Updated: Nov 02, 2023 Department of Civil Engineering, University of Engineering and Technology Peshawar, Pakistan



Lecture 03

Design of Two-way Slab Systems with Beams

By: Prof. Dr. Qaisar Ali Civil Engineering Department UET Peshawar

drqaisarali@uetpeshawar.edu.pk www.drqaisarali.com

Prof. Dr. Qaisar Ali

CE 416: Reinforced Concrete Design - II



Lecture Contents

- General
- Analysis of Two-way Slabs
- ACI Code Provisions for Two-way Slabs
- Design Examples
- Homework
- References



Learning Objectives

□ At the end of this lecture, students will be able to

- **Classify** one-way and two-way slab systems
- *Employ* ACI coefficient method for two-way slab analyses
- Analyze and Design two-way slabs for flexure
- **Compare** manual and Finite Element Analysis (FEA) results
- **Design** a typical house & regular building for gravity loads





Introduction

- When the ratio of long to short span in a slab supported on all sides is less than 2, then the bending is in two directions. Such a slab is termed as a two-way slab.
- In two-way slabs, the shorter side receives more demand than the longer side.



Slab supported on all sides, but $\beta = l_b/l_a < 2$



□ Bending Behavior of Two-way Slabs





□ Bending Behavior of Two-way Slabs

• Consider the typical floor plan as shown below





□ Bending Behavior of Two-way Slabs

Short Direction Moments



CE 416: Reinforced Concrete Design - II



□ Bending Behavior of Two-way Slabs

Long Direction Moments





- The Moment Coefficient Method, first introduced in the ACI Code in 1963, is applicable to two-way slabs with walls, steel beams, and relatively deep, stiff edge beams supporting each slab panel on its four sides ($h = 3h_f$).
- Although, not included in 1977 and later versions of ACI code, its continued use is permissible under the ACI 318-19 Code provision, Section, 8.2.1.
- The procedure for using this method is explained in the following slides.



Moment Coefficient Method

• The figure below shows the four critical locations where bending moments for a two-way slab panel are calculated.





Moment Coefficient Method

- These four bending moments are calculated in the following manner.
 - 1) $M_{a,neg} = C_a w_u l_a^2$
 - 2) $M_{b,neg} = C_b w_u l_b^2$
 - 3) $M_{a,pos} = C_{a,dl} w_{u,dl} l_a^2 + C_{a,ll} w_{u,ll} l_a^2$
 - 4) $M_{b,pos} = C_{b,dl} w_{u,dl} l_b^2 + C_{b,ll} w_{u,ll} l_b^2$

where;

 $w_u = \text{total factored load}$ $w_{u,dl} = \text{total factored dead load}$ $w_{u,ll} = \text{total factored live load}$ $C_a, C_{b,}C_{a,dl}, C_{a,ll}, C_{b,dl}, C_{b,ll} = \text{coefficients obtained from ACI Tables.}$



- * Various Cases of Slab Panel
 - Depending on the support conditions, several cases are possible





- Various Cases of Slab Panel
 - Depending on the support conditions, several cases are possible





- Various Cases of Slab Panel
 - Depending on the support conditions, several cases are possible





- Various Cases of Slab Panel
 - Depending on the support conditions, several cases are possible





Moment Coefficient Method

* ACI Moment Coefficients Tables

Table	Table A1: Coefficients (C _{a, Negative}) For Negative Moment in Slab along Short Direction										
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9		
0.50	0.000	0.086	0.000	0.094	0.090	0.097	0.000	0.089	0.088		
0.55	0.000	0.084	0.000	0.092	0.089	0.096	0.000	0.085	0.086		
0.60	0.000	0.081	0.000	0.089	0.088	0.095	0.000	0.080	0.085		
0.65	0.000	0.077	0.000	0.085	0.087	0.093	0.000	0.074	0.083		
0.70	0.000	0.074	0.000	0.081	0.086	0.091	0.000	0.068	0.081		
0.75	0.000	0.069	0.000	0.076	0.085	0.088	0.000	0.061	0.078		
0.80	0.000	0.065	0.000	0.071	0.083	0.086	0.000	0.055	0.075		
0.85	0.000	0.060	0.000	0.066	0.082	0.083	0.000	0.049	0.072		
0.90	0.000	0.055	0.000	0.060	0.080	0.079	0.000	0.043	0.068		
0.95	0.000	0.050	0.000	0.055	0.079	0.075	0.000	0.038	0.065		
1.00	0.000	0.045	0.000	0.050	0.075	0.071	0.000	0.033	0.061		



Moment Coefficient Method

* ACI Moment Coefficients Tables

Table	Table A2: Coefficients (C _{b, Negative}) For Negative Moment in Slab along Long Direction										
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9		
0.50	0.000	0.006	0.022	0.006	0.000	0.000	0.014	0.010	0.003		
0.55	0.000	0.007	0.028	0.008	0.000	0.000	0.019	0.014	0.005		
0.60	0.000	0.010	0.035	0.011	0.000	0.000	0.024	0.018	0.006		
0.65	0.000	0.014	0.043	0.015	0.000	0.000	0.031	0.024	0.008		
0.70	0.000	0.017	0.050	0.019	0.000	0.000	0.038	0.029	0.011		
0.75	0.000	0.022	0.056	0.024	0.000	0.000	0.044	0.036	0.014		
0.80	0.000	0.027	0.061	0.029	0.000	0.000	0.051	0.041	0.017		
0.85	0.000	0.031	0.065	0.034	0.000	0.000	0.057	0.046	0.021		
0.90	0.000	0.037	0.070	0.040	0.000	0.000	0.062	0.052	0.025		
0.95	0.000	0.041	0.072	0.045	0.000	0.000	0.067	0.056	0.029		
1.00	0.000	0.045	0.076	0.050	0.000	0.000	0.071	0.061	0.033		



Moment Coefficient Method

* ACI Moment Coefficients Tables

Table A3: Coefficients (C _{a, dl}) For Dead Load Positive Moment in Slab along Short Direction									
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.095	0.037	0.080	0.059	0.039	0.061	0.089	0.056	0.023
0.55	0.088	0.035	0.071	0.056	0.038	0.058	0.081	0.052	0.024
0.60	0.081	0.034	0.062	0.053	0.037	0.056	0.073	0.048	0.026
0.65	0.074	0.032	0.054	0.050	0.036	0.054	0.065	0.044	0.028
0.70	0.068	0.030	0.046	0.046	0.035	0.051	0.058	0.040	0.029
0.75	0.061	0.028	0.040	0.043	0.033	0.048	0.051	0.036	0.031
0.80	0.056	0.026	0.034	0.039	0.032	0.045	0.045	0.032	0.029
0.85	0.050	0.024	0.029	0.036	0.031	0.042	0.040	0.029	0.028
0.90	0.045	0.022	0.025	0.033	0.029	0.039	0.035	0.025	0.026
0.95	0.040	0.020	0.021	0.030	0.028	0.036	0.031	0.022	0.024
1.00	0.036	0.018	0.018	0.027	0.027	0.033	0.027	0.020	0.023



Moment Coefficient Method

* ACI Moment Coefficients Tables

Table A4: Coefficients (C _{b, dl}) For Dead Load Positive Moment in Slab along Long Direction										
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	
0.50	0.006	0.002	0.007	0.004	0.001	0.003	0.007	0.004	0.002	
0.55	0.008	0.003	0.009	0.005	0.002	0.004	0.009	0.005	0.003	
0.60	0.010	0.004	0.011	0.007	0.003	0.006	0.012	0.007	0.004	
0.65	0.013	0.006	0.014	0.009	0.004	0.007	0.014	0.009	0.005	
0.70	0.016	0.007	0.016	0.011	0.005	0.009	0.017	0.011	0.006	
0.75	0.019	0.009	0.018	0.013	0.007	0.013	0.020	0.013	0.007	
0.80	0.023	0.011	0.020	0.016	0.009	0.015	0.022	0.015	0.010	
0.85	0.026	0.012	0.022	0.019	0.011	0.017	0.025	0.017	0.013	
0.90	0.029	0.014	0.024	0.022	0.013	0.021	0.028	0.019	0.015	
0.95	0.033	0.016	0.025	0.024	0.015	0.024	0.031	0.021	0.017	
1.00	0.036	0.018	0.027	0.027	0.018	0.027	0.033	0.023	0.020	



Moment Coefficient Method

* ACI Moment Coefficients Tables

Table A5: Coefficients (C _{a, ll}) For Live Load Positive Moment in Slab along Short Direction										
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	
0.50	0.095	0.066	0.088	0.077	0.067	0.078	0.092	0.076	0.067	
0.55	0.088	0.062	0.080	0.072	0.063	0.073	0.085	0.070	0.063	
0.60	0.081	0.058	0.071	0.067	0.059	0.068	0.077	0.065	0.059	
0.65	0.074	0.053	0.064	0.062	0.055	0.064	0.070	0.059	0.054	
0.70	0.068	0.049	0.057	0.057	0.051	0.060	0.063	0.054	0.050	
0.75	0.061	0.045	0.051	0.052	0.047	0.055	0.056	0.049	0.046	
0.80	0.056	0.041	0.045	0.048	0.044	0.051	0.051	0.044	0.042	
0.85	0.050	0.037	0.040	0.043	0.041	0.046	0.045	0.040	0.039	
0.90	0.045	0.034	0.035	0.039	0.037	0.042	0.040	0.035	0.036	
0.95	0.040	0.030	0.031	0.035	0.034	0.038	0.036	0.031	0.032	
1.00	0.036	0.027	0.027	0.032	0.032	0.035	0.032	0.028	0.030	



Moment Coefficient Method

* ACI Moment Coefficients Tables

Table A6: Coefficients (C _{b, ll}) For Live Load Positive Moment in Slab along Long Direction										
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	
0.50	0.006	0.004	0.007	0.005	0.004	0.005	0.007	0.005	0.007	
0.55	0.008	0.006	0.009	0.007	0.005	0.006	0.009	0.007	0.006	
0.60	0.010	0.007	0.011	0.009	0.007	0.008	0.011	0.009	0.007	
0.65	0.013	0.010	0.014	0.011	0.009	0.010	0.014	0.011	0.009	
0.70	0.016	0.012	0.016	0.014	0.011	0.013	0.017	0.014	0.011	
0.75	0.019	0.014	0.019	0.016	0.013	0.016	0.020	0.016	0.013	
0.80	0.023	0.017	0.022	0.020	0.016	0.019	0.023	0.019	0.017	
0.85	0.026	0.019	0.024	0.023	0.019	0.022	0.026	0.022	0.020	
0.90	0.029	0.022	0.027	0.026	0.021	0.025	0.029	0.024	0.022	
0.95	0.033	0.025	0.029	0.029	0.024	0.029	0.032	0.027	0.025	
1.00	0.036	0.027	0.032	0.032	0.027	0.032	0.035	0.030	0.08	



Moment Coefficient Method

* ACI Moment Coefficients Tables

Table	Table A7: Ratio of Load "w" in Short Direction for Shear in Slab and Load on Supports										
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9		
0.50	0.94	0.94	0.76	0.94	0.99	0.97	0.86	0.89	0.97		
0.55	0.92	0.92	0.69	0.92	0.98	0.96	0.81	0.85	0.95		
0.60	0.89	0.89	0.61	0.89	0.97	0.95	0.76	0.80	0.94		
0.65	0.85	0.85	0.53	0.85	0.96	0.93	0.69	0.74	0.92		
0.70	0.81	0.81	0.45	0.81	0.95	0.91	0.62	0.68	0.89		
0.75	0.76	0.76	0.39	0.76	0.94	0.88	0.56	0.61	0.86		
0.80	0.71	0.71	0.33	0.71	0.92	0.86	0.49	0.55	0.83		
0.85	0.66	0.66	0.28	0.66	0.90	0.83	0.43	0.49	0.79		
0.90	0.60	0.60	0.23	0.60	0.88	0.79	0.38	0.43	0.75		
0.95	0.55	0.55	0.20	0.55	0.86	0.75	0.33	0.38	0.71		
1.00	0.50	0.50	0.17	0.50	0.83	0.71	0.29	0.33	0.67		



ACI Code Provisions for Two-way Slabs

Minimum Slab Thickness (8.3.1.2)

• The Minimum thickness of two-way slabs with beams spanning between supports on all sides shall be as per ACI Table 8.3.1.2.

α_{fm}		Minimum h, in.
$\alpha_{fm} \leq 0.2$	Pro	ovisions of One-way slabs apply
$0.2 \le \alpha_{fm} \le 2.0$	Greater of:	$\frac{l_n \left(0.8 + \frac{f_y}{200,000}\right)}{36 + 5\beta (\alpha_{fm} - 0.2)}$ 5
$\alpha_{fm} > 2.0$	Greater of:	$\frac{l_n \left(0.8 + \frac{f_y}{200,000}\right)}{36 + 9\beta}$ 3.5
• a is the everage v		became an address of a papel $\alpha - E = L/E - L$

• α_{fm} is the average value of α_f for all beams on edges of a panel. $\alpha_f = E_{cb} I_b / E_{cs} I_s$

- l_n is the clear span in the long direction, measured face-to-face of beams (in.).
- β is the ratio of clear spans in long to short directions of slab.



ACI Code Provisions for Two-way Slabs

Minimum Slab Thickness (8.3.1.2)

• If the beams are stiff enough, then α_{fm} can be taken greater than 2 and hence the third condition of the Table 8.3.1.2 would be governed.

$$h_{min} = max \left[\frac{l_n \left(0.8 + \frac{f_y}{200,000} \right)}{36 + 9\beta} , 3.5'' \right]$$

Setting $l_n = l_b$ and $\beta = l_b/l_a$ the equation becomes

$$h_{min} = max \left[\frac{l_b \left(0.8 + \frac{f_y}{200,000} \right)}{36 + 9(l_b/l_a)} , 3.5'' \right]$$

• This equation will be used onward to calculate minimum slab thickness.



ACI Code Provisions for Two-way Slabs

Minimum Flexural Reinforcement (8.6.1)

 The minimum reinforcement requirement for two-way slabs is identical to that of one-way slabs.

 $A_{s,min} = 0.0018A_g$

□ Spacing of Flexural Reinforcement (8.7.1)

- Maximum spacing *s* shall be the lesser of:
 - 2h and 18 in. at critical sections
 - 3h and 18 in. at other sections



- Stepwise Procedure
- Calculate minimum slab depth
- Calculate loads
- Decide about case of slab
- Use tables to pick moment coefficients
- Calculate Moments
- Determine required reinforcement
- Apply reinforcement checks

Updated: Nov 02, 2023 Department of Civil Engineering, University of Engineering and Technology Peshawar, Pakistan



Design Example 4.1

Design of Typical Single Story House

Prof. Dr. Qaisar Ali

CE 416: Reinforced Concrete Design – II



Problem Statement

A single – story house with two bedrooms and a verandah is shown in figure on the next slide. The story height is 12 feet, and the thickness of the masonry wall is 9 inches. The slab is insulated by providing four inches of mud layer and two inches of tile brick. According to ASCE 7 -10, the expected uniform service live load for the residential buildings is 40psf.Material strengths are $f_c' = 3$ ksi and $f_y = 60$ ksi.

Design the slab, beam B1 and column



Problem Statement





Given Data

Dimensions of Rooms: 16' x 12' (interior)

Story height, h = 12'

SDL: 4" Mud layer and 2" Tile layer

Live load: 40psf

$$f_c' = 3 \text{ ks}$$

$$f_y = 60$$
 ksi

 $q_a = 2.204 \text{ ksf}$

Required Data

Design the Slab, Beam B1 and Column





Solution

 As can be seen from the figure that the slab over the rooms is a twoway slab case whereas the verandah slab is a one-way slab.



CE 416: Reinforced Concrete Design - II



Solution

- ✤ Slab Design
- Step 1: Selection of Sizes
 - For Two-way Slabs: Assuming $\alpha_{fm} > 2.0$, the minimum thickness is given by:

$$h_{min} = max \left[\frac{l_n \left(0.8 + \frac{f_y}{200,000} \right)}{36 + 9\beta} , 3.5'' \right]$$

Substituting values, we get

$$h_{min} = max \left[\frac{16 \left(0.8 + \frac{60,000}{200,000} \right)}{36 + 9 \left(\frac{16}{12} \right)} , 3.5'' \right] = 4.4''$$





Solution

- ✤ Slab Design
- Step 1: Selection of Sizes
 - For One-way Slab: For one-end continuous slabs, we have

$$h_{min} = \frac{l}{24} \left(0.4 + \frac{f_y}{100000} \right)$$

Substituting values, we get

$$h_{min} = \frac{8.875}{24} \left(0.4 + \frac{60,000}{100,000} \right) = 4.4''$$

Finally take h = 5''



Section A-A



Solution

✤ Slab Design

Step 2: Calculation of loads

Material	Thickness h (in.)	Unit weight γ (kcf)	W = h x γ (ksf)
Concrete Slab	5	0.15	(5/12) × 0.15 = 0.0625
Mud	4	0.12	(4/12) × 0.12 = 0.04
Tile	2	0.12	(2/12) × 0.12= 0.02
То	tal dead load =	0.1225 ksf	

 $w_{u,dl} = 1.2D = 1.2 \times 0.1225 = 0.147$ ksf

 $w_{u,ll} = 1.6 \times 0.04 = 0.064 \text{ ksf}$

 $w_u = 0.147 + 0.064 = 0.211 \text{ ksf}$



Solution

- ✤ Slab Design
- Step 3: Analysis
 - The given system consists of both one way and two-way slabs. A system where a two-way slab is continuous with a one-way slab or vice versa can be called as a mixed slab system.
 - Strictly speaking, the ACI approximate analysis methods are not suitable for mixed systems.
 - In case of one-way slabs, the ACI approximate analysis is applicable where a one-way slab is continuous with a one-way slab.
 - In case of two-way slabs, the moment coefficient tables are applicable where a two-way slab is continuous with a two-way slab.



Solution

- ✤ Slab Design
- Step 3: Analysis
 - The best approach to analyze a mixed system is to use Finite Element software.
 - However, such a system can also be analyzed manually by making certain approximations.
 - We will analyze this system using both methods.


Solution

- ✤ Slab Design
- Step 3: Analysis (two-way slab)
 - Below are the Finite Element Analysis (FEA) results obtained using SAFE.



Moments in Long Direction

Moments in Short Direction

Two-way	Slab Mome	nts (in-kip/ft)	for Rooms	One-way Slab Moment (ir	n-kip/ft) for Verandah
$M_{a(+)}$	$M_{b(+)}$	$M_{a(-)}$	$M_{b(-)}$	$M_{ver(+)}$	$M_{ver,ext(-)}$
19.0	14.0	25.2	20.0	13.2	4.6



Solution

- ✤ Slab Design
- Step 3: Analysis (two-way slab)
- Select slab case

From the figure, the slab case is 4

$$m = \frac{l_a}{l_b} = \frac{12}{16} = 0.750$$

Now, with m = 0.75 and slab case 4, pickup the moment coefficients from the relevant Tables





Given Solution

- ✤ Slab Design
- Step 3: Analysis (two-way slab)

Moment Coefficients

 $C_{a,neg} = 0.076$

	Table A1:	Coefficien	ts (C _{a, Negative}) For Nega	tive Mome	nt in Slab a	long Short	Direction	
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.000	0.086	0.000	0.094	0.090	0.097	0.000	0.089	0.088
0.55	0.000	0.084	0.000	0.092	0.089	0.096	0.000	0.085	0.086
0.60	0.000	0.081	0.000	0.089	0.088	0.095	0.000	0.080	0.085
0.65	0.000	0.077	0.000	0.085	0.087	0.093	0.000	0.074	0.083
0.70	0.000	0.074	0.000	0.081	0.086	0.091	0.000	0.068	0.081
0.75	0.000	0.069	0.000	0.076	0.085	0.088	0.000	0.061	0.078
0.80	0.000	0.065	0.000	0.071	0.083	0.086	0.000	0.055	0.075
0.85	0.000	0.060	0.000	0.066	0.082	0.083	0.000	0.049	0.072
0.90	0.000	0.055	0.000	0.060	0.080	0.079	0.000	0.043	0.068
0.95	0.000	0.050	0.000	0.055	0.079	0.075	0.000	0.038	0.065
1.00	0.000	0.045	0.000	0.050	0.075	0.071	0.000	0.033	0.061



Solution

✤ Slab Design

Step 3: Analysis (two-way slab)

Moment Coefficients

 $C_{a,neg} = 0.076$

$$C_{b,neg}=0.024$$

	Table A2:	Coefficien	ts (C _{b, Negative}	_e) For Nega	tive Mome	nt in Slab a	long Short	Direction	
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.000	0.006	0.022	0.006	0.000	0.000	0.014	0.010	0.003
0.55	0.000	0.007	0.028	0.008	0.000	0.000	0.019	0.014	0.005
0.60	0.000	0.010	0.035	0.011	0.000	0.000	0.024	0.018	0.006
0.65	0.000	0.014	0.043	0.015	0.000	0.000	0.031	0.024	0.008
0.70	0.000	0.017	0.050	0.019	0.000	0.000	0.038	0.029	0.011
0.75	0.000	0.022	0.056	0.024	0.000	0.000	0.044	0.036	0.014
0.80	0.000	0.027	0.061	0.029	0.000	0.000	0.051	0.041	0.017
0.85	0.000	0.031	0.065	0.034	0.000	0.000	0.057	0.046	0.021
0.90	0.000	0.037	0.070	0.040	0.000	0.000	0.062	0.052	0.025
0.95	0.000	0.041	0.072	0.045	0.000	0.000	0.067	0.056	0.029
1.00	0.000	0.045	0.076	0.050	0.000	0.000	0.071	0.061	0.033



Solution

✤ Slab Design

Step 3: Analysis (two-way slab)

Moment Coefficients

 $C_{a,neg} = 0.076$

 $C_{b,neg}=0.024$

$$C_{a,pos,dl} = 0.043$$

Та	ble A3: Co	efficients (C _{a, dl}) For D	ead Load F	ositive Mo	ment in Sla	b along Sh	ort Direction	on
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.095	0.037	0.080	0.059	0.039	0.061	0.089	0.056	0.023
0.55	0.088	0.035	0.071	0.0 <mark>56</mark>	0.038	0.058	0.081	0.052	0.024
0.60	0.081	0.034	0.062	0.0 <mark>5</mark> 3	0.037	0.056	0.073	0.048	0.026
0.65	0.074	0.032	0.054	0.0 <mark>5</mark> 0	0.036	0.054	0.065	0.044	0.028
0.70	0.068	0.030	0.046	0.046	0.035	0.051	0.058	0.040	0.029
0.75	0.061	0.028	0.040	0.043	0.033	0.048	0.051	0.036	0.031
0.80	0.056	0.026	0.034	0.039	0.032	0.045	0.045	0.032	0.029
0.85	0.050	0.024	0.029	0.036	0.031	0.042	0.040	0.029	0.028
0.90	0.045	0.022	0.025	0.033	0.029	0.039	0.035	0.025	0.026
0.95	0.040	0.020	0.021	0.030	0.028	0.036	0.031	0.022	0.024
1.00	0.036	0.018	0.018	0.027	0.027	0.033	0.027	0.020	0.023



Solution

✤ Slab Design

Step 3: Analysis (two-way slab)

Moment Coefficients

 $C_{a,neg} = 0.076$

 $C_{b,neg}=0.024$

$$C_{a,pos,dl} = 0.043$$

$$C_{a,pos,ll} = 0.052$$

Т	able A4: Co	oefficients	(C _{a, II}) For L	ive Load P	ositive Mor	nent in Sla	b along Sh	ort Directio	n
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.095	0.066	0.088	0.077	0.067	0.078	0.092	0.076	0.067
0.55	0.088	0.062	0.080	0.072	0.063	0.073	0.085	0.070	0.063
0.60	0.081	0.058	0.071	0.067	0.059	0.068	0.077	0.065	0.059
0.65	0.074	0.053	0.064	0.062	0.055	0.064	0.070	0.059	0.054
0.70	0.068	0.049	0.057	0.057	0.051	0.060	0.063	0.054	0.050
0.75	0.061	0.045	0.051	0.052	0.047	0.055	0.056	0.049	0.046
0.80	0.056	0.041	0.045	0.048	0.044	0.051	0.051	0.044	0.042
0.85	0.050	0.037	0.040	0.043	0.041	0.046	0.045	0.040	0.039
0.90	0.045	0.034	0.035	0.039	0.037	0.042	0.040	0.035	0.036
0.95	0.040	0.030	0.031	0.035	0.034	0.038	0.036	0.031	0.032
1.00	0.036	0.027	0.027	0.032	0.032	0.035	0.032	0.028	0.030



Solution

✤ Slab Design

Step 3: Analysis (two-way slab)

Moment Coefficients

 $C_{a,neg} = 0.076$

 $C_{b,neg}=0.024$

 $C_{a,pos,dl} = 0.043$

$$C_{a,pos,ll} = 0.052$$

$$C_{b,pos,dl} = 0.013$$

Та	ble A5: Co	efficients (C _{b, dl}) For D	ead Load F	ositive Mo	ment in Sla	ab along Lo	ong Directio	on
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.006	0.002	0.007	0.004	0.001	0.003	0.007	0.004	0.002
0.55	0.008	0.003	0.009	0.005	0.002	0.004	0.009	0.005	0.003
0.60	0.010	0.004	0.011	0.007	0.003	0.006	0.012	0.007	0.004
0.65	0.013	0.006	0.014	0.009	0.004	0.007	0.014	0.009	0.005
0.70	0.016	0.007	0.016	0.011	0.005	0.009	0.017	0.011	0.006
0.75	0.019	0.009	0.018	0.013	0.007	0.013	0.020	0.013	0.007
0.80	0.023	0.011	0.020	0.016	0.009	0.015	0.022	0.015	0.010
0.85	0.026	0.012	0.022	0.019	0.011	0.017	0.025	0.017	0.013
0.90	0.029	0.014	0.024	0.022	0.013	0.021	0.028	0.019	0.015
0.95	0.033	0.016	0.025	0.024	0.015	0.024	0.031	0.021	0.017
1.00	0.036	0.018	0.027	0.027	0.018	0.027	0.033	0.023	0.020



Solution

✤ Slab Design

Step 3: Analysis (two-way slab)

Moment Coefficients

 $C_{a,neg} = 0.076$ $C_{b,neg} = 0.024$

 $C_{a,pos,dl} = 0.043$

$$C_{a,pos,ll} = 0.013$$

$$C_{b,pos,dl} = 0.052$$

$$C_{b,pos,ll} = 0.016$$

т	able A6: Co	oefficients	(C _{b, //}) For L	ive Load P	ositive Mo	ment in Sla	b along Lo	ng Directio	n
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.006	0.004	0.007	0.005	0.004	0.005	0.007	0.005	0.007
0.55	0.008	0.006	0.009	0.007	0.005	0.006	0.009	0.007	0.006
0.60	0.010	0.007	0.011	0.009	0.007	0.008	0.011	0.009	0.007
0.65	0.013	0.010	0.014	0.011	0.009	0.010	0.014	0.011	0.009
0.70	0.016	0.012	0.016	0.014	0.011	0.013	0.017	0.014	0.011
0.75	0.019	0.014	0.019	0.016	0.013	0.016	0.020	0.016	0.013
0.80	0.023	0.017	0.022	0.020	0.016	0.019	0.023	0.019	0.017
0.85	0.026	0.019	0.024	0.023	0.019	0.022	0.026	0.022	0.020
0.90	0.029	0.022	0.027	0.026	0.021	0.025	0.029	0.024	0.022
0.95	0.033	0.025	0.029	0.029	0.024	0.029	0.032	0.027	0.025
1.00	0.036	0.027	0.032	0.032	0.027	0.032	0.035	0.030	0.08



Solution

- ✤ Slab Design
- Step 3: Analysis (two-way slab)
 - Calculate bending Moments

$$W_{u,dl} = 0.147 ksf$$
, $W_{u,dl} = 0.064 ksf$, $W_u = 0.211 ksf$, $l_a = 12'$ and $l_b = 16'$

Coefficients	Moment formulae	Moment Values (in.kip)
$C_{a,neg} = 0.076$	$M_{a,neg} = C_{a,neg} W_u {l_a}^2$	27.7
$C_{b,neg}=0.024$	$M_{b,neg} = C_{b,neg} W_u {l_b}^2$	15.6
$C_{a,pos,dl} = 0.043$	$M = C \qquad M \qquad l^2 + C \qquad M \qquad l^2$	16 7
$C_{a,pos,ll} = 0.052$	$M_{a,pos} = C_{a,pos,dl} W_{u,dl} L_a + C_{a,pos,ll} W_{u,ll} L_a$	10.7
$C_{b,pos,dl} = 0.013$	$M = C \qquad M \qquad l^2 + C \qquad M \qquad l^2$	0.0
$C_{b,pos,ll} = 0.016$	$M_{b,pos} = C_{b,pos,dl} W_{u,dl} l_b + C_{b,pos,ll} W_{u,ll} l_b$	9.0

CE 416: Reinforced Concrete Design - II



Solution

- * Slab Design
- Step 4: Analysis (one-way slab)

$$M_{ver,int(-)} = \frac{w_u l_n^2}{9} = 18.0$$
 in. kip/ft

$$M_{ver(+)} = \frac{w_u {l_n}^2}{14} = 11.6$$
 in. kip/ft

$$M_{ver,ext(-)} = \frac{w_u l_n^2}{24} = 6.8$$
 in. kip/ft

Note:

For negative moment above the long wall common to rooms and veranda, maximum moment will be picked from both analyses.

Moment of 2.31 from two-way slab analysis is more than 1.5, therefore we will design for 2.31.



Prof. Dr. Qaisar Ali

CE 416: Reinforced Concrete Design - II



Solution

- ✤ Slab Design
- Step 3: Analysis (comparison of results)

Analysis	Two – w	ay Slab M (Roo	loments (oms)	in-kip/ft)	One-way Slab Moment (in-kip/ft) (Verandah)			
Method	$M_{a(+)}$	$M_{b(+)}$	$M_{a(-)}$	$M_{b(-)}$	$M_{ver,int(-)}$	$M_{ver(+)}$	$M_{ver,ext(-)}$	
FEA (SAFE)	19.0	14.0	25.2	20.0	25.2	13.2	4.6	
Manual	16.7	9.0	27.7	15.6	18.0	11.6	6.8	

- Analysis results from both approaches are almost similar.
- Hence the intelligent use of manual analysis yields reasonable results in most cases.



Solution

- ✤ Slab Design
- Step 4: Determination of Flexural Steel Area

The minimum reinforcement is given by

 $A_{s,min} = 0.0018bh = 0.0018(12)(5) = 0.108 \text{ in}^2/\text{ft}$

Using #3 bars with $A_b = 0.11 \text{ in}^2$

$$S = \frac{12A_b}{A_s} = \frac{12 \times 0.11}{0.108} = 12.2''c/c$$

Calculated spacing shall not exceed S_{max} which is given by

$$S_{max} = \min(2h, 18") \Rightarrow \min(2 \times 5, 18") = 10"$$

Calculated spacing of 12.2" exceeds 10". Finally Provide #3@10" c/c.



Solution

- ✤ Slab Design
- > Step 4: Determination of Flexural Steel Area

With #3@10" c/c, calculate moment capacity

$$A_{s,min} = \frac{12A_b}{S} = \frac{12(0.11)}{10} = 0.132 \text{ in}^2/\text{ft}$$

$$a = \frac{A_{s,min} f_y}{0.85 f'_c b} = \frac{0.132 \times 60}{0.85 \times 3 \times 12} = 0.26 \text{ in}$$

Now,

$$\emptyset M_n = 0.9 \times 0.132 \times 60 \left(4 - \frac{0.26}{2}\right) = 27.6 \text{ in. kip/ft}$$



Solution

- ✤ Slab Design
- Step 4: Determination of Flexural Steel Area

The flexural design summary is provided below.

Location	Moments (in.kip/ft)	<i>M_{n,min}</i> (in. kip/ft)	A _s (in²)	<i>S</i> using #3 bar (<i>in</i>)
M _{a,neg}	27.71		$\approx A_{s,min}$ governs	10
$M_{b,neg}$	15.56		<i>A_{s,min}</i> governs	10
M _{a,pos}	16.67	27.60	<i>A_{s,min}</i> governs	10
M _{b,pos}	9.02		<i>A_{s,min}</i> governs	10
M _{+,Ver}	11.52		<i>A_{s,min}</i> governs	10



Solution

- ✤ Slab Design
- Step 5: Determination of temperature/shrinkage reinforcement

For one-way slab, the temperature reinforcement is given by $A_{s+T} = A_{min} = 0.108 \text{ in}^2/\text{ft}$

$$S = \frac{12A_b}{A_s} = \frac{12 \times 0.11}{0.108} = 12.2''c/c$$

Maximum spacing for shrinkage reinforcement is given by

$$s_{max} = min[5(5) \text{ or } 18''] = 18'' \to OK!$$

Finally, provide #3 @10 in. c/c



Solution

- ✤ Slab Design
- Step 5: Determination of temperature/shrinkage reinforcement
 - Reinforcement at discontinuous ends
 - Reinforcement at discontinuous ends in a two way slab is 1/3 of the positive reinforcement.
 - Positive reinforcement at midspan in this case is #3 @ 10" c/c. Therefore, reinforcement at discontinuous end may be provided @ 30" c/c.
 - However, in field practice, the spacing of reinforcement at discontinuous ends seldom exceeds 18" c/c. The same is provided here as well.



Given Solution

- ✤ Slab Design
- Step 6: Drafting





Solution

- **Beam Design** *
- Step 1: Selection of sizes
 - Assume $b_w = 12"$

$$h_{min,40} = \frac{16.75}{18.5} \times 12 = 10.9" \rightarrow Take h = 18'$$

Now,

$$b_{f,L} = least of \int_{b_w}^{b_w} + 6h_f = 12 + 16 \times 5 = 92"$$

$$b_w + \frac{S_w}{2} = Not \ applicable$$

$$b_w + \frac{l_n}{12} = 12 + \frac{15.875}{12} \times 12 = 27.9"$$

$$b_{e,L} = 27.9"$$



'J ,L



Solution

- ✤ Beam Design
- Step 2: Calculation of loads

Self weight of beam is given by

$$SW = \frac{12(18-5)}{144} \times 0.150 = 0.163$$
 k/ft

Now,

$$W_{u,beam} = w_{u,slab} \times b_t + 1.2SW$$

By putting values

$$W_{u,beam} = 0.211 \times 5 + 1.2(0.163)$$

$$W_{u,beam} = 1.25 \text{ k/ft}$$







Solution

- ✤ Beam Design
- Step 3: Analysis



CE 416: Reinforced Concrete Design - II



Solution

- ✤ Beam Design
- Step 4: Determination of Flexural Reinforcement

Flexural Design Summary										
M _u (in-kip)	d (in.)	b (in.)	A _s (in²)	A _{smin} (in²)	A _{smax} (in²)	A _s	Detailing			
343.66 (+)	15	27.875	0.43	0.60	2.42	0.60	3 - #4			
420.03 (-)	15	12	0.44	0.60	2.42	0.60	3 - #4			

Shear Design Summary									
Location	V _u (@ d)(kip)	ΦV _c (kips)	S _{max} (in)	Detailing					
Exterior	8.34	14.78	7.5″	2-legged #3 @7.5" c/c					
Interior	9.61	14.78	7.5″	2-legged #3 @7.5" c/c					



Solution

- ✤ Beam Design
- Step No 5: Drafting



Prof. Dr. Qaisar Ali

CE 416: Reinforced Concrete Design - II



Solution

- ✤ Column Design
- Step 1: Selection of Sizes

Assume column size = 12" × 12"

Step 2: Calculation of Loads

P_u = 11.41 × 2 = 22.82 kip

> Step 3: Longitudinal Reinforcement

Assuming the column as concentric, the axial capacity is given by:

 $\alpha \emptyset P_n = 0.8 \times 0.65 \left[0.85 f_c' \left(A_g - A_{st} \right) - A_{st} f_y \right]$







Solution

- Column Design
- Step 3: Longitudinal Reinforcement

Assuming $A_{st} = 0.01A_g = 0.01 \times 144 = 1.44 \ in^2$ $\alpha \emptyset P_n = 0.8 \times 0.65 [0.85 \times 3(144 - 1.44) - 1.44 \times 60]$ $= 144.10 > P_u = 22.82 \ \text{kip} \rightarrow 0\text{K}$

Using #4 bar with $A_b = 0.20$ in²

No. of bars = $1.44/0.20 = 7.2 \approx 8$

Hence, Provide 8-#4 bars.



Solution

- Column Design
- Step 4: Determination of Spacing for Shear Reinforcement

Using #3 bar with $A_b = 0.11 in^2$, S_{max} is the least of:

i.
$$\frac{A_v f_y}{50b}$$
 = 0.22 x 60,000/ (50x12) = 22.0"

ii.
$$\frac{A_v f_y}{0.75\sqrt{f_c'b}} = 0.22 \times 60,000 / (0.75\sqrt{3000} \times 12) = 26.8''$$

- iii. $16d_b$ of longitudinal bar = $16 \times 4/8 = 8''$
- iv. 48 d_b of tie bar = 48 x 3/8 = 18"
- v. Smallest dimension of member = 12"

 S_{max} = 8". Provide #3 ties @ 8" c/c







Crack in slab in village house due to absence of negative reinforcement



Updated: Nov 02, 2023 Department of Civil Engineering, University of Engineering and Technology Peshawar, Pakistan



Design Example 4.2

Design of Three-story Commercial Building

Prof. Dr. Qaisar Ali

CE 416: Reinforced Concrete Design – II



Problem Statement

- A 100' x 60' three-story 4 by 3 bay commercial building is shown below . The 7 in-thick floors are subjected to uniform service live load of 144psf. Taking $f_c' = 3ksi$ and $f_y = 40 ksi$.
- **Design** slab and beams of one of the floors.







Given Data

Dimensions of floor : $100' \times 60'$ (center – to – center)

Story height, h = 12'

All beams are 14" x 20"

All columns are 14" x 14"

Superimposed Dead load : Nil

Live load: 144 psf

 $f_c' = 3 \ ksi \ \& \ f_y = 40 \ ksi$

Required Data

Design slab and beams of one of the floors





□ Solution

- ✤ Slab Design
- Step 1 and 2: Selection of Structural Configuration and Sizes

Structural configuration and slab thickness are given, $h_f = 7$ "

Step 2: Calculation of Loads

Self weight of slab =
$$\frac{7}{12} \times 0.150 = 0.0875 \ ksf$$

$$W_{u,dl} = 1.2(0.0875) = 0.105 \, ksf$$

$$W_{u,ll} = 1.6(0.144) = 0.230 \, ksf$$

$$W_u = 0.105 + 0.2304 = 0.335 \, ksf$$



□ Solution

- ✤ Slab Design
- Step 3: Analysis

Complete analysis of the slab is done by analyzing four panels





Solution

- ✤ Slab Design
- Step 3: Analysis
 - Moment Coefficients

 $C_{a,neg} = 0.071$

Panel – I

	Table A1: Coefficients (C _{a, Negative}) For Negative Moment in Slab along Short Direction									
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	
0.50	0.000	0.086	0.000	0.094	0.090	0.097	0.000	0.089	0.088	
0.55	0.000	0.084	0.000	0.092	0.089	0.096	0.000	0.085	0.086	
0.60	0.000	0.081	0.000	0.089	0.088	0.095	0.000	0.080	0.085	
0.65	0.000	0.077	0.000	0.085	0.087	0.093	0.000	0.074	0.083	
0.70	0.000	0.074	0.000	0.081	0.086	0.091	0.000	0.068	0.081	
0.75	0.000	0.069	0.000	0.076	0.085	0.088	0.000	0.061	0.078	
0.80	0.000	0.005	0.000	0.071	0.083	0.086	0.000	0.055	0.075	
0.85	0.000	0.060	0.000	0.066	0.082	0.083	0.000	0.049	0.072	
0.90	0.000	0.055	0.000	0.060	0.080	0.079	0.000	0.043	0.068	
0.95	0.000	0.050	0.000	0.055	0.079	0.075	0.000	0.038	0.065	
1.00	0.000	0.045	0.000	0.050	0.075	0.071	0.000	0.033	0.061	

 $m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$ Slab Case = 4

 $W_{u,dl} = 0.105 ksf W_{u,ll} = 0.230 ksf$ and $W_u = 0.335 ksf$

CE 416: Reinforced Concrete Design – II



Solution

- ✤ Slab Design
- Step 3: Analysis
 - Moment Coefficients

 $C_{a,neg} = 0.071$

$$C_{b,neg} = 0.029$$

Panel – I

Table A2: Coefficients (C _{b, Negative}) For Negative Moment in Slab along Short Direction										
m	Case 1	Case 2	Case 3	Case	4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.000	0.006	0.022	0.00	ô	0.000	0.000	0.014	0.010	0.003
0.55	0.000	0.007	0.028	0.0	3	0.000	0.000	0.019	0.014	0.005
0.60	0.000	0.010	0.035	0.0 ¹⁷	1	0.000	0.000	0.024	0.018	0.006
0.65	0.000	0.014	0.043	0.015	5	0.000	0.000	0.031	0.024	0.008
0.70	0.000	0.017	0.050	0.019	9	0.000	0.000	0.038	0.029	0.011
0.75	0.000	0.022	0.056	0.024	4	0.000	0.000	0.044	0.036	0.014
0.80	0.000	0.027	0.061	0.029	9	0.000	0.000	0.051	0.041	0.017
0.85	0.000	0.031	0.065	0.034	1	0.000	0.000	0.057	0.046	0.021
0.90	0.000	0.037	0.070	0.040	C	0.000	0.000	0.062	0.052	0.025
0.95	0.000	0.041	0.072	0.045	5	0.000	0.000	0.067	0.056	0.029
1.00	0.000	0.045	0.076	0.050)	0.000	0.000	0.071	0.061	0.033

 $m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$ Slab Case = 4

 $W_{u,dl} = 0.105 ksf W_{u,ll} = 0.230 ksf$ and $W_u = 0.335 ksf$

CE 416: Reinforced Concrete Design – II



Solution

- ✤ Slab Design
- Step 3: Analysis
 - Moment Coefficients

 $C_{a,neg} = 0.076$

 $C_{b,neg}=0.029$

 $C_{a,pos,dl} = 0.039$

Panel – I

Table A3: Coefficients (C _{a, dl}) For Dead Load Positive Moment in Slab along Short Direction									
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.095	0.037	0.080	0.059	0.039	0.061	0.089	0.056	0.023
0.55	0.088	0.035	0.071	0.056	0.038	0.058	0.081	0.052	0.024
0.60	0.081	0.034	0.062	0.053	0.037	0.056	0.073	0.048	0.026
0.65	0.074	0.032	0.054	0.050	0.036	0.054	0.065	0.044	0.028
0.70	0.068	0.030	0.046	0.046	0.035	0.051	0.058	0.040	0.029
0.75	0.061	0.028	0.040	0.043	0.033	0.048	0.051	0.036	0.031
0.80	0.050	0.020	0.034	0.039	0.032	0.045	0.045	0.032	0.029
0.85	0.050	0.024	0.029	0.036	0.031	0.042	0.040	0.029	0.028
0.90	0.045	0.022	0.025	0.033	0.029	0.039	0.035	0.025	0.026
0.95	0.040	0.020	0.021	0.030	0.028	0.036	0.031	0.022	0.024
1.00	0.036	0.018	0.018	0.027	0.027	0.033	0.027	0.020	0.023

 $m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$ Slab Case = 4

 $W_{u,dl} = 0.105 ksf W_{u,ll} = 0.230 ksf$ and $W_u = 0.335 ksf$

CE 416: Reinforced Concrete Design - II



Given Solution

- ✤ Slab Design
- Step 3: Analysis
 - Moment Coefficients

 $C_{a,neg} = 0.071$

 $C_{b,neg} = 0.029$

 $C_{a,pos,dl} = 0.039$

$$C_{a,pos,ll} = 0.048$$

Panel – I

Table A4: Coefficients ($C_{a, ll}$) For Live Load Positive Moment in Slab along Short Direction									
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.095	0.066	0.088	0.077	0.067	0.078	0.092	0.076	0.067
0.55	0.088	0.062	0.080	0.072	0.063	0.073	0.085	0.070	0.063
0.60	0.081	0.058	0.071	0.067	0.059	0.068	0.077	0.065	0.059
0.65	0.074	0.053	0.064	0.062	0.055	0.064	0.070	0.059	0.054
0.70	0.068	0.049	0.057	0.0 57	0.051	0.060	0.063	0.054	0.050
0.75	0.061	0.045	0.051	0.052	0.047	0.055	0.056	0.049	0.046
0.80	0.050	0.041	0.045	0.048	0.044	0.051	0.051	0.044	0.042
0.85	0.050	0.037	0.040	0.043	0.041	0.046	0.045	0.040	0.039
0.90	0.045	0.034	0.035	0.039	0.037	0.042	0.040	0.035	0.036
0.95	0.040	0.030	0.031	0.035	0.034	0.038	0.036	0.031	0.032
1.00	0.036	0.027	0.027	0.032	0.032	0.035	0.032	0.028	0.030

 $m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$ Slab Case = 4

 $W_{u,dl} = 0.105 ksf W_{u,ll} = 0.230 ksf$ and $W_u = 0.335 ksf$


□ Solution

- ✤ Slab Design
- Step 3: Analysis
 - Moment Coefficients

 $C_{a,neg}=0.071$

 $C_{b,neg} = 0.029$

 $C_{a,pos,dl} = 0.039$

 $C_{a,pos,ll} = 0.048$

 $\overline{C_{b,pos,dl}} = 0.016$

Panel – I

Ta	Table A5: Coefficients (C _{b, dl}) For Dead Load Positive Moment in Slab along Long Direction								
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.006	0.002	0.007	0.004	0.001	0.003	0.007	0.004	0.002
0.55	0.008	0.003	0.009	0.005	0.002	0.004	0.009	0.005	0.003
0.60	0.010	0.004	0.011	0.007	0.003	0.006	0.012	0.007	0.004
0.65	0.013	0.006	0.014	0.009	0.004	0.007	0.014	0.009	0.005
0.70	0.016	0.007	0.016	0.011	0.005	0.009	0.017	0.011	0.006
0.75	0.019	0.009	0.018	0.013	0.007	0.013	0.020	0.013	0.007
0.80	0.023	0.011	0.020	0.016	0.009	0.015	0.022	0.015	0.010
0.85	0.026	0.012	0.022	0.019	0.011	0.017	0.025	0.017	0.013
0.90	0.029	0.014	0.024	0.022	0.013	0.021	0.028	0.019	0.015
0.95	0.033	0.016	0.025	0.024	0.015	0.024	0.031	0.021	0.017
1.00	0.036	0.018	0.027	0.027	0.018	0.027	0.033	0.023	0.020

 $m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$ Slab Case = 4

 $W_{u,dl} = 0.105 ksf W_{u,ll} = 0.230 ksf$ and $W_u = 0.335 ksf$



□ Solution

- ✤ Slab Design
- Step 3: Analysis
 - Moment Coefficients

 $C_{a,neg} = 0.071$

 $C_{b,neg} = 0.029$

 $C_{a,pos,dl} = 0.039$

 $C_{a,pos,ll} = 0.048$

 $C_{b,pos,dl} = 0.016$

 $\overline{C_{b,pos,ll}} = 0.020$

Panel – I

т	Table A6: Coefficients (C _{b, //}) For Live Load Positive Moment in Slab along Long Direction								
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.006	0.004	0.007	0.005	0.004	0.005	0.007	0.005	0.007
0.55	0.008	0.006	0.009	0.007	0.005	0.006	0.009	0.007	0.006
0.60	0.010	0.007	0.011	0.009	0.007	0.008	0.011	0.009	0.007
0.65	0.013	0.010	0.014	0.011	0.009	0.010	0.014	0.011	0.009
0.70	0.016	0.012	0.016	0.014	0.011	0.013	0.017	0.014	0.011
0.75	0.019	0.014	0.019	0.016	0.013	0.016	0.020	0.016	0.013
0.80	0.023	0.017	0.022	0.020	0.016	0.019	0.023	0.019	0.017
0.85	0.026	0.019	0.024	0.023	0.019	0.022	0.026	0.022	0.020
0.90	0.029	0.022	0.027	0.026	0.021	0.025	0.029	0.024	0.022
0.95	0.033	0.025	0.029	0.029	0.024	0.029	0.032	0.027	0.025
1.00	0.036	0.027	0.032	0.032	0.027	0.032	0.035	0.030	0.08

 $m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$ Slab Case = 4 $W_{u,dl} = 0.105ksf W_{u,ll} = 0.230ksf$ and $W_u = 0.335ksf$



Solution

- ✤ Slab Design
- Step 3: Analysis (Panel I)

 $m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$

Slab Case = 4



$$W_{u,dl} = 0.105 ksf W_{u,ll} = 0.230 ksf$$
 and $W_u = 0.335 ksf$

Coefficients	Moment formulae	Moment Values (in.kip)
$C_{a,neg} = 0.071$	$M_{a,neg} = C_{a,neg} W_u {l_a}^2$	101.32
$C_{b,neg} = 0.029$	$M_{b,neg} = C_{b,neg} W_u {l_b}^2$	66.28
$C_{a,pos,dl} = 0.039$	$M = C \qquad M \qquad l^2 + C \qquad M \qquad l^2$	64.49
$C_{a,pos,ll} = 0.048$	$M_{a,pos} = C_{a,pos,dl} W_{u,dl} \iota_a + C_{a,pos,ll} W_{u,ll} \iota_a$	04.40
$C_{b,pos,dl} = 0.016$	$M = C \qquad W \qquad l^2 + C \qquad W \qquad l^2$	12.05
$C_{b,pos,ll} = 0.02$	$M_{b,pos} = C_{b,pos,dl} W_{u,dl} L_b + C_{b,pos,ll} W_{u,ll} L_b$	42.05



□ Solution

- ✤ Slab Design
- Step 3: Analysis (Panel II)

 $m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$

Slab Case = 9



$$W_{u,dl} = 0.105 ksf W_{u,ll} = 0.230 ksf$$
 and $W_u = 0.335 ksf$

Coefficients	Moment formulae	Moment Values (in.kip)
$C_{a,neg} = 0.075$	$M_{a,neg} = C_{a,neg} W_u {l_a}^2$	107.03
$C_{b,neg} = 0.017$	$M_{b,neg} = C_{b,neg} W_u {l_b}^2$	38.85
$C_{a,pos,dl} = 0.029$	$M = C \qquad M \qquad l^2 + C \qquad M \qquad l^2$	E4 12
$C_{a,pos,ll} = 0.042$	$M_{a,pos} = C_{a,pos,dl} W_{u,dl} \iota_a + C_{a,pos,ll} W_{u,ll} \iota_a$	54.15
$C_{b,pos,dl} = 0.01$	$M = C \qquad W \qquad l^2 + C \qquad W \qquad l^2$	22.05
$C_{b,pos,ll} = 0.017$	$M_{b,pos} = C_{b,pos,dl} W_{u,dl} L_b + C_{b,pos,ll} W_{u,ll} L_b$	33.85



□ Solution

- ✤ Slab Design
- Step 3: Analysis (Panel III)

 $m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$

Slab Case = 8



$$W_{u,dl} = 0.105 ksf W_{u,ll} = 0.230 ksf$$
 and $W_u = 0.335 ksf$

Coefficients	Moment formulae	Moment Values (in.kip)
$C_{a,neg} = 0.055$	$M_{a,neg} = C_{a,neg} W_u {l_a}^2$	78.49
$C_{b,neg} = 0.041$	$M_{b,neg} = C_{b,neg} W_u {l_b}^2$	93.71
$C_{a,pos,dl} = 0.032$	$M = C \qquad M \qquad l^2 + C \qquad M \qquad l^2$	57 42
$C_{a,pos,ll} = 0.044$	$M_{a,pos} = C_{a,pos,dl} W_{u,dl} \iota_a + C_{a,pos,ll} W_{u,ll} \iota_a$	57.45
$C_{b,pos,dl} = 0.015$	$M = C \qquad W \qquad l^2 + C \qquad W \qquad l^2$	40.56
$C_{b,pos,ll} = 0.019$	$M_{b,pos} = C_{b,pos,dl} W_{u,dl} L_b + C_{b,pos,ll} W_{u,ll} L_b$	40.56



□ Solution

- ✤ Slab Design
- Step 3: Analysis (Panel IV)

 $m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$

Slab Case = 2



$$W_{u,dl} = 0.105 ksf W_{u,ll} = 0.230 ksf$$
 and $W_u = 0.335 ksf$

Coefficients	Moment formulae	Moment Values (in.kip)
$C_{a,neg} = 0.065$	$M_{a,neg} = C_{a,neg} W_u {l_a}^2$	92.76
$C_{b,neg} = 0.027$	$M_{b,neg} = C_{b,neg} W_u {l_b}^2$	61.71
$C_{a,pos,dl} = 0.026$	$M = C \qquad M \qquad l^2 + C \qquad M \qquad l^2$	51 01
$C_{a,pos,ll} = 0.041$	$M_{a,pos} = C_{a,pos,dl} W_{u,dl} \iota_a + C_{a,pos,ll} W_{u,ll} \iota_a$	51.01
$C_{b,pos,dl} = 0.011$	$M = C \qquad W \qquad l^2 + C \qquad W \qquad l^2$	24 56
$C_{b,pos,ll} = 0.017$	$M_{b,pos} = C_{b,pos,dl} W_{u,dl} C_{b,pos,ll} W_{u,ll} C_{b,pos,ll} W_{u,ll} C_{b,pos,ll} W_{u,ll} C_{b,pos,dl} C_{b,po$	34.30



□ Solution

- ✤ Slab Design
- Step 3: Analysis

Summary of Analysis						
Location	Moment Values (in. kip/ft)					
	Panel – I	Panel - II	Panel - III	Panel - IV		
M _{a,neg}	101.32	107.03	78.49	92.76		
M _{b,neg}	66.28	38.85	93.71	61.71		
M _{a,pos}	64.48	54.13	57.43	51.81		
M _{b,pos}	42.85	33.85	40.56	34.56		



□ Solution

- ✤ Slab Design
- Step 3: Analysis



NOTE:

- All values are in in.kip/ft
- White values: Long Direction Moments
- Yellow values: Short Direction Moments



□ Solution

- ✤ Slab Design
- Step 4: Determination of Steel Area
 - As there are several bending moments, making it exceedingly timeconsuming and lengthy to calculate the steel area for each one.
 - As a result, we shall use the "Unity Rule" in this situation.
 - Calculate the area of steel for a unit moment and multiply it by the actual moments to obtain the necessary area of steel as described on the next slide.



□ Solution

- ✤ Slab Design
- Step 4: Determination of Steel Area
 - For $M_u = 1$ in. kip

$$a = d - \sqrt{d^2 - \frac{2.614M_u}{f_c'b}} = 6 - \sqrt{6^2 - \frac{2.614 \times (1)}{3 \times 12}} = 6.05 \times 10^{-3} in.$$

Now,

$$A_{s} = \frac{M_{u}}{0.9f_{y}\left(d - \frac{a}{2}\right)} = \frac{1}{0.9 \times 40\left(6 - \frac{6.05 \times 10^{-3}}{2}\right)} = 0.005 \ in^{2}/ft$$



□ Solution

✤ Slab Design

Step 4: Determination of Steel Area

For Positive Moments in Short Directions

$$A_{s,min} = 0.0018(12)(7) = 0.151in^2/ft_{-}$$

M_u (in.kip/ft)	$A_s = 0.005M_u$ (in^2/ft)	<i>S_{req}</i> using #4	Final S
64.48	0.32> <i>A</i> _{s,min}	7.5″	7″
54.13	0.27> <i>A</i> _{s,min}	8.9″	7″
57.43	0.29> <i>A</i> _{s,min}	8.3″	7″
51.81	0.26> <i>A</i> _{s,min}	9.2″	7″





□ Solution

✤ Slab Design

Step 4: Determination of Steel Area

For Negative Moments in Short Directions

$$A_{s,min} = 0.0018(12)(7) = 0.151in^2/ft_{-}$$

M_u (in.kip/ft)	$A_s = 0.005M_u$ (in^2/ft)	<i>S_{req}</i> using #4	Final S
107.03	0.54> <i>A</i> _{s,min}	4.4"	4″
92.76	0.46> <i>A</i> _{s,min}	5.2″	4″





□ Solution

✤ Slab Design

Step 4: Determination of Steel Area

For Positive Moments in Long Directions

$$A_{s,min} = 0.0018(12)(7) = 0.151in^2/ft_{-}$$

M_u (in.kip/ft)	$A_s = 0.005M_u$ (in^2/ft)	<i>S_{req}</i> using #4	Final S
42.85	0.21> <i>A</i> _{s,min}	11.4″	10″
40.56	0.202> <i>A</i> _{s,min}	11.88″	10″
33.85	0.17> <i>A</i> _{s,min}	14.1″	10″





□ Solution

✤ Slab Design

Step 4: Determination of Steel Area

For Negative Moments in Long Directions

$$A_{s,min} = 0.0018(12)(7) = 0.151in^2/ft_{-}$$

M_u (in.kip/ft)	$A_s = 0.005M_u$ (in^2/ft)	<i>S_{req}</i> using #4	Final S
93.73	0.47> <i>A</i> _{s,min}	5.1″	4″
61.71	0.31> <i>A</i> _{s,min}	8.1″	7″





□ Solution

✤ Slab Design

Step 5: Reinforcement Detailing

Finally, three set of spacings have been provided as shown.

- A = #4 @ 10"c/cB = #4 @ 7"c/c
- C = #4 @ 4'' c / c

Note:

- Reinforcement at discontinuous ends have been provided equal to 1/3 of the Positive reinforcement as per ACI Code.
- Yellow values: Short Direction White values : Long direction





□ Solution

- ✤ Beam Design
 - In the following session, only the mechanism of load transfer from slabs to beams will be discussed; the analysis and design portions can be completed as usual.



□ Solution

- ✤ Beam Design
 - Load Transfer Mechanism





- In case of one-way slab system the entire slab load is transferred in short direction.
- Load transfer in short direction = $(w_u \times l/2 \times 1) + (w_u \times l/2 \times 1)$
- Load transfer in long direction = $w_u \times l/2 \times 0$



□ Solution

✤ Beam Design

• Load Transfer Mechanism

 In case of two way slab system, entire slab load is NOT transferred in shorter direction.
4 spans @ 25'-0"



 $W_b = 1 - W_a$; where W_a is NOT equal to 1. It is specified by ACI Table



□ Solution

- ✤ Beam Design
 - Load Transfer Mechanism
 - The load transfer to B1 from Panels I and II in the short direction can be calculated as follows;

 $W_{a,Pan} = 0.71, \quad W_{a,Panel} = 0.83 \quad and \quad w_u = 0.336$

Now,

$$\frac{w_u l}{2} W_{a,Panel-I} + \frac{w_u l}{2} W_{a,Panel-II} = \frac{0.336 \times 20}{2} \times 0.71 + \frac{0.336 \times 20}{2} \times 0.83$$

Load on $B_1 = 5.17k/ft$



□ Solution

✤ Beam Design

• Load On Beams from coefficient tables

Panel I





□ Solution

✤ Beam Design

• Load On Beams from Coefficient Tables

Panel II





□ Solution

✤ Beam Design

• Load On Beams from Coefficient Tables

Panel III





□ Solution

✤ Beam Design

• Load On Beams from Coefficient Tables

Panel IV





□ Solution

✤ Beam Design





□ Solution

✤ Beam Design





□ Solution







□ Solution

Beam Design (after including self-weight)





Pictures of a Multi-story Commercial Building





Homework

Example 4.3

• Design the given slab system using the data provided.



• All beams are 18" x 24"

• All columns are 18" x 18"



References

- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-19)

