Lecture 07

Idealized Structural Modeling of RC Structures

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- Design Cycle
- Idealization of Physical Structure
- Models for Analysis
- Deficiencies in Idealization Process
- Live Load Placement
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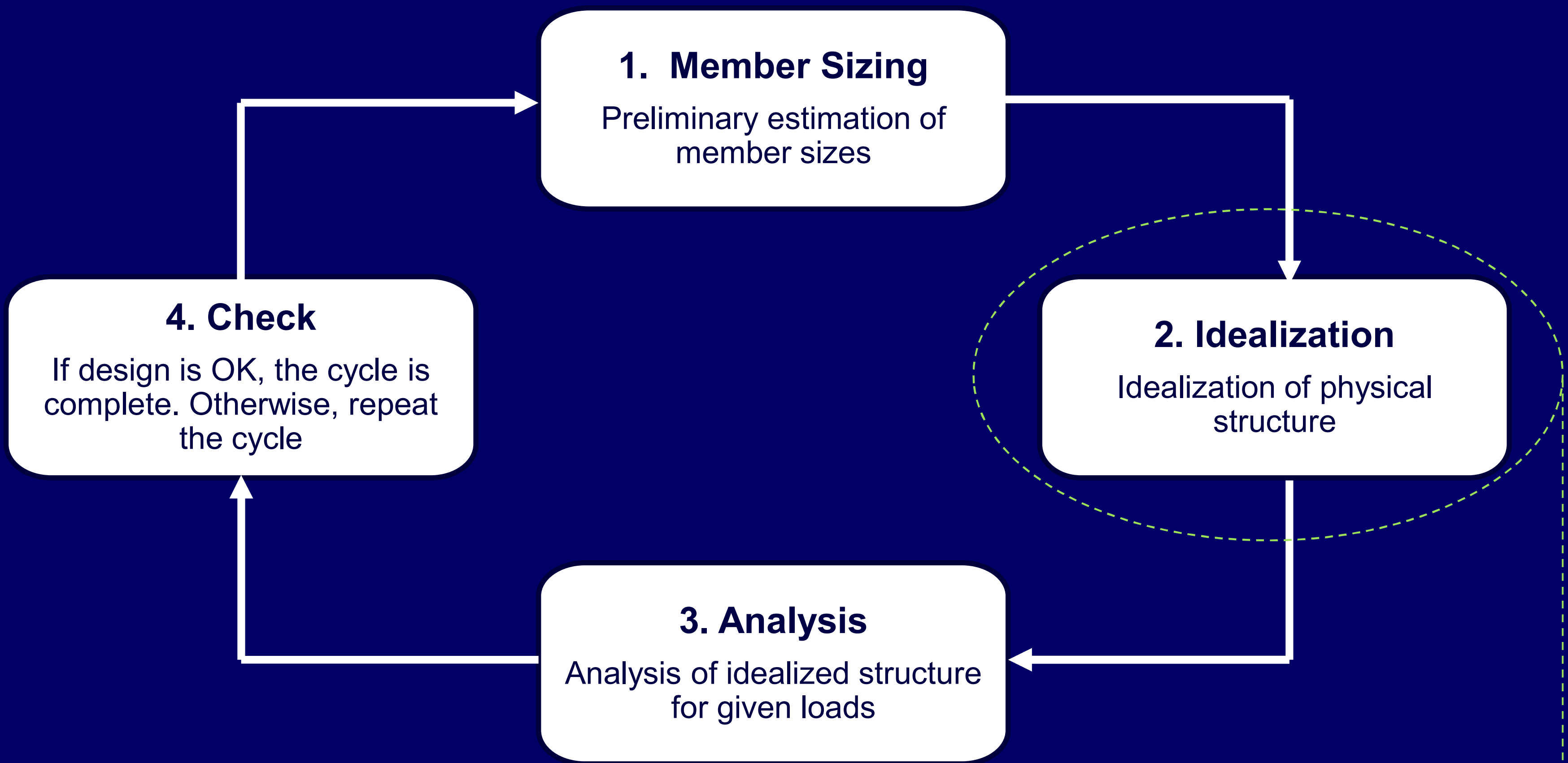
- Closing Note

Design Cycle

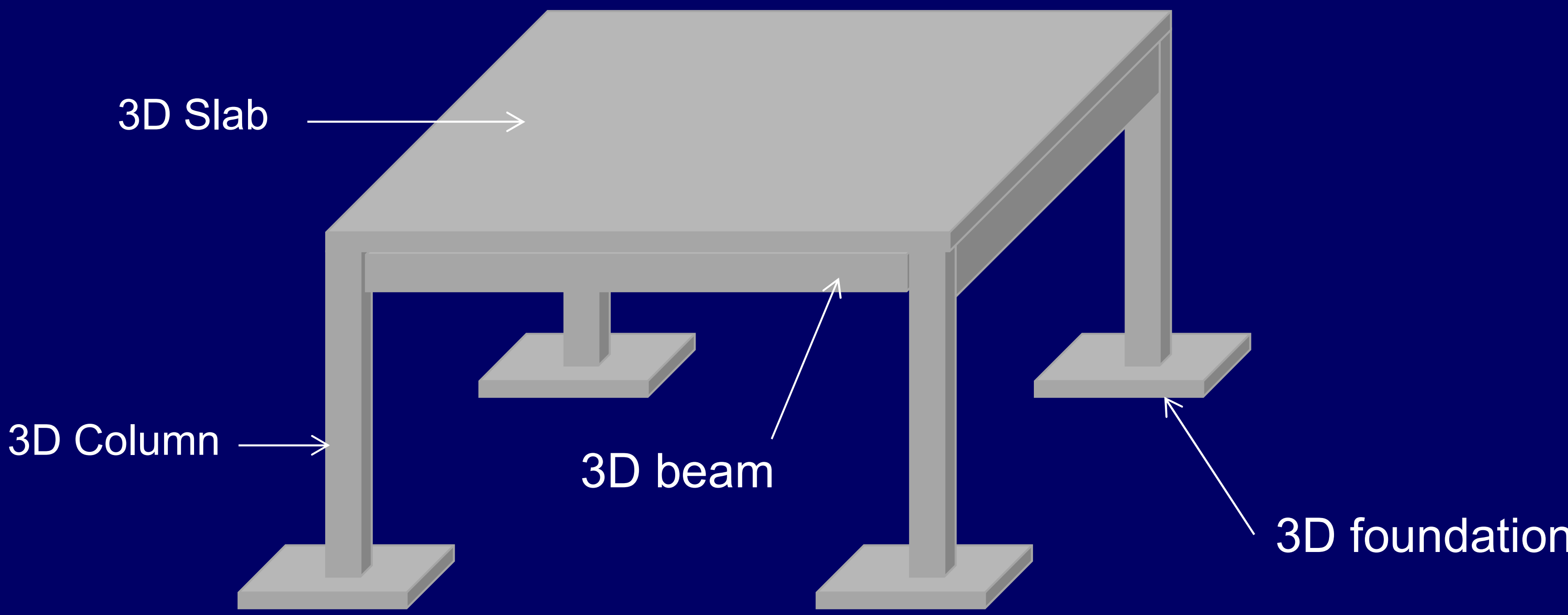


Idealization of Physical Structure will be discussed in this lecture.

Idealization of Physical Structure

❑ Physical Structure

- Every structure in the universe is basically a 3D structure and is composed of 3D members.



Idealization of Physical Structure

❑ Idealized Structure

- For analysis purpose, a physical structure is transformed to an idealized structure.

- While transforming a physical structure to its equivalent idealized form, following characteristics of the idealized structure shall be selected as closely as possible to the actual structure:
  - Span length,
  - Supports,
  - Stiffness,
  - Loads, etc.

Idealization of Physical Structure

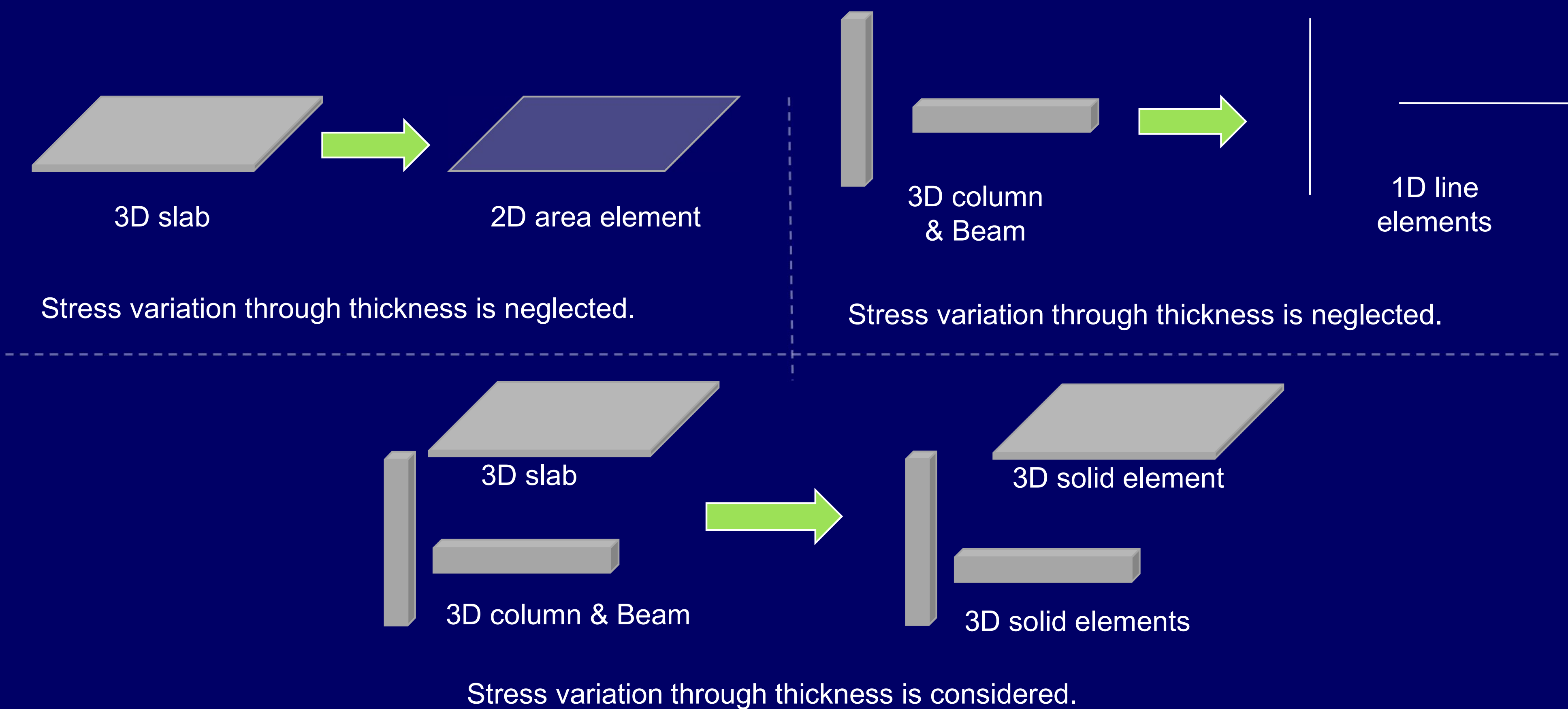
Idealized Structure

- A simple case of idealization is when a beam or column is represented by a line with valid boundary conditions and the relevant geometric properties of the physical members are assigned to the line.
- The idealization of structural members, joints, supports and loads in a RC structure is discussed in the following slides.

Idealization of Physical Structure

❑ Idealization of Structural Members

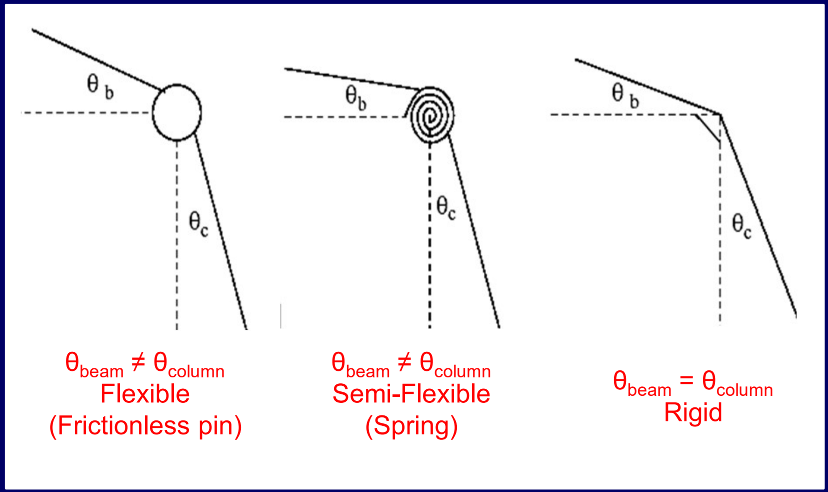
- Following model elements are used for idealization of structural members.



Idealization of Physical Structure

❑ Idealization of Joints

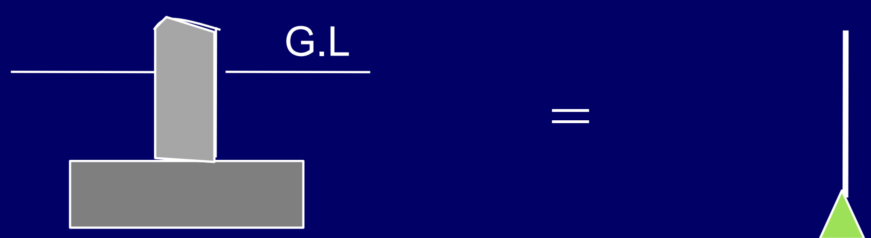
- Joints are those locations where two or more members are connected. Depending on the connectivity of members, several types of joints are possible, as shown in the figure.
- In RC structure, as members are monolithically connected, joints are generally modeled as rigid joints. The design of RC joints is covered under ACI 352.



Idealization of Physical Structure

❑ Idealization of Restraints (Supports)

- A shallow foundation with relatively smaller dimensions can be idealized as a hinge support.



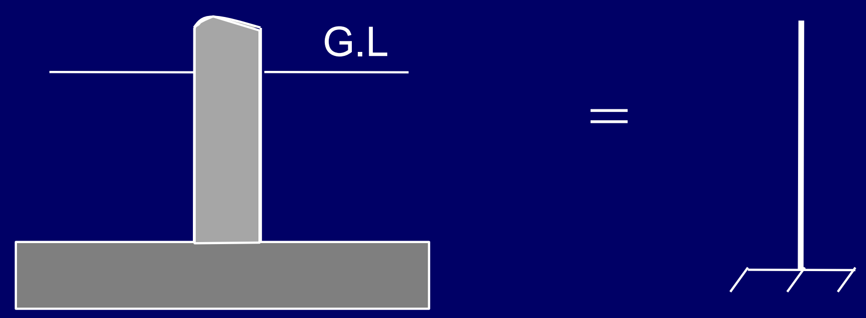
Shallow foundation with relatively smaller dimensions

Hinge

Idealization of Physical Structure

❑ Idealization of Restraints (Supports)

- A relatively deep foundation with relatively larger dimensions (e.g., continuous and raft footing) can be idealized as a fixed support.



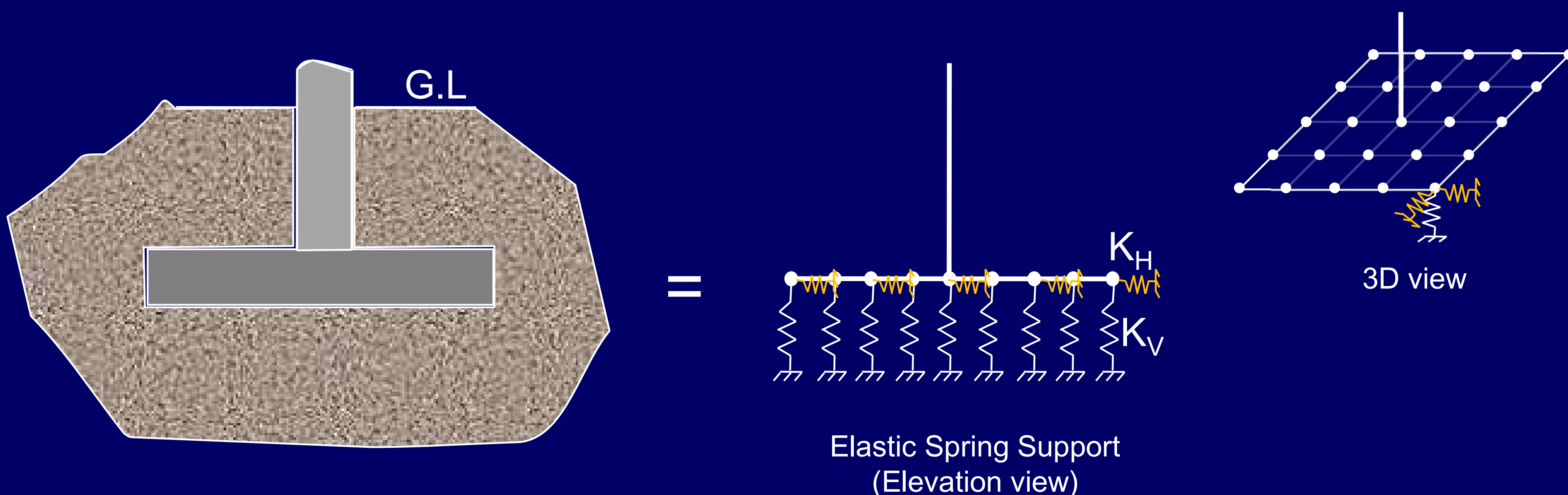
Relatively deep foundation with large dimensions resting on solid stratum

Fixed

Idealization of Physical Structure

❑ Idealization of Restraints (Supports)

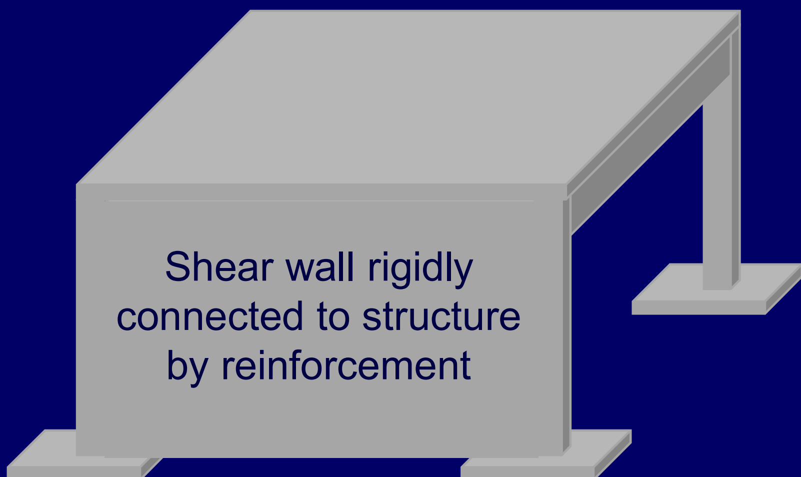
- However, a foundation may be more truly represented by modeling it as an area or solid element with soil modeled as spring elements etc.



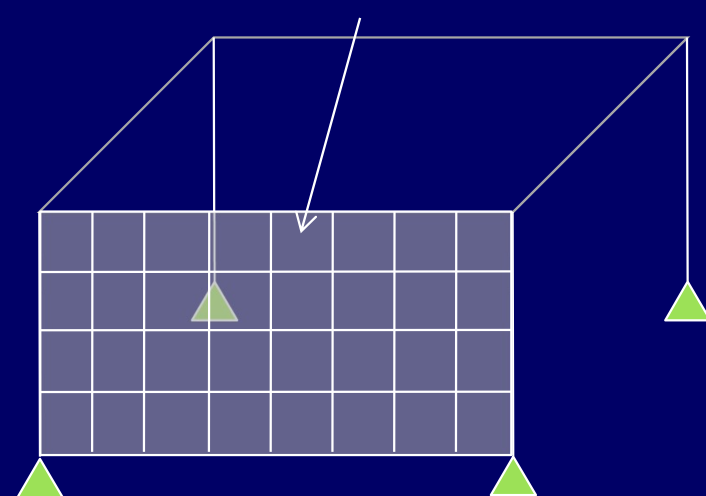
Idealization of Physical Structure

❑ Idealization of Walls

❖ RC Shear Walls



3D structure

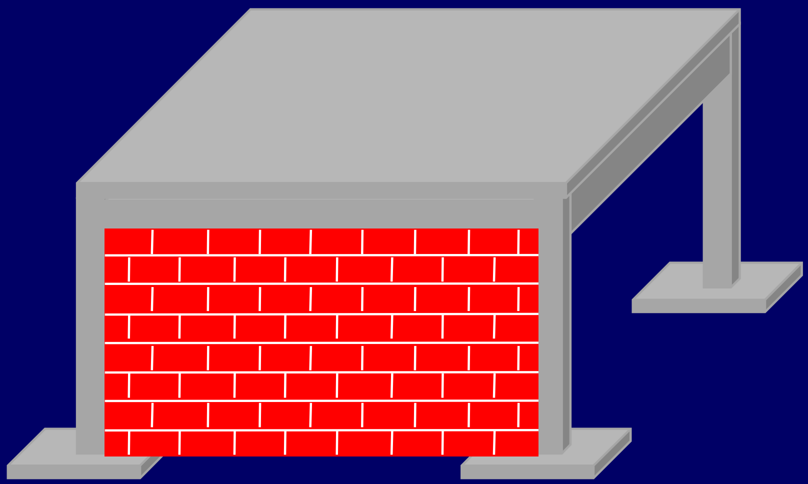


3D Line Model

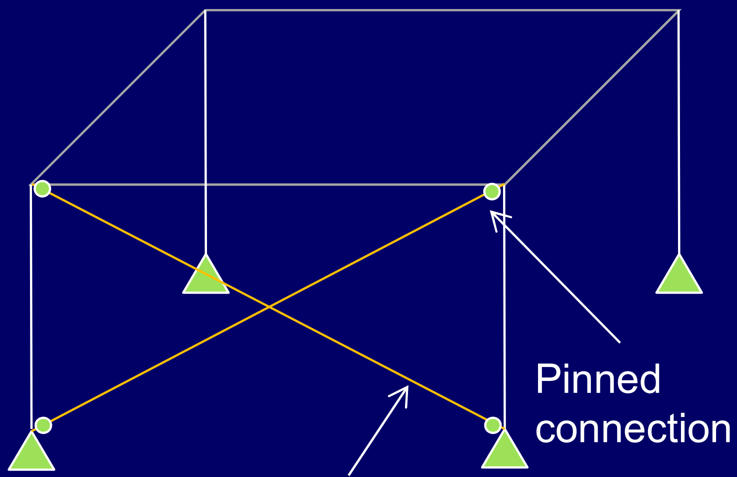
Idealization of Physical Structure

❑ Idealization of Walls

Masonry Walls



3D structure

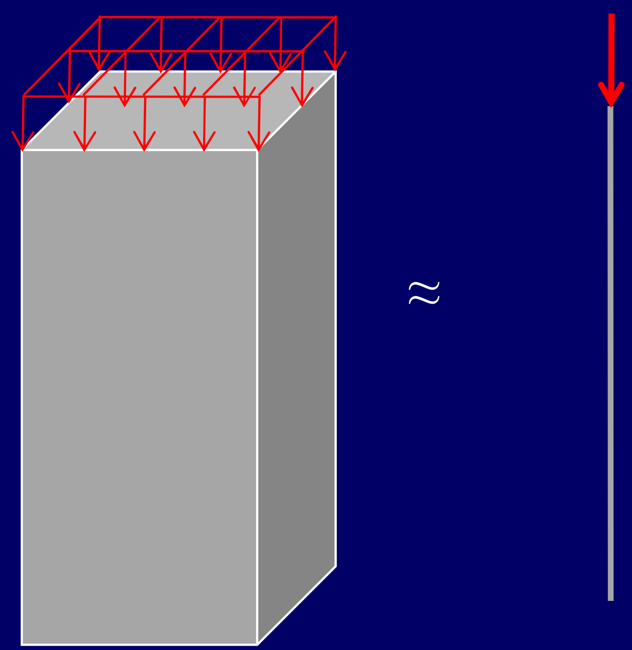


3D Line Model

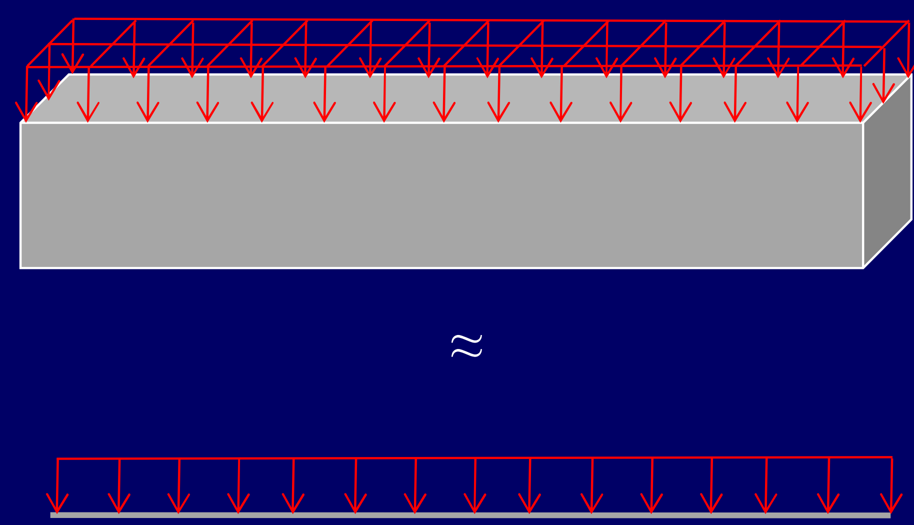
- Masonry wall modeled as equivalent diagonal bracing.
- Brace member is rectangular and has thickness equal to thickness of wall.
- Brace has depth equal to 0.25 times length of diagonal length.
- Brace is connected to structure through pinned connection.
- Other models are also available in the literature.

Idealization of Physical Structure

- Idealization of Loads



Column



Beam

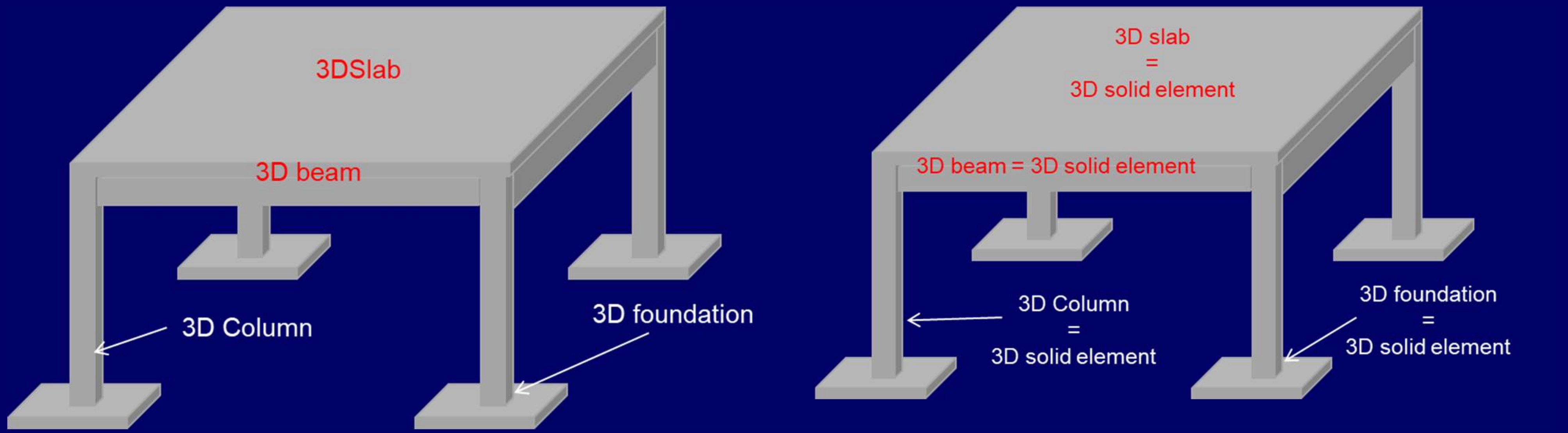
Idealization of Physical Structure

Models for Analysis

- Depending on the nature of problem and available tools-, one-, two- or three-dimensional idealized models of the physical structure can be prepared for the purpose of analysis. Some of the options are listed below.
  - Three-Dimensional analysis using solid elements.
  - Three-Dimensional analysis using line and area elements.
  - Two-Dimensional analysis using line elements.
  - One-Dimensional analysis using line elements.

Models for Analysis

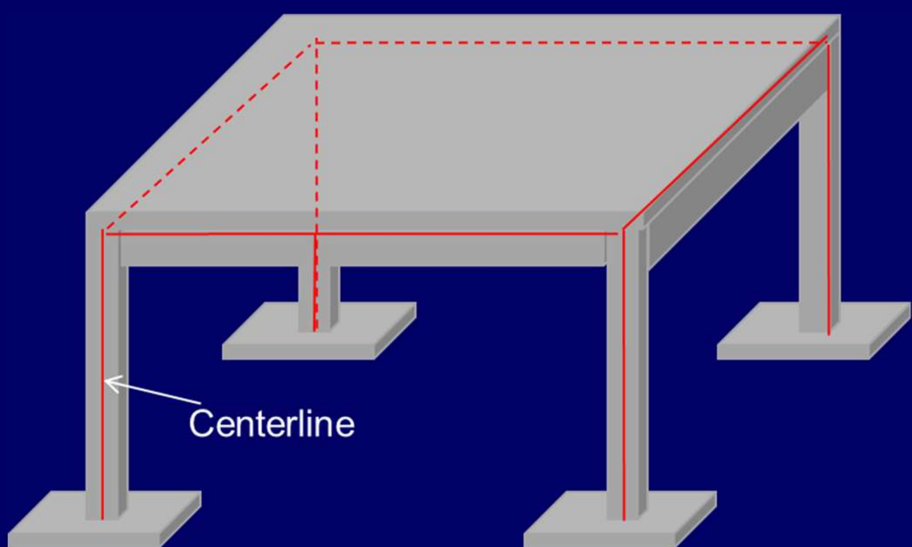
- Three-Dimensional Analysis using Solid Elements

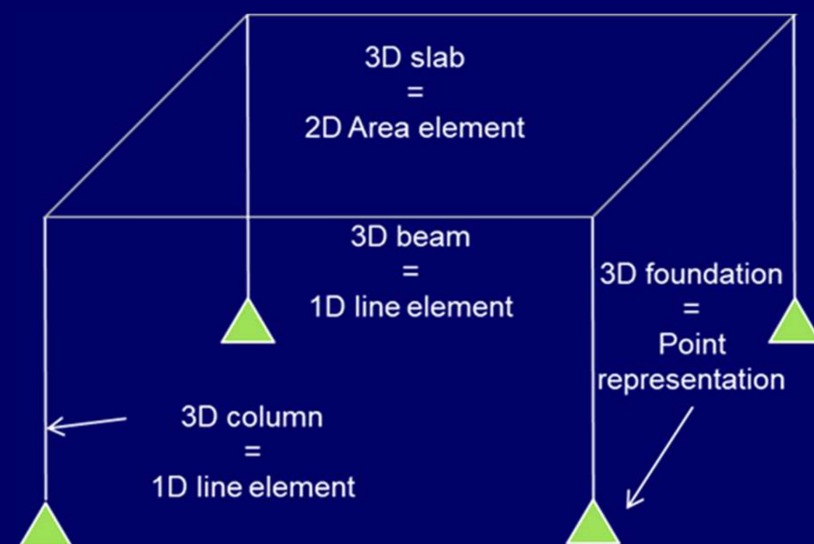


Models for Analysis

❑ Three-Dimensional Analysis using line and area elements

- In the case of 3D analysis using line and area elements, the model is idealized on the centerlines of 3D members





Models for Analysis

Two-Dimensional Analysis using Line Elements

Conversion of 3D model to 2D

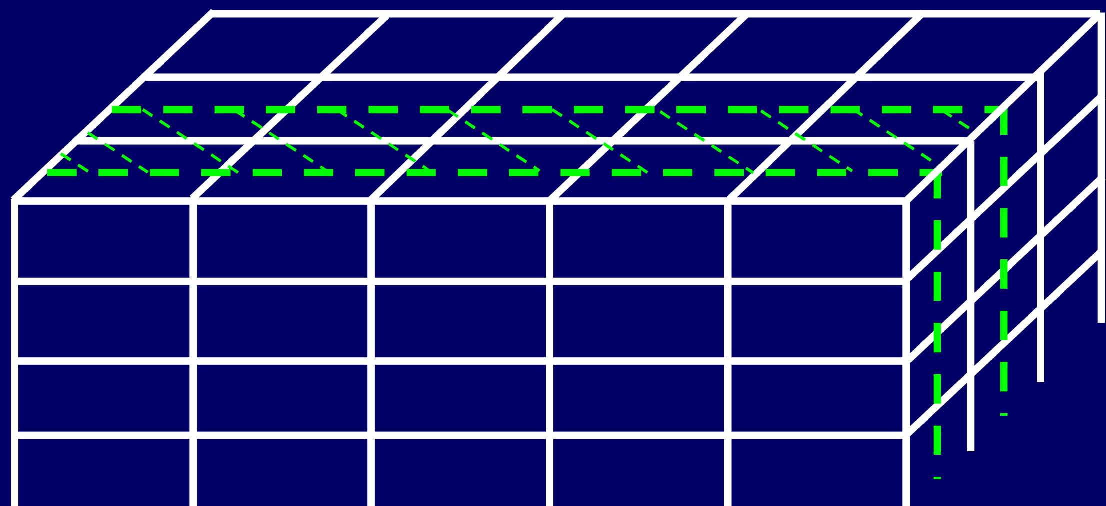
- Due to various constraints (software availability, software efficiency, time consumption, high skills required etc.), one would further like to transform a 3D model to a 2D model.
- Several methods of such transformation are available. Care must be taken not to alter (too much) the properties of 3D structure during transformation process.
- Most common method for this purpose is Equivalent Frame Method.

Models for Analysis

❑ Two-Dimensional Analysis using Line Elements

❖ Equivalent Frame Method

- In Equivalent frame method, an equivalent 2D system of a 3D structure is extracted by considering stiffnesses of relevant members e.g., slab and beams.



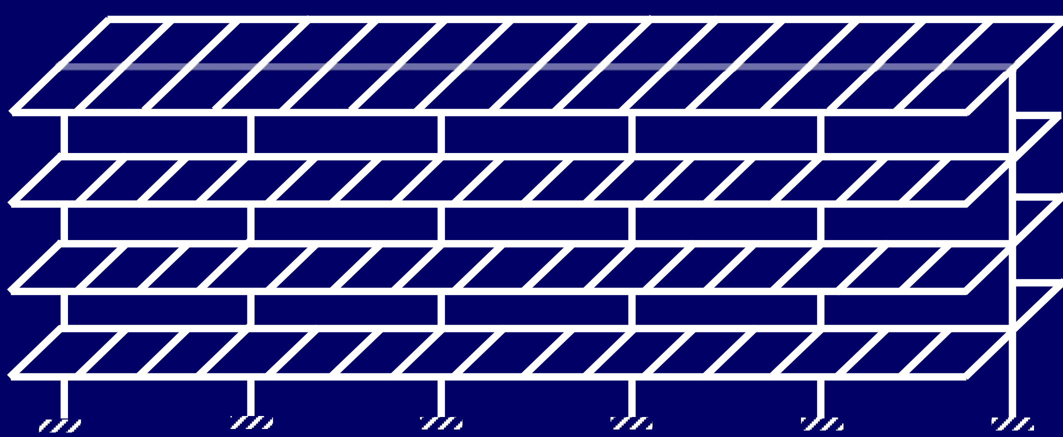
1. 3D line model

Models for Analysis

❑ Two-Dimensional Analysis using Line Elements

- **Equivalent Frame Method**

- Extraction of 3D frame from structure



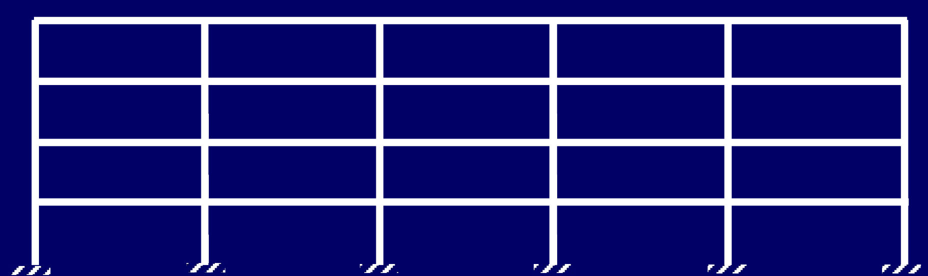
2. 3D frame extracted from structure

Models for Analysis

Two-Dimensional Analysis using Line Elements

Equivalent Frame Method

- Once the 2D system is extracted, any method can be used for analysis.

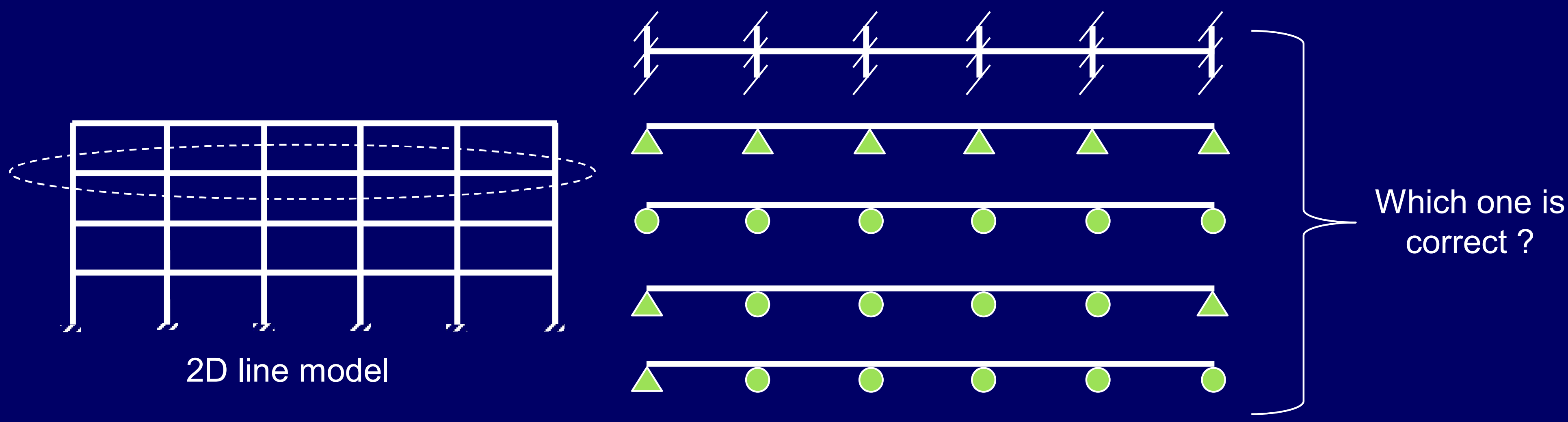


3. Converted 2D line model

Models for Analysis

❑ Two-Dimensional Analysis using Line Elements

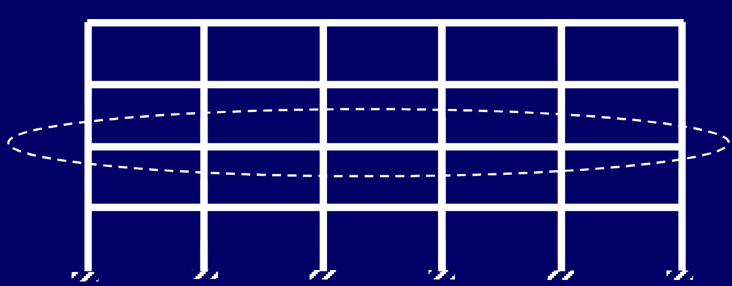
- 2D structures can be further idealized as simple 1D line models.
- End conditions must be selected with good structural engineering judgment for correct structural representation.



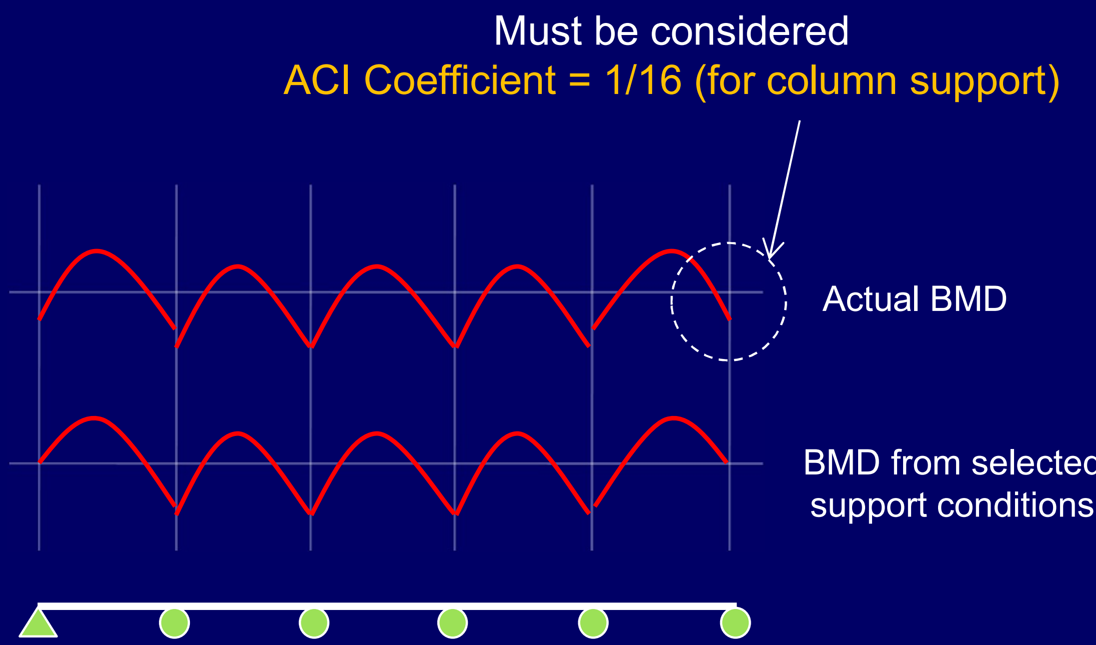
Models for Analysis

❑ Two-Dimensional Analysis using Line Elements

- 2D structures can be further idealized as simple 1D line models.
- End conditions must be selected with good structural engineering judgment for correct structural representation.



2D frame



Models for Analysis

❑ Deficiencies in Idealization Process

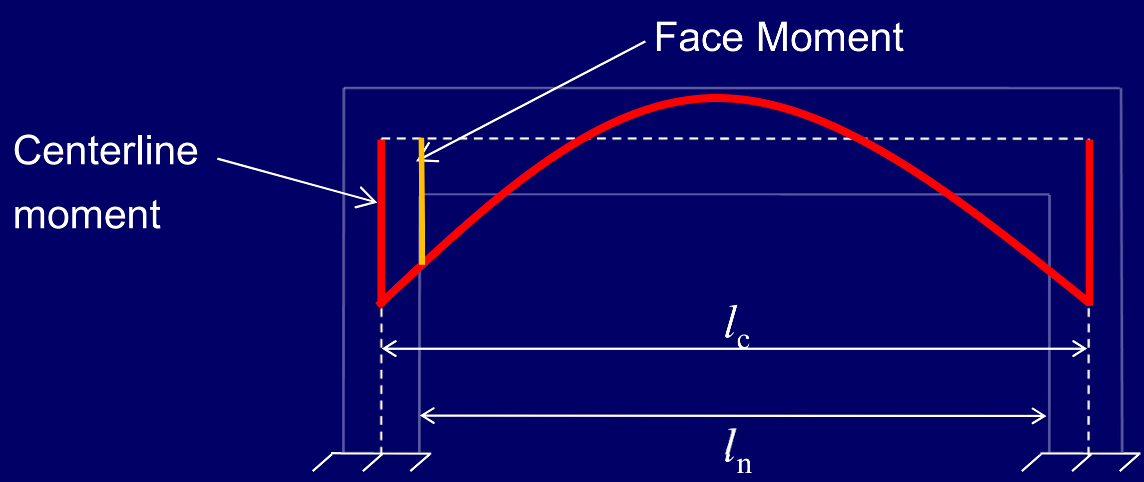
- As idealized structure is an approximate representation of physical structure, multiple problems are encountered during the process resulting into creating a number of deficiencies in the idealized model.
- Only some of these deficiencies are discussed in the next few slides.

Deficiencies in Idealization Process

❑ Deficiencies in due to Line Modeling

❖ Use of Centerline Length Instead of Clear Length

- The use of center line length instead of actual length of members would result in larger moments and hence unnecessarily large section.



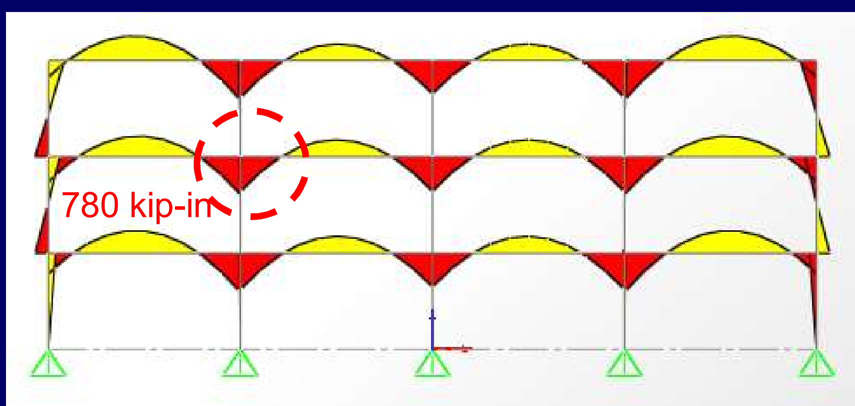
Center line moment for beam is larger than face moment.

Deficiencies in Idealization Process

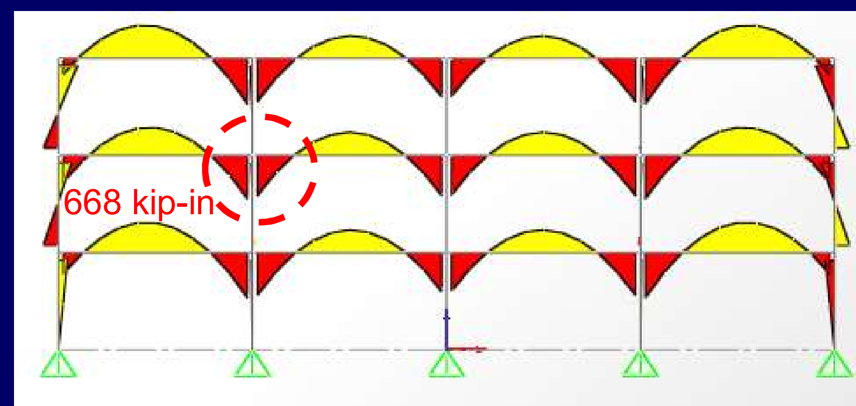
❑ Deficiencies in due to Line Modeling

❖ Use of Centerline Length Instead of Clear Length

- Centerline and face (offset) moments in a beam for a given case are compared in the following figures.



Centerline Moment:
Beam: 12″ × 18″; Column size: 12″ × 12″



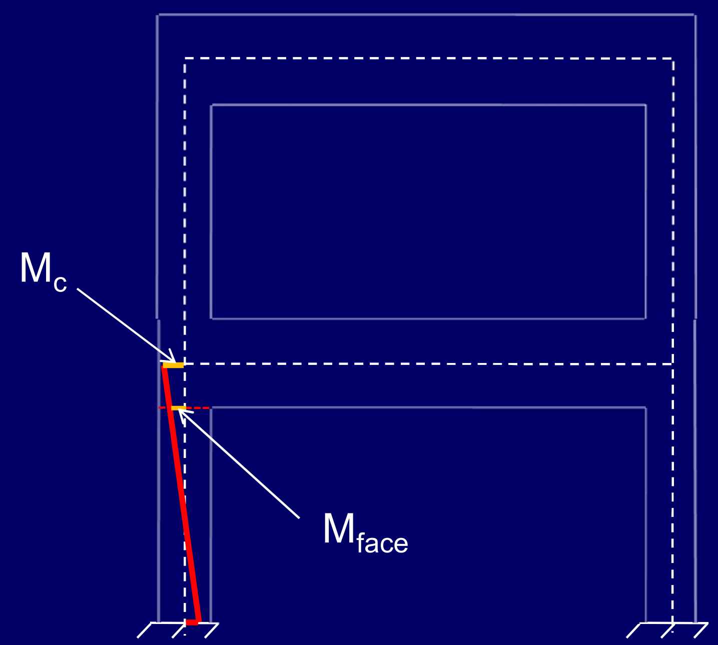
Offset Moment:
Beam: 12″ × 18″; Column size: 12″ × 12″

Deficiencies in Idealization Process

❑ Deficiencies in due to Line Modeling

❖ Use of Centerline Length Instead of Clear Length

- In the case of columns, as moment gradient is not very steep, the difference between M_{face} and M_c is small.



Deficiencies in Idealization Process

Deficiencies in due to Line Modeling

Use of Centerline Length Instead of Clear Length

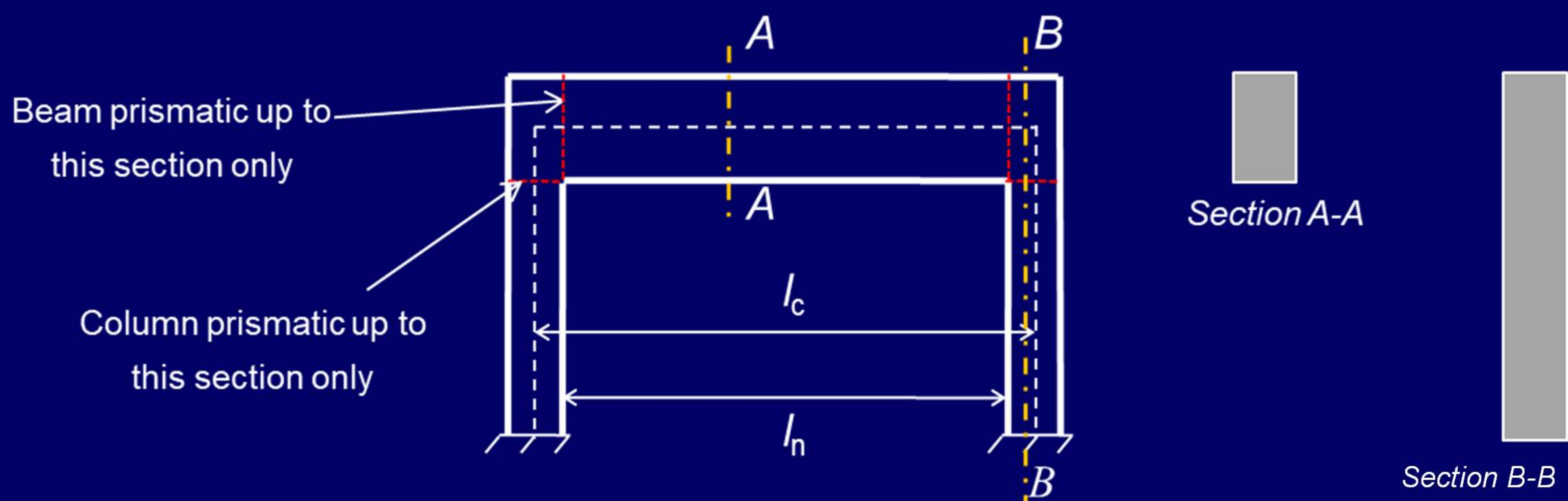
- For analysis of frames, ACI permits the use of center-to-center lengths.
- For design, ACI 9.4.2.1 permits the use of bending moment at face of the support.

Deficiencies in Idealization Process

Deficiencies in due to Line Modeling

Use of Prismatic Instead of Non-prismatic Members

- The usual assumption in frame analysis is that the members are prismatic, with constant moment of inertia between centerlines. This is not strictly correct as shown below.

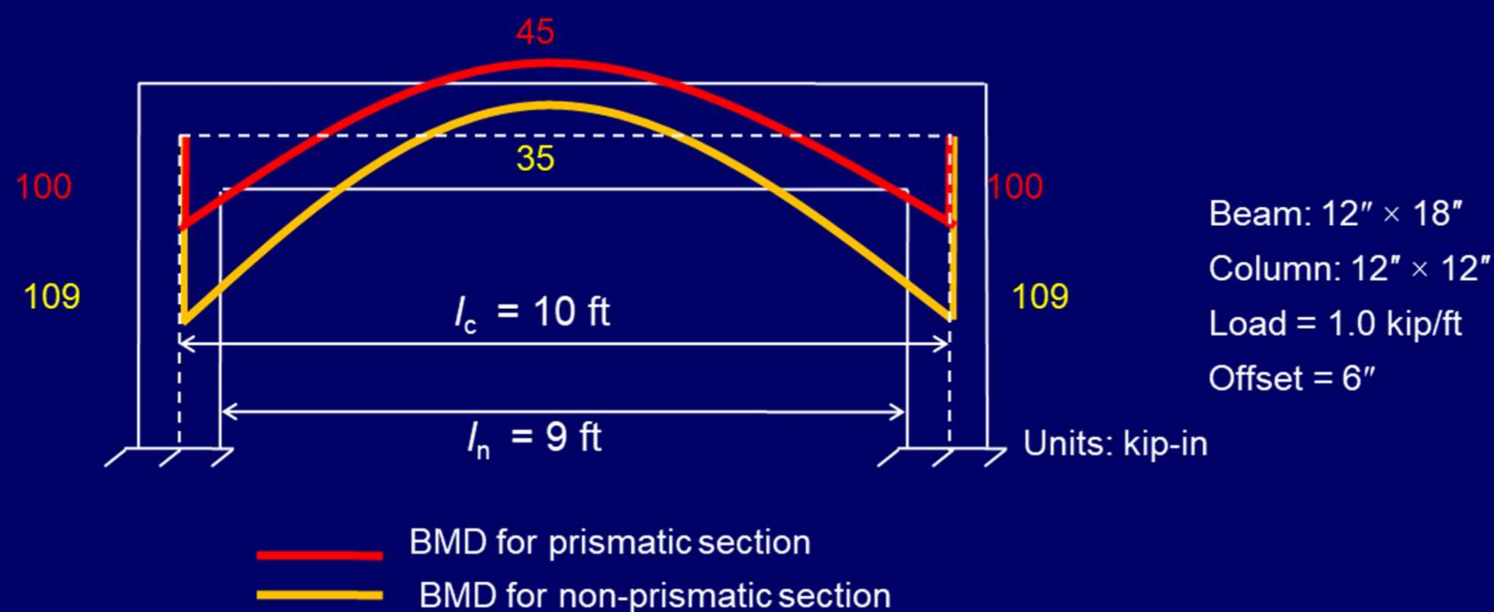


Deficiencies in Idealization Process

❑ Deficiencies in due to Line Modeling

❖ Use of Prismatic Instead of Non-prismatic Members

- As sectional dimensions of the members are very small as compared to their length, the variation in bending moment resulting from such deficient modeling will normally be not significant.



Deficiencies in Idealization Process

❑ Effects of Cracking on Moment of Inertia of Members

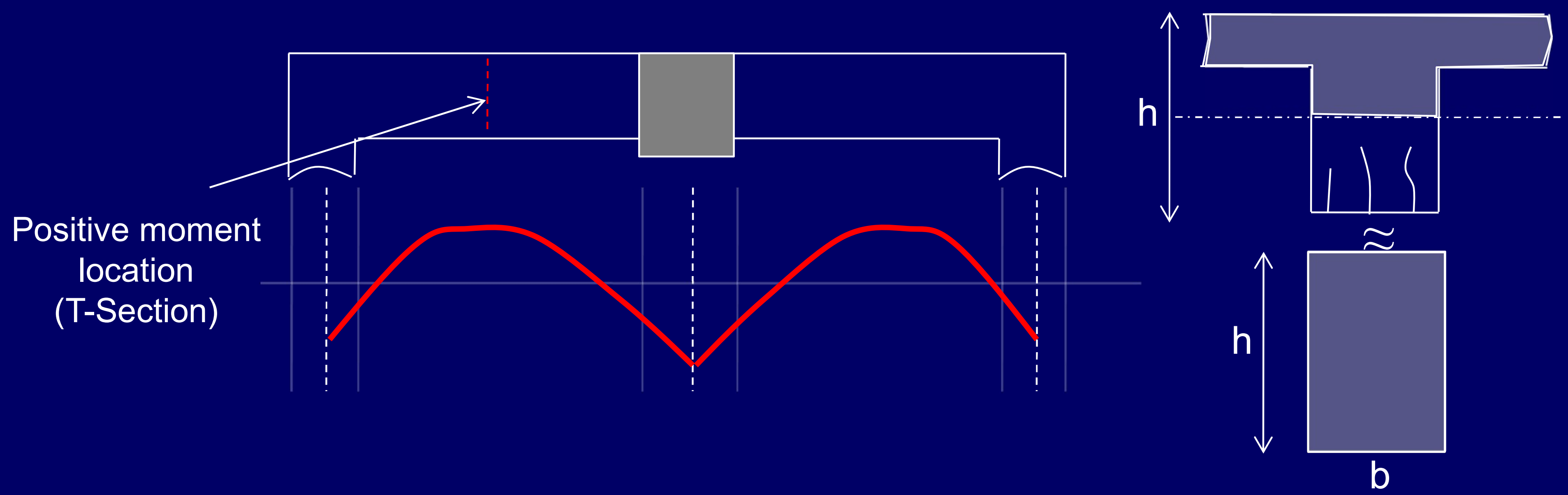
- RC members in a physical structure crack well below the ultimate load. Therefore, using full moment of inertia of the member in the idealized model is not justified.
- Though the use of full moment of inertia of members will not affect the total value of bending (positive plus negative), the distribution between the members may change considerably.
- It is not the absolute moment of inertia that matters, rather it is the relative stiffness of members that must be considered.

Deficiencies in Idealization Process

❑ Effects of Cracking on Moment of Inertia of Members

❖ Beam Section

- At positive moment locations where T-section is effective, the cracked T-section moment of inertia is almost equal to a rectangular section with the same dimensions.

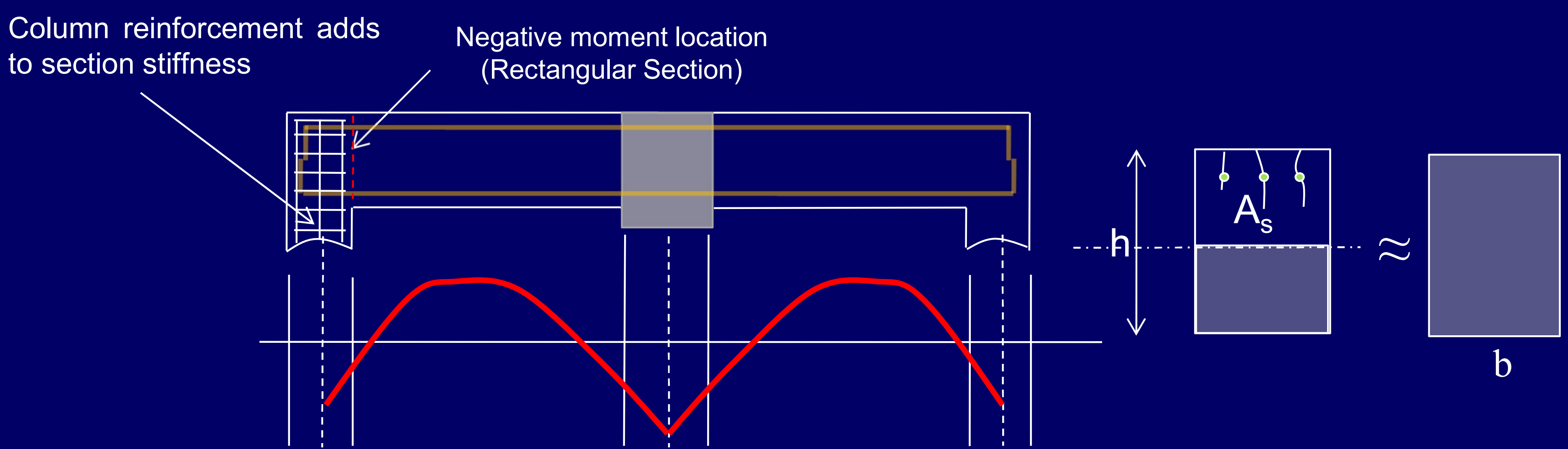


Deficiencies in Idealization Process

❑ Effects of Cracking on Moment of Inertia of Members

Beam Section

- At negative moment locations where rectangular section is effective, the cracked rectangular section moment of inertia is almost equal to rectangular section with same dimensions due to additional stiffness provided by beam and column reinforcements.



Deficiencies in Idealization Process

❑ Effects of Cracking on Moment of Inertia of Members

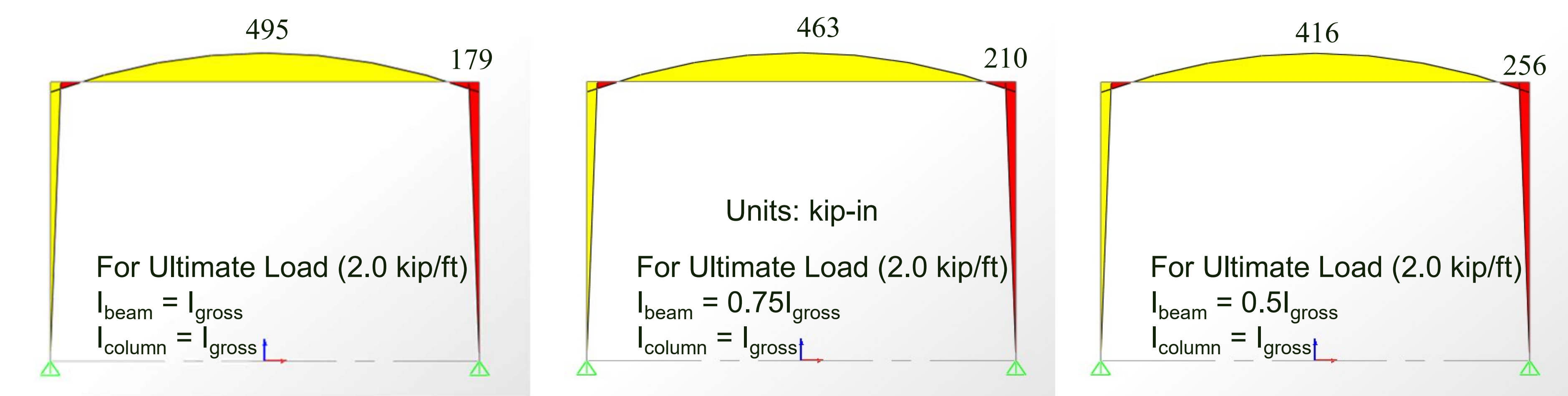
❖ Column Section

- The effect of cracks on variation of moment of inertia of columns is relatively less due to axial load on column.

Deficiencies in Idealization Process

❑ Effects of Cracking on Moment of Inertia of Members

- The figure shows distribution of bending moments in beam and column at different *I*.



Beam: 12″ × 18″; Column: 12″ × 12″; DL = 1 kip/ft; LL = 0.5 kip/ft

Deficiencies in Idealization Process

Effects of Cracking on Moment of Inertia of Members

ACI Code on Stiffness

- ACI 6.3.1.1 states that use of any set of reasonable assumptions shall be permitted for computing relative flexural and torsional stiffnesses of columns, walls, floors, and roof systems. The assumptions adopted shall be consistent throughout analysis.

- ACI R6.3.1.1 states that relative values of stiffness are important. Common assumptions are:
  - Gross EI values for all members.
  - Half the gross EI of the beam stem for beams and the gross EI for the columns.

Deficiencies in Idealization Process

❑ Effects of Cracking on Moment of Inertia of Members

❖ ACI Code on Stiffness

- Additional guidance is given in ACI Code 6.6.3.1.1, which specifies the section properties to be used for frames subject to side-sway.

Table 6.6.3.1.1(a)—Moments of inertia and cross-sectional areas permitted for elastic analysis at factored load level

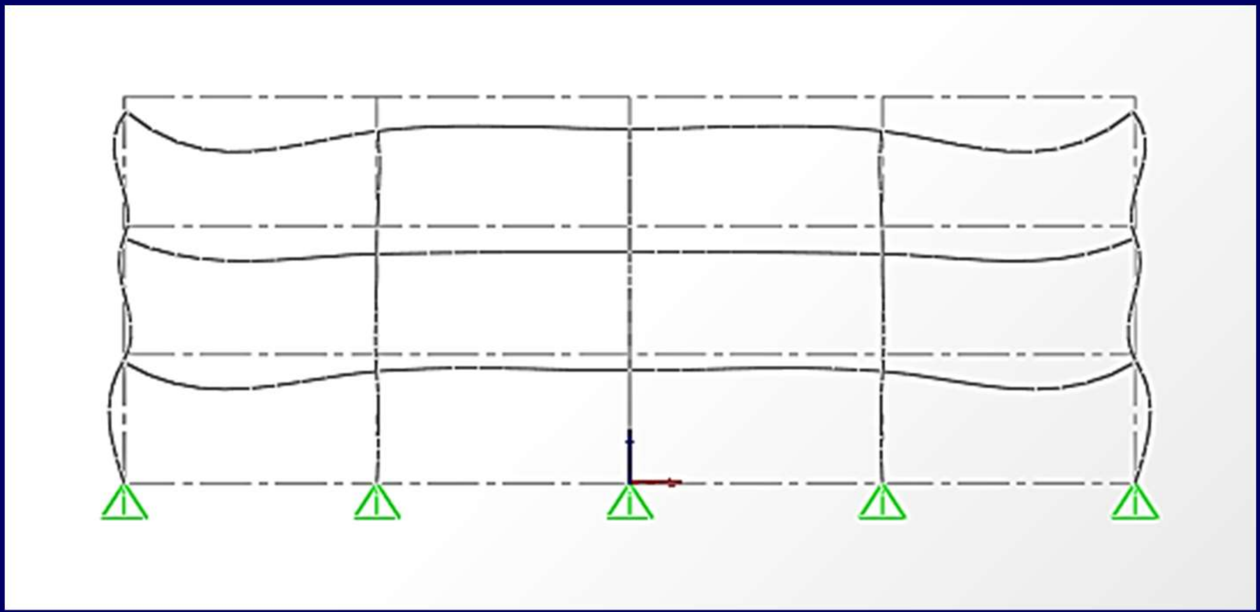
Member and condition		Moment of inertia	Cross-sectional area for axial deformations	Cross-sectional area for shear deformations
Columns		$0.70I_g$	$1.0A_g$	b_wh
Walls	Uncracked	$0.70I_g$		
	Cracked	$0.35I_g$		
Beams		$0.35I_g$		
Flat plates and flat slabs		$0.25I_g$		

Note: *Effective moment of inertia shall be used for calculation of deflection at a particular load stage.*

Placement of Live Load

❑ Placement of Loads

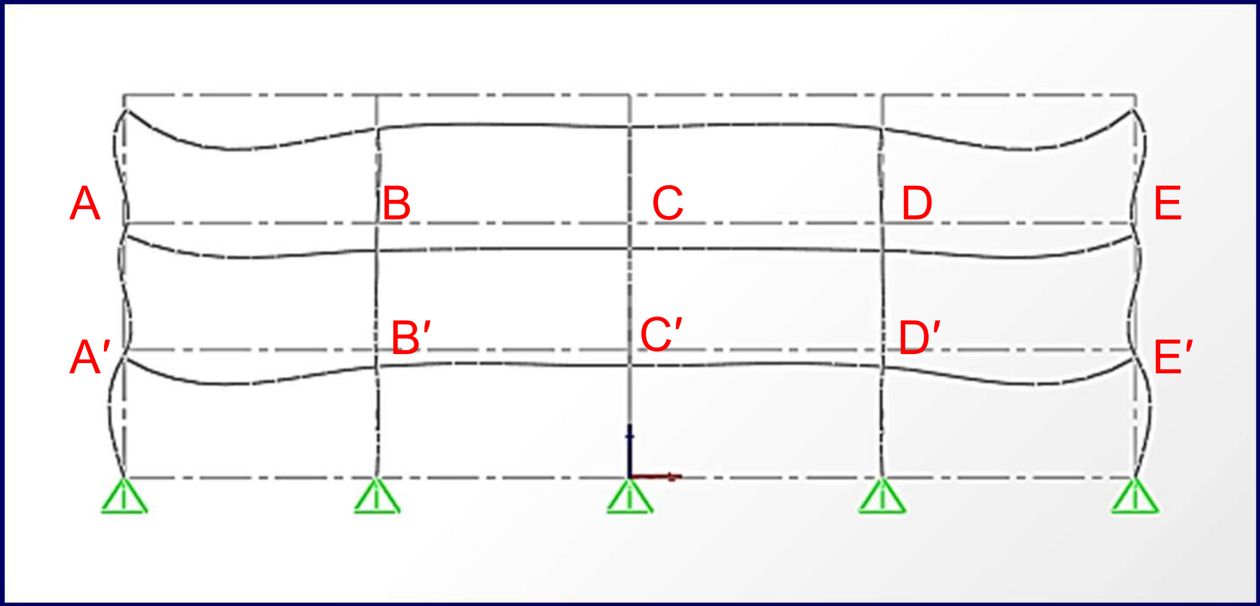
- Consider a 2D frame with rigid joints.
- The frame subjected to dead load only will have deflected shape as shown below.



Placement of Live Load

❑ Placement of Loads

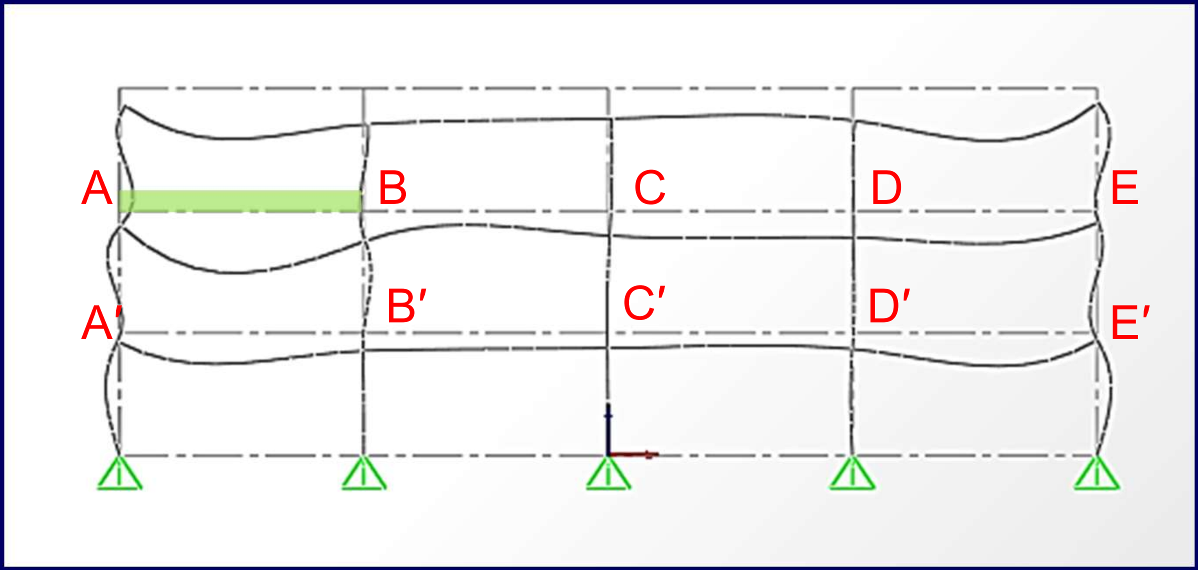
- Live load may or may not occupy the complete floor of the structure at the same time. It is possible that only span AB or any other span is fully loaded at a time.
- Several arrangements of live load are possible.



Placement of Live Load

❑ Placement of Loads

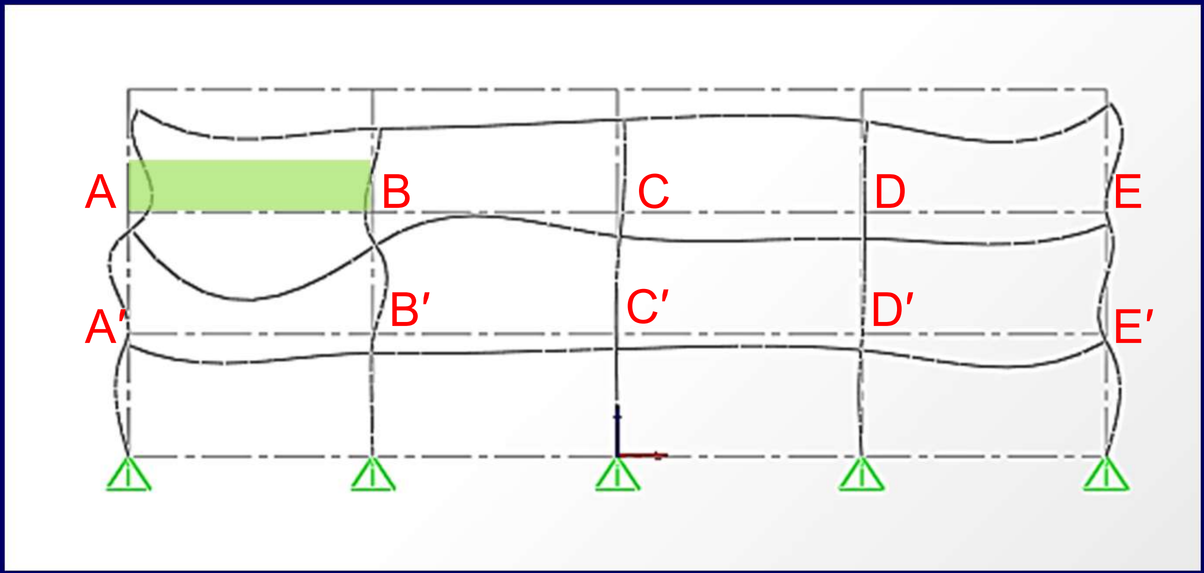
- Consider the frame below. Due to application of uniform live load on span AB, the deflection of beam and adjacent columns will increase.



Placement of Live Load

❑ Placement of Loads

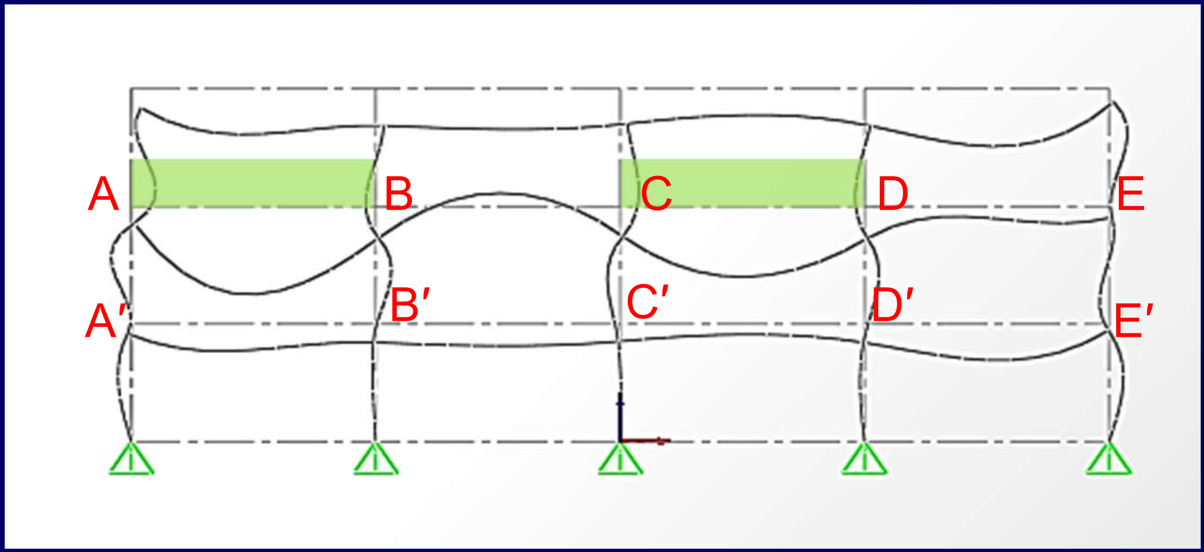
- On further increase in live load, the span BC curvature can reverse, bringing tension in upper fibers.
- Similarly, the column AA′ converts from double to single curvature.



Placement of Live Load

❑ Placement of Loads

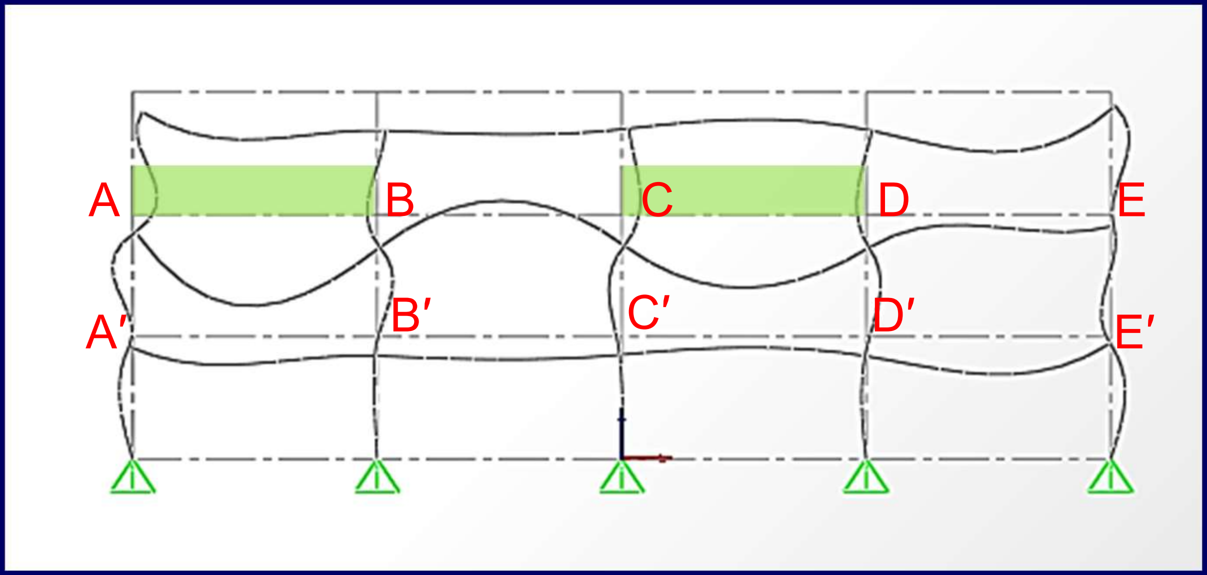
- If span CD is loaded as well with live load, the deflections in span AB will further increase. Also span BC will reach reverse curvature earlier.



Placement of Live Load

❑ Placement of Loads

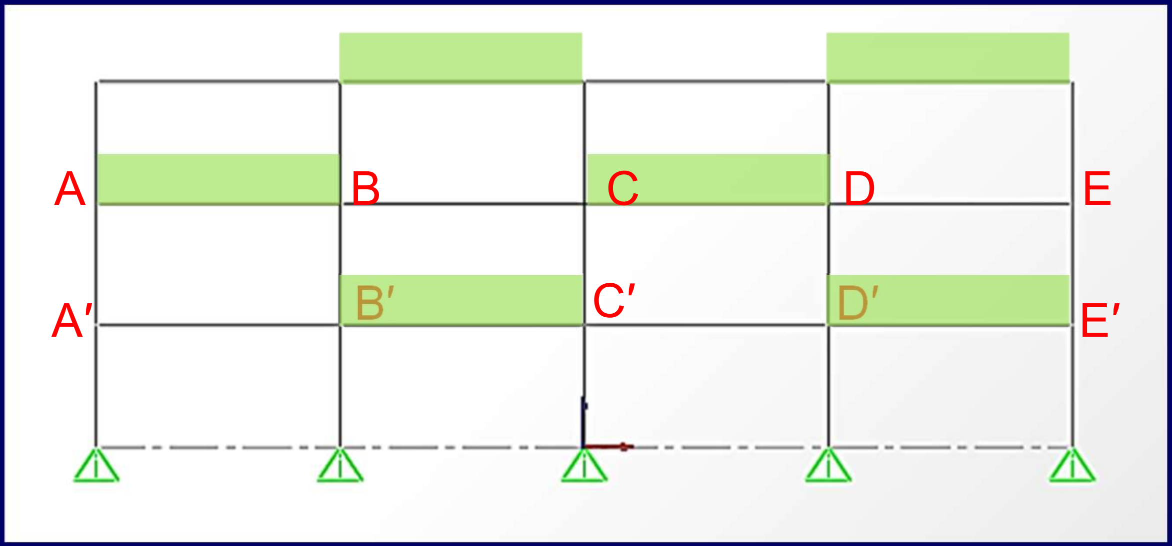
- So, it is not the full live load on floor ABCD that causes more bending in span AB, rather it is the arrangement or pattern shown below that causes more bending in span AB.



Placement of Live Load

❑ Pattern for Maximum Mid Span Moment

- With pattern 1, max. positive moments in spans AB and CD are obtained. Maximum mid span negative moment in BC is also obtained.

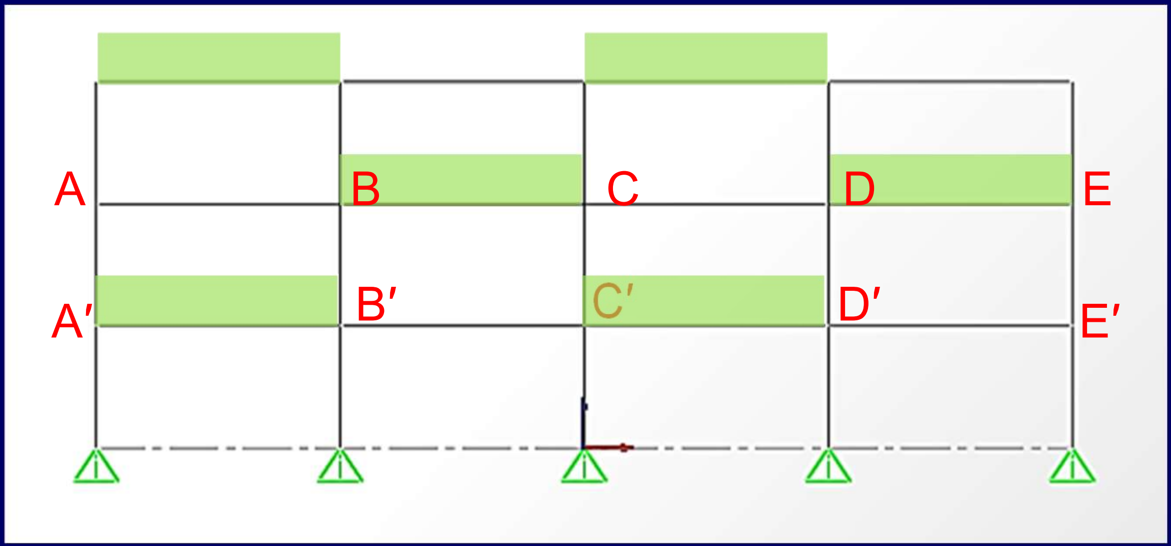


Pattern 1

Placement of Live Load

❑ Pattern for Maximum Mid Span Moment

- With pattern 2, maximum positive moment in spans BC is obtained.
- Maximum mid span negative moment in AB and CD is also obtained.

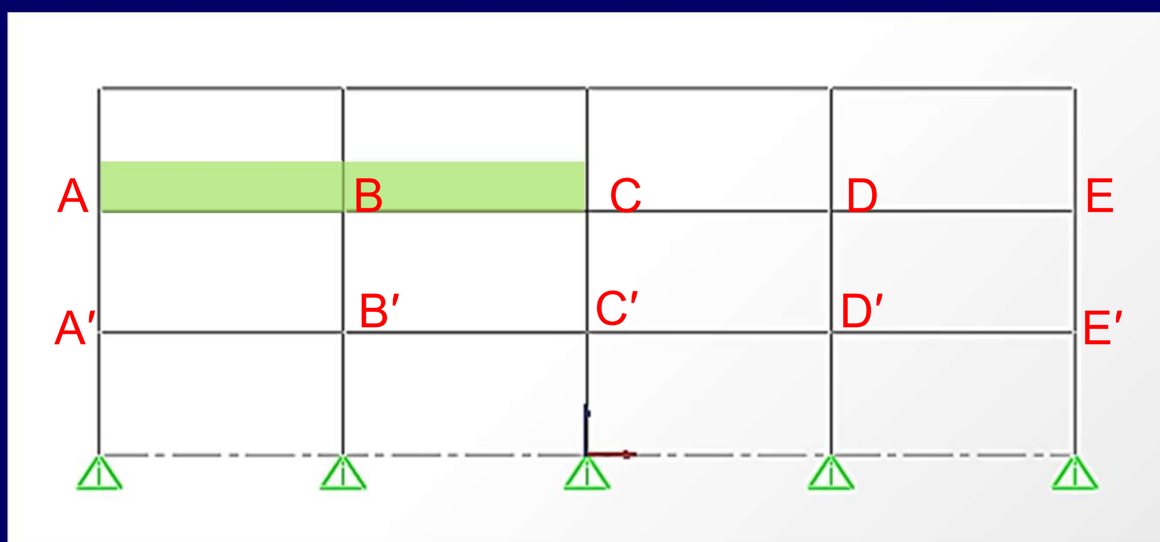


Pattern 2

Placement of Live Load

❑ Pattern for Maximum Support Moment

- Maximum support moment (at support B for example) is obtained for the given frame from pattern 3.

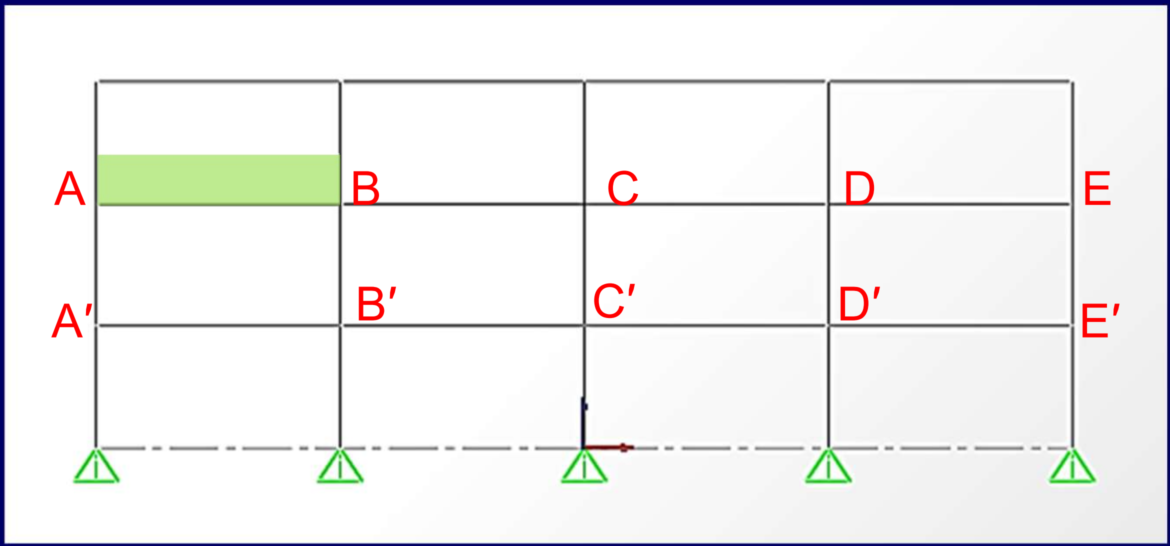


Pattern 3

Placement of Live Load

❑ Pattern for Maximum Column Moment

- Maximum column moments (for column AA′ for example) is obtained for the given frame from pattern 4.

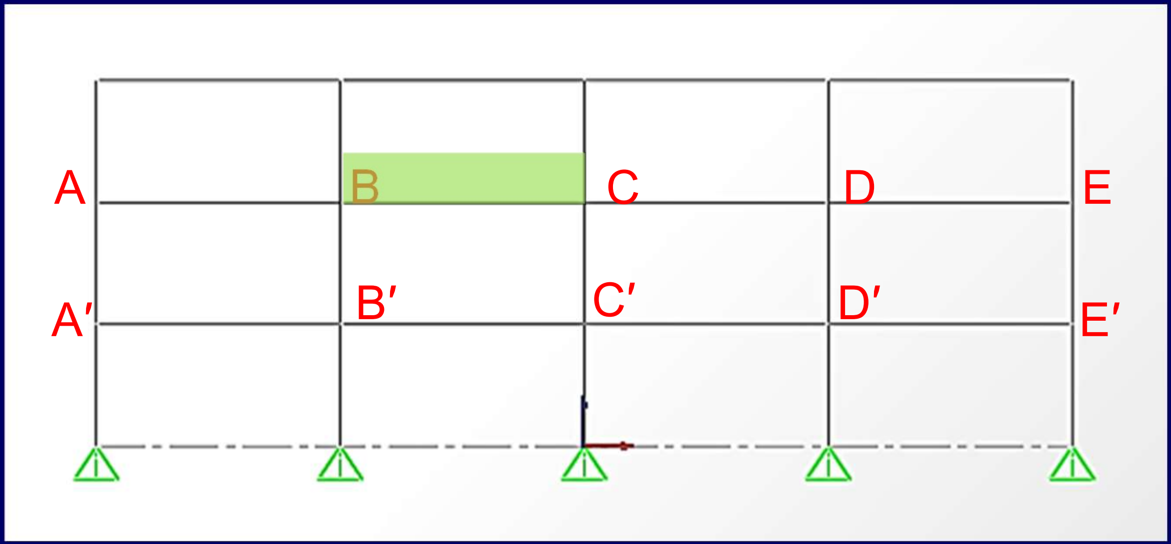


Pattern 4

Placement of Live Load

❑ Pattern for Maximum Column Moment

- Maximum column moments (for column CC′ for example) is obtained for the given frame from pattern 5.

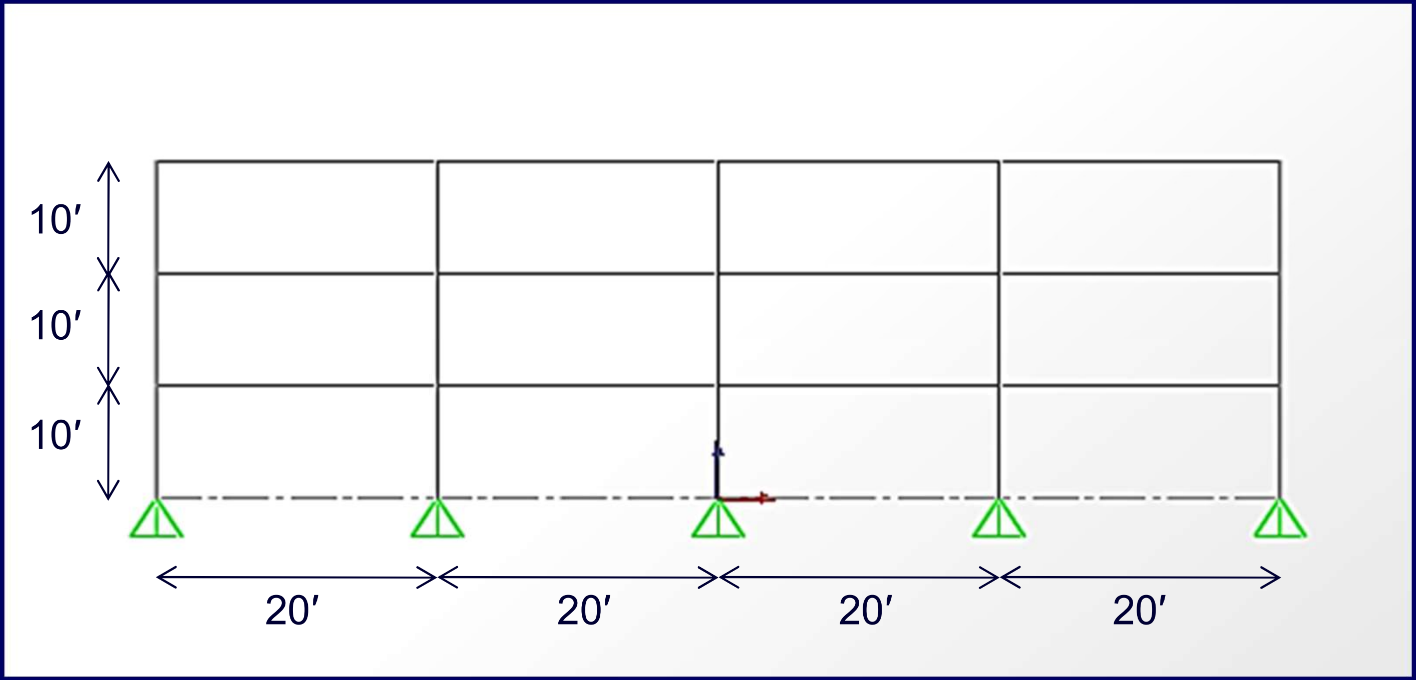


Pattern 5

Case Study

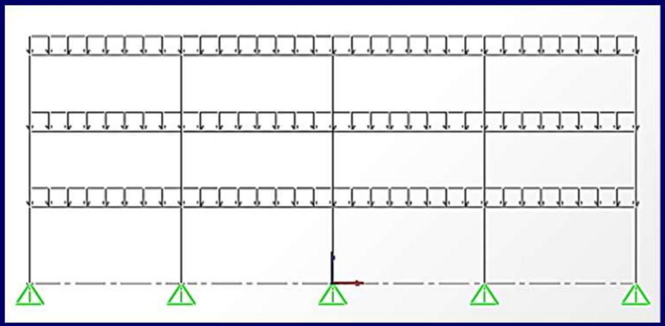
❑ Effect of Pattern Load on Bending Moments

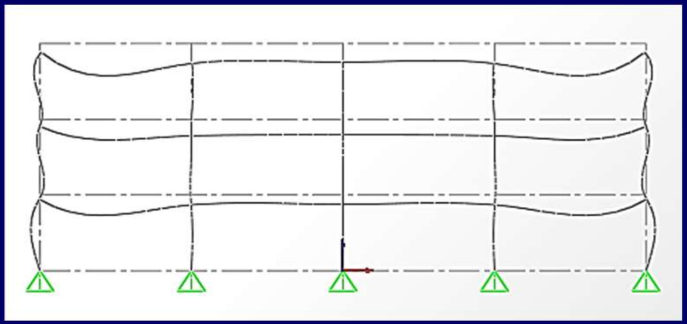
- Factored DL = 1.8 kip/ft (for 6" thick slab and 40 psf SDL)
- Factored LL = 1.152 kip/ft (for 60 psf LL on 12′ width)

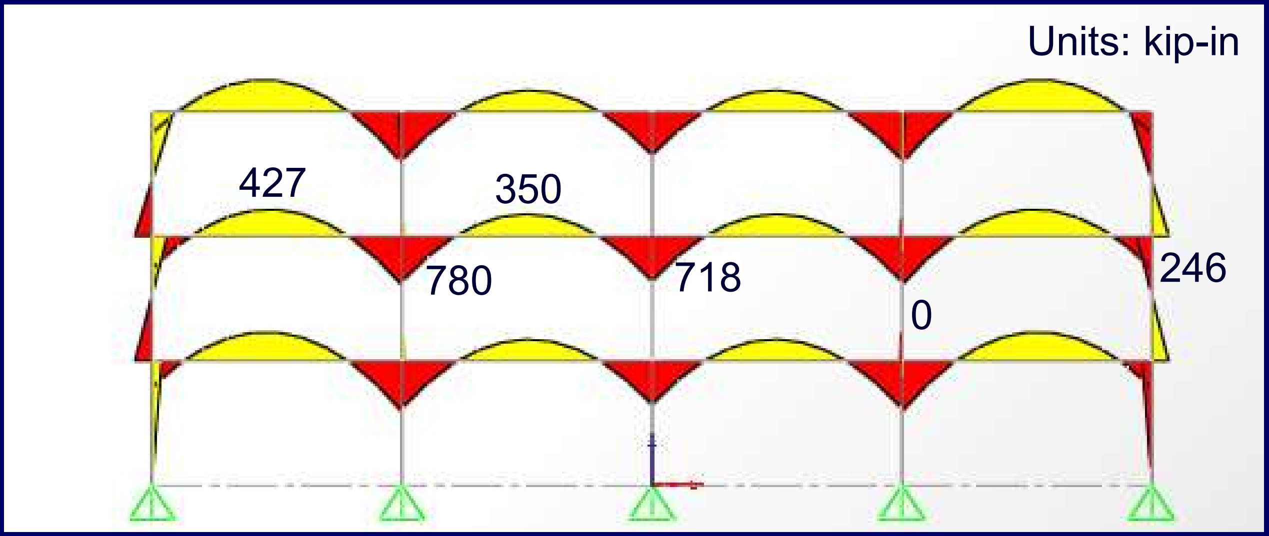


Case Study

❑ Effect of Pattern Load on Bending Moments

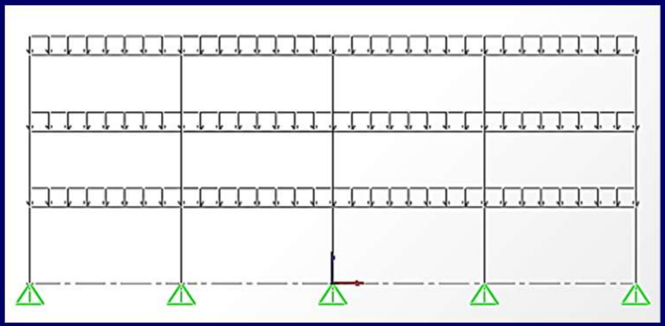

Load: Full dead only (1.80 kip/ft)


Deflected shape

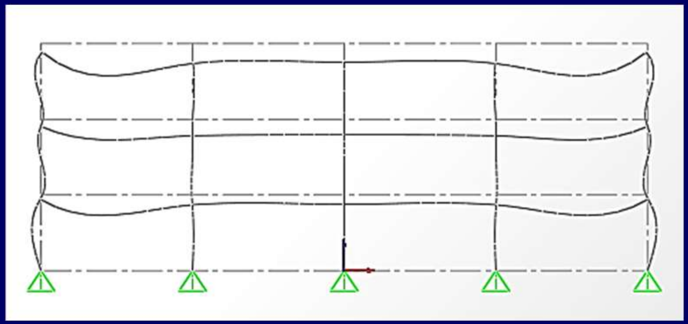


Case Study

❑ Effect of Pattern Load on Bending Moments



Load: Full dead only (1.80 kip/ft)



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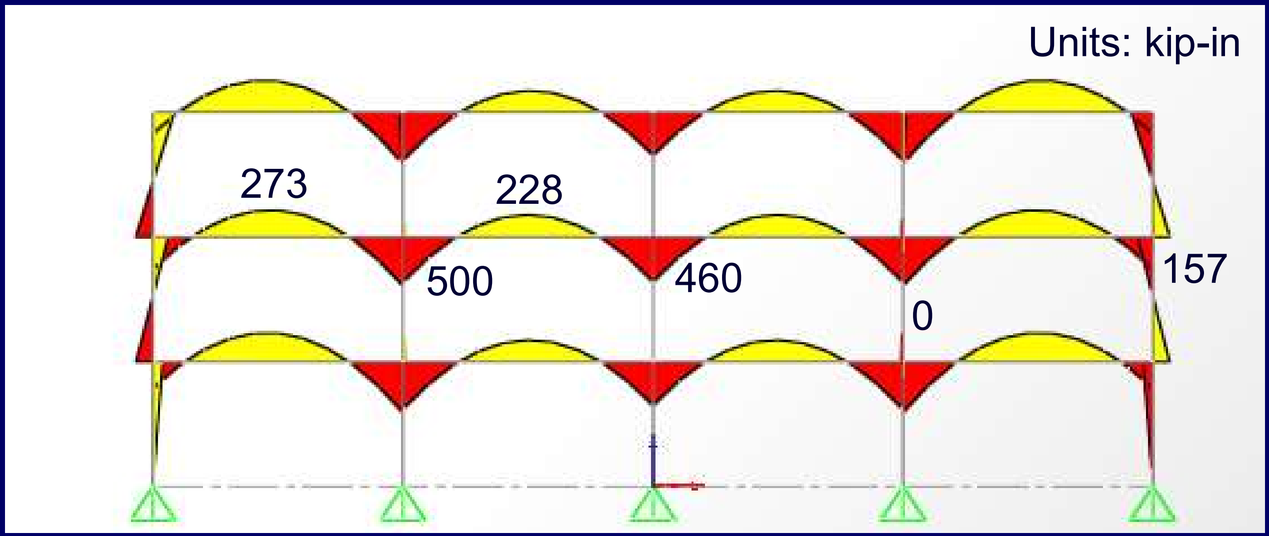
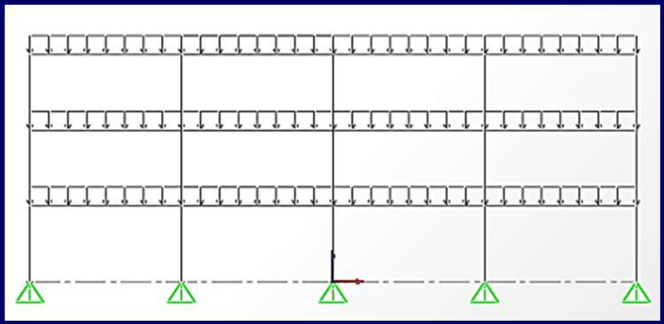
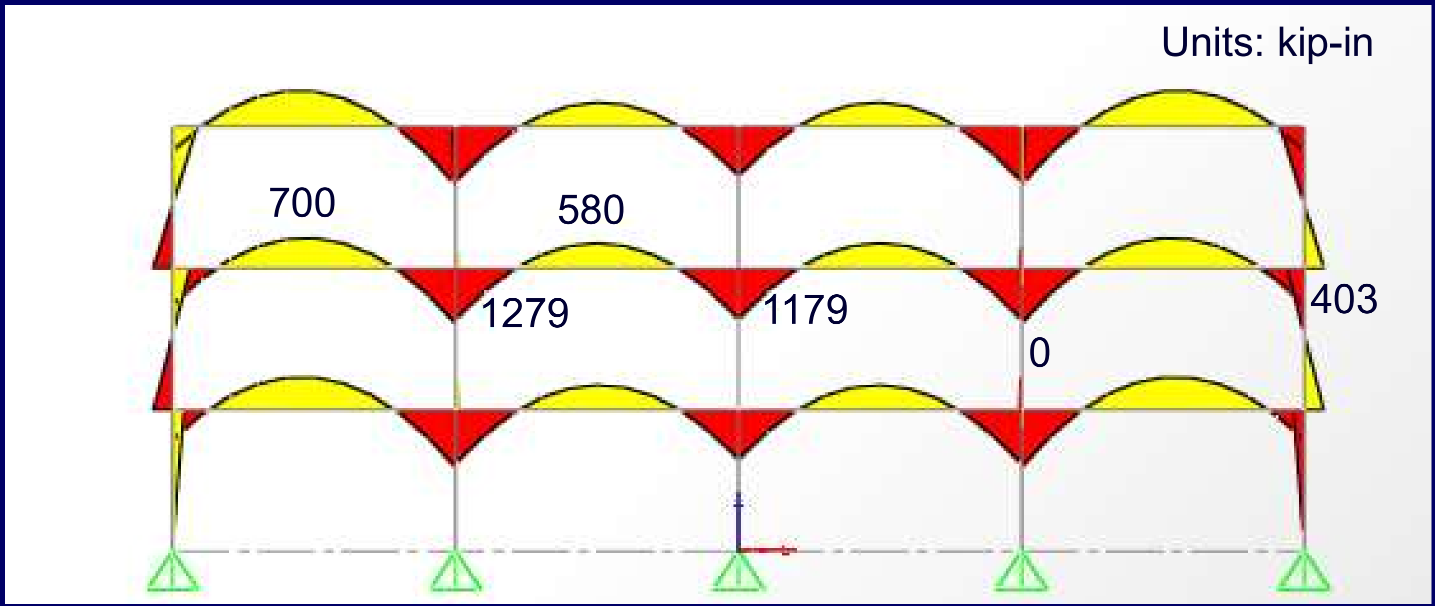


Case Study

❑ Effect of Pattern Load on Bending Moments

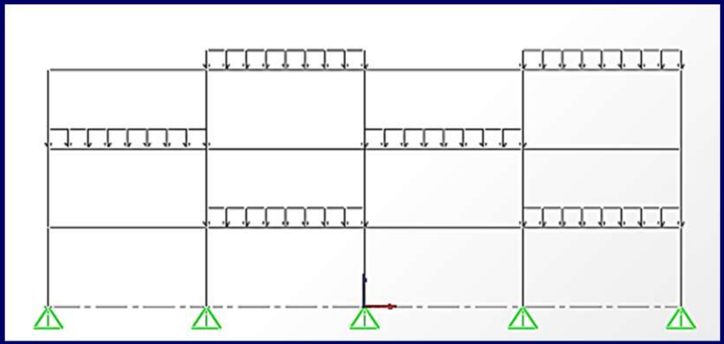




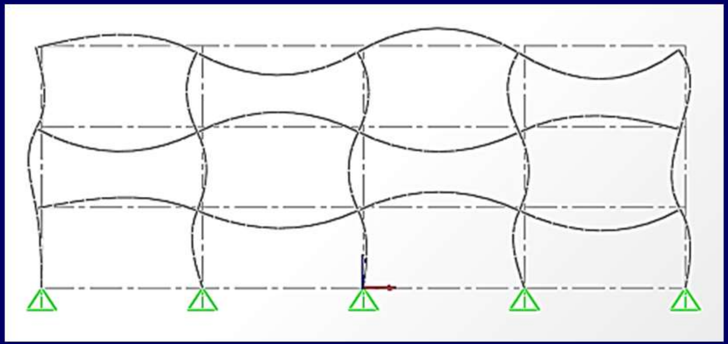
Load: Full dead + live {1.80 +1.152} kip/ft

Case Study

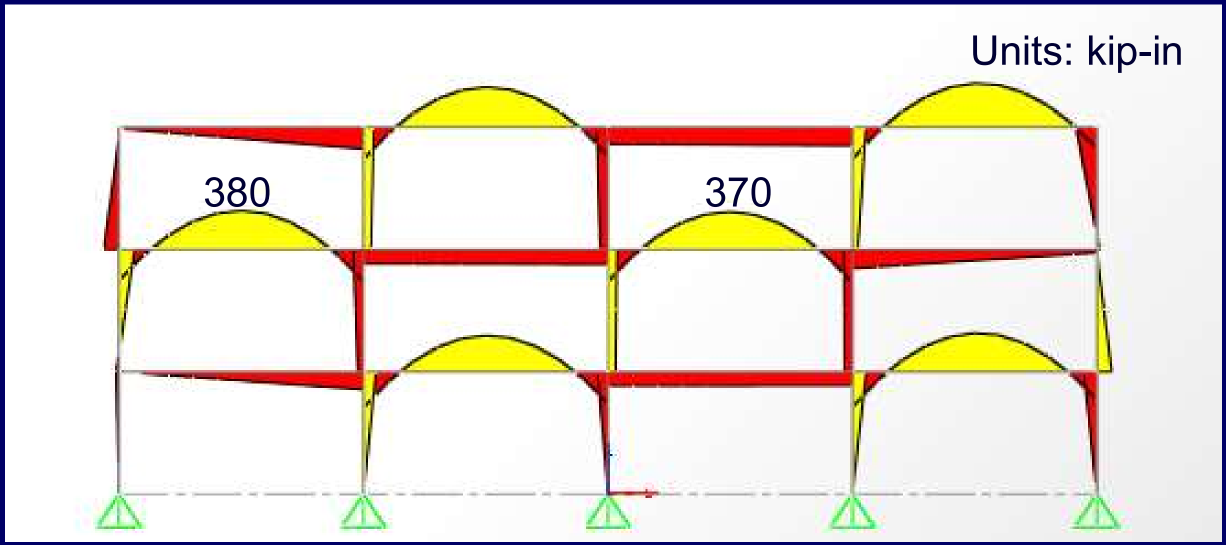
❑ Effect of Pattern Load on Bending Moments



Load: Pattern Live Only
(1.152 kip/ft) for $M_{mid+, max}$

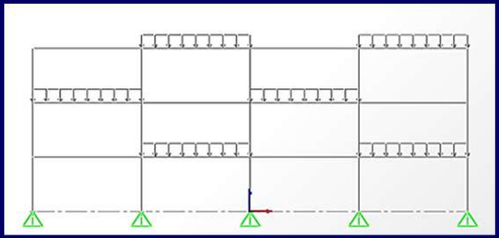


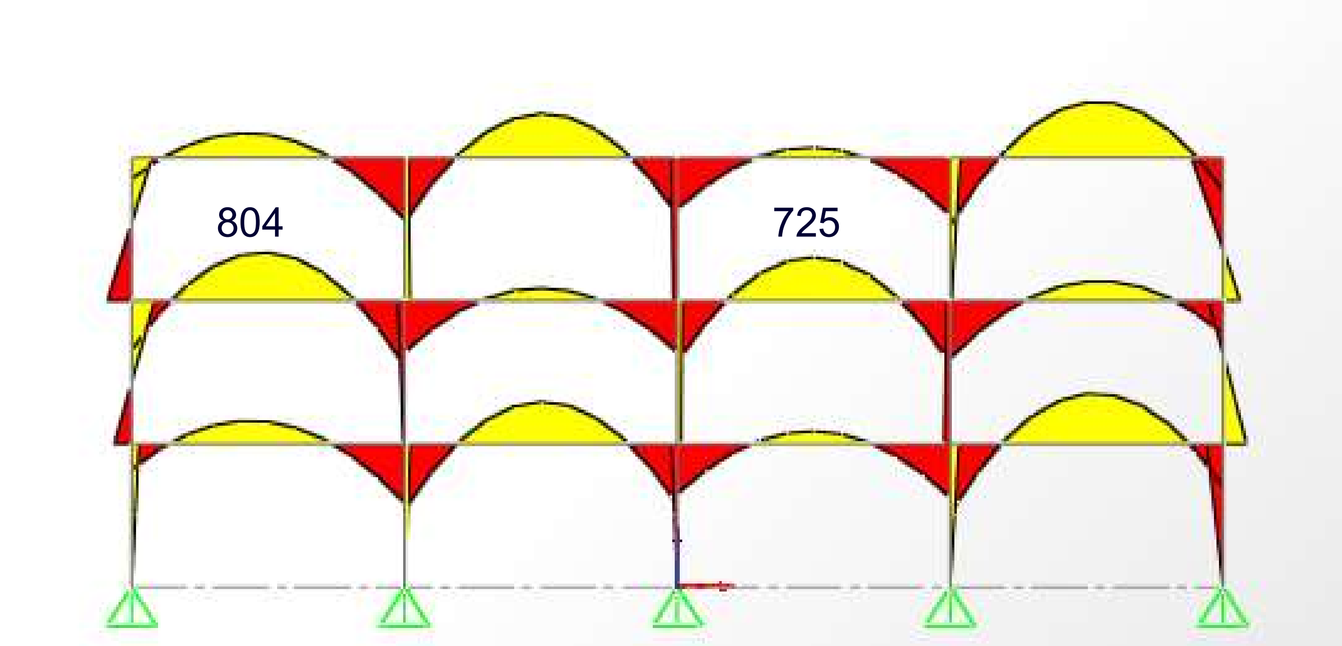
Deflected shape



Case Study

❑ Effect of Pattern Load on Bending Moments

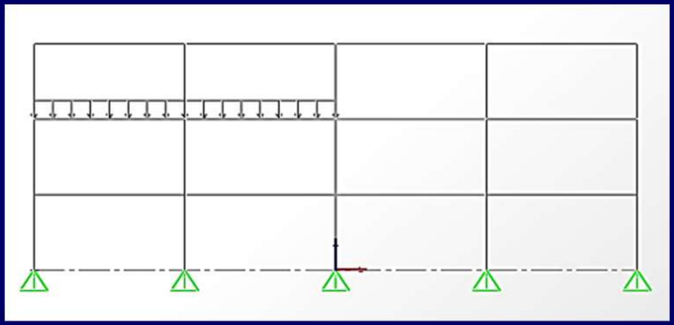




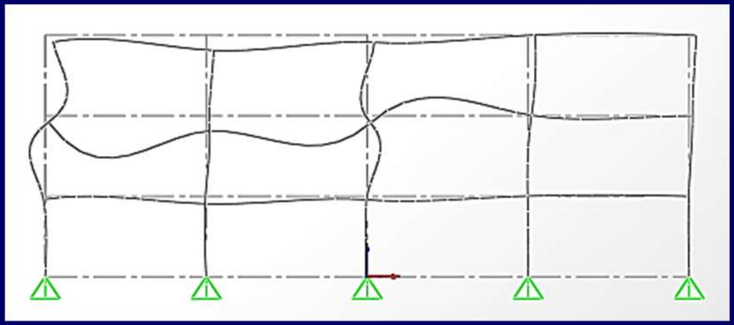
Load: Dead + Pattern Live (1.152 kip/ft) for $M_{mid+,max}$

Case Study

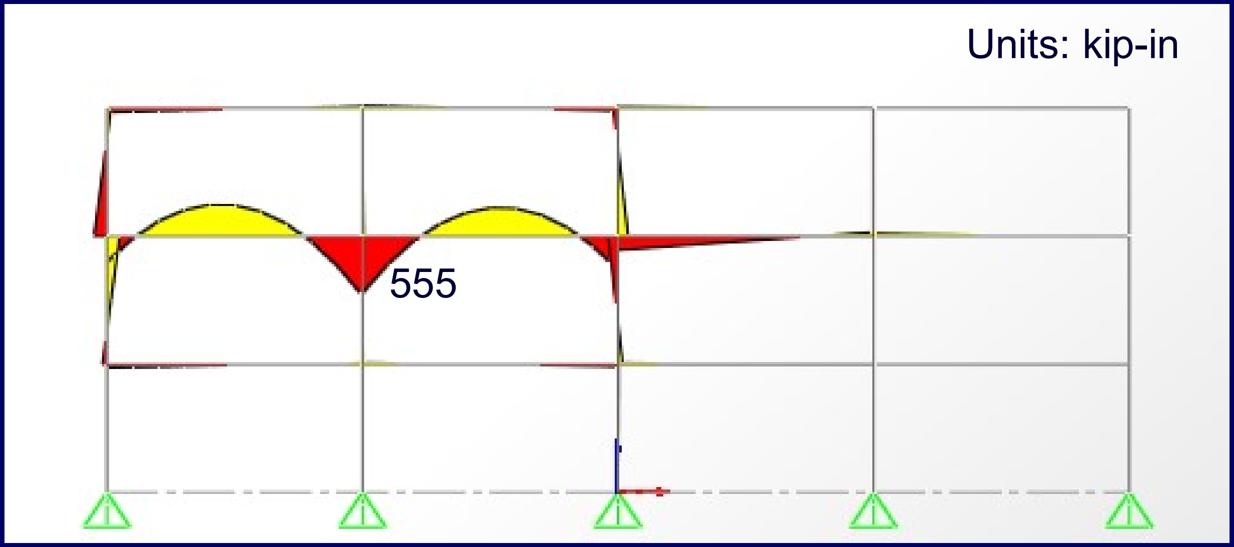
❑ Effect of Pattern Load on Bending Moments



Load: Pattern Live Only (1.152 kip/ft) for $M_{1st\ int\ supp}$ *(negative moment in first interior support)*

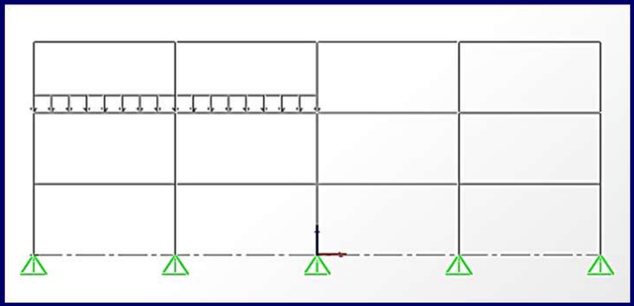


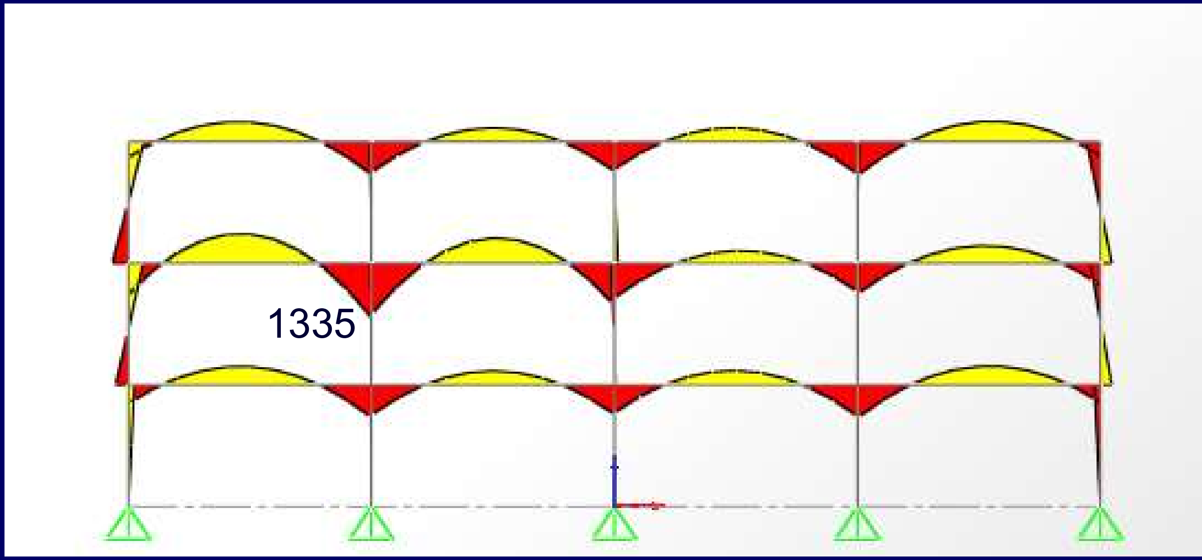
Deflected shape



Case Study

❑ Effect of Pattern Load on Bending Moments

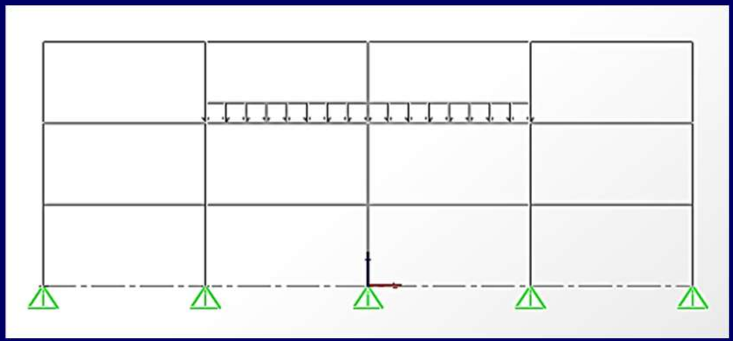




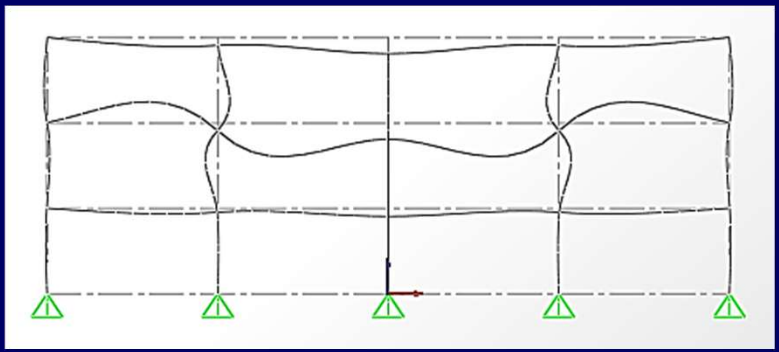
Load: Dead + Pattern Live (1.152 kip/ft) for $M_{1st\ int\ supp\ -}$

Case Study

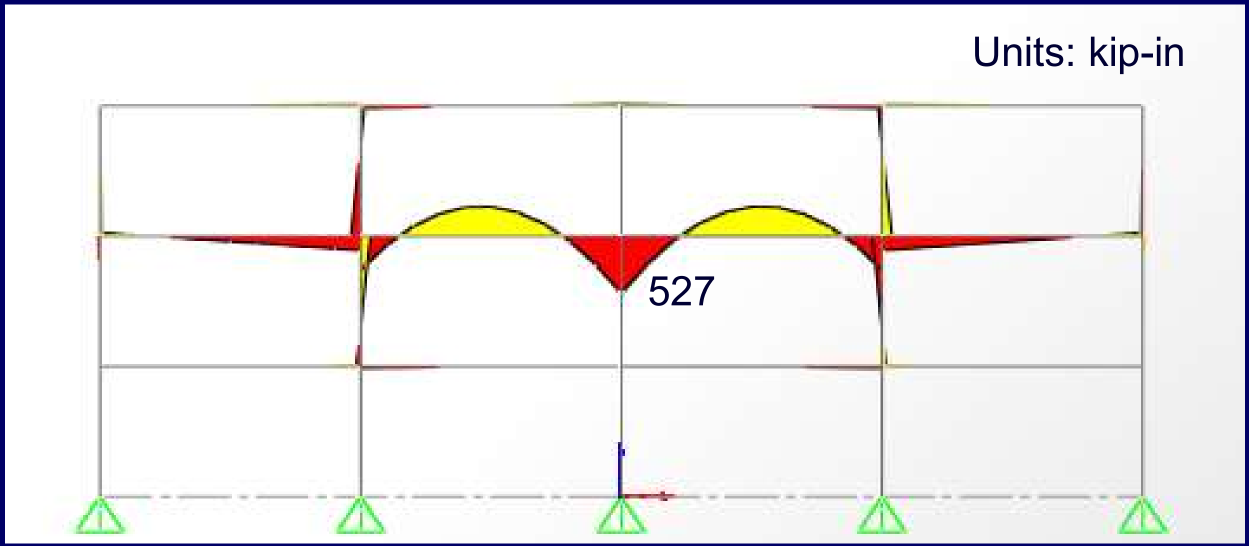
❑ Effect of Pattern Load on Bending Moments



Load: Pattern Live Only
(1.152 kip/ft) for $M_{int\ supp\ -}$

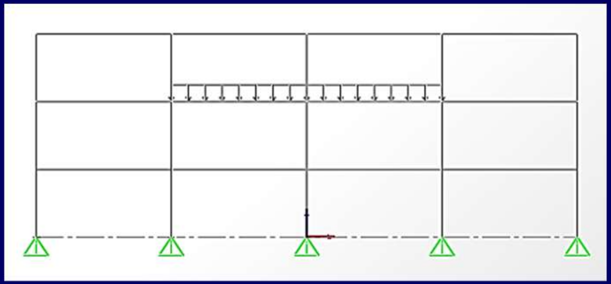


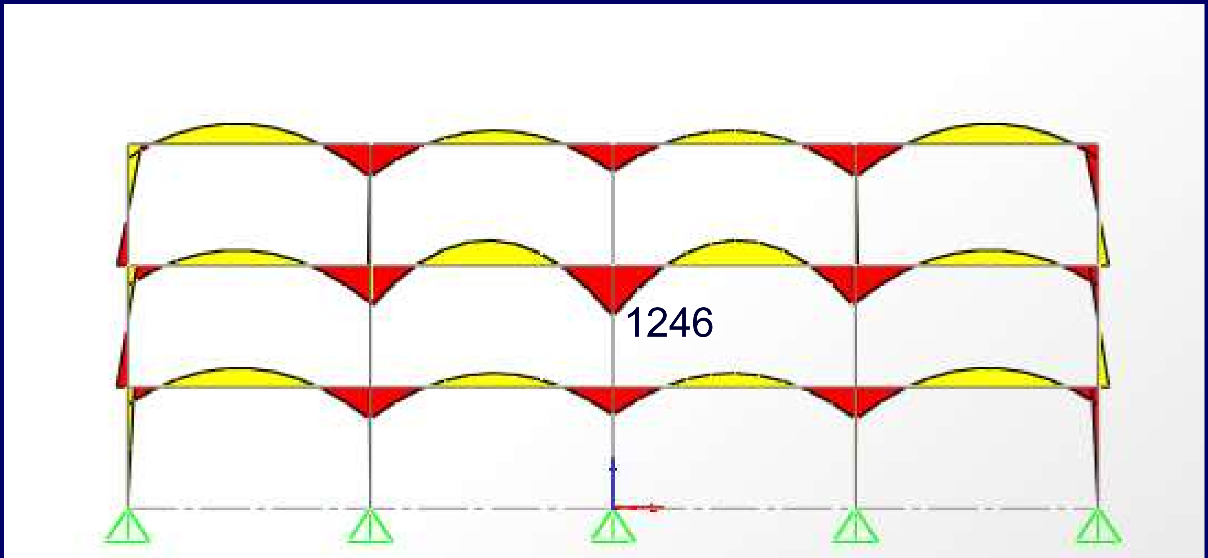
Deflected shape



Case Study

❑ Effect of Pattern Load on Bending Moments

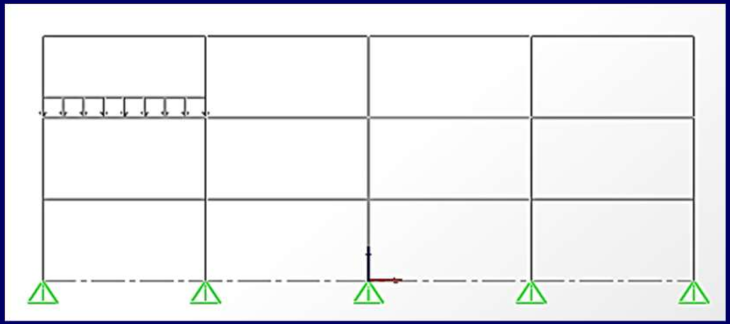




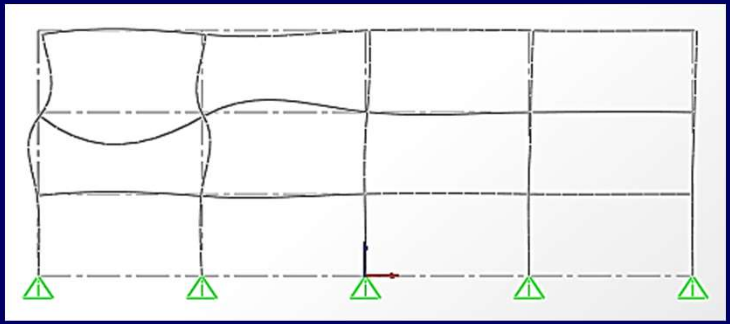
Load: Dead + Pattern Live (1.152 kip/ft) for $M_{int\ supp\ -}$

Case Study

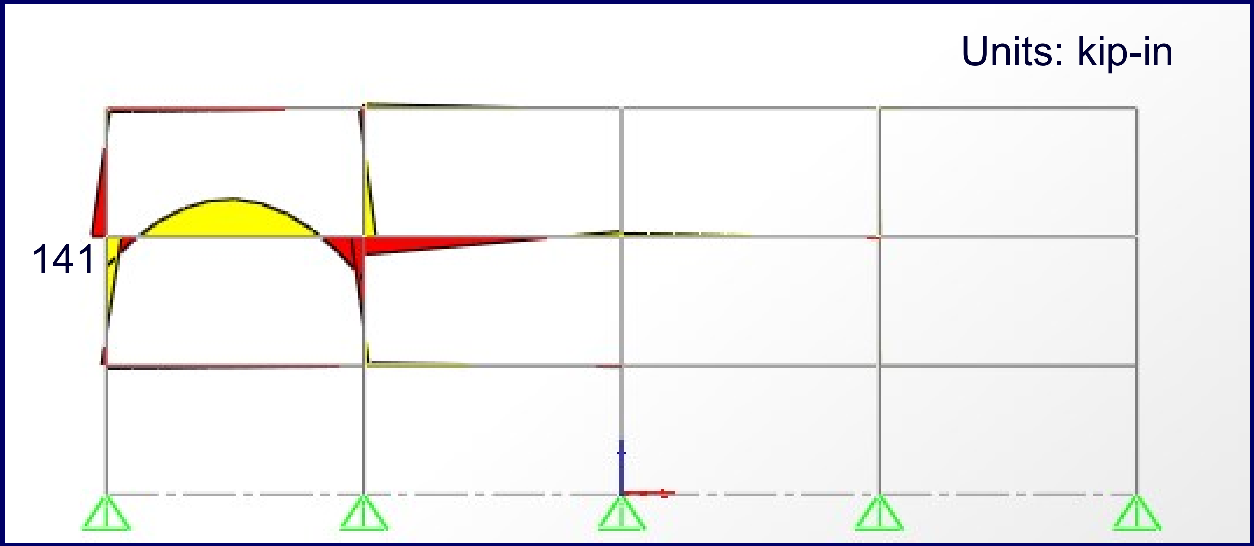
❑ Effect of Pattern Load on Bending Moments



Load: Pattern Live Only
(1.152 kip/ft) for $M_{ext\ col}$

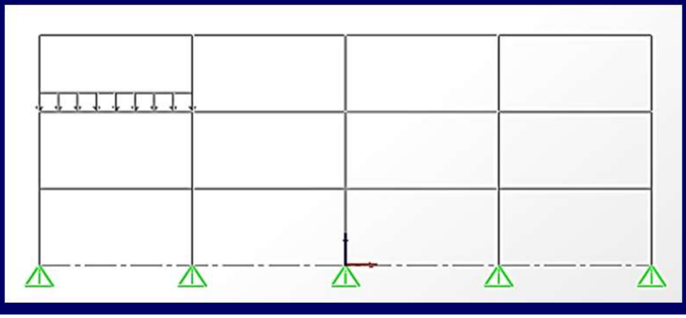


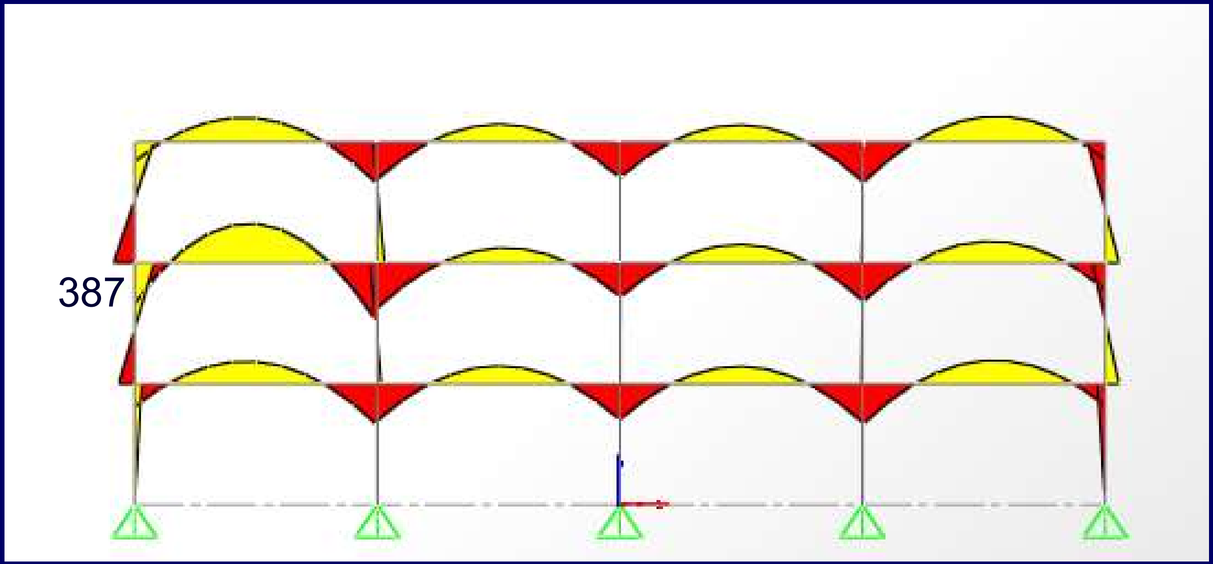
Deflected shape



Case Study

❑ Effect of Pattern Load on Bending Moments

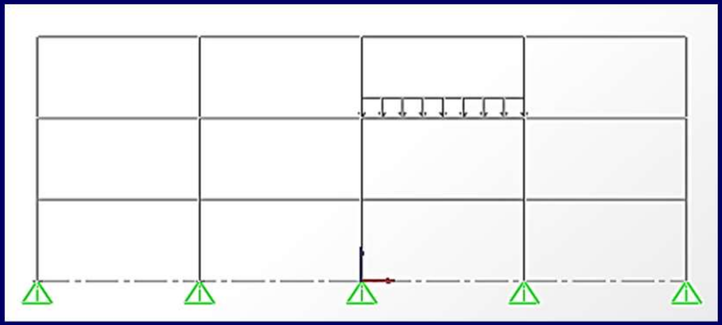




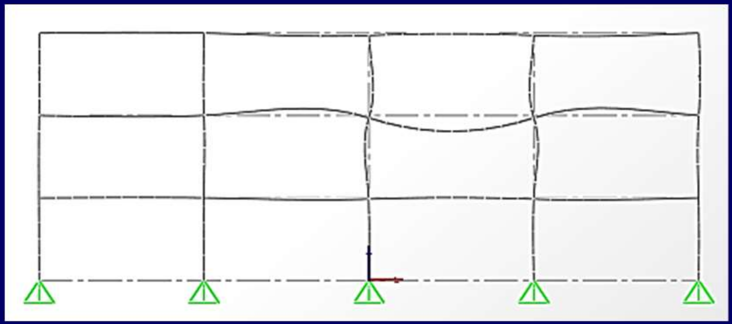
Load: Dead + Pattern Live (1.152 kip/ft) for $M_{ext\ col}$

Case Study

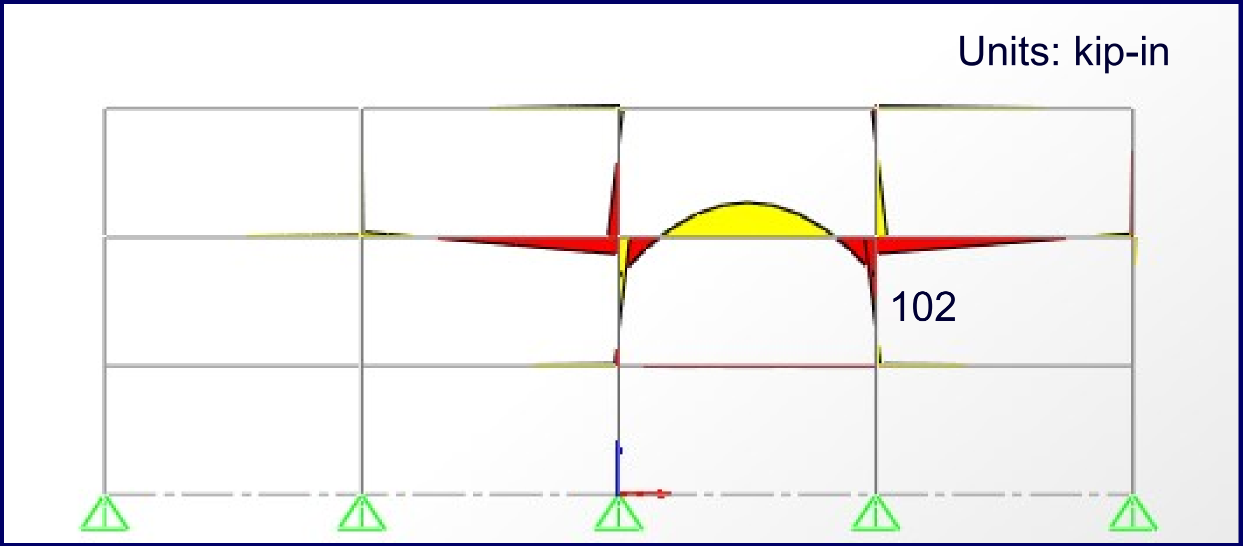
❑ Effect of Pattern Load on Bending Moments



Load: Pattern Live Only
(1.152 kip/ft) for $M_{int\ col}$

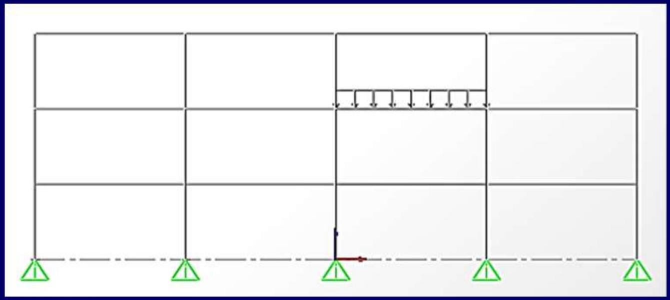


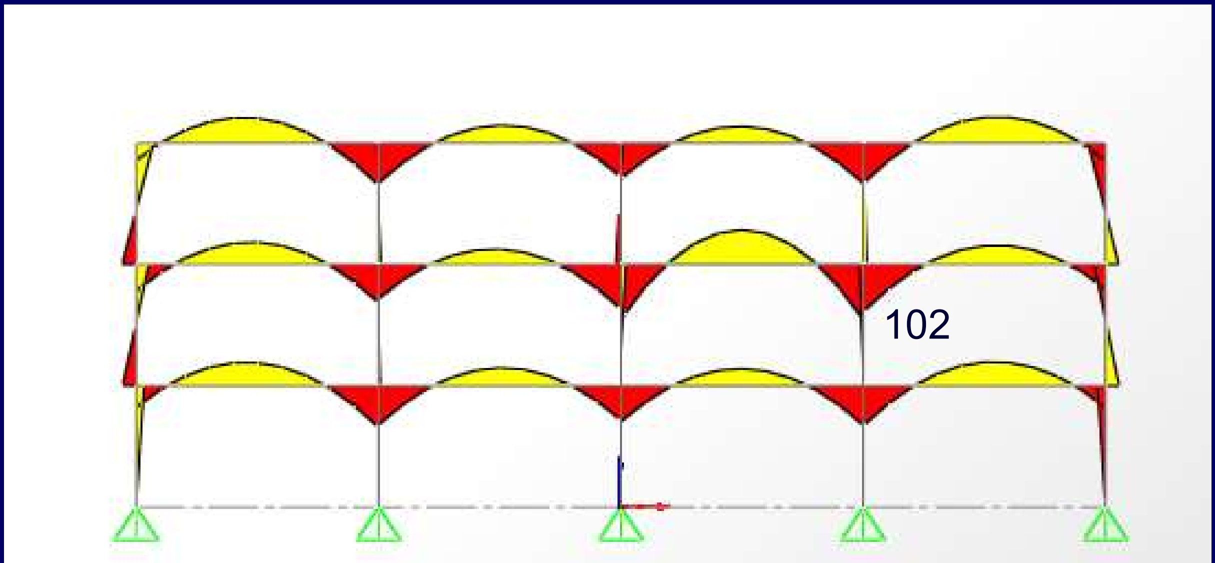
Deflected shape



Case Study

❑ Effect of Pattern Load on Bending Moments



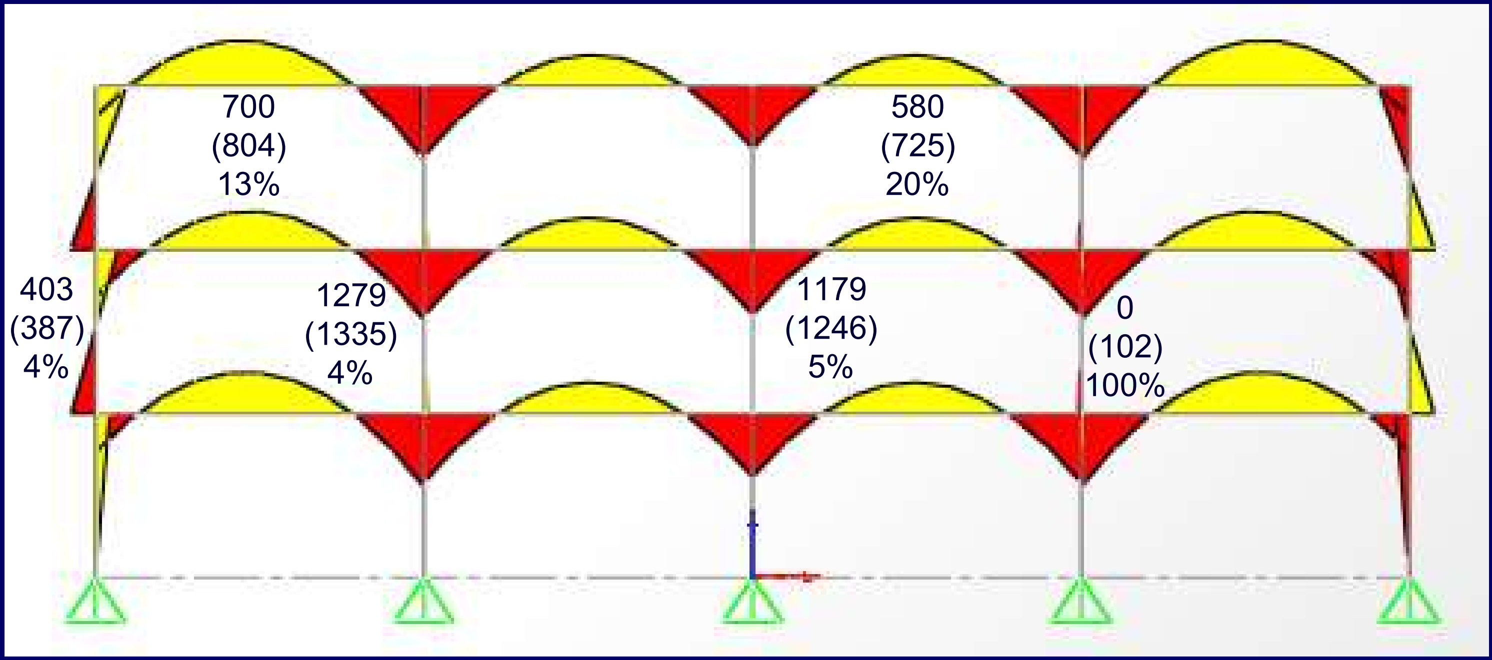


Load: Dead + Pattern Live (1.152 kip/ft) for $M_{int\ col}$

Case Study

❑ Effect of Pattern Load on Bending Moments

- All maximum load effects can be combined on a single diagram giving envelop. Comparison with full dead and live load is also shown.



Full (Pattern) %age difference

Case Study

❑ Effect of Pattern Load on Bending Moments

❖ Concluding Remarks

- To calculate the maximum possible moments at all critical points of a frame, live load must be placed in a great variety of different schemes.
- In most practical cases, however, consideration of the relative magnitude of effects will permit limitation of analysis to a small number of significant cases.

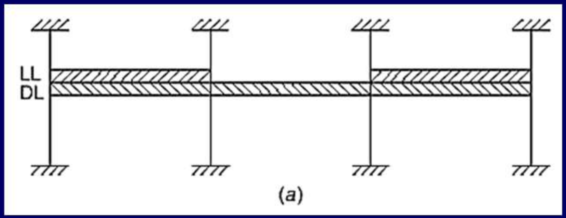
Live Load Arrangement as per ACI

❑ Section 6.4.2

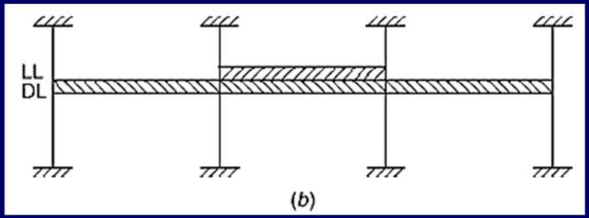
- For one-way slabs and beams, it shall be permitted to assume (a) and (b):

  a) Maximum positive M_u near mid-span occurs with factored L on the span and on alternate spans (fig a and b below)

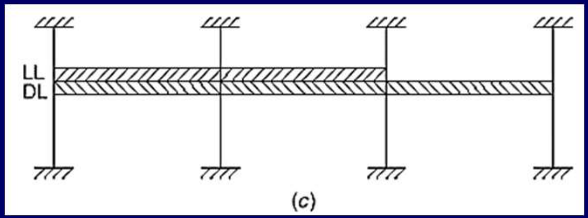
  b) Maximum negative M_u at a support occurs with factored L on adjacent spans only (fig c)



Alternate spans loaded (a)



Alternate spans loaded (b)



Adjacent spans loaded (c)

Closing Note

- The characteristics of the idealized structural model should be as close as possible to the physical structure.
- The engineer must not only accept the uncertainties of load placement, magnitude, and duration typical of any structural analysis, but must also cope with other complications that are unique to reinforced concrete such as effect of cracking, creep and shrinkage.

Closing Note

- The student may well despair of accurate calculation of the internal forces for which the members of a reinforced concrete frame must be designed.
- However mainly because of plastic flow, a concrete structure tries with admirable ductility to adapt itself to our calculations.

Closing Note

- As very correctly said by Halvard Birklands

"…the structure, in many instances, will accept our rash assumptions and our imperfect mathematical models to such an extent that the structure will exhaust all means of standing before it decides to fall "

Closing Note

- However, too great a deviation from the actual distribution of internal forces can result in serviceability problems associated with cracking and deflection and can even result in premature failure.
- But it is reassuring to know that, if good judgment is used in assigning internal forces to critical sections, the wisdom of the structure will prevail.

References

- Reinforced Concrete - Mechanics and Design (7th Ed.) by James MacGregor.
- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-19)
- SAP2000 FEM based Software.
- Case Studies by Dr. Qaisar Ali and Engr. Umer