Lecture 05

Serviceability Requirements & Development of Reinforcement

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Section – I

Deflections

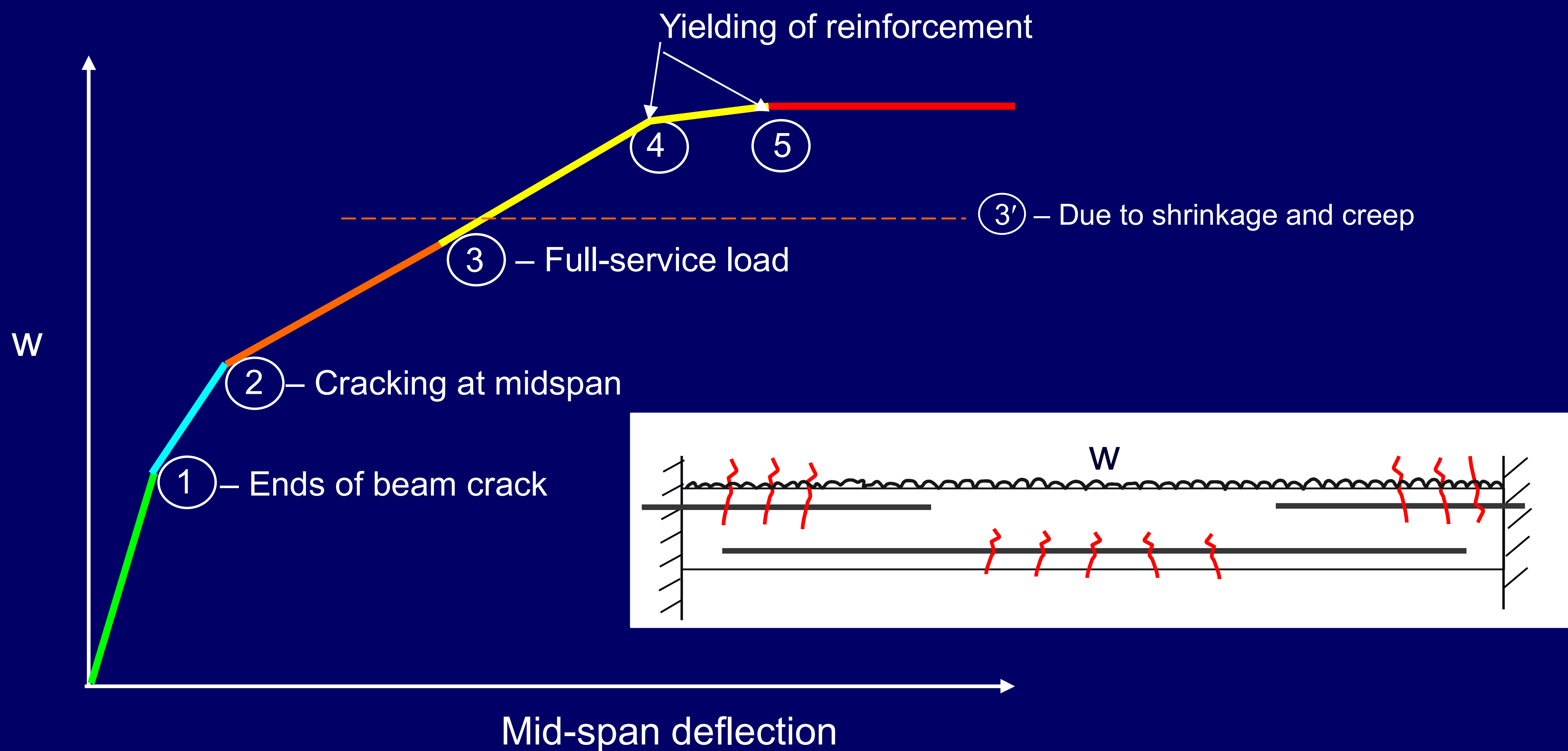
General

❑ Background

- In previous lectures, we have dealt primarily with the strength design of reinforced concrete beams.
- Methods have been developed to ensure that beams will have a proper safety margin against failure in flexure or shear, or due to inadequate bond and anchorage of the reinforcement.
- However, in addition to safety, serviceability requirements must also be ensured so that the structure performs well under service load conditions.
- Deflection control is an important serviceability consideration in the structural design of concrete buildings.

Introduction

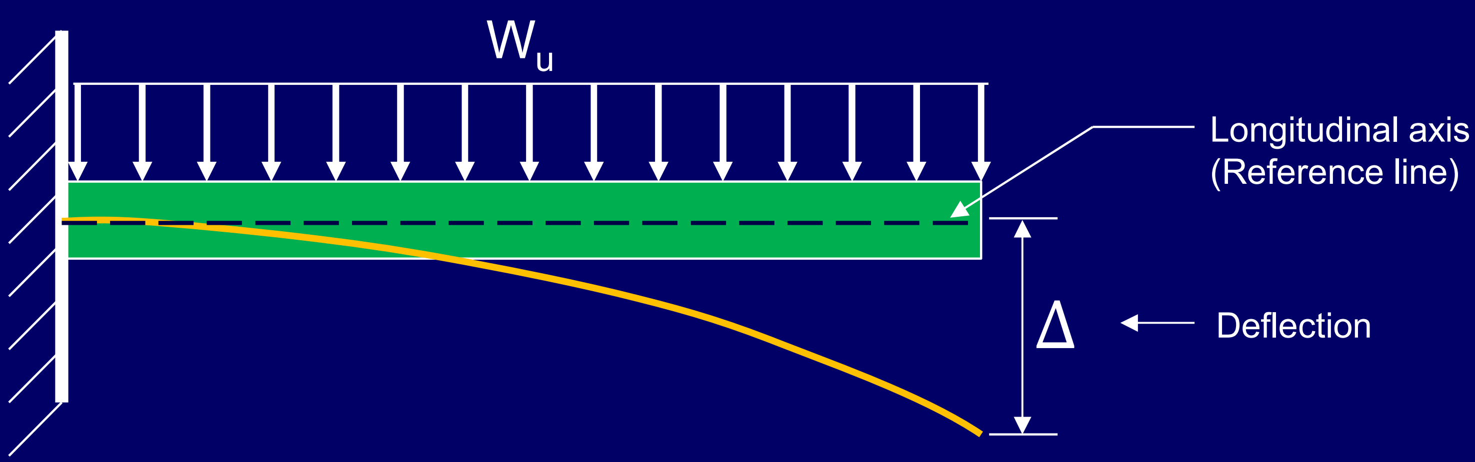
- Deflection history of fixed ended beam subjected to UDL



Introduction

❑ Definition of Deflection

- Deflection is defined as the movement of a point on a structure or structural element, usually measured as a linear displacement or as succession displacements transverse to a reference line or axis (ACI Concrete Terminology) .



Introduction

❑ Deflection Effects

- It is important to maintain control of deflections so that members designed mainly for strength at prescribed overloads will also perform well in normal service.
- Excessive deflections can lead to cracking of supported walls and partitions, ill-fitting doors and windows, poor roof drainage, misalignment of sensitive machinery and equipment, and visually offensive sag etc.

Introduction

Types of Deflections

1. **Short-term deflections**
   - The immediate deflection after casting and application of partial or full-service loads.
2. **Long-term deflections**
   - Deflection that occurs over time as a result of shrinkage and creep of concrete.

Both types are discussed in the subsequent slides.

Deflection in One-way Slabs and Beams

❑ Immediate (short-term) Deflections

- Immediate deflection at a given load in a structure is calculated using equations of elastic deflection.

$$\Delta = \frac{f(loads, spans, supports)}{EI}$$

Where;

- $f(loads, spans, supports)$ is a function of load, span and support arrangement and
- EI is the flexural rigidity.

Deflection in One-way Slabs and Beams

❑ Immediate (short-term) Deflections

- Elastic equation in a general form for deflection of cantilever, simple and continuous beams subjected to uniform loading can be written as

$$\Delta_i = K\frac{5M_a l^2}{48E_c I_e}$$

Where;

- M_a = the mid-span moment (when K is so defined) for simple and continuous beams. For cantilever beams, M_a will be the support moment.
- l = span length.
- K = deflection coefficient that can be obtained from PCA Table (shown next)
- I_e = effective moment of inertia

Deflection in One-way Slabs and Beams

❑ Immediate (short-term) Deflections

❖ Determination of K

- Theoretical values of deflection coefficient "K" for uniformly distributed loading w.

		K
1.	Cantilevers (deflection due to rotation at supports not included)	2.40
2.	Simple beams	1.0
3.	Continuous beams	$1.2 - 0.2\, M_o/M_a$
4.	Fixed-hinged beams (midspan deflection)	0.80
5.	Fixed-hinged beams (maximum deflection using maximum moment)	0.74
6.	Fixed-fixed beams	0.60

M_o = Simple span moment at midspan $\left(\dfrac{w\ell^2}{8}\right)$

M_a = Net midspan moment.

Reference: PCA Notes

Deflection in One-way Slabs and Beams

Immediate (short-term) Deflections

Determination of E_c

- As per ACI 318-19, Section 19.2.2.1, Modulus of elasticity E_c for concrete shall be accordance with (a) or (b):
  - For values of w_c between 90 and 160 lb/ft^3

$$E_c = w_c^{1.5} 33\sqrt{f_c'} \quad (psi) \qquad (19.2.2.1.a)$$

  - For Normalweight concrete

$$E_c = 57000\sqrt{f_c'} \quad (psi) \qquad (19.2.2.1.b)$$

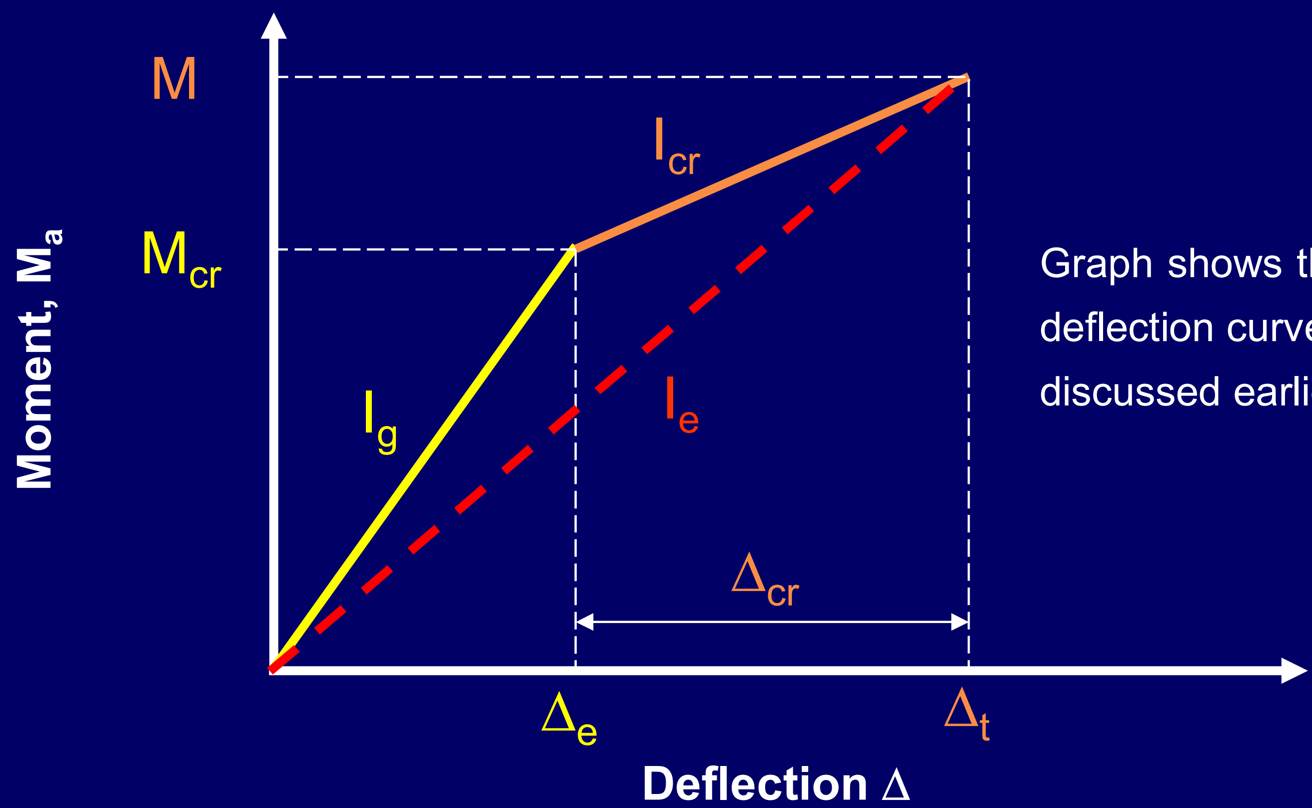
- w_c in equation (a) is the equilibrium density of concrete mixture.

Deflection in One-way Slabs and Beams

❑ Immediate (short-term) Deflections

- **Determination of I_e**

- *Idealized short term deflection of RC beam*



Graph shows the portion of load deflection curve up to stage 3 as discussed earlier

Deflection in One-way Slabs and Beams

❑ Immediate (short-term) Deflections

❖ Determination of I_e

- The effective moment of inertia for no prestressed concrete members is calculated in accordance with ACI Table 24.2.3.5.

Table 24.2.3.5—Effective Moment of Inertia, I_e

Service Moment	Effective moment of inertia I_e, in^4	
$M_a \le (2/3)\, M_{cr}$	I_g	(a)
$M_a > (2/3)\, M_{cr}$	$\dfrac{I_{cr}}{1-\left(\dfrac{(2/3)M_{cr}}{M_a}\right)^2\left(1-\dfrac{I_{cr}}{I_g}\right)}$	(b)

M_a = Maximum service load moment for which deflections are being considered.

M_{cr} = Cracking Moment = $f_r I_g / y_t$

I_g = Gross moment of inertia

I_{cr} = Moment of inertia of cracked section

Deflection in One-way Slabs and Beams

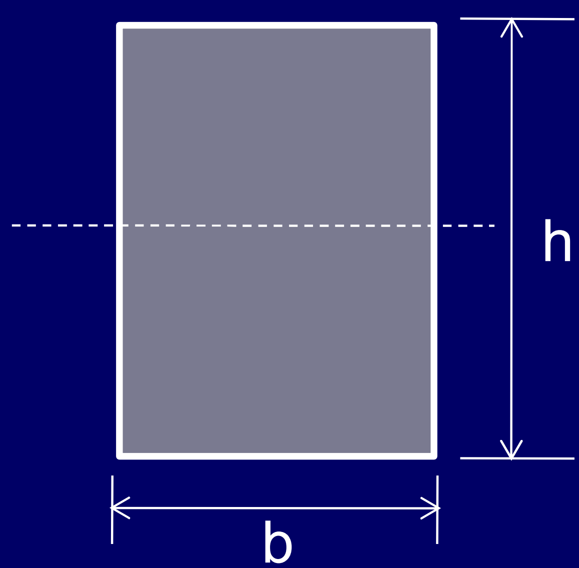
Immediate (short-term) Deflections

Determination of I_e

- **Un-cracked section moment of Inertia [about centroid]**

- For an un-cracked concrete section, full section will be effective so that:

$$I_e = I_g = \frac{bh^3}{12}$$



Deflection in One-way Slabs and Beams

❑ Immediate (short-term) Deflections

❖ Determination of I_e

- **Un-cracked section moment of Inertia [about base]**

- For determination of moment of inertia about base of the section, transfer formula may be used.

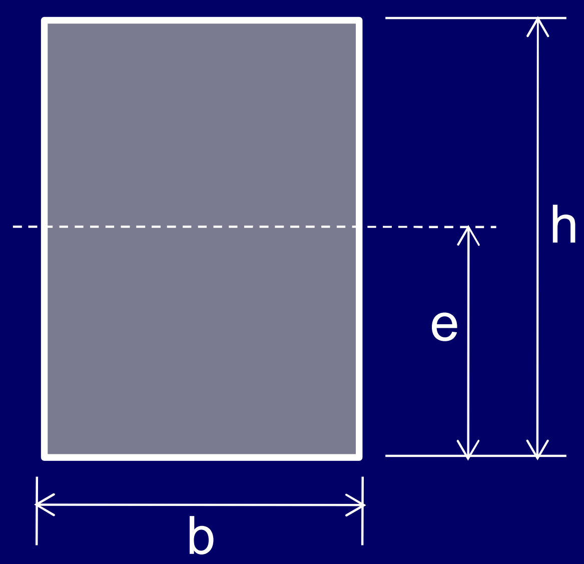
$$I_{base} = I_{c.a} + Ae^2$$

For the given case;

$$\frac{bh^3}{12}\ , A = bh \quad and \quad e = \frac{h}{2}$$

Substituting values, we get

$$I_{base} = \frac{bh^3}{12} + bh\left(\frac{h}{2}\right)^2 = \frac{bh^3}{3}$$



Deflection in One-way Slabs and Beams

Immediate (short-term) Deflections

Determination of I_e

- **Cracked section moment of Inertia [about neutral axis]**

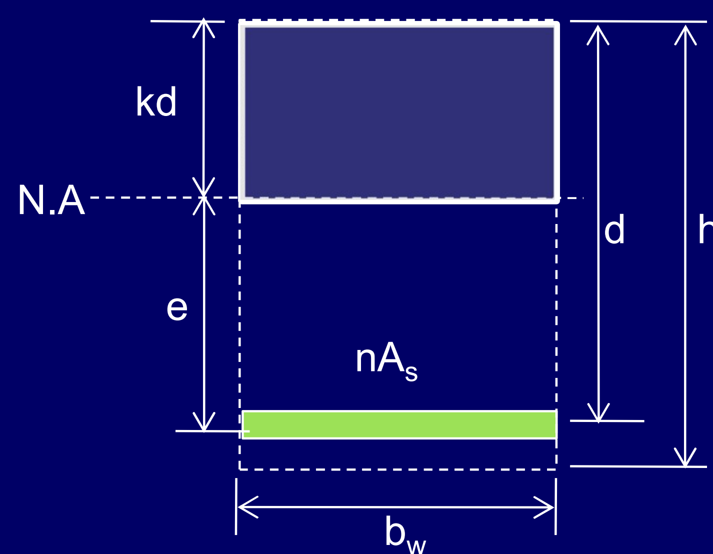
- Similarly, moment of inertia of cracked section about neutral axis can be determined through transfer formulae ($I = bh^3/3 + Ae^2$)

For the section shown;

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d - kd)^2$$

kd in above equation is unknown.

$$n = \frac{E_s}{E_c}$$



Deflection in One-way Slabs and Beams

Immediate (short-term) Deflections

Determination of I_e

- **Cracked section moment of Inertia [about neutral axis]**

- For determination of kd we take moments of area about the neutral axis

$$kd \times b_w \times \frac{kd}{2} = nA_s \times (d - kd)$$

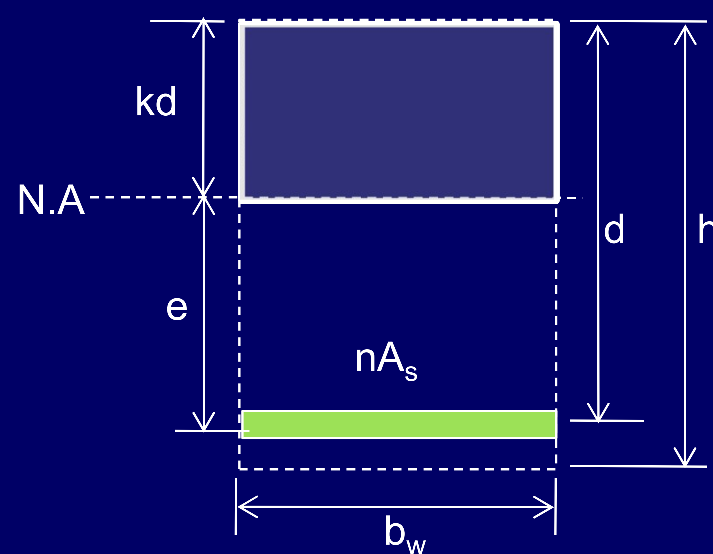
$$\frac{(kd)^2 b_w}{2nA_s} = d - kd$$

Let $B = b_w / nA_s$, then

$$(kd)^2 B/2 = d - kd$$

On simplification, we get

$$kd = \frac{\sqrt{1 + 2Bd} - 1}{B}$$



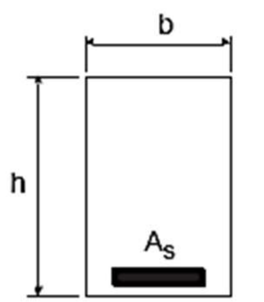
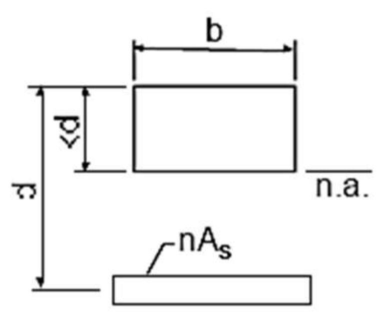
Deflection in One-way Slabs and Beams

❑ Immediate (short-term) Deflections

❖ Determination of I_e

- **Cracked section moment of Inertia [about neutral axis]**

- Alternatively, gross and cracked moment of inertia of rectangular and flanged sections can be calculated from the given tables.

Gross Section	Cracked Transformed Section	Gross and Cracked Moment of Inertia
	 Without compression steel	$n = \frac{E_s}{E_c}$ $B = \frac{b}{(nA_s)}$ $I_g = \frac{bh^3}{12}$ Without compression steel $kd = \left(\sqrt{2dB+1}-1\right)/B$ $I_{cr} = b(kd)^3/3 + nA_s(d-kd)^2$

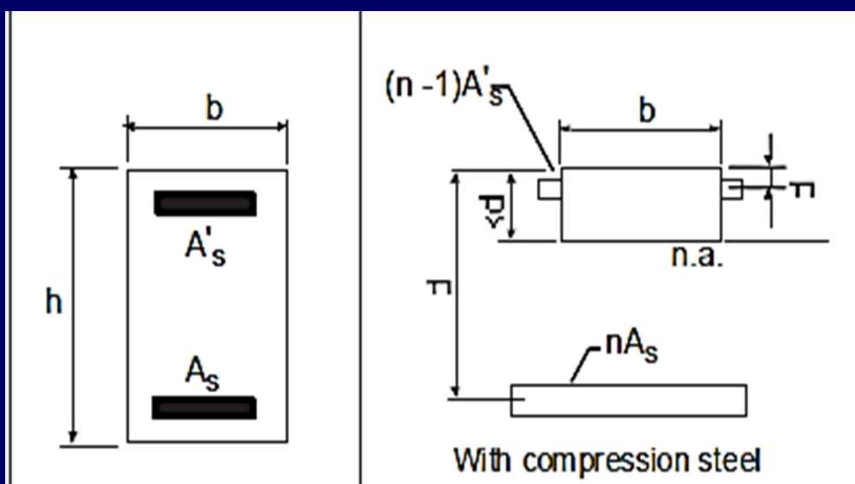
Deflection in One-way Slabs and Beams

❑ Immediate (short-term) Deflections

❖ Determination of I_e

- **Cracked section moment of Inertia [about neutral axis]**

- Alternatively, gross and cracked moment of inertia of rectangular and flanged sections can be calculated from the given tables.



With compression steel

$$r = (n-1)A'_s/(nA_s)$$

$$kd = \left[\sqrt{2dB(1+rd'/d)+(1+r)^2} - (1+r)\right]/B$$

$$I_{cr} = b(kd)^3/3 + nA_s(d-kd)^2 + (n-1)A'_s(kd-d')^2$$

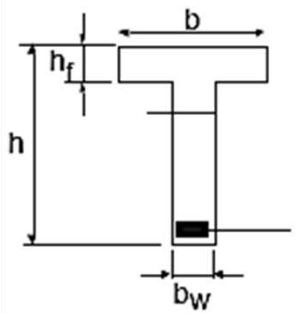
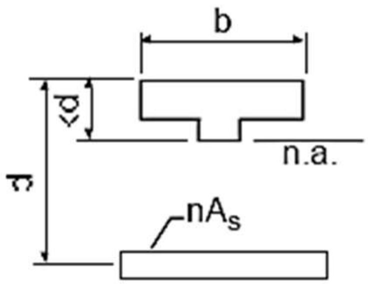
Deflection in One-way Slabs and Beams

❑ Immediate (short-term) Deflections

❖ Determination of I_e

- **Cracked section moment of Inertia [about neutral axis]**

- Alternatively, gross and cracked moment of inertia of rectangular and flanged sections can be calculated from the given tables.

Gross Section	Cracked Transformed Section	Gross and Cracked Moment of Inertia
	 Without compression steel	$n = \frac{E_s}{E_c}$ $C = b_w/(nA_s)$, $f = h_f(b-b_w)/(nA_s)$, $y_t = h - 1/2\left[(b-b_w)h_f^2 + b_w h^2\right]/\left[(b-b_w)h_f + b_w h\right]$ $I_g = (b-b_w)h_f^3/12 + b_w h^3/12 + (b-b_w)h_f(h-h_f/2-y_t)^2 +$ $b_w h(y_t - h/2)^2$ Without compression steel $kd = \left[\sqrt{C(2d+h_f f)+(1+f)^2} - (1+f)\right]/C$ $I_{cr} = (b-b_w)h_f^3/12 + b_w(kd)^3/3 + (b-b_w)h_f(kd-h_f/2)^2 +$ $nA_s(d-kd)^2$

Note: Effective width of T-section as per ACI 6.3.2.1.

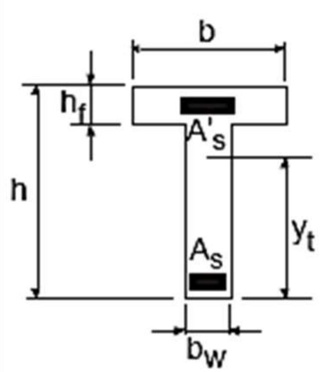
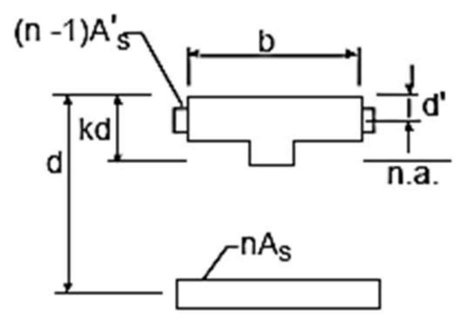
Deflection in One-way Slabs and Beams

❑ Immediate (short-term) Deflections

Determination of I_e

- **Cracked section moment of Inertia [about neutral axis]**

- Alternatively, gross and cracked moment of inertia of rectangular and flanged sections can be calculated from the given tables.

Gross Section	Cracked Transformed Section	Gross and Cracked Moment of Inertia
	 With compression steel	With compression steel $kd = \left[\sqrt{C(2d + h_f f + 2rd') + (f + r + 1)^2} - (f + r + 1)\right]/C$ $I_{cr} = (b - b_w)h_f^3/12 + b_w(kd)^3/3 + (b - b_w)h_f(kd - h_f/2)^2 + nA_s(d - kd)^2 + (n - 1)A'_s(kd - d')^2$

Note: Effective width of T-section as per ACI 6.3.2.1.

Deflection in One-way Slabs and Beams

Immediate (short-term) Deflections

Application of I_e for Simple and Continuous Members

- We have seen that I_e depends on M_a , which will be different at different locations . Therefore, according to (ACI R 24.2.3.7) for :

a) **Simply supported Beams**

$$I_e = I_{e,midspan}$$

b) **Beams with One end Continuous**

$$I_{e,avg} = 0.85 I_{e,midspan} + 0.15 I_{e,cont.\,end}$$

c) **Beams with Both ends Continuous**

$$I_{e,avg} = 0.70 I_{e,midspan} + 0.15 (I_{e1} + I_{e2})$$

Where, I_{e1} and I_{e2} refer to I_e at the respective beam ends. (ACI R 24.2.3.7)

Deflection in One-way Slabs and Beams

Long-term Deflections

- Shrinkage and creep due to sustained loads cause additional Long-term deflections over and above those which occur when loads are first placed on the structure.
- Such deflections are influenced by:
  - Temperature,
  - Humidity,
  - Curing conditions,
  - Age at the time of loading,
  - Quantity of compression reinforcement, and
  - Magnitude of the sustained load.

Deflection in One-way Slabs and Beams

❑ Long-term Deflections

- Additional Long-term deflection resulting from the combined effect of creep and shrinkage is determined by multiplying the immediate deflection caused by the sustained load with the factor ID as given in ACI Table 24.2.4.1.3.

$$\Delta_{(cp+sh)} = \lambda_{\Delta} (\Delta_i)_{sus}$$

Where;

$$\lambda_{\Delta} = \frac{\xi}{1 + 50A'_s/bd}$$

Table 24.2.4.1.3	
Sustained load duration $(\Delta_i)_{sus}$, (months)	**Time dependent factor, ξ**
3	1.0
6	1.2
12	1.4
60 or more	2.0

Deflection in One-way Slabs and Beams

❑ Long-term Deflections

- It is important to note here that long term deflections are function of immediate deflections due to sustained load only i.e.

  $\Delta_{(cp+sh)} = \lambda_{\Delta}(\Delta_i)_{sus}$

- Sustained loads are loads that are permanently applied on the structure e.g., dead loads, superimposed dead loads and live loads kept on the structure for long period.

Deflection in One-way Slabs and Beams

Deflection Control according to the ACI Code

1. Direct Approach

- In direct approach, the deflections are controlled by restricting their magnitude to the permitted limits recommended by ACI 318 Code.
- Deflections are said to be within limits if the combined effect of immediate and Long-term deflections does not exceed the limits specified in ACI table 24.2.2 (shown on next slide).

Deflection in One-way Slabs and Beams

❑ Deflection Control according to the ACI Code

1. Direct Approach

<table>
<tr><th colspan="5">Maximum Permissible Calculated Deflections (ACI Table 24.2.2)</th></tr>
<tr><th>Member</th><th colspan="2">Condition</th><th>Deflection to be considered</th><th>Deflection limitation</th></tr>
<tr><td>Flat roofs</td><td colspan="2" rowspan="2">Not supporting or attached to nonstructural elements likely to be damaged by large deflections Immediate deflection due to L</td><td>Immediate deflection due to maximum of Lr, S, and R</td><td>$l/180$</td></tr>
<tr><td>Floors</td><td>Immediate deflection due to L</td><td>$l/360$</td></tr>
<tr><td rowspan="2">Roofs or floors</td><td rowspan="2">Supporting or attached to non-structural elements</td><td>Likely to be damaged by large deflections</td><td rowspan="2">That part of the total deflection occurring after attachment of nonstructural elements, which is the sum of the time-dependent deflection due to all sustained loads and the immediate deflection due to any additional live load</td><td>$l/480$</td></tr>
<tr><td>Not likely to be damaged by large deflections</td><td>$l/240$</td></tr>
<tr><td colspan="5">Where, L = live load, Lr = reduced live load, S = Snow Load and R = Rain Load</td></tr>
</table>

Deflection in One-way Slabs and Beams

❑ Deflection Control according to the ACI Code

2. Indirect Approach

- Deflections can also be controlled indirectly by limiting the thickness (depth) of structural members.
- Deflections are said to be within limits if the thickness of beams and one-way slabs are greater than the minimum requirements given in ACI table 7.3.1.1 and 9.3.1.1 (as shown on next slide).
- However, this method is applicable only to the cases of loadings and spans commonly experienced in buildings and cannot be used for unusually large values of loading and span.

Deflection in One-way Slabs and Beams

❑ Deflection Control according to the ACI Code

2. Indirect Approach

Minimum thickness of solid one-way slabs and beams (ACI Tables 7.3.1 and 9.3.1.1)		
Support condition	Minimum depth h	
	One-way Slabs	*Beams*
Simply supported	$l/20$	$l/16$
One end continuous	$l/24$	$l/18.5$
Both ends continuous	$l/28$	$l/21$
Cantilever	$l/10$	$l/8$

- $l =$ center-to-center length for interior spans, clear projection for cantilever.
- for f_y other than 60,000 psi, the expressions in Table 7.3.1.1 and 9.3.1.1 shall be multiplied by (0.4 + f_y /100,000)

Deflection in RC Two-way Slabs

Immediate (short-term) Deflections

- The calculation of deflections for two-way slabs is complicated even if linear elastic behavior can be assumed.
- For immediate deflections 2D structural analysis is required in which the values of E_c and I_e for one-way slabs and beams discussed earlier may be used.

Deflection in RC Two-way Slabs

❑ Long-term Deflections

- The additional Long-term deflection for two-way construction is required to be computed using the multipliers given in ACI 24.2.4.1.

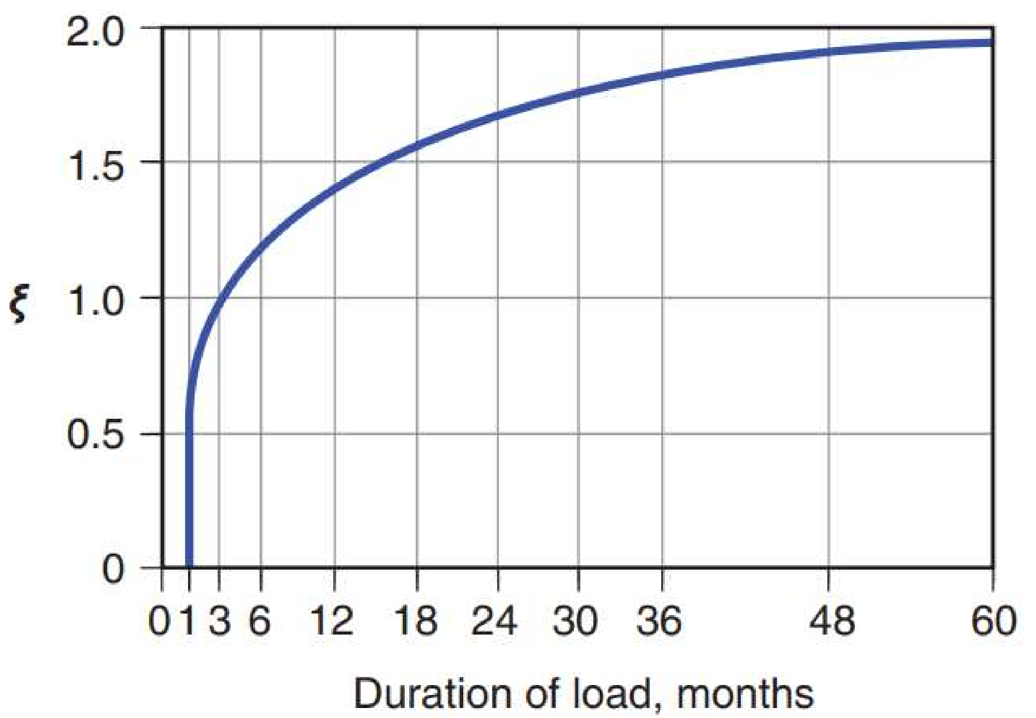


***Fig. R24.2.4.1**—Multipliers for time-dependent deflections.*

Deflection in RC Two-way Slabs

❑ Deflection Control according to the ACI Code

- Deflections may be controlled directly by limiting computed deflections (see Table 24.2.2) or indirectly by means of minimum thickness, table 8.3.1.1 and table 8.3.1.2 for two-way systems.

Table 8.3.1.1 —Minimum thickness of nonprestressed two-way slabs without interior beams (in.)

f_y (psi)	Without drop panels			With drop panels		
	Exterior Panels		Interior panels	Exterior Panels		Interior panels
	Without edge beams	With edge beams		Without edge beams	With edge beams	
40,000	$l_n/33$	$l_n/36$	$l_n/36$	$l_n/36$	$l_n/40$	$l_n/40$
60,000	$l_n/30$	$l_n/33$	$l_n/33$	$l_n/33$	$l_n/36$	$l_n/36$
80,000	$l_n/27$	$l_n/30$	$l_n/30$	$l_n/30$	$l_n/33$	$l_n/33$

Deflection in RC Two-way Slabs

❑ Deflection Control according to the ACI Code

<table>
<tr><th colspan="3">Table 8.3.1.2—Minimum thickness of nonprestressed two-way slabs with beams spanning between supports on all sides</th></tr>
<tr><th>α_{fm}</th><th colspan="2">Minimum h, in.</th></tr>
<tr><td>$\alpha_{fm} \leq 0.2$</td><td colspan="2">Provisions of One-way slabs apply</td></tr>
<tr><td rowspan="2">$0.2 \leq \alpha_{fm} \leq 2.0$</td><td rowspan="2">Greater of:</td><td>$\dfrac{l_n\left(0.8 + \dfrac{f_y}{200{,}000}\right)}{36 + 5\beta(\alpha_{fm} - 0.2)}$</td></tr>
<tr><td>5</td></tr>
<tr><td rowspan="2">$\alpha_{fm} > 2.0$</td><td rowspan="2">Greater of:</td><td>$\dfrac{l_n\left(0.8 + \dfrac{f_y}{200{,}000}\right)}{36 + 9\beta}$</td></tr>
<tr><td>3.5</td></tr>
</table>

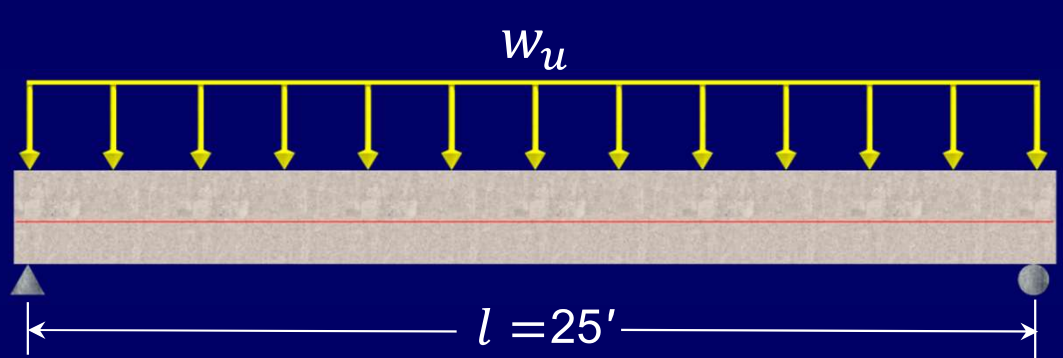
- α_{fm} is the average value of α_f for all beams on edges of a panel. $\alpha_f = E_{cb} I_b / E_{cs} I_s$
- l_n is the clear span in the long direction, measured face-to-face of beams (in.).
- β is the ratio of clear spans in long to short directions of slab.

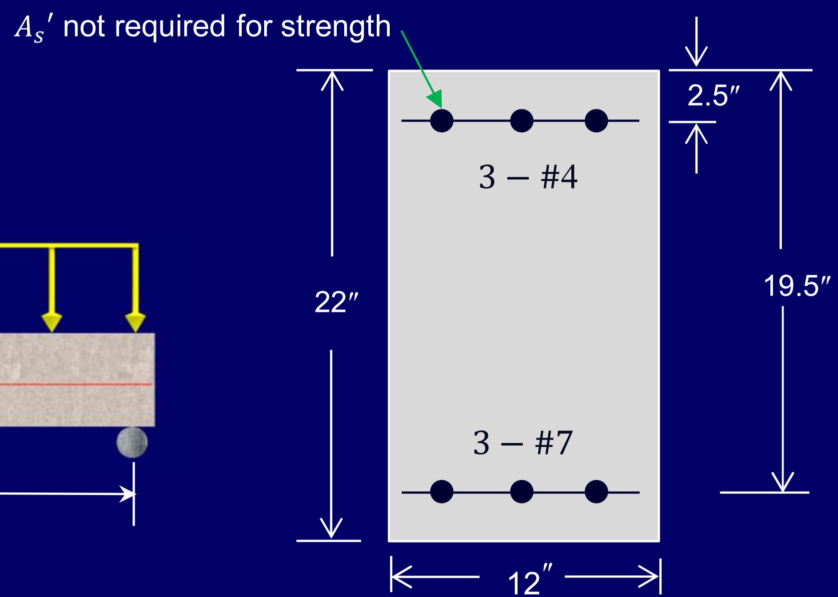
Example 5.1

❑ Problem Statement

A 25ft long simply supported RC beam is subjected to superimposed dead load (excluding selfweight) of 120 lb/ft and live load of 300 lb/ft (50% sustained). Material strengths are $f_c' = 3\ ksi$ and $f_y = 40\ ksi$.

Analyze the beam for short-term and Long-term deflections at ages 3 months and 5 years.





Example 5.1

❑ Solution

Given Data

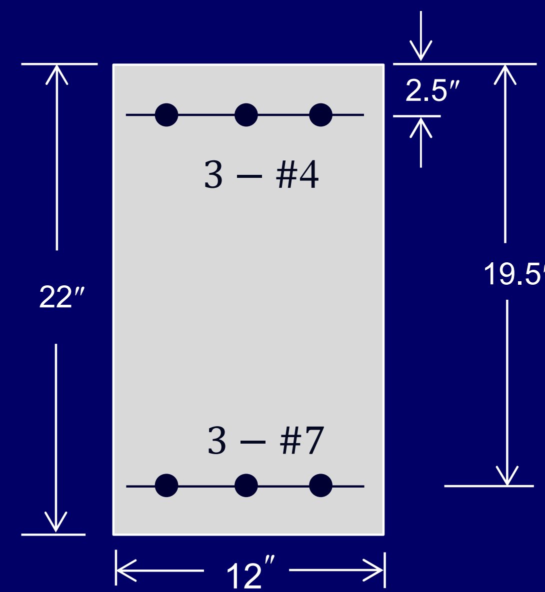
$b = 12'', h = 22'', d = 19.5'' \text{ and } d' = 2.5''$

$A_s = 3 \times 0.60 = 1.80 \; in^2$

$\rho = \dfrac{A_s}{bd} = 0.0077$

$A_s' = 3 \times 0.20 = 0.60 \; in^2$

$\rho' = \dfrac{A_s'}{bd} = 0.0026$



Required Data

Short-term and long-term deflections

Example 5.1

❑ Solution

➢ Step 1: Check Minimum Depth

$$h_{min} = \frac{l}{16}\left(0.4 + \frac{f_y}{100000}\right)$$

$$h_{min} = \frac{25 \times 12}{16}\left(0.4 + \frac{40000}{100000}\right) = 15''$$

The given depth of 22″ is O.K.

Example 5.1

❑ Solution

➢ Step 2: Calculation of Bending Moments

Calculate moments due to dead, live and sustained loads

$$w_d = DL + SW = 0.120 + \frac{12 \times 22}{144} \times 0.150 = 0.395 \text{ kip/ft}$$

$$M_d = \frac{w_d l^2}{8} = \frac{0.395 \times 25^2}{8} = 370.31 \text{ in. kip}$$

$$M_l = \frac{w_l l^2}{8} = \frac{0.300 \times 25^2}{8} = 281.25 \text{ in. kip}$$

$$M_{d+l} = M_d + M_l = 370.31 + 281.25 = 651.56 \text{ in. kip}$$

Now,

$$M_{sus} = M_d + 0.5M_l = 370.31 + 0.5(281.25) = 510.94 \text{ in. kip}$$

Example 5.1

❑ Solution

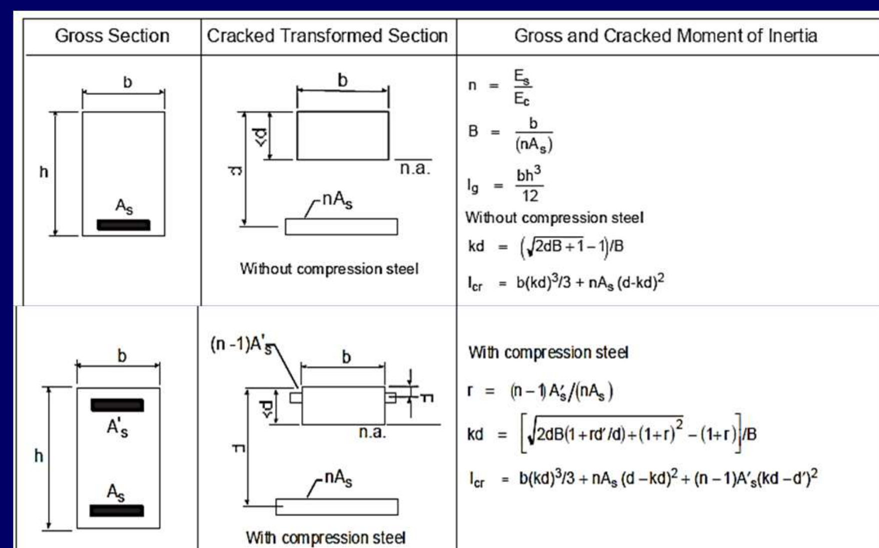
➢ Step 3: Determination of Cracked Moment of Inertia

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.29$$

$$I_g = \frac{bh^3}{12} = \frac{12 \times 22^3}{12} = 10648\ in^4$$

$$B = \frac{b}{nA_s} = \frac{12}{9.29 \times 1.80} = 0.72''$$

$$r = \frac{(n-1)A'_s}{nA_s} = \frac{(9.29-1)0.60}{9.29 \times 1 \times 1.8} = 0.30$$



Example 5.1

❑ Solution

➢ Step 3: Determination of Cracked Moment of Inertia

$$kd = \frac{1}{B}\left[\sqrt{2dB\left(1+\frac{rd'}{d}\right)+(1+r)^2} - (1+r)\right]$$

$$kd = \frac{1}{0.72}\left[\sqrt{2\times 19.5\times 0.72\left(1+\frac{0.30\times 2.5}{19.5}\right)+(1+0.30)^2} - (1+0.30)\right]$$

On solving , we get

$$kd = 5.91''$$

Example 5.1

❑ Solution

➢ Step 3: Determination of Cracked Moment of Inertia

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d - kd)^2 + (n-1)A_s'(kd - d')^2$$

Substituting values, we get

$$I_{cr} = \frac{12(5.91)^3}{3} + 9.29 \times 1.8(19.5 - 5.91)^2 + (9.29 - 1)0.60(5.91 - 2.5)^2$$

$$I_{cr} = 3971.89\ in^4$$

Example 5.1

Solution

Step 4: Determination of Effective Moment of Inertia

$$M_{cr} = \frac{7.5\sqrt{f_c'}I_g}{y} = \frac{7.5 \times \sqrt{3000} \times 10648}{22/2} \times \frac{1}{1000} = 397.65 \text{ in. kip}$$

a) Under Dead Loads Only

$$M_a = M_d = 370.31 \text{ in. kip} \quad ; \quad (2/3)M_{cr} = 265.1 \text{ in. kip}$$

Since $M_a > (2/3)M_{cr}$ therefore,

$$I_{e,d} = \frac{I_{cr}}{1 - \left(\frac{(2/3)M_{cr}}{M_a}\right)^2 \left(1 - \frac{I_{cr}}{I_g}\right)} = \frac{3971.89}{1 - \left(\frac{265.1}{370.31}\right)^2 \left(1 - \frac{3971.89}{10648}\right)} = 5852.42\ in^4$$

Example 5.1

Solution

Step 4: Determination of Effective Moment of Inertia

b) Under Dead and Live Both

$M_a = M_{d+l} = 651.56 \text{ in. kip} \quad ; (2/3)M_{cr} = 265.1 \text{ in. kip}$

Since $M_a > (2/3)M_{cr}$ therefore,

$$I_{e,d+l} = \frac{I_{cr}}{1 - \left(\frac{(2/3)M_{cr}}{M_a}\right)^2 \left(1 - \frac{I_{cr}}{I_g}\right)}$$

$$I_{e,d+l} = \frac{3971.89}{1 - \left(\frac{265.1}{651.56}\right)^2 \left(1 - \frac{3971.89}{10648}\right)} = 4431.89\ in^4$$

Example 5.1

❑ Solution

➢ Step 4: Determination of Effective Moment of Inertia

c) Under Sustained Load

$M_a = M_{sus} = 510.94 \text{ in. kip} \;\; ; (2/3)M_{cr} = 265.1 \text{ in. kip}$

Since $M_a > (2/3)M_{cr}$ therefore,

$$I_{e,sus} = \frac{I_{cr}}{1 - \left(\frac{(2/3)M_{cr}}{M_a}\right)^2 \left(1 - \frac{I_{cr}}{I_g}\right)}$$

$$I_{e,sus} = \frac{3971.89}{1 - \left(\frac{265.1}{510.94}\right)^2 \left(1 - \frac{3971.89}{10648}\right)} = 4778.42 \; in^4$$

Example 5.1

Solution

Step 5: Short-term Deflection

Taking K = 1 for simply supported spans

$$\Delta_{i,d} = K\frac{5M_d l^2}{48E_c I_{e,d}} = \frac{5(370.31)(25 \times 12)^2}{48 \times 3122 \times 5852.42} = 0.190''$$

$$\Delta_{i,d+l} = K\frac{5M_{d+l} l^2}{48E_c I_{e,d+l}} = \frac{5(651.56)(25 \times 12)^2}{48 \times 3122 \times 4431.89} = 0.441''$$

$$\Delta_{i,l} = \Delta_{i,d+l} - \Delta_{i,d} = 0.441 - 0.190 = 0.251''$$

$$\Delta_{i,sus} = K\frac{5M_{sus} l^2}{48E_c I_{e,sus}} = \frac{5(510.94)(25 \times 12)^2}{48 \times 3122 \times 4778.42} = 0.321''$$

Note:
$\Delta_{i,l}$ cannot be directly calculated using equation because the live load is imposed after the dead load.
Therefore, directly computing $\Delta_{i,l}$ would inherently include the deflection caused by the dead load as well.

Example 5.1

❑ Solution

➢ Step 5: Short-term Deflection

❖ Allowable Deflection

a) For flat roofs not supporting and not attached to nonstructural elements likely to be damaged by large deflections:

$$\Delta_{allowable} \leq \frac{l}{180} = \frac{25 \times 12}{180} = 1.67''$$

$$\Delta_{i,l} = 0.251'' < 1.67'' \rightarrow OK$$

b) Floors not supporting and not attached to nonstructural elements likely to be damaged by large deflections:

$$\Delta_{allowable} \leq \frac{l}{360} = \frac{25 \times 12}{360} = 0.83''$$

$$\Delta_{i,l} = 0.251'' < 0.83'' \rightarrow OK$$

Example 5.1

❑ Solution

➢ Step 6: Long-term Deflection

Duration	ξ	$\lambda = \frac{\xi}{1+50\rho'}$	$\Delta_{i,sus}$	$\Delta_{(cr+sh)} = \lambda\Delta_{i,sus}$	$\Delta_{i,l}$
5 years	2.0	1.77	0.321″	0.568″	0.251″
3 months	1.0	0.89	0.321″	0.286″	0.251″

- Which computed deflection should be considered for this case ?
- The code states that this should include "Sum of the Long-term deflection due to all sustained loads and the immediate deflection due to any additional live load".

Example 5.1

❑ Solution

➢ Step 6: Long-term Deflection

- As the partition will be installed after the immediate deflection due to dead load has occurred, total deflection (which will affect the partition) would include:

  1. Only long-term dead load deflection (not including dead load immediate deflection)
  2. Both Short and long-term sustained live load portion
  3. Additional live load immediate deflection. This is immediate deflection due to total live load minus sustained live load.

Example 5.1

❑ Solution

➢ Step 6: Long-term Deflection

Therefore, the total deflection for this case would be equal to:

$$\Delta_{total} = \lambda\Delta_d + \Delta_{i,sus\ live} + \lambda\Delta_{i,sus\ live} + \Delta_{i,additional\ live}$$

$$\Delta_{total} = (\lambda\Delta_d + \lambda\Delta_{i,sus\ live}) + (\Delta_{i,sus\ live} + \Delta_{i,additional\ live})$$

$$\Delta_{total} = \lambda\left(\Delta_d + \Delta_{i,sus\ live}\right) + \Delta_{i,l\ total}$$

Setting $\left(\Delta_d + \Delta_{i,sus\ live}\right) = \Delta_{i,sus}$ and $\Delta_{i,l\ total} = \Delta_{i,l}$ we get,

$$\boldsymbol{\Delta_{total} = \lambda\Delta_{i,sus} + \Delta_{i,l}}$$

Example 5.1

❑ Solution

➢ Step 6: Long-term Deflection

Duration	$\lambda\Delta_{i,sus}$	$\Delta_{i,l}$	$\Delta_{total} = \lambda\Delta_{i,sus} + \Delta_{i,l}$
5 years	0.568″	0.251″	0.819″
3 months	0.286″	0.251″	0.537″

Allowable Deflection

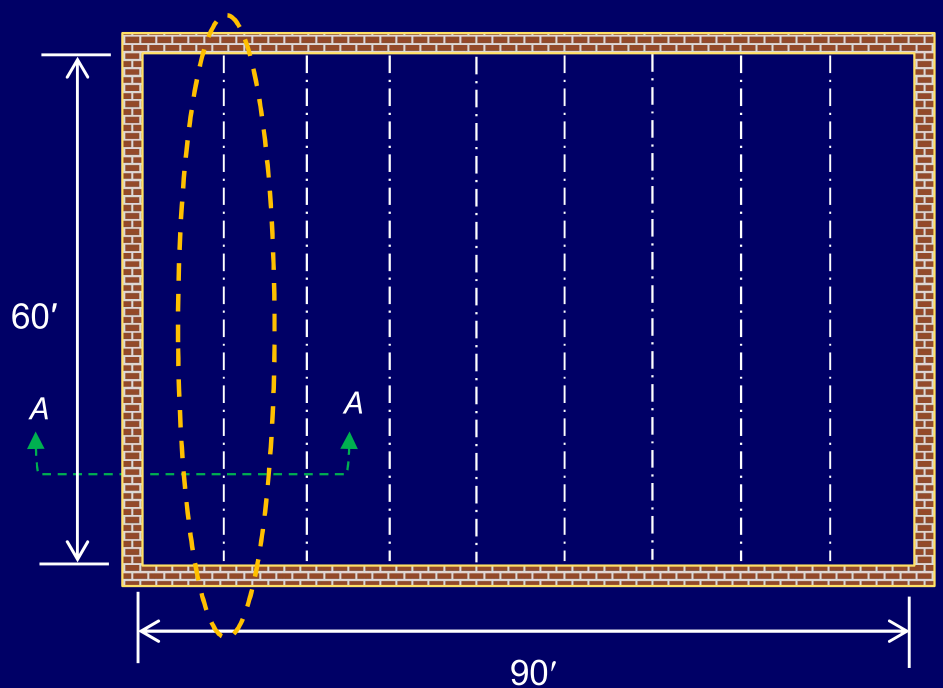
- Δ_{total} should be ≤ $l/480$; where $l/480$ = 300/480 = 0.63″
- Δ_{total} = 0.819″ > 0.63″ → Not OK!

What is the way forward ?

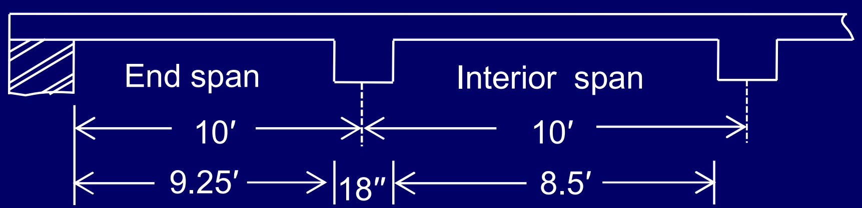
Example 5.2

❑ Problem Statement

- Find the deflection under full-service load for the highlighted beam of the hall shown below.



Wall thickness = 18 in
Beam dimensions = 18 in x 60 in



Section A-A

Data:

$f_c' = 3\ ksi\ (normal\ weight)\ and\ f_y = 40\ ksi$

$w_d = 2.26$ kip/ft; $w_l = 0.40$ kip/ft

$W_s = w_d + w_l = 2.66$ kip/ft

No live load is sustained.

Flexural design: 12 - #8 bars

Example 5.2

❑ Solution

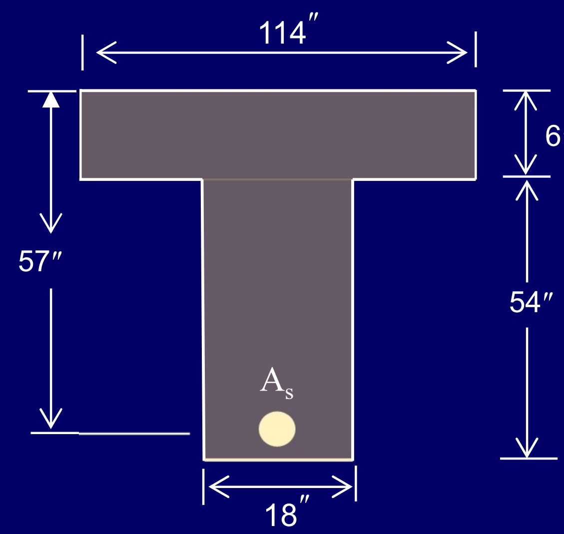
➢ Step 1: Sizes

$$h_{min} = \frac{61.5}{16}\left(0.4 + \frac{40{,}000}{100{,}000}\right) = 36.9'' \text{ ; Provided depth of } 60'' \text{ is OK}$$

The effective flange width b_f is the least of:

- $b_w + 16h_f = 18 + 16(6) = 114"$
- $b_w + s_w = 18 + 8.5 \times 12 = 120"$
- $b_w + \frac{l_n}{4} = 18 + \frac{60}{4} \times 12 = 198"$

Therefore, $b_f = 114"$



Example 5.2

❑ Solution

➢ Step 2: Calculation of Moments

$w_d = 2.26 \text{ kip/ft} \;\; and \;\; w_l = 0.40 \text{ kip/ft}$

$$M_d = \frac{w_d l^2}{8} = \frac{2.26 \times 61.5^2}{8} \times 12 = 12821.82 \text{ in. kip}$$

$$M_l = \frac{w_l l^2}{8} = \frac{0.40 \times 61.5^2}{8} \times 12 = 2269.35 \text{ in. kip}$$

$$M_{d+l} = M_d + M_l = 15091 \text{ in. kip}$$

Example 5.2

Solution

Step 3: Short-term Deflection

Calculate necessary parameter for calculating effective moment of inertia.

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^9}{57000\sqrt{3000}} = 9.289$$

$$C = \frac{b_w}{nA_s} = \frac{18}{9.289 \times (12 \times 0.79)} = 0.204$$

$$f = \frac{h_f\left(b_f - b_w\right)}{nA_s} = \frac{6(114 - 18)}{9.289 \times (12 \times 0.79)} = 6.54$$

$$y_t = h - \frac{0.5\left[\left(b_f - b_w\right)h_f^2 + b_w h^2\right]}{\left(b_f - b_w\right)h_f + b_w h} = 60 - \frac{0.5[(114 - 18)6^2 + 18 \times 60^2]}{(114 - 18)6 + 18 \times 60} = 39.39''$$

Example 5.2

❑ Solution

➢ Step 3: Short-term Deflection

The gross moment of inertia can be determined as;

$$I_g = \frac{(b_f - b_w)h_f^3}{12} + \frac{b_w(h)^3}{12} + (b_f - b_w)h_f\left(h - \frac{h_f}{2} - y_t\right)^2 + b_w h\left(y_t - \frac{h}{2}\right)^2$$

Subsisting the relevant values

$$I_g = \frac{(114 - 18)6^3}{12} + \frac{18(60)^3}{12} + (114 - 18)6\left(60 - \frac{6}{2} - 39.39\right)^2 + 18 \times 60\left(39.39 - \frac{60}{2}\right)^2$$

On solving we get;

$$I_g = 599578.4\ in^4$$

Example 5.2

❑ Solution

➢ Step 3: Short-term Deflection

$$M_{cr} = \frac{7.5\sqrt{f_c'}I_g}{y_t} = \frac{7.5\sqrt{3000} \times 599578.4}{39.39}$$

$M_{cr} = 6252.9 \text{ in. kip} < M_d = 12821.82 \text{ in. kip} \rightarrow$ section is cracked for dead load

Now, *kd* without compression steel is given by

$$kd = \frac{1}{C}\left[\sqrt{C\left(2d + h_f f\right) + (1 + f)^2} - (1 + f)\right]$$

$$kd = \frac{1}{0.204}\left[\sqrt{0.204(2 \times 57 + 6 \times 6.54) + (1 + 6.54)^2} - (1 + 6.54)\right]$$

$$kd = 9.05$$

Example 5.2

❑ Solution

➢ Step 3: Short-term Deflection

Determine the cracked moment of inertia using the following formula

$$I_{cr} = \frac{(b_f - b_w)h_f^3}{12} + \frac{b_w(kd)^3}{3} + (b_f - b_w)h_f\left(kd - \frac{h_f}{2}\right)^2 + nA_s(d - kd)^2$$

Substituting the relevant values

$$I_{cr} = \frac{(114 - 18)6^3}{12} + \frac{18(9.05)^3}{3} + 6(114 - 18)\left(9.05 - \frac{6}{2}\right)^2 + 9.289 \times 9.48(57 - 9.05)^2$$

On solving, we get

$$I_{cr} = 229725.47\ in^4$$

Example 5.2

❑ Solution

➢ Step 3: Short-term Deflection

Determine effective moment of inertia for dead and live loads.

❖ For Dead Load Only

$M_a = M_d = 12821.82 \text{ in. kip}$

Since $M_a > \frac{2}{3}M_{cr}$ therefore,

$$I_{e,d} = \frac{I_{cr}}{1 - \left(\frac{(2/3)M_{cr}}{M_a}\right)^2 \left(1 - I_{cr}/I_g\right)}$$

$$= \frac{229725.47}{1 - \left(\frac{(2/3)6252.9}{12821.82}\right)^2 \left(1 - \frac{229725.47}{599578.4}\right)} = 245748.91\ in^4$$

Example 5.2

❑ Solution

➢ Step 3: Short-term Deflection

Determine effective moment of inertia for dead and live loads.

❖ **For Dead and Live Load**

$M_a = M_{d+l} = 15091 \text{ in.kip}$

Since $M_a > \frac{2}{3}M_{cr}$ therefore,

$$I_{e,d+l} = \frac{I_{cr}}{1 - \left(\frac{(2/3)M_{cr}}{M_a}\right)^2 \left(1 - I_{cr}/I_g\right)}$$

$$= \frac{229725.47}{1 - \left(\frac{(2/3)6252.9}{15091}\right)^2 \left(1 - \frac{229725.47}{599578.4}\right)} = 241072.31 \, in^4$$

Example 5.2

❑ Solution

➢ Step 3: Short-term Deflection

Taking K = 1 for simply supported spans

$$\Delta_{i,d} = K\frac{5M_d l^2}{48E_c I_{e,d}} = \frac{(1)5(12821)(61.5 \times 12)^2}{48 \times 3122 \times 245748.91} = 0.948''$$

$$\Delta_{i,d+l} = K\frac{5M_{d+l} l^2}{48E_c I_{e,d+l}} = \frac{(1)5(15091)(61.5 \times 12)^2}{48 \times 3122 \times 241072.31} = 1.137''$$

$$\Delta_{i,l} = \Delta_{i,d+l} - \Delta_{i,d} = 1.137 - 0.948 = 0.189''$$

Note that in this case, $\Delta_{i,sus} = \Delta_{i,d}$

Example 5.2

Solution

Step 4: Long-term Deflection

Duration	ξ	$\lambda = \dfrac{\xi}{1+50\rho'}$	$\Delta_{i,sus}$	$\lambda\Delta_{i,sus}$	$\Delta_{i,l}$	$\lambda\Delta_{i,sus} + \Delta_{i,l}$
5 years	2.0	2	0.948″	1.896″	0.189″	2.085″
3 months	1.0	1	0.948″	0.948″	0.189″	1.137″

- **Allowable Deflection**

Case – I : With no false ceiling attachments, the case for "Roof and floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections" applies: $\lambda\Delta_{i,sus} + \Delta_{i,l} \leq l/240$

$$\Delta_{allowable} \leq \frac{l}{240} = \frac{61.5 \times 12}{240} = 3.08''$$

Calculated deflection of 2.085″ < 3.08″→ OK

Example 5.2

❑ Solution

➢ Step 4: Long-term Deflection

❖ Allowable Deflection

Case – II : With false ceiling attachments, the case for "Roof and floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections" applies: $\lambda\Delta_{i,sus} + \Delta_{i,l} \leq l/480$

$$\Delta_{allowable} \leq \frac{l}{480} = \frac{61.5 \times 12}{480} = 1.54''$$

Calculated deflection of 2.085″ > 1.54″→ Not OK

Example 5.2

❑ Solution

➢ Deflections Vs Depth of Beam

- The table provided below shows the deflections due to live loads for the beam in question, spanning depths from the minimum required by the ACI code up to 60 inches.

Depth (in.)	$\lambda\Delta_{i,sus} + \Delta_{i,l}$ at 5 years (in.)	ACI limit (in.)	Remarks
36.9 (h_{min} of ACI)	6.501	3.08	Not Governing
40	5.379	3.08	Not Governing
50	3.214	3.08	Not Governing
60	2.085	3.08	OK

- From the above table, it is concluded that minimum depth requirements of ACI does not govern for unusually large values of loading and/or span.

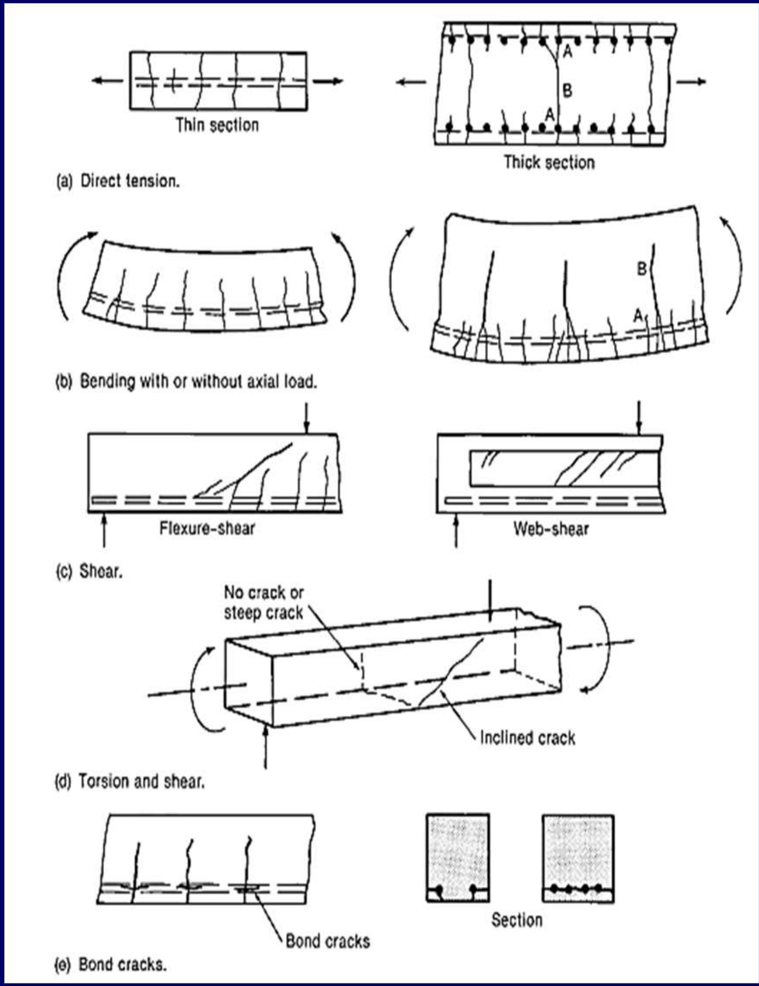
Section – II

Cracking in RC Members

General

❑ Crack Formation

- Tensile stresses induced by loads, moments, shears, and torsion cause distinctive crack patterns.
- Cracks also develop in response to imposed deformations, such as differential settlements, shrinkage, temperature differentials and corrosion of reinforcement.



General

Crack Formation

- All RC beams crack, generally starting at loads well below service level, and possibly even prior to loading due to restrained shrinkage.
- In a well-designed beam, flexural cracks are fine, so-called hairline cracks, almost invisible to a casual observer, and they permit little if any corrosion to the reinforcement.
- As loads are gradually increased above the cracking load, both the number and width of cracks increase, and at service load level a maximum width of crack of about 0.016 inch (0.40 mm) is typical.
- If loads are further increased, crack widths increase further, although the number of cracks do not increase substantially.

General

❑ Crack Formation

- In ACI code prior to 1995, the limitation on crack width for interior and exterior exposure was 0.016 and 0.013 inch respectively.
- Research in later years has shown that corrosion is not clearly correlated with surface crack widths in the range normally found with reinforcement stresses at service load levels. For this reason, the former distinction between interior and exterior exposure has been eliminated in later codes.
- Therefore, the limiting value of crack width both for interior and exterior exposures is now taken as 0.016 inch.

General

❑ Crack Formation

- Because of complexity of the problem, present methods for predicting crack widths are based primarily on test observations.
- Most equations that have been developed predict the probable maximum crack width, which usually means that about 90 % of the crack widths in the member are below the calculated value.

General

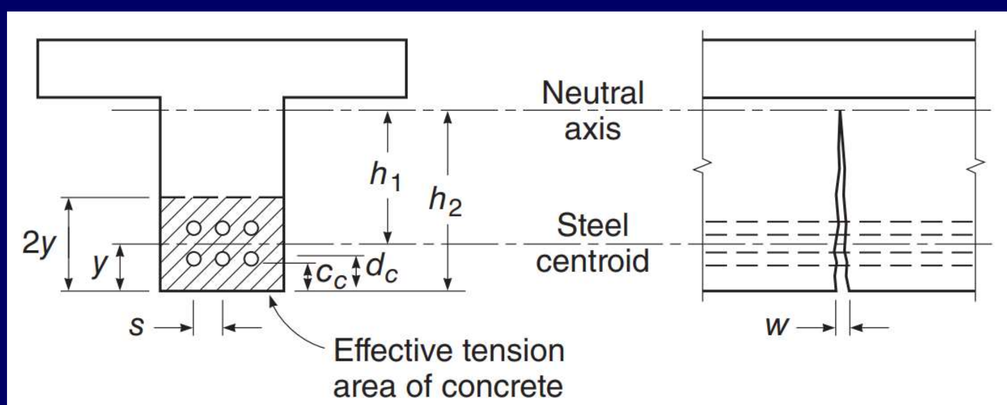
❑ Reason for Crack Width Control

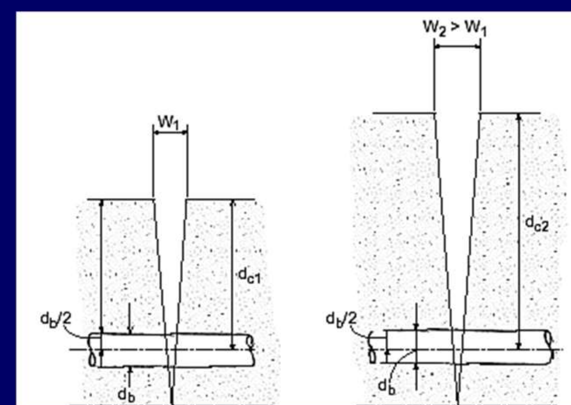
1. **Appearance:** important for concrete exposed to view such as wall panels.
2. **Corrosion:** significant for concrete exposed to aggressive environments.
3. **Water tightness:** may be required for marine/sanitary structures.

Parameters Affecting Crack Width

❑ Concrete Cover

- Experiments have shown that both crack spacing, and crack width are related to the concrete cover distance (dc), measured from center of the bar to the face of concrete. Increasing the concrete cover increases the spacing of cracks and increases crack width.



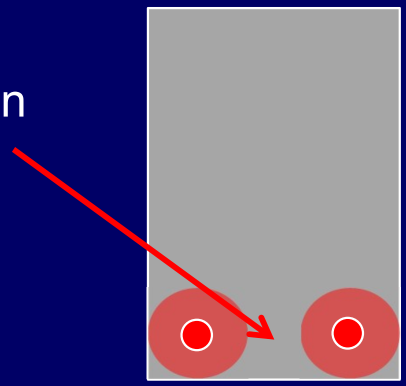


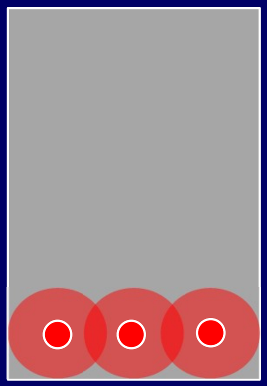
Parameters Affecting Crack Width

❑ Distribution of Reinforcement in Tension Zone of Beam

- Generally, to control cracking, it is better to use a larger number of smaller diameter bars to provide the required As than to use the minimum number of larger bars, and the bars should be well distributed over the tensile zone of the concrete.

This portion may crack





Smaller diameter bars distributed over tension zone

Parameters Affecting Crack Width

Stress in Reinforcement (crack width ∝ f_s^n)

- Where fs is the steel stress and "n" is an exponent that varies in the range of 1.0 to 1.4. For steel stress in the range of practical interest, say from 20 to 36 ksi, n may be taken equal to 1.0.
- Steel stress may be computed based on elastic cracked section analysis.
- Alternatively, f_s may be taken equal to (2/3)f_y (ACI 24.3.2.1).

Parameters Affecting Crack Width

❑ Bar Deformations

- Beams with smooth round bars will display a relatively small number of wide cracks in service, while beams with bars having proper surface deformations will show a larger number of very fine, almost invisible cracks.

ACI Code Provisions for Crack Control

❑ Direct Approach

- Expected crack width is calculated and is compared with the maximum permissible limit provided by the code.

❑ Indirect Approach

- Cracking in RC member can be controlled by imposing limit on the maximum bar spacing.

ACI Code Provisions for Crack Control

Indirect Approach

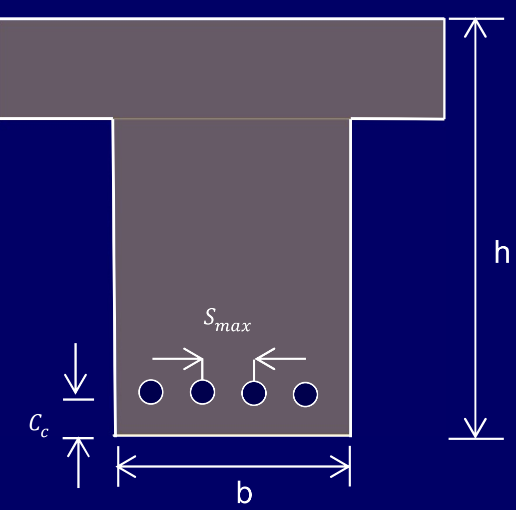
- The maximum center-to-center spacing between the adjacent bars shall not exceed the limits in ACI Table 24.3.2.
- For deformed bars or wires, maximum spacing s_{max} is given by;

$$s_{max} = Least\ of\ \ 15\left(\frac{40{,}000}{f_s}\right) - 2.5C_c \quad and \quad 12\left(\frac{40{,}000}{f_s}\right)$$

Where;

f_s = the bar stress in psi under service condition.

C_c = the clear cover in inches from the nearest surface in tension to the surface of the flexural tension reinforcement.



ACI Code Provisions for Crack Control

❑ Indirect Approach

- The stress f_s is calculated by dividing the service load moment by the product of area of reinforcement and the internal moment arm.

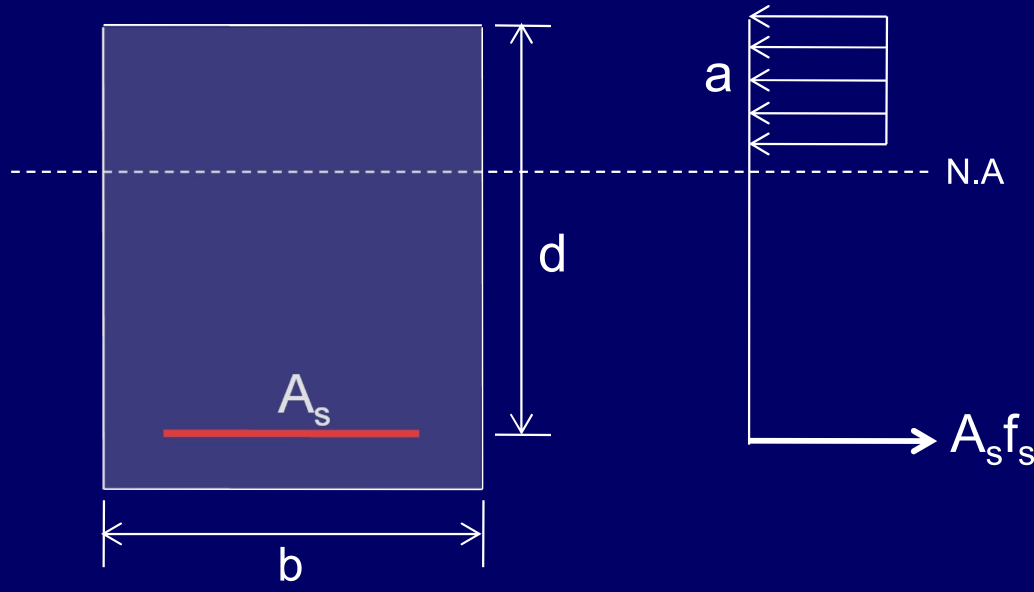
For equilibrium,

$$M_s = A_s f_s \left(d - \frac{a}{2}\right)$$

$$f_s = \frac{M_s}{A_s \left(d - \frac{a}{2}\right)}$$

Where;

M_s = service load moment



ACI Code Provisions for Crack Control

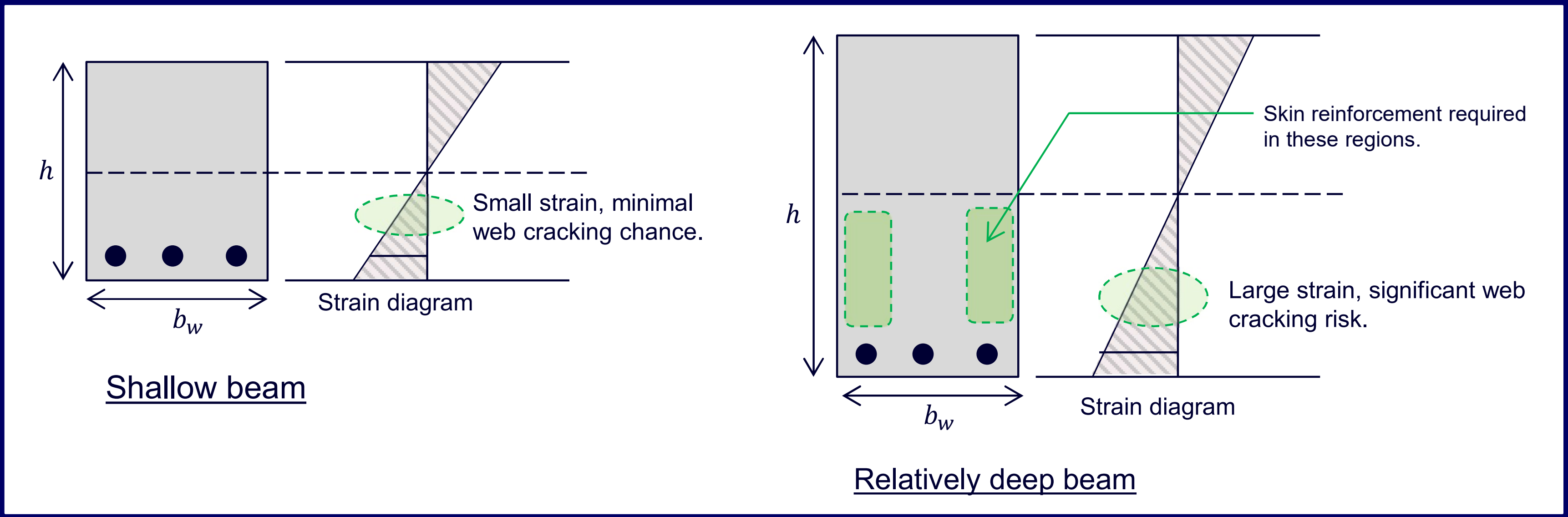
❑ Indirect Approach

- Alternatively, the ACI Code Section 24.3.2.1 permits f_s to be taken as $2/3$ of the specified yield strength f_y (because seldom will be reinforcing bars stressed greater than about 67% of f_y at service loads).
- Note that the condition $f_s = (2/3)f_y$ is for full-service load condition. For loading less than that, f_s shall be calculated.

ACI Code Provisions for Crack Control

❑ Crack Control Reinforcement in Deep Flexural Members

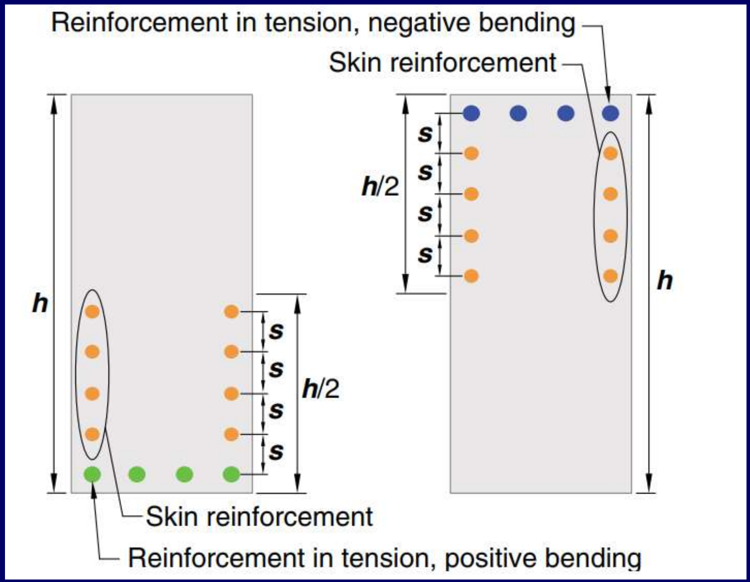
- For relatively deep beams, some reinforcement should be placed near the vertical faces of the tension zone to control cracking in the web (ACI R9.7.2.3). This is termed as *skin reinforcement*.



ACI Code Provisions for Crack Control

❑ Crack Control Reinforcement in Deep Flexural Members

- For beams with depth exceeding 36 inches ($h \geq 36''$), longitudinal skin reinforcement shall be uniformly distributed on both side faces of the beam for a distance h/2 from the tension face (ACI 9.7.2.3).
- The size of the skin reinforcement is not specified.
- Research indicates that bar sizes No. 3 to No.5, with a minimum area of 0.1 in^2 per foot of depth (0.1h), are typically provided (ACI R9.7.2.3).

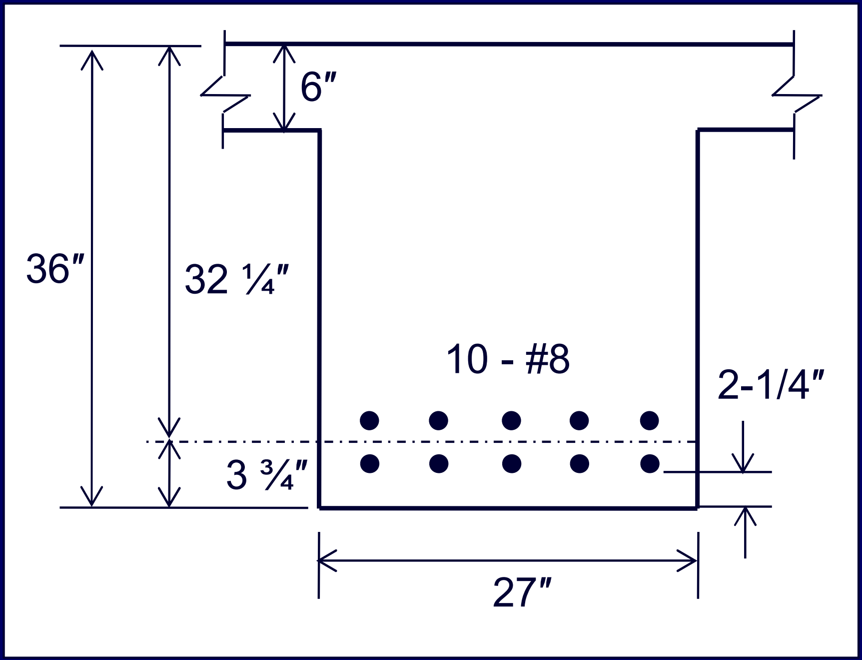


Skin Reinforcement for beams with h > 36″

Example 5.3

❑ Problem Statement

- Figure shows the main flexural reinforcement at mid span for a T girder in a high rise building that carries a service load moment of 8630 in-kips. ***Determine*** if the beam meets the crack control criteria in the ACI Code.



Example 5.3

❑ Solution

- Since the depth of the web is less than 36 inches, skin reinforcement is not needed.
- To check the bar spacing criteria, the steel stress can be estimated closely by taking the internal lever arm equal to distance $d - h_f/2$.

$$f_s = \frac{M_s}{A_s\left(d - \frac{h_f}{2}\right)} = \frac{8630}{7.9\left(32.25 - \frac{6}{2}\right)} = 37.35\ ksi$$

Or alternatively,

$$f_s = \frac{2}{3} f_y = \frac{2}{3} \times 60 = 40\ ksi$$

Example 5.3

❑ Solution

The center-to-center spacing between adjacent bars shall not exceed s_{max}

$$s_{max} = \text{Least of } 15\left(\frac{40{,}000}{f_s}\right) - 2.5C_c \quad and \;\; 12\left(\frac{40{,}000}{f_s}\right)$$

Substituting $f_s = 37.35 \times 1000 = 3750\, psi$ and $C_c - 2.25''$, we get

$$s_{max} = \text{Least of } 15\left(\frac{40{,}000}{37350}\right) - 2.5(2.25) \;\; and \;\; 12\left(\frac{40{,}000}{40{,}000}\right)$$

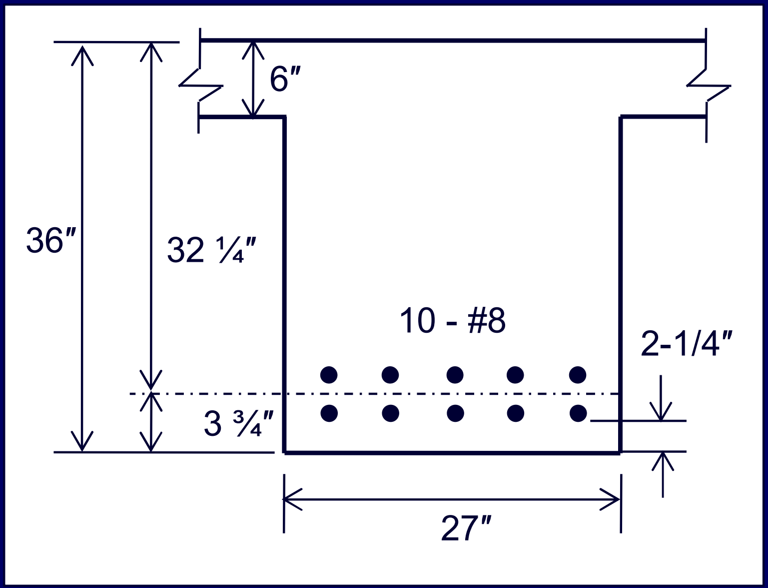
$$s_{max} = 10.4''$$

Example 5.3

❑ Solution

Now calculate the actual (provided) spacing from given figure.

$$s = \frac{b_w - 2(C_c) - d_b}{No.of\ spaces} \text{ (for the given case)}$$



Substituting the values

$$s = \frac{27 - 2(2.25) - 1}{4}$$

$$s = 5.4'' < S_{max}$$

Therefore, the crack control criteria of ACI code is satisfied. If the results had been unfavorable, a redesign using a larger number of smaller-diameter bars would have been indicated.

Equations for Maximum Crack Width

❑ Alternative Approach

- As alternate to ACI crack width equation, crack width can be calculated using following equations. These equations have been used in deriving the ACI Code equation on crack control.

❖ Gergely and Lutz Equation

$$w = 0.076\beta f_s (d_c A)^{1/3}$$

❖ Frosch Equation

$$w = \frac{2000\beta f_s \sqrt{d_c^2 + (s/2)^2}}{E_s}$$

w = maximum width of crack, thousandth inch

f_s = steel stress at loads for which crack width is to be determined, ksi

E_s = modulus of elasticity of steel, ksi

d_c = thickness of concrete cover measured from tension face to center of bar closest to that face, inch

s = maximum bar spacing, inch

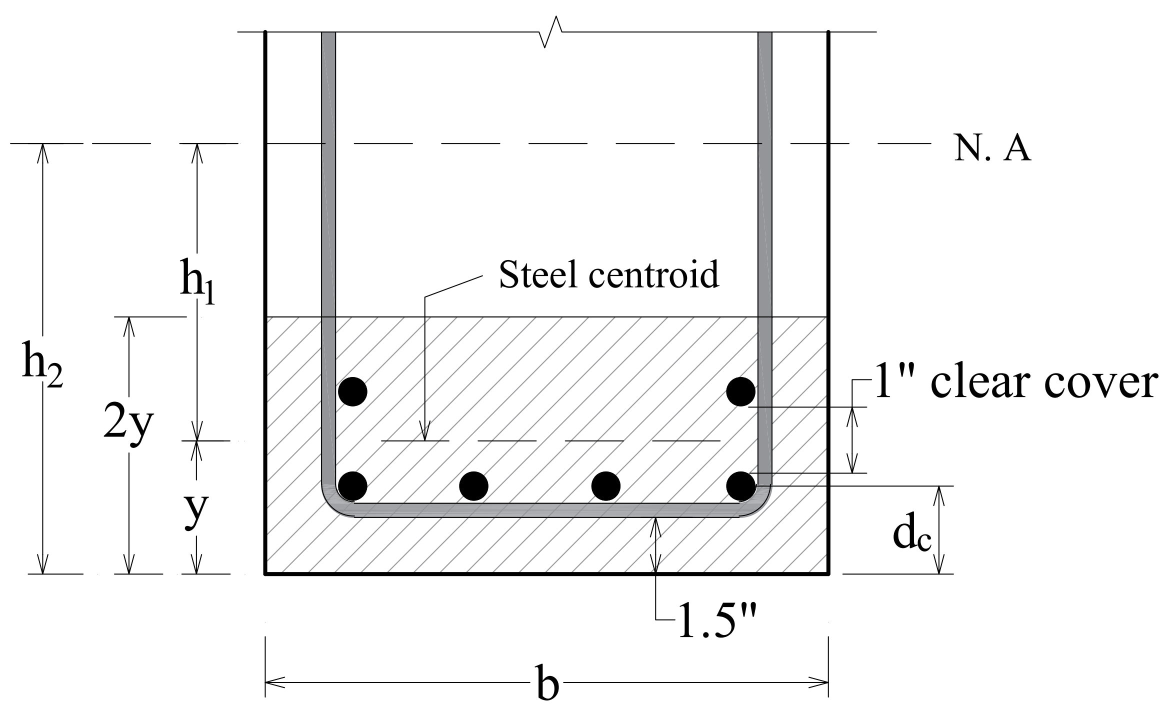
β = ratio of distance from tension face to neutral axis to distance from steel centroid to neutral axis

A = concrete area surrounding one bar which is equal to total effective tension area of concrete surrounding total reinforcement divided by number of bars, in^2

Equations for Maximum Crack Width

❑ Alternative Approach

❖ Determination of β *and* A

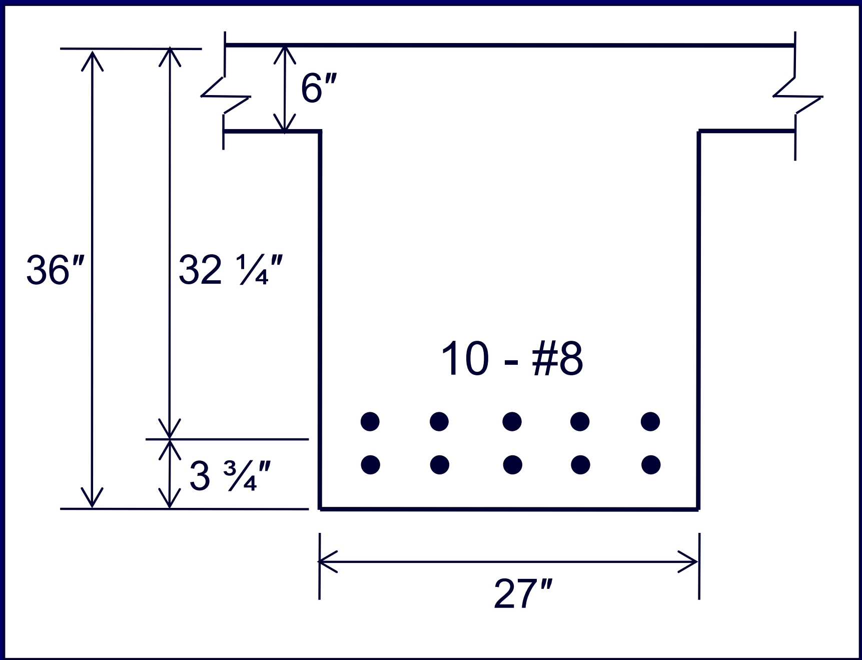


$$\beta = \frac{h_2}{h_1} \quad and \quad A = 2yb/n$$ (where n = number of bars)

Example 5.4

❑ Problem Statement

- Using Gergely & Lutz and Frosch equations, ***Check*** the maximum crack width of the beam shown below. The beam is subjected to service load moment of 8630 in kips. The clear cover on the side and bottom of the beam stem is 2 ¼ inches. The material strengths are f_c' = 4 ksi and f_y = 60 ksi.



Example 5.4

❑ Solution

Gergely & Lutz Equation

$$w = 0.076\beta f_s(d_c A)^{1/3}$$

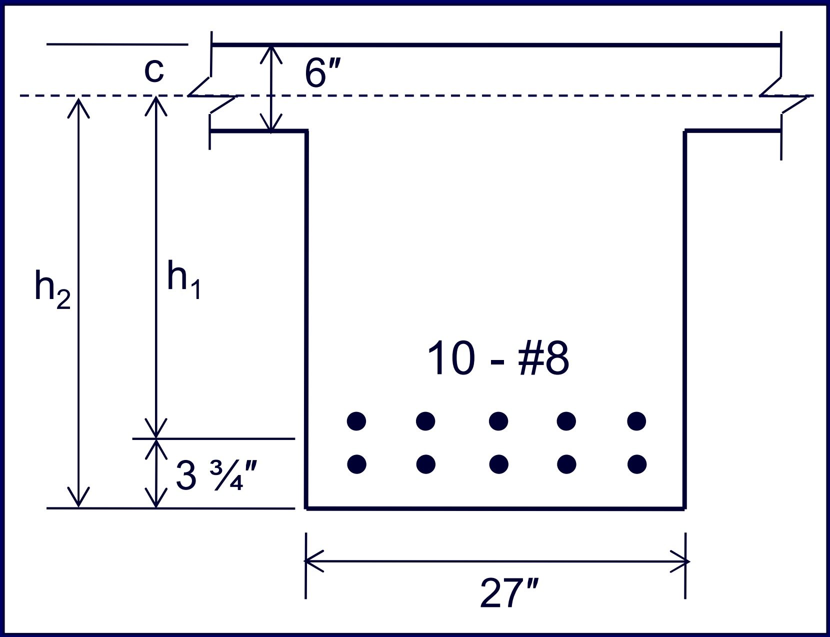
$$f_s = \frac{2}{3}f_y = \frac{2}{3}(60) = 40\ ksi$$

Calculate β

$$a = \frac{A_s f_s}{0.85 f_c' b} = \frac{(10 \times 0.79)40}{0.85 \times 4 \times 27} = 3.44''$$

$$c = \frac{a}{\beta_1} = \frac{3.44}{0.85} = 4.05''$$

$$\beta = \frac{h_2}{h_1} = \frac{36 - 4.05}{32.25 - 4.05} = 1.13$$



Example 5.4

❑ Solution

❖ Gergely & Lutz Equation

$y = 2.75 + 1 = 3.75''$

$$A = \frac{2yb}{n} = \frac{2(3.75) \times 27}{10} = 20.25 \text{ in}^2$$

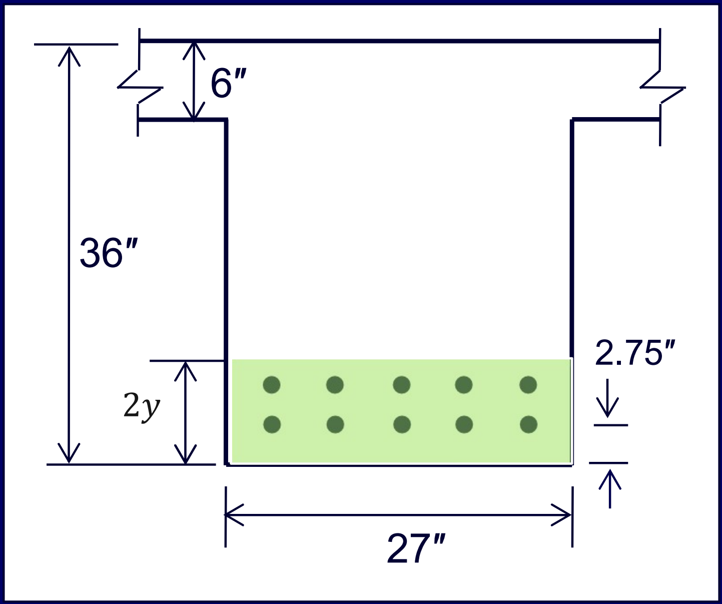
Now substituting the values

$w = 0.076\beta f_s (d_c A)^{1/3}$

$w = 0.076(1.13)(40)\ (2.75 \times 20.25)^{1/3}$

$w = 13.12$ thousandth inch OR

$w = 0.013''(0.33 \text{ mm})$



Example 5.4

❑ Solution

Frosch Equation

$$w = \frac{2000\beta f_s \sqrt{d_c^2 + (s/2)^2}}{E_s}$$

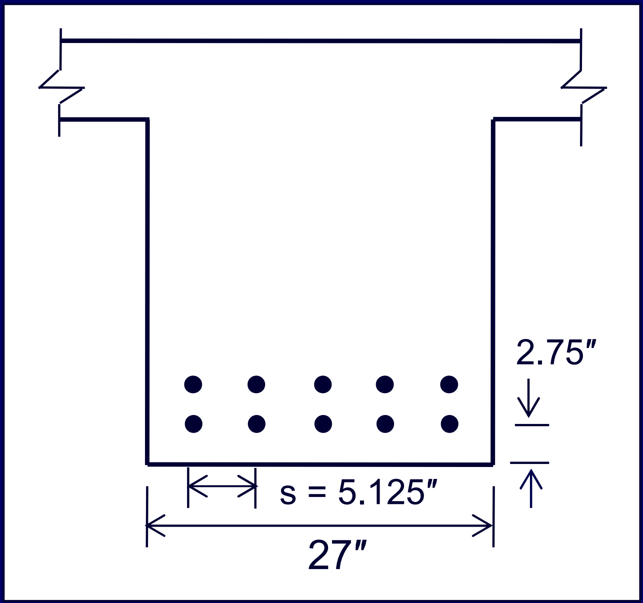
c/c spacing between bars, s = 5.125″

Substituting values, we get

$$w = \frac{2000(1.13)(40)\sqrt{2.75^2 + (5.125/2)^2}}{29000}$$

$w = 11.72$ thousandth inch OR

$w = 0.012''(0.31 \text{ mm})$



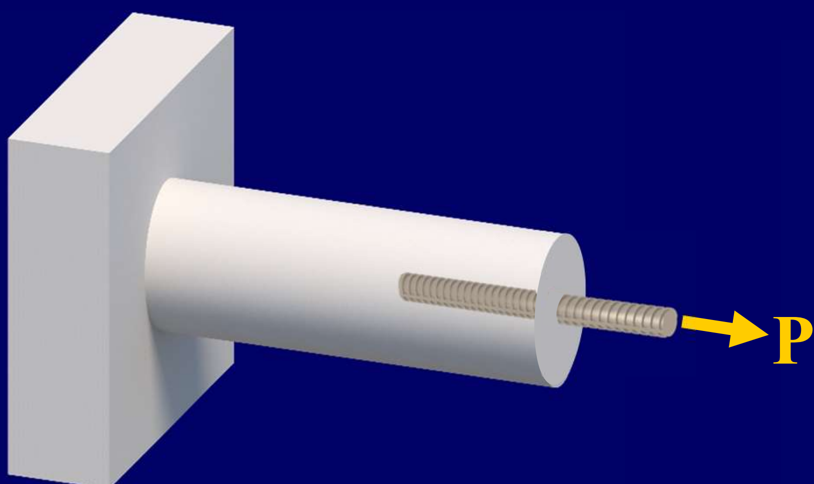
Section – III

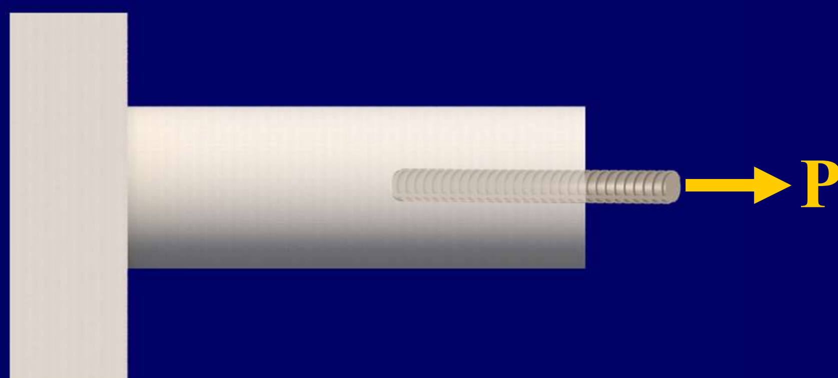
Development & Splices of Reinforcement

Development Length

❑ Introduction

- Consider a steel bar embedded in a concrete block; if force P is gradually increased, depending on the embedment length, either the bar will come out of the concrete block, or the steel will yield.

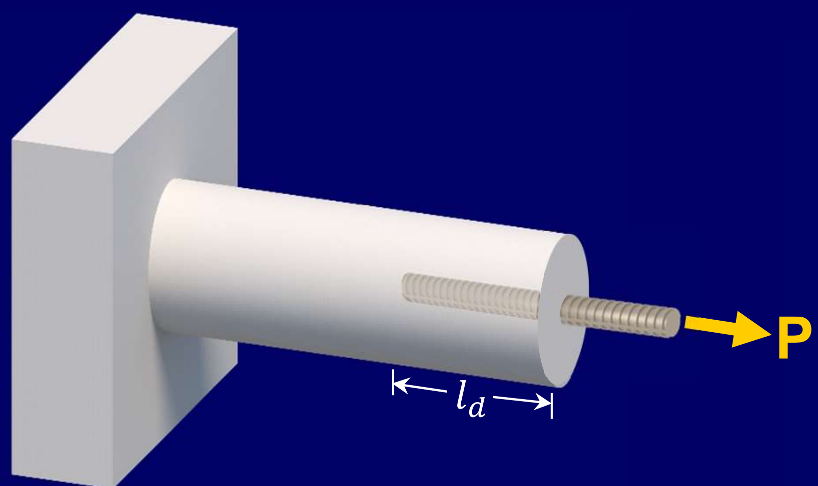


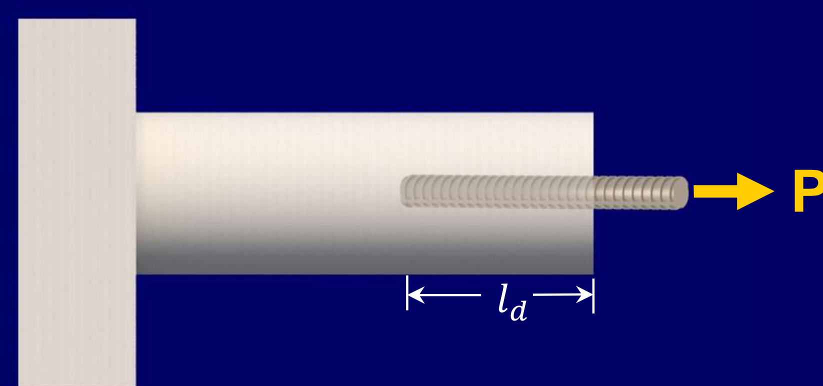


Development Length

❑ Introduction

- Development length refers to "the minimum length of the bar that must be embedded in the concrete block so that it yields but does not pull out due to bond failure".
- Bond failure occurs when the provided embedment length l is less than the development length l_d.





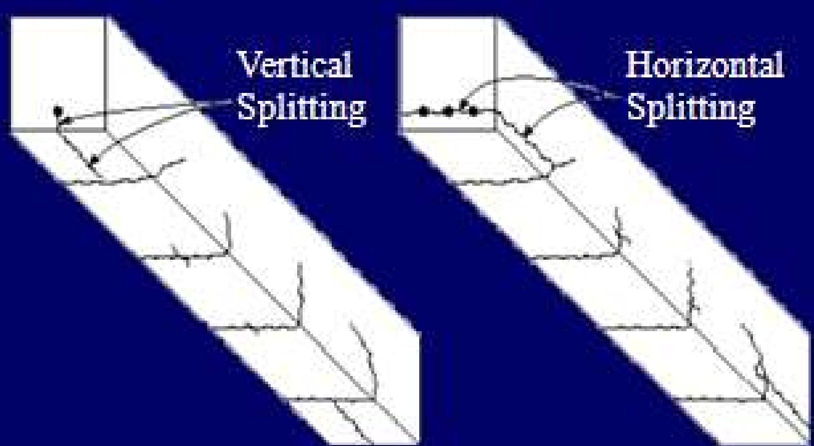
Development Length

❑ Bond Failure

- There are two types of bond failure

1. **Direct pullout of reinforcement:** Direct pullout of reinforcement occurs in members subjected to direct tension.

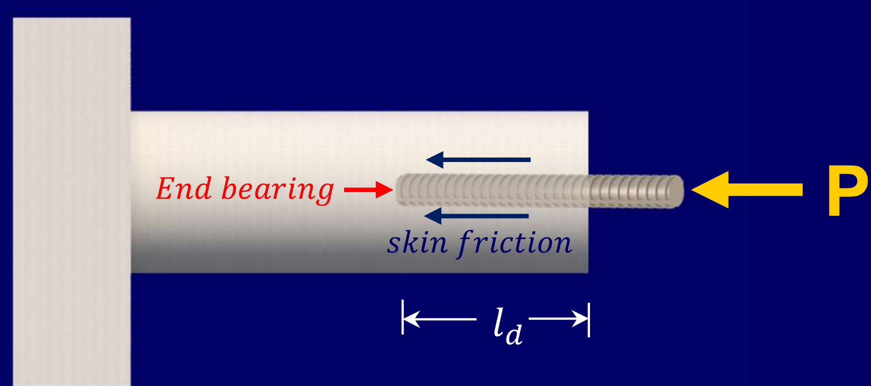
2. **Splitting of concrete:** In members subjected to tensile flexural stresses, the reinforcement causes splitting of concrete as shown.



Development Length

❑ Development Length of Compression Members

- In the case of bars in compression, a part of the total force is transferred by bond along the embedded length, and a part is transferred by end bearing of the bars on the concrete.
- As the surrounding concrete is relatively free of fractures and because of the beneficial effect of end bearing, compression bars can have shorter basic development lengths than tension bars.



Development Length

❑ Development Length of Compression Members

- The development length of tension reinforcement will be discussed in the next section of the lecture only, because it is the governing criteria in most reinforced concrete structures.
- For more details refer to section 5.8 of Design of Concrete Structures 14th Ed. by Nilson, Darwin and Dolan.

Development Length

❑ Development Length of Tension Reinforcement

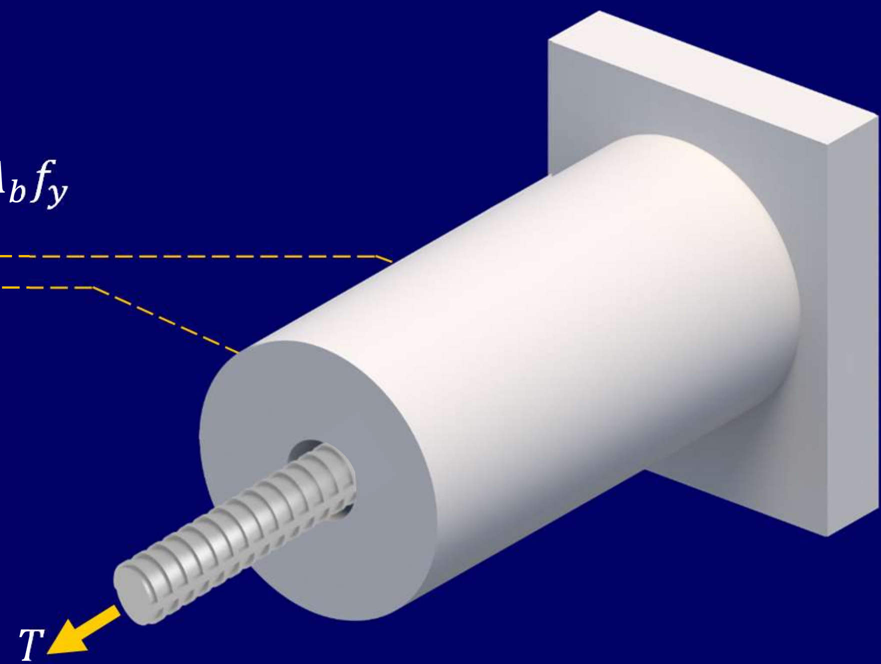
- Tensile force T, acting on bar A_b having yield strength f_y , tends to pullout bar embedded in concrete member with bond strength f_b
- Resistance R against the force T, offered by skin friction around bar is given as;

$$R \geq T \quad \Rightarrow A_{sk} f_b \geq A_b f_y$$

$$Length \times circumference \times f_b \geq A_b f_y$$

$$l_d \times \pi d_b \times \alpha\sqrt{f_c'} \geq \frac{\pi (d_b)^2}{4} f_y$$

$$l_d = \left(\frac{f_y}{4\alpha\sqrt{f_c'}}\right) d_b$$



Development Length

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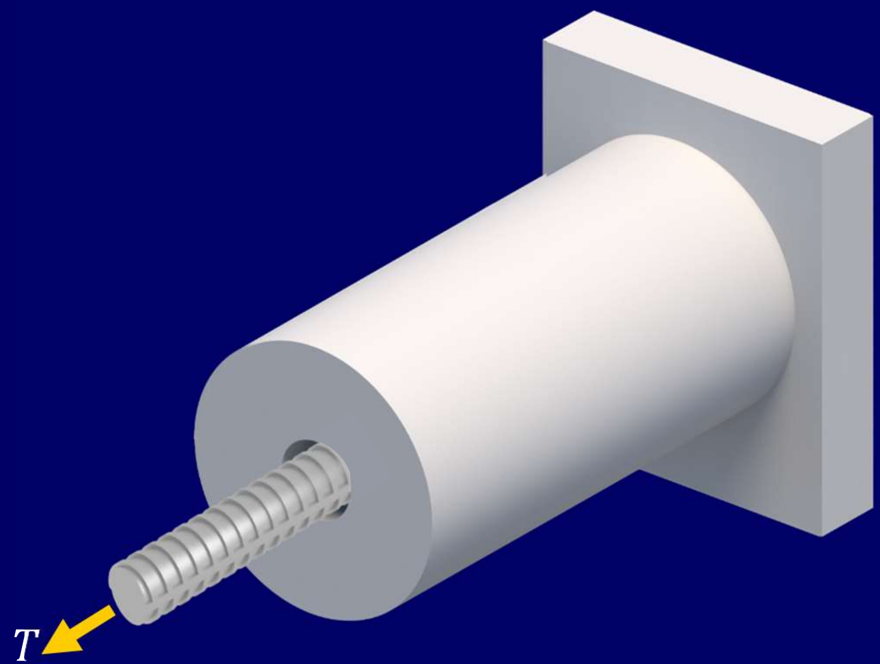

Development Length

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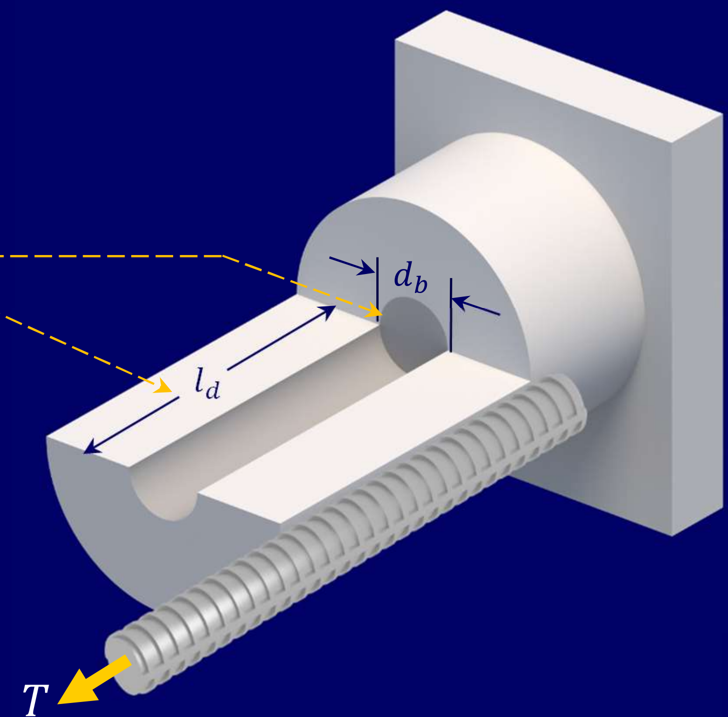
$$Length \times circumference \times f_b \geq A_b f_y$$

$$l_d \times \pi d_b \times \alpha\sqrt{f_c'} \geq \frac{\pi (d_b)^2}{4} f_y$$

$$l_d = \left(\frac{f_y}{4\alpha\sqrt{f_c'}}\right) d_b$$



Development Length

❑ ACI 318 Code Provisions for Development of Tension Reinforcement

- As per ACI 318, section 25.4.2.4, for deformed bars or deformed wires, l_d shall be calculated by:

$$l_d = \left[\frac{3}{40}\frac{f_y}{\lambda\sqrt{f_c'}}\frac{\Psi_t\Psi_e\Psi_s\Psi_g}{\left(\frac{c_b + K_{tr}}{d_b}\right)}\right] d_b$$

Where;

- $\lambda, \Psi_t, \Psi_e, \Psi_s, \Psi_g$ are called modification factors
- C_b is a factor that represents the least of the side cover, the concrete cover to the bar.
- K_{tr} is a factor that represents the contribution of confinement reinforcement across potential splitting planes. It shall be permitted to take $K_{tr} = 0$ for design simplification.
- The term $\left(\frac{c_b+K_{tr}}{d_b}\right)$ shall not exceed 2.5. For practical cases, it is taken as 1.5.

Development Length

❑ ACI 318 Code Provisions for Development of Tension Reinforcement

Table 25.4.2.5 —Modification factors for development of deformed bars in tension

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Normal weight concrete	1
Reinforcement grade Ψ_g	Grade 40 or Grade 60	1
	Grade 80	1.15
	Grade 100	1.3
Epoxy Ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement with clear cover less than 3db or clear spacing less than 6db	1.5
	Epoxy-coated or zinc and epoxy dual-coated reinforcement for all other conditions	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1
Size Ψ_s	No. 7 and larger bars	1
	No. 6 and smaller bars and deformed wires	0.8
Casting position Ψ_t	More than 12 in. of fresh concrete placed below horizontal reinforcement	1.3
	Other	1

Development Length

❑ ACI 318 Code Provisions for Development of Tension Reinforcement

- If the values of λ , Ψ_t , Ψ_e , Ψ_g are taken equal to 1, and $\left(\frac{c_b + K_{tr}}{d_b}\right) = 1.5$ then the previous equation reduces to;

$$l_d\ (in.) = \left(\frac{f_y}{20\sqrt{f_c{}'}}\right) d_b$$ (For No. 7 and larger bars, Ψ_s = 1)

and

$$l_d\ (in.) = \left(\frac{f_y}{25\sqrt{f_c{}'}}\right) d_b$$ (For No.6 and smaller bars, Ψ_s = 0.8)

Development Length

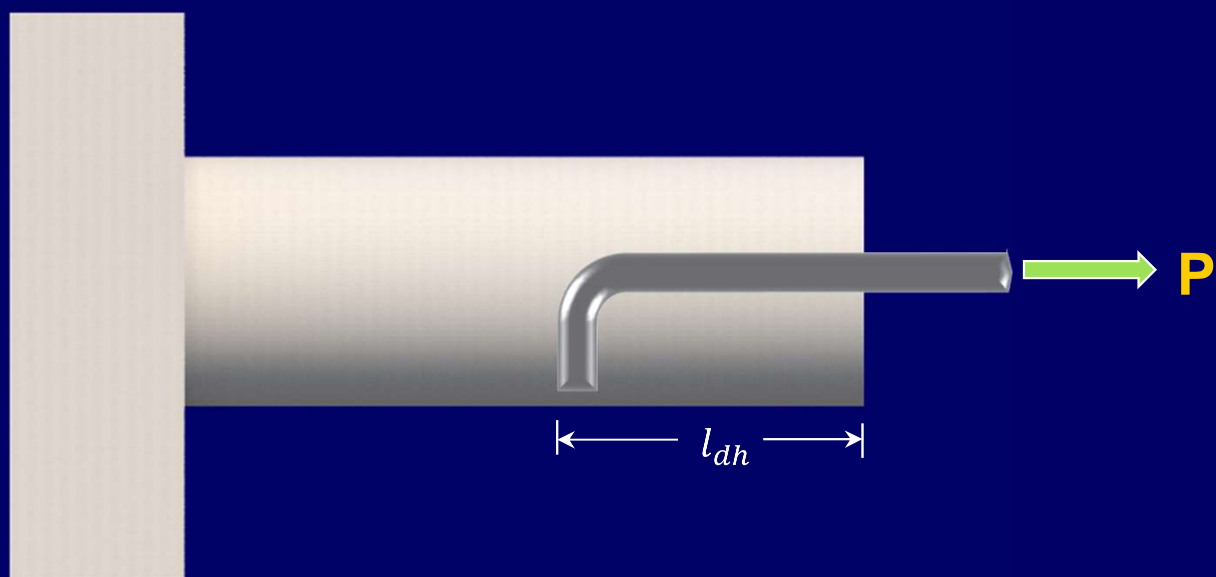
- ACI 318 Code Provisions for Development of Tension Reinforcement

Development Lengths for Normal weight Concrete of 3000psi			
Bar No.	Equation	Development length l_d	
		Grade 40	Grade 60
#3	$l_d(in) = \left(\frac{f_y}{25\sqrt{f_c'}}\right) d_b$	11″	17″
#4		15″	22″
#5		19″	28″
#6		22″	33″
#7	$l_d(in) = \left(\frac{f_y}{20\sqrt{f_c'}}\right) d_b$	32″	48″
#8		37″	55″
#9		41″	62″

Development Length

❑ ACI 318 Code Provisions for Development of Standard hook in Tension

- If a hook is provided at the end of the embedded bar, the requirement on the straight length portion of embedded bar is reduced.



Development Length

❑ ACI 318 Code Provisions for Development of Standard hook in Tension

- As per ACI 318 -19, section 18.8.5.1, for bar sizes No. 3 through No. 11 terminating in a standard hook, l_{dh} shall be calculated as follows;

Concrete type	Standard hook length l_{dh} (in)
Normalweight concrete	$Larger\ of\ \left(\frac{f_y}{65\sqrt{f_c'}}\right) d_b, 8d_b, 6"$
Lightweight concrete	$Larger\ of\ \left(\frac{f_y}{48.75\sqrt{f_c'}}\right) d_b, 10d_b, 7.5"$

Development Length

❑ ACI 318 Code Provisions for Development of Standard hook in Tension

Standard hook in Tension for Normal-weight concrete of 3000psi		
Bar No.	Development length l_{dh}	
	Grade 40	Grade 60
#3	5″	7″
#4	6″	9″
#5	7″	11″
#6	9″	13″
#7	10″	15″
#8	12″	17″
#9	13″	19″

Development Length

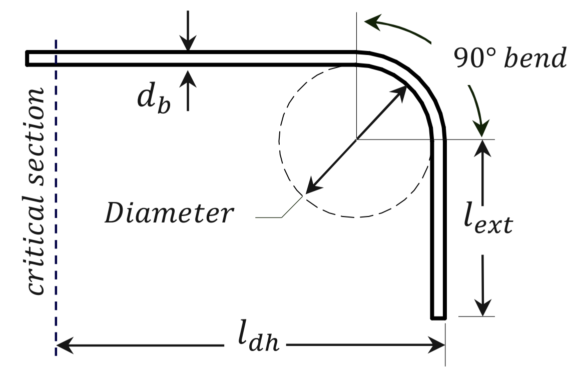
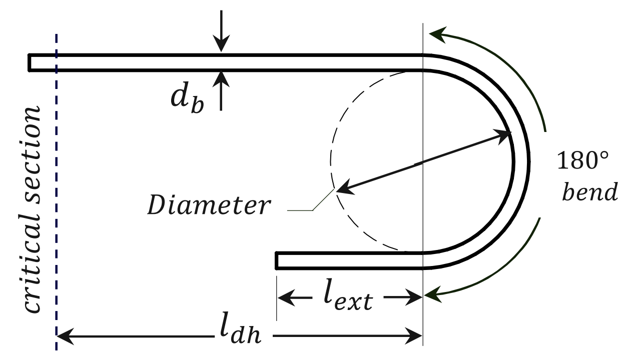
- ACI 318 Code Provisions for Development of Standard hook in Tension

Comparison between l_d and l_{dh} for normalweight concrete of 3000psi

Bar No.	Grade 40		Grade 60	
	l_d	l_{dh}	l_d	l_{dh}
#3	11″	5″	17″	7″
#4	15″	6″	22″	9″
#5	19″	7″	28″	11″
#6	22″	9″	33″	13″
#7	32″	10″	48″	15″
#8	37″	12″	55″	17″
#9	41″	13″	62″	19″

Development Length

❑ Dimensions and bends for standard hooks (Table 25.3.1)

Type of standard hook	Bar size	Min. inside bend diameter (in.)	Straight extension[1] l_{ext}, in.	Type of standard hook
90 – degree hook	#3 through #8	$6d_b$	$10d_b$	
180 – degree hook	#3 through #8	$6d_b$	Greater of $4d_b$ and 2.5″	

[1]A standard hook for deformed bars in tension includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

Development Length

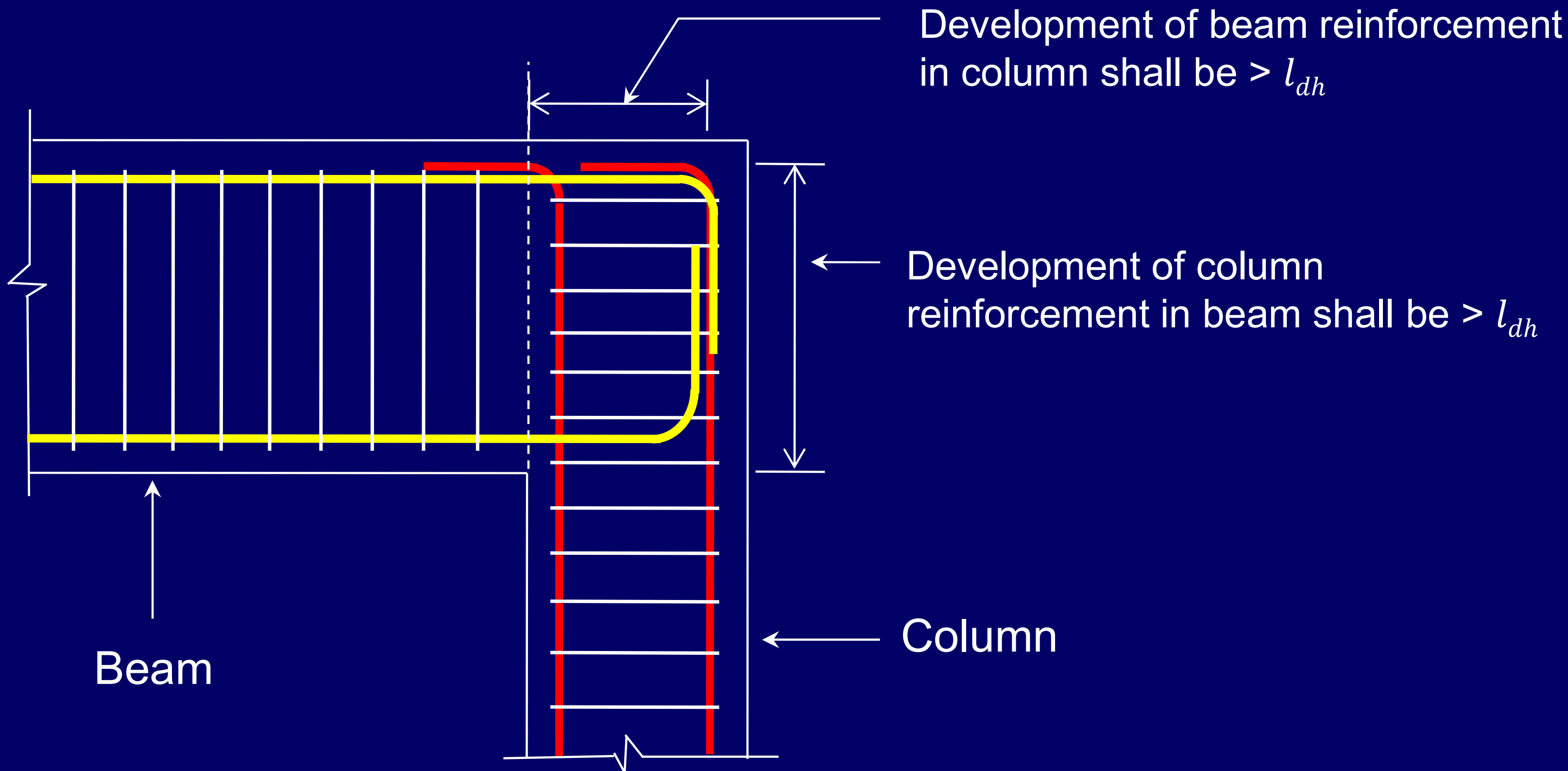
Critical Sections for Development Length

- Development length of a rebar is measured from critical section.
- A critical section is the location where Load effects such as Bending moment, Shear or Torsion are maximum.
- Critical sections for various cases are shown below.

Development Length

❑ Critical Sections for Development Length

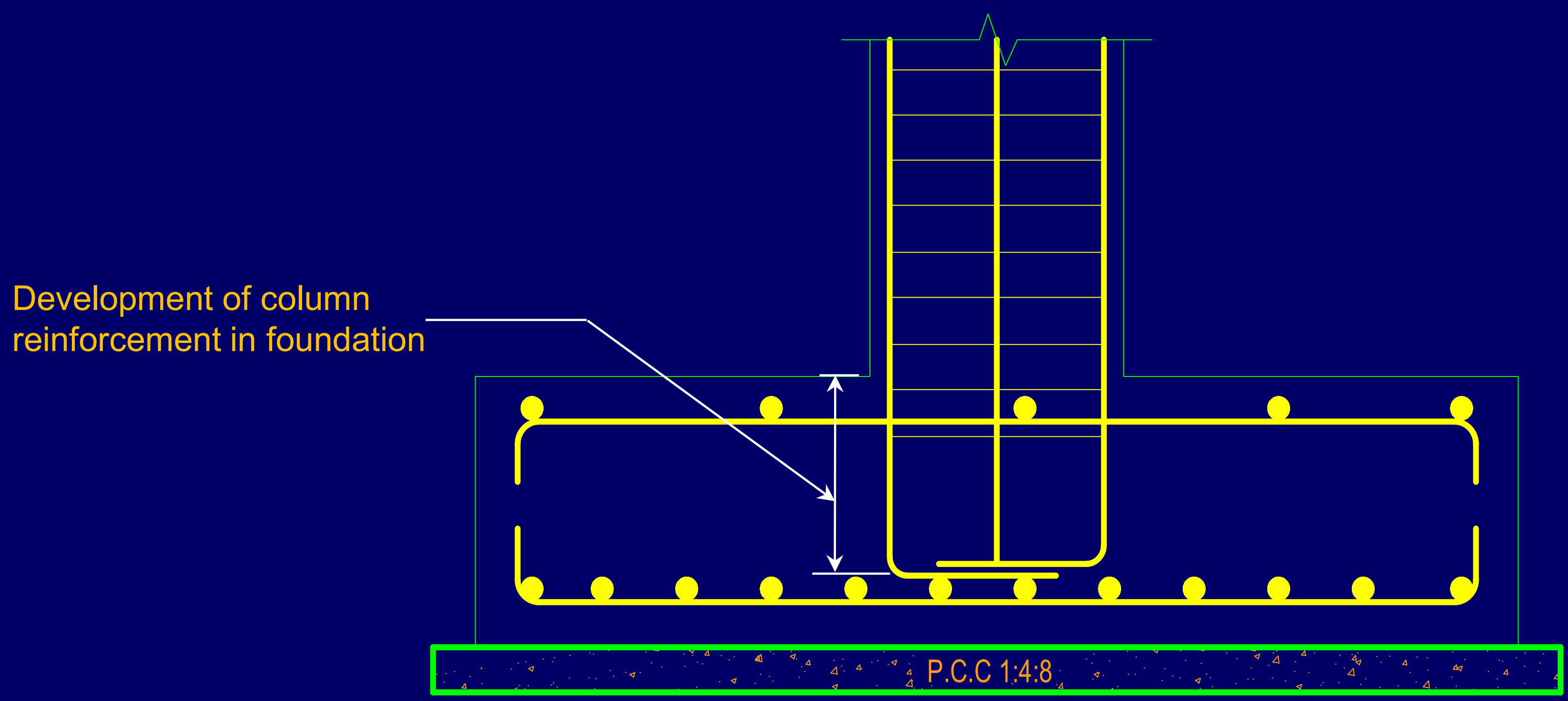
1. Beam Column Joint



Development Length

❑ Critical Sections for Development Length

2. Development of column reinforcement in foundation



Splices of Deformed Bars

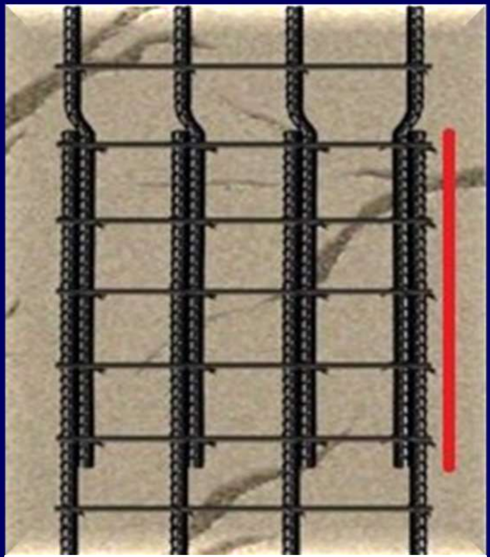
Introduction

- Splice means "to join".
- In general, reinforcing bars are stocked by supplier in lengths up to 60ft.
- For this reason, and because it is often more convenient to work with shorter bar lengths, it is frequently necessary to splice bars.
- Splices in the reinforcement at points of maximum stress should be avoided.
- Splices should be staggered.

Splices of Deformed Bars

❑ Various Types of Splices

- Bar splicing can be done in three ways:



1. Lap Splice

2. Mechanical Splice

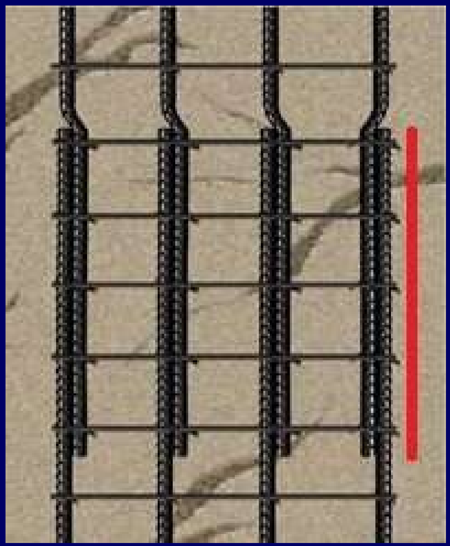
3. Weld Splice

Splices of Deformed Bars

❑ Various Types of Splices

1. Lap Splice

- Splices for #11 bars and smaller are usually made simply lapping the bars by a sufficient distance to transfer stress by bond from one bar to the other.



Splices of Deformed Bars

❑ Various Types of Splices

1. Lap Splice

- The lapped bars are usually placed in contact and lightly wired so that they stay in position as the concrete is placed.
- According to ACI 318-19, Section 25.5.1.3, bars spliced by noncontact lap splices in flexural members shall not be spaced transversely farther apart than one-fifth the required lap splice length, nor 6 inches.

Splices of Deformed Bars

❑ Various Types of Splices

1. Lap Splice

- According to ACI 318-19, Section 25.5.2.1, minimum length of lap for tension lap splices shall be as required for Class A or B splice, but not less than 12 inches, where:
  - Class A splice .... 1.0 l_d
  - Class B splice ....1.3 l_d

  Where l_d as per ACI 25.4 (discussed earlier).
- As per ACI Code, Lap splices in general must be class B splices.

Splices of Deformed Bars

❑ Various Types of Splices

2. Mechanical Splice

- In this method of splicing, the bars in direct contact are mechanically connected through couplers.

Splices of Deformed Bars

❑ Various Types of Splices

2. Mechanical Splice

- According to ACI 318-19, Section 25.5.7.1, a full mechanical splice shall develop in tension or compression, as required, at least 125% of specified yield strength f_y of the bar.
- This ensures that the overloaded spliced bar would fail by ductile yielding in the region away from the splice, rather than at the splice where brittle failure is likely.

Splices of Deformed Bars

❑ Various Types of Splices

3. Welded splice

- Bars in direct contact are welded and the stresses are transferred by weld rather than bond.
- According to ACI 25.5.7.1, a full welded splice shall develop at least 125% of the specified yield strength f_y of the bar.

Splices of Deformed Bars

❑ Splice Location

- The splicing should be avoided in the critical locations, such as at the maximum bending moment locations and at the shear critical locations.

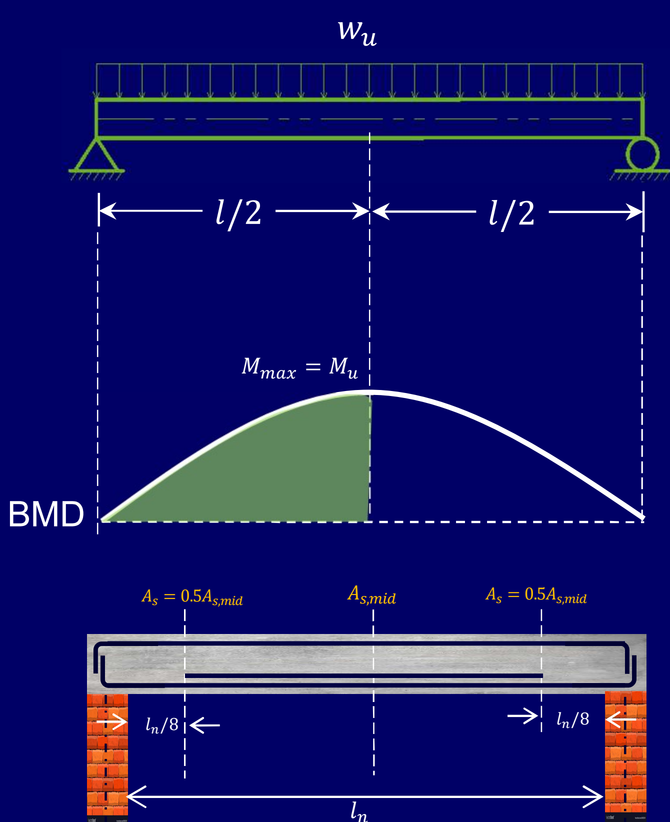
Curtailment of Reinforcement

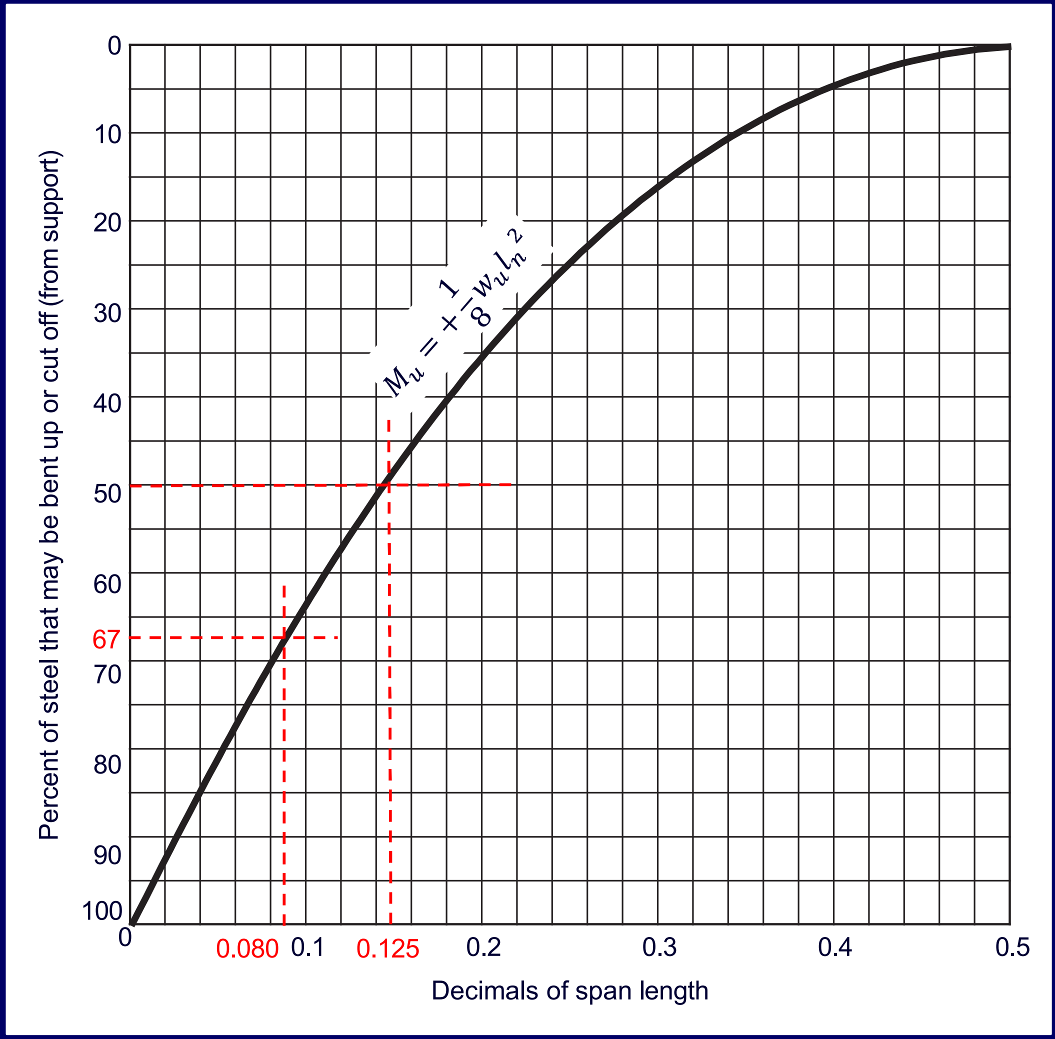
Introduction

- It is common practice to cut off bars where they are no longer needed to resist stress.
- In the case of simply supported beams, the figure on the next slide shows cut off locations for various percentages of reinforcement curtailment.

Curtailment of Reinforcement

- ❑ **Cut off locations for Various Percentages of Reinforcement Curtailment.**

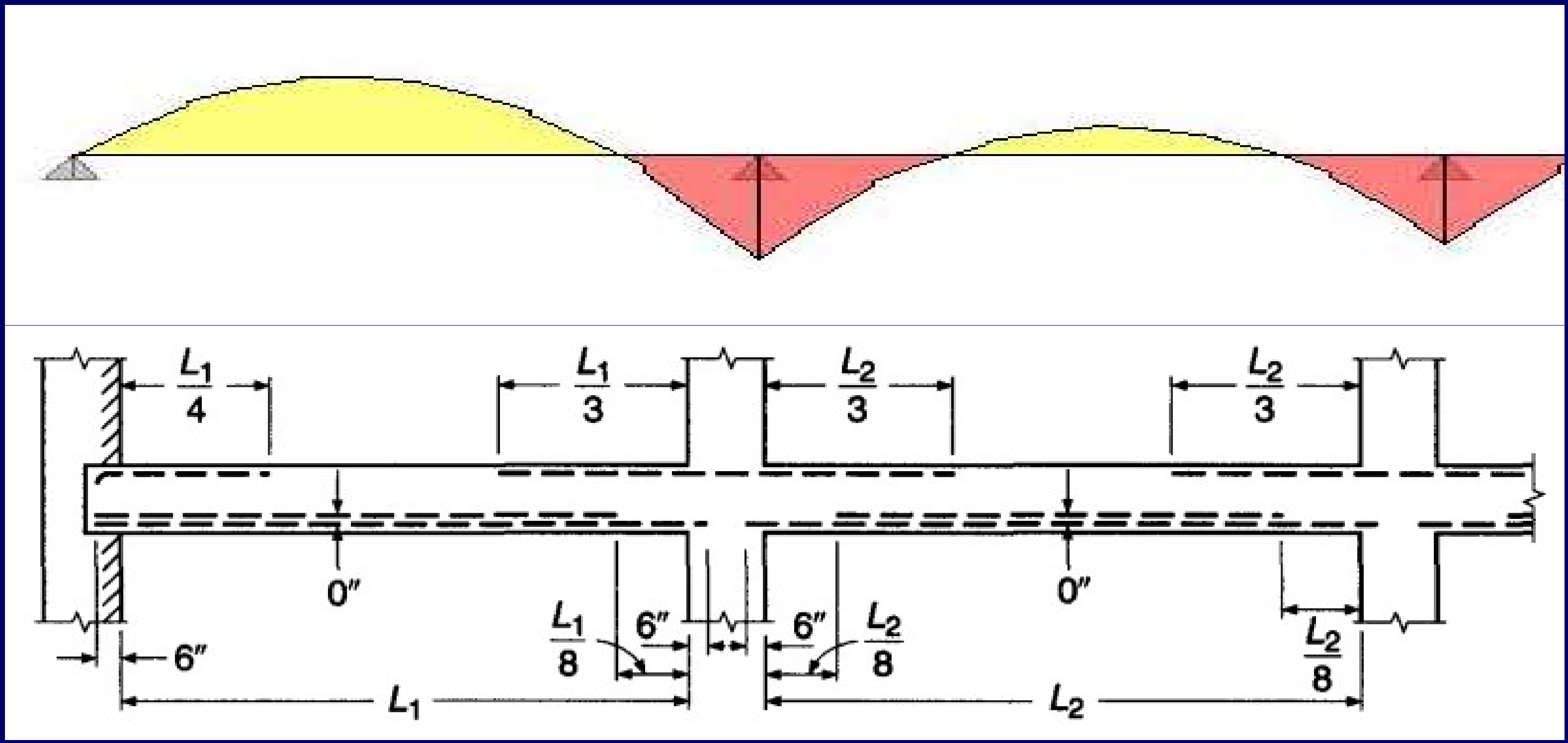




Curtailment of Reinforcement

Cut off locations for a Typical Continuous Beam

- For nearly equal spans, uniformly loaded, in which not more than about one-half the tensile steel is to be cut off, the locations shown in figure are satisfactory.



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