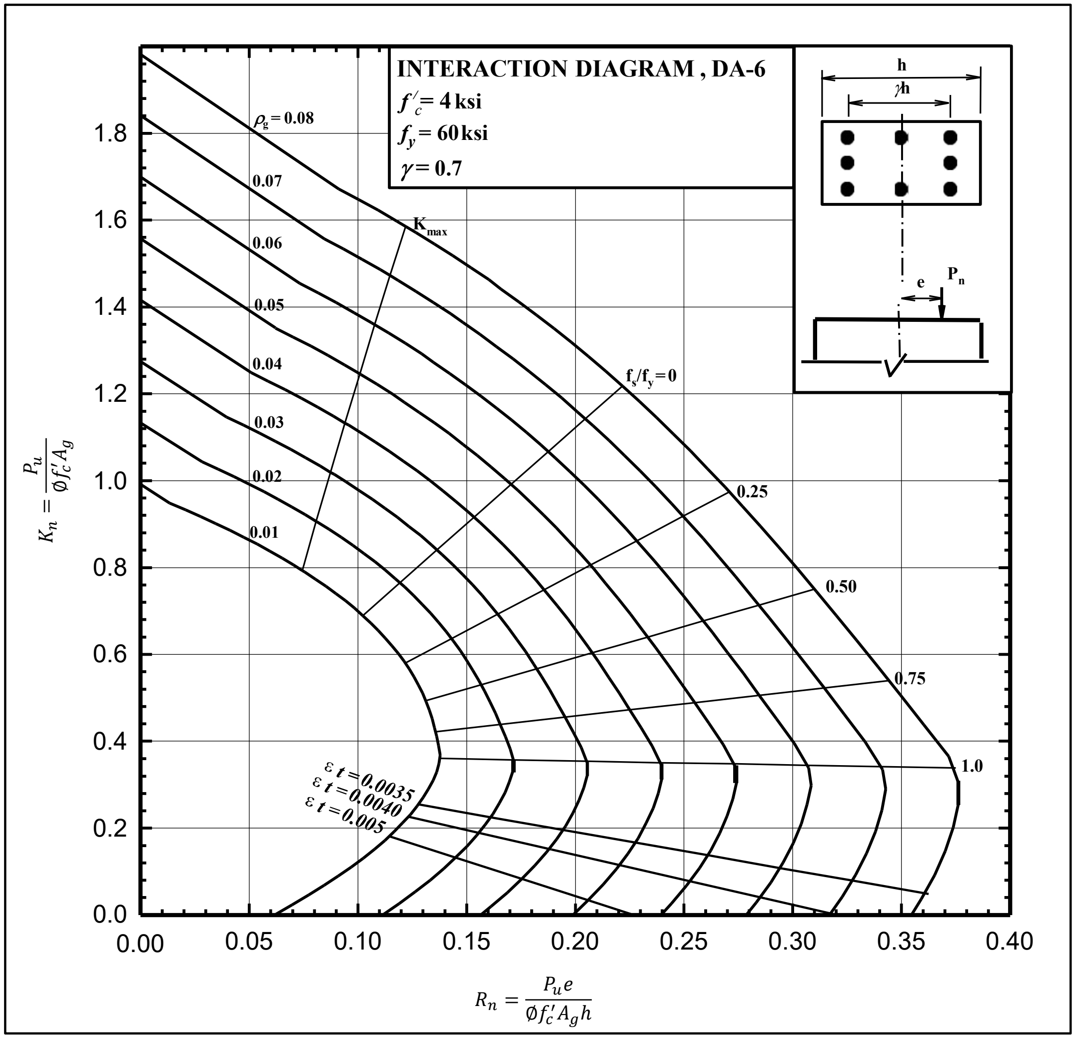
Appendix

❑ DESIGN AIDS (DA-5)



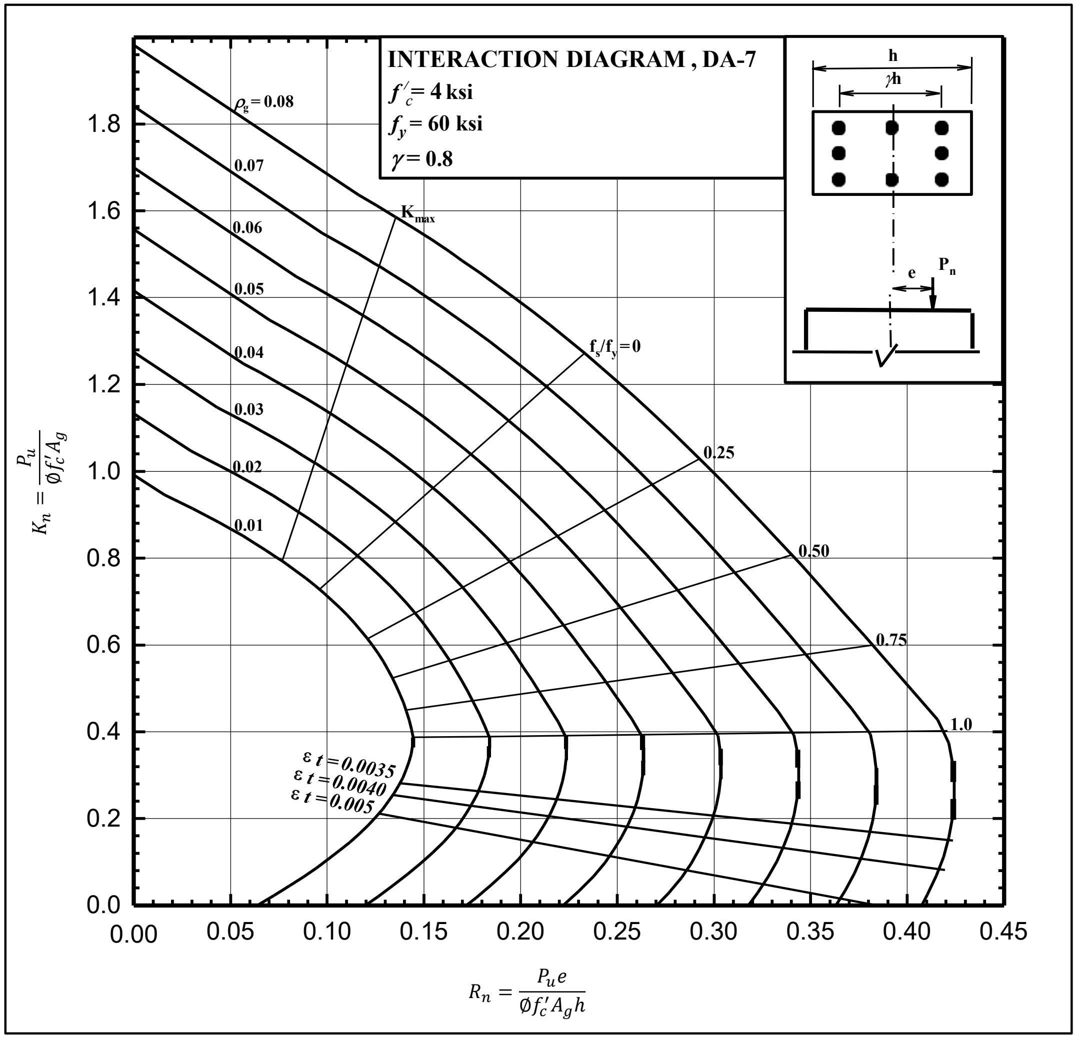
Appendix

❑ DESIGN AIDS (DA-6)



Appendix

❑ DESIGN AIDS (DA-7)



Appendix

❑ DESIGN AIDS (DA-8)