Lecture 08

Design of Reinforced Concrete Columns

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Lecture Contents

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Learning Outcomes

- **At the end of this lecture, students will be able to;**
  - *Explain* the importance of longitudinal and lateral reinforcement in RC columns
  - *Develop* interaction diagrams for square RC columns
  - *Design* concentric and uniaxially eccentric RC columns

General

❑ Introduction

- A structural member (usually vertical) , used primarily to support axial compressive load is called column.
- However, columns would generally carry bending moments as well, about one or both axes of the cross section.

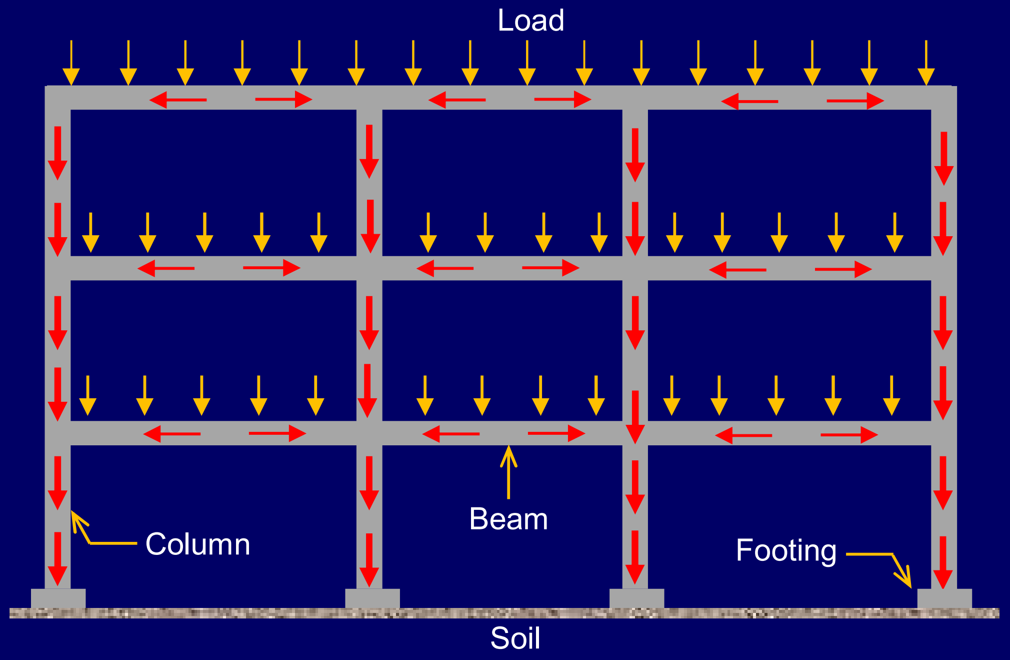

Column for 60-story Bank of America Corporate Center, North Carolina


Column for 20-story Dominion Mall & Apartments, Pakistan (Designed by DQA).

General

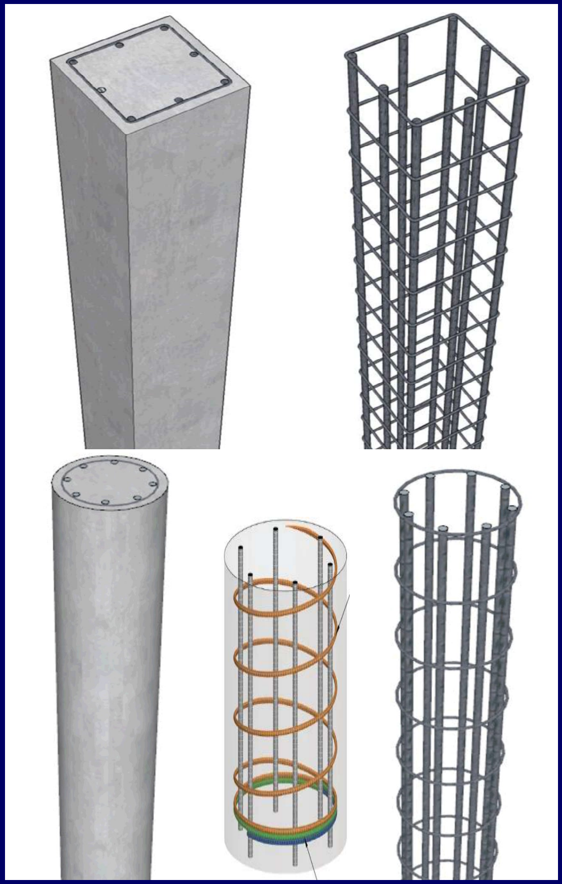
❑ Introduction

- Columns transmit loads from upper floor levels to the lower floor levels and ultimately to the ground through the foundations.
- Unlike beams and slabs that carry the load of a single floor, columns bear the load of multiple floors above them, resulting in an accumulation of load.



General

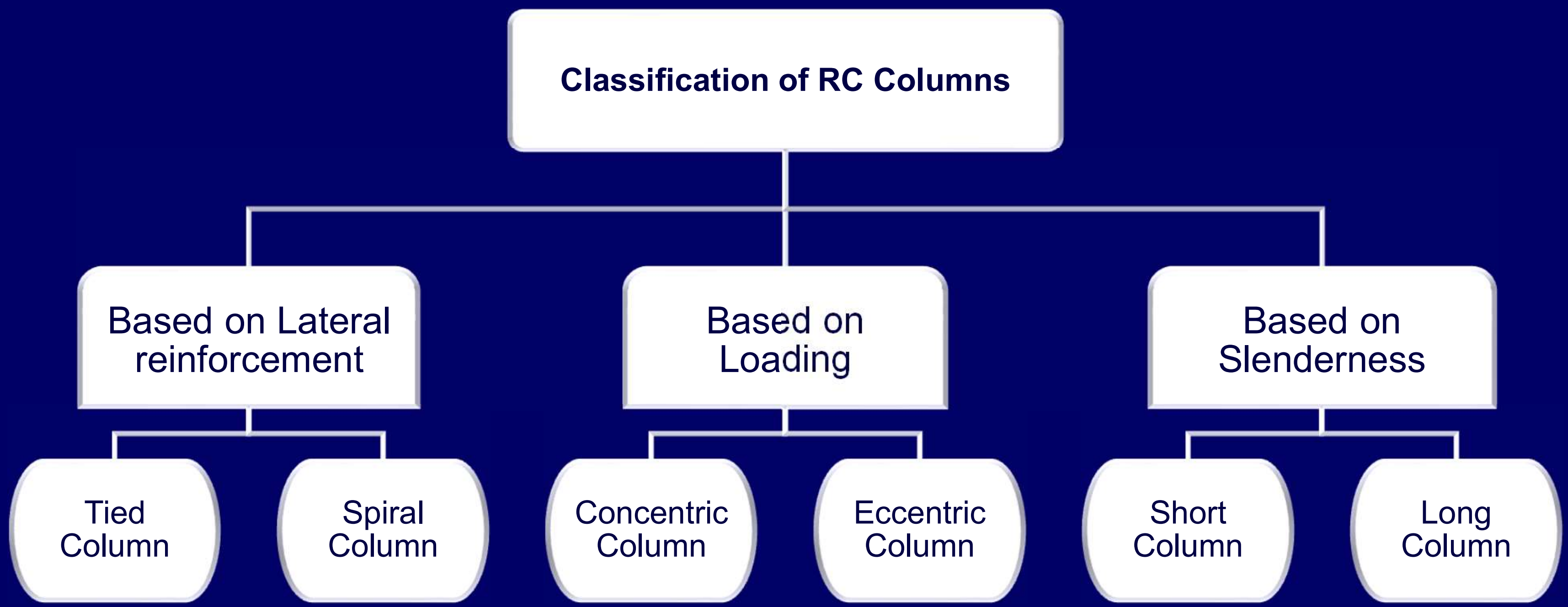
- **Reinforcement in RC columns**
  - **Longitudinal Reinforcement**
    - They are provided parallel to the direction of the load to resist the Bending moment as well as the Compression.
  - **Lateral Reinforcement**
    - The lateral reinforcement is provided in the form of ties or continuous spiral to resist Shear and to hold the longitudinal bars.



General

❑ Classification of RC Columns

- RC columns can be classified on various bases as shown below.



General

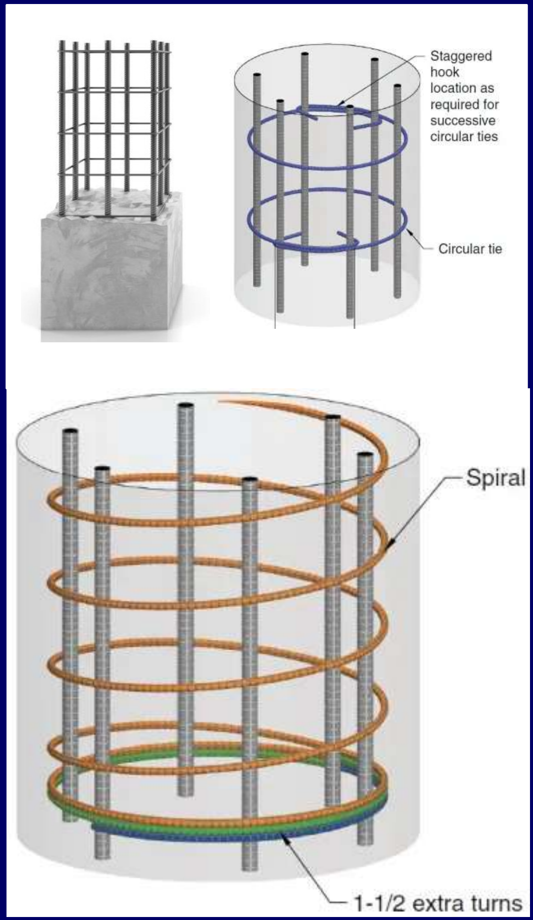
❑ Types of RC Columns (based on lateral reinforcement)

1. **Tied Columns**

    - Columns (of any shape) with closely spaced lateral ties/hoops.

2. **Spiral Columns**

    - Columns (of any shape) with continuous spiral reinforcement wound in a helical pattern.
    - They are generally more efficient than tied columns.



General

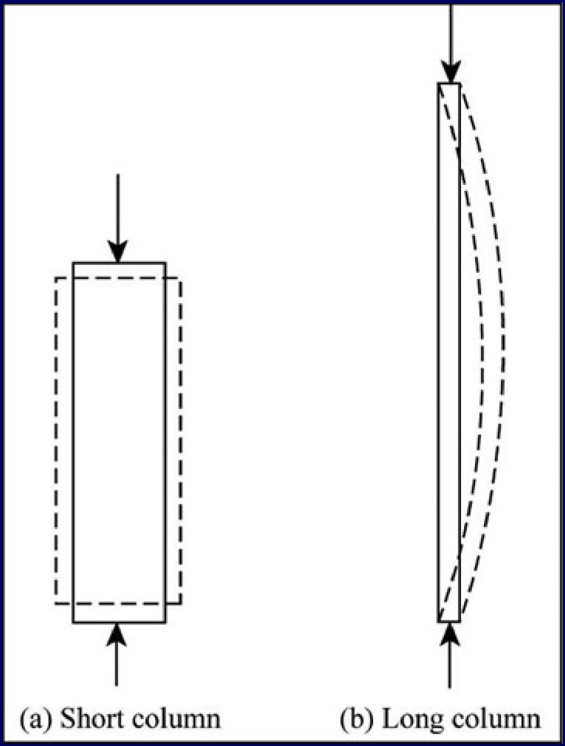
❑ Types of RC Columns (based on slenderness)

1. Short Columns

- Columns that fail due to the failure of materials are called short columns.
- Most of the concrete columns fall in this category.

2. Long /Slender columns

- Columns in which failure occurs due to geometric instability (buckling) are called long columns.



General

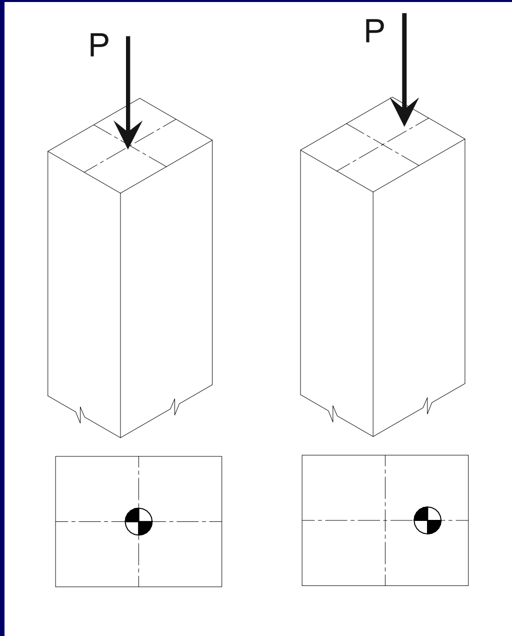
Types of RC Columns (based on loading)

1. Concentric Columns

- Columns in which applied load is aligned with its central axis, resulting in uniform compression throughout the column's cross-section.

2. Eccentric Columns

- Columns in which applied load does not coincide with its central axis, causing an uneven distribution of compression forces across the column's cross-section. They can be
  1. Uniaxially eccentric
  2. Biaxially eccentric



a) Concentric column b) Eccentric column

General

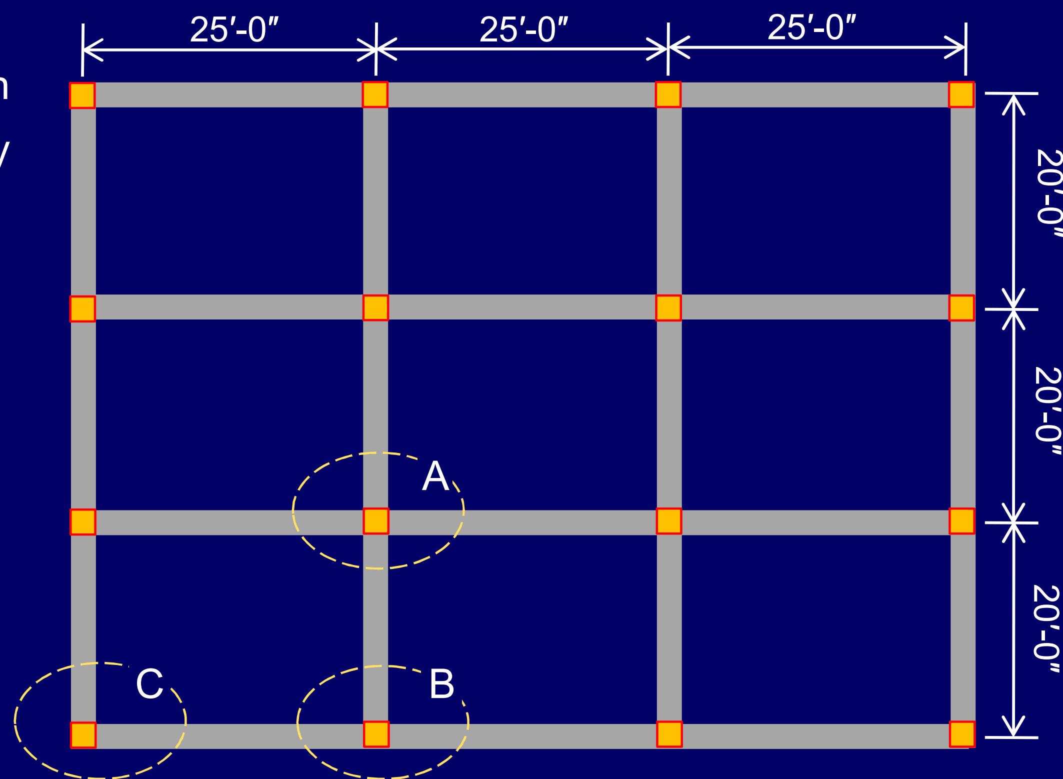
❑ Types of RC Columns (based on loading)

When the spans are equal in both directions and the loading is uniformly distributed then

A) **Interior columns ⇒ Concentric**

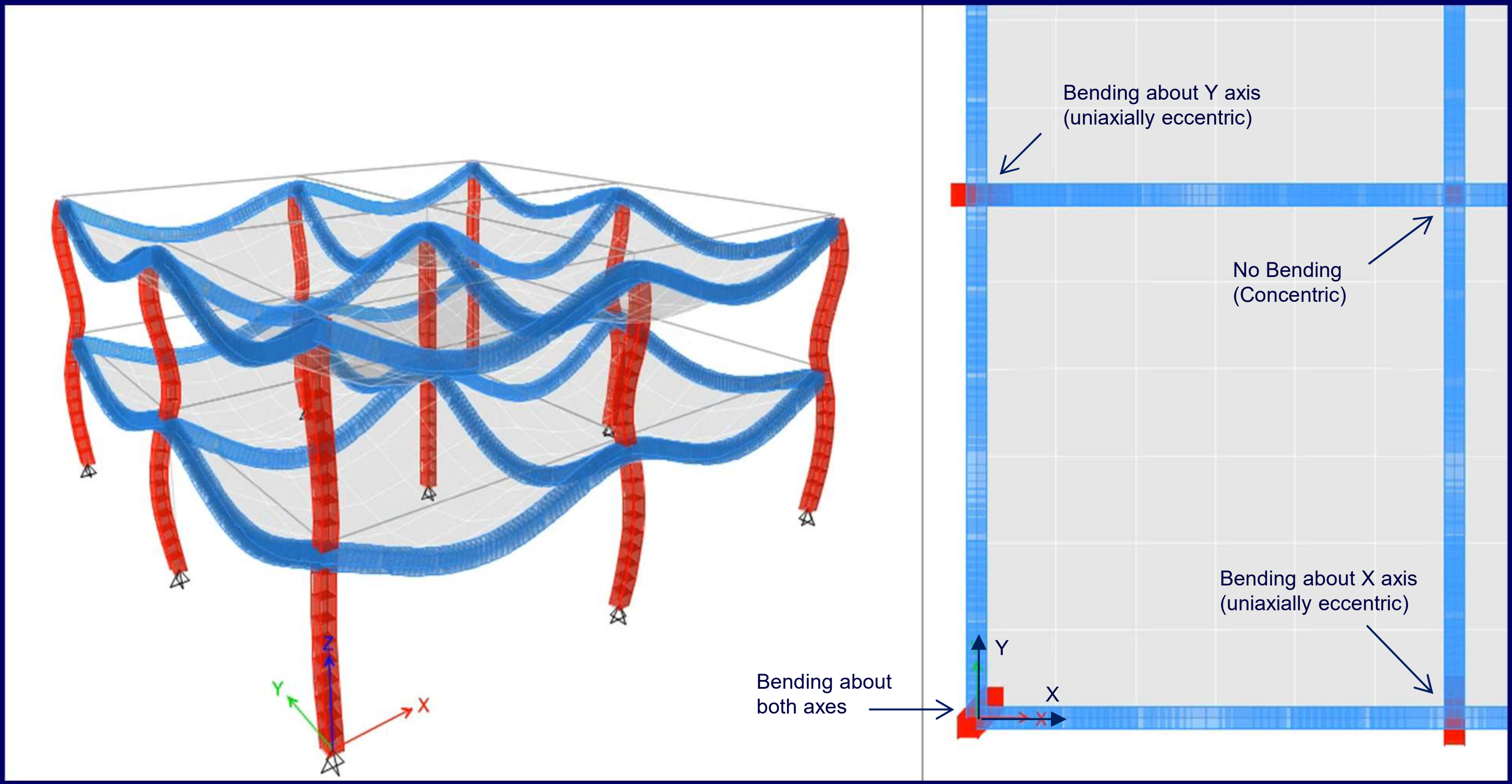
B) **Edge columns ⇒ Uniaxially eccentric**

C) **Corner Columns ⇒ Biaxially eccentric**



General

- Types of RC Columns (based on loading)



ACI 318 Code Provisions for RC Columns

Dimensional Limits

- According to ACI Code 18.7.2, column shall be at least 12 in.

Reinforcement Limits

a) **Longitudinal reinforcement limits (ACI 10.6.1.1)**

- Area of longitudinal reinforcement shall be at least $0.01A_g$ but shall not exceed $0.08A_g$.
- Minimum Reinforcement is necessary to provide resistance to bending, and to reduce the effects of creep and shrinkage of the concrete under sustained compressive stresses.

ACI 318 Code Provisions for RC Columns

Reinforcement Limits

a) Longitudinal reinforcement limits

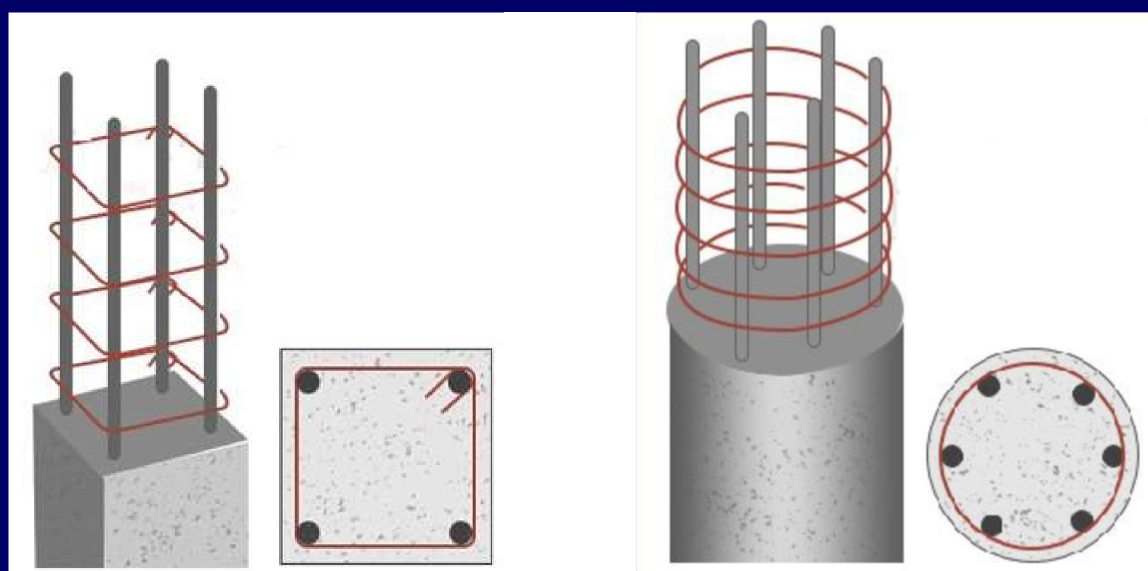
- Maximum amount of longitudinal reinforcement is limited to ensure that concrete can be effectively consolidated around the bars
- Longitudinal reinforcement in columns usually does not exceed 4 percent as the lap splice zone will have twice as much reinforcement, if all lap splice occur at the same location.

ACI 318 Code Provisions for RC Columns

❑ Reinforcement Limits

a) Longitudinal reinforcement limits

- Minimum diameter ⇒ #4 (ACI 10.7.3)
- Minimum number of bars ⇒ 4 for rectangular columns
  6 for circular columns.



ACI 318 Code Provisions for RC Columns

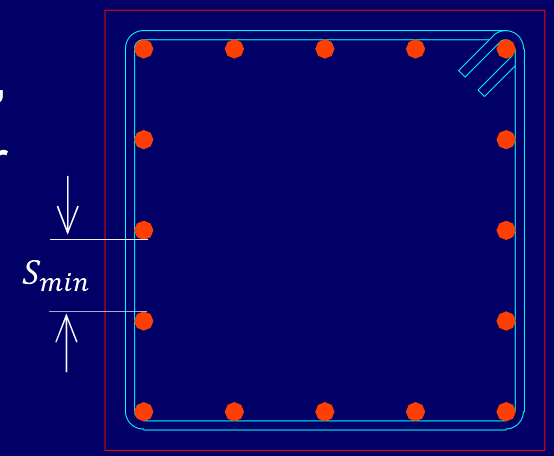
❑ Reinforcement Limits

a) Longitudinal reinforcement limits

❖ Minimum spacing between longitudinal bars (ACI 25.2.3)

- Clear spacing between longitudinal bars shall be at least the greatest of; 1.5 in. and 1.5d_b (where d_b is the diameter of longitudinal bar).

- However, to ensure proper concreting, it is better to maintain a minimum clear spacing of 3 inches.



ACI 318 Code Provisions for RC Columns

Reinforcement Limits

b) Shear reinforcement limits

- Maximum spacing of lateral ties (ACI 25.7.2.1)
  - Maximum spacing S_{max} shall not exceed the least of;

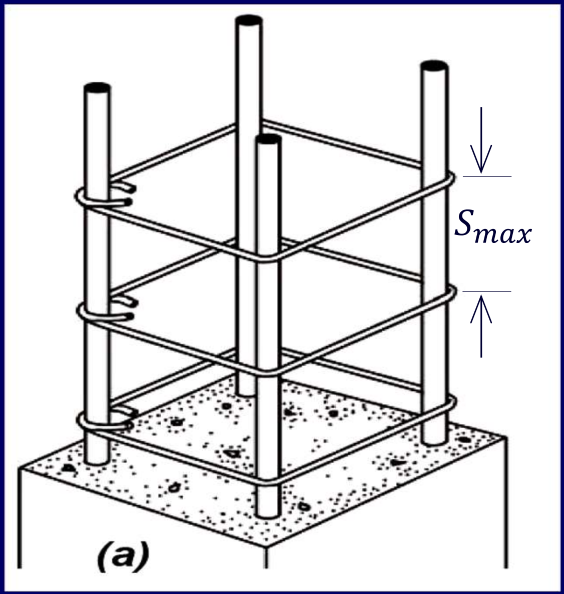
i. $\dfrac{A_v f_y}{50b}$

ii. $\dfrac{A_v f_y}{0.75\sqrt{f_c'}b}$

iii. $16d_b$ of longitudinal bar

iv. $48d_h$ of hoop/tie bar

v. Smallest dimension of member



ACI 318 Code Provisions for RC Columns

❑ Reinforcement Limits

b) Shear reinforcement limits

- Minimum diameter of lateral ties (ACI 25.7.2.2)
  - Diameter of tie bar shall be at least:
    i. #3 for longitudinal bars having size up to #10.
    ii. #4 for longitudinal bars having size larger than #10.

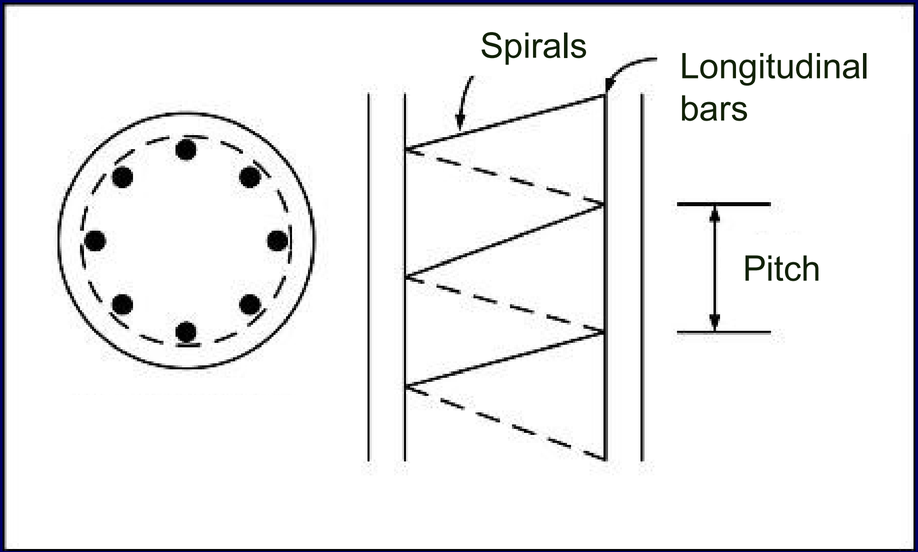
ACI 318 Code Provisions for RC Columns

❑ Reinforcement Limits

b) Shear reinforcement limits

❖ Diameter and spacing of spiral reinforcement (ACI 25.7.3)

- The minimum spiral reinforcement size is 3/8 in.
- Spacing/pitch of spiral must not be less than 1 in. and greater than 3 in.



Part - I

Design of Concentric RC Columns

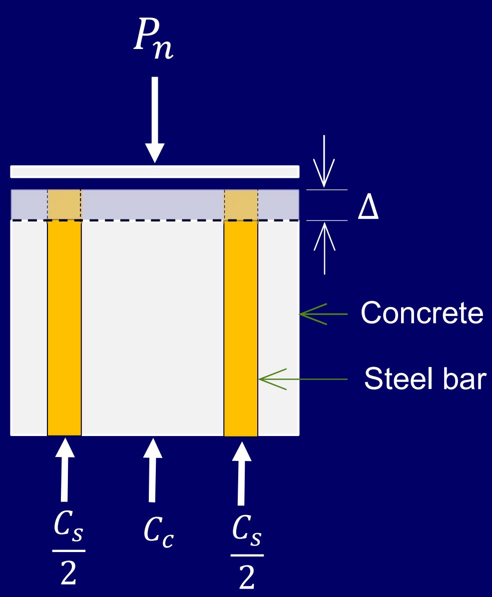
Mechanics

❑ Axial Capacity

From the figure shown below, we have

$P_n = C_c + C_s = f_c A_c + f_s A_s$

Because of the perfect bonding between concrete and steel bars, the strain in both materials will be identical. As a result, steel bars with a grade of 80 or lower will yield at the ultimate stage ($\epsilon_u = 0.003$).



$f_c = 0.85 f_c'$ and $f_s = f_y$ $(for\ f_y \leq 80ksi)$

$$\epsilon_{y,40} = \frac{f_y}{E_s} = \frac{40}{29000} = 0.0014 < \epsilon_u = 0.003$$

$$\epsilon_{y,60} = \frac{60}{29000} = 0.0021 < \epsilon_u$$

$$\epsilon_{y,80} = \frac{80}{29000} = 0.0028 < \epsilon_u$$

so,

$P_n = 0.85 f_c' A_c + f_y A_s$

Mechanics

Axial Capacity

Area of concrete A_c can be found by subtracting steel area from the gross area of the section. Taking $A_c = A_g - A_s$, the preceding equation becomes

$$P_n = 0.85f_c{}'(A_g - A_s) + f_yA_s$$

From which the design axial capacity is determined as;

$$\emptyset P_n = \emptyset\left[0.85f_c{}'(A_g - A_s) + f_yA_s\right]$$

Where ;

$\emptyset = 0.65$ for tied column and 0.75 for spiral column (ACI Table 21.2.2).

Mechanics

❑ Axial Capacity

According to ACI 318, R22.4.2.1, an additional reduction factor 'α' is used to account for accidental eccentricities not considered in the analysis that may exist in a compression member, and to recognize that concrete strength may be less than f_c' under sustained high loads. Finally, we get

$$\alpha \emptyset P_n = 0.80 \times 0.65\left[0.85 f_c'(A_g - A_{st}) + f_y A_{st}\right]$$ (for tied column)

and

$$\alpha \emptyset P_n = 0.85 \times 0.75\left[0.85 f_c'(A_g - A_s t) + f_y A_{st}\right]$$ (for spiral column)

Mechanics

❑ Axial Capacity

For no failure;

$\alpha \emptyset P_n \geq P_u$

Taking $\alpha \emptyset P_n = P_u$

$$0.80 \times 0.65\left[0.85 f_c{}'(A_g - A_{st}) + f_y A_{st}\right] = P_u$$ ---- (8.1) (for tied column)

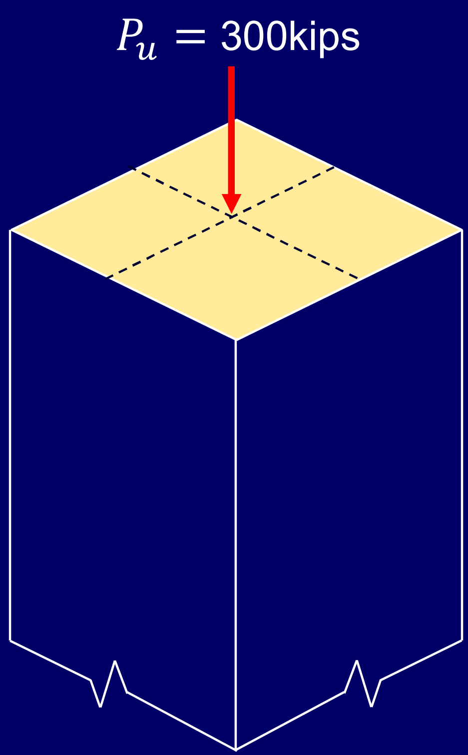
And

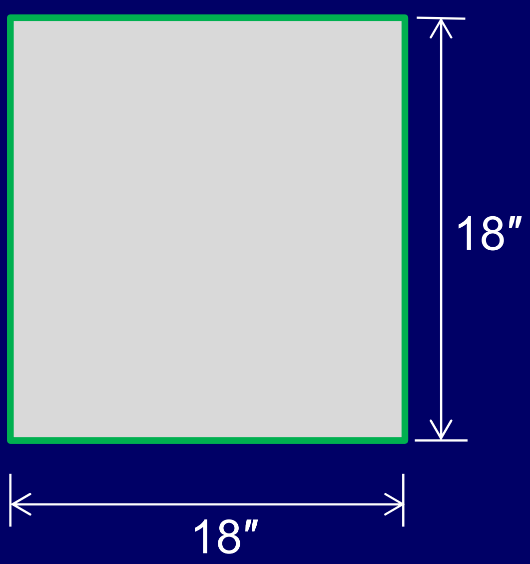
$$0.85 \times 0.75\left[0.85 f_c{}'(A_g - A_{st}) + f_y A_{st}\right] = P_u$$ ---- (8.2) (for spiral column)

Design of Tied Column

❑ Example 8.1

- ***Design*** an 18″ × 18″ tied column for a factored axial compressive load of 300 kips. Take $f_c' = 3ksi$ and $f_y = 40ksi$





Design of Tied Column

Solution

- **Given Data**

$b = 18''$

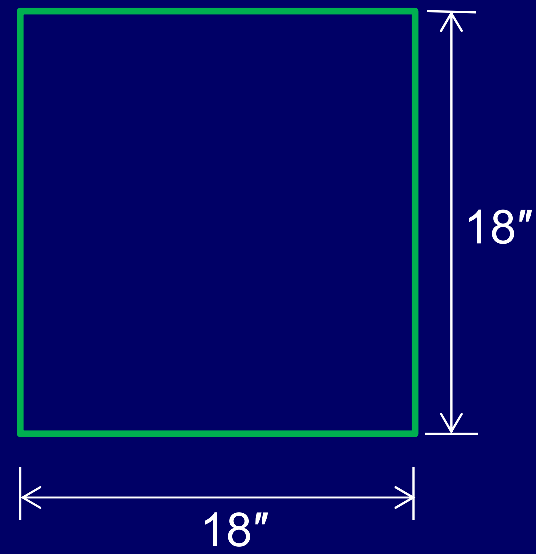
$h = 18''$

$A_g = 18'' \times 18'' = 324in^2$

$P_u = 300\ kip$

$f_c' = 3ksi$

$f_y = 40\ ksi$



- **Required Data**

Design the column for the given axial load

Design of Tied Column

Solution

Step 1: Determination of Longitudinal Reinforcement

From eq.(8.1), we have

$$P_u = 0.80 \times 0.65[0.85 f_c{}'(A_g - A_{st}) + f_y A_{st}]$$

Substituting values in the above equation gives

$$300 = 0.80 \times 0.65[0.85 \times 3\ (324 - A_{st}) + (40)A_{st}]$$

On solving for A_{st} we get

$A_{st} = -6.66 in^2 \rightarrow$ negative sign shows no reinforcement is required.

Thus, provide minimum steel , $A_{st,min} = 0.01 A_g$

$$A_{st,min} = 0.01(324) = 3.24 in^2$$

Design of Tied Column

Solution

Step 1: Determination of Longitudinal Reinforcement

- Alternative approach:

Calculate design axial capacity of column by assuming 1% steel area and compare the calculated capacity with demand axial load

$$A_{st} = 0.01A_g$$

$$\alpha \emptyset P_n = 0.80 \times 0.65\left[0.85 \times 3\ (A_g - 0.01A_g) + (40)0.01A_g\right] = 1.521A_g$$

$\alpha \emptyset P_n = 1.521(324) = 492.8kip > P_u \rightarrow$ OK!

Therefore, $A_{st} = 0.01A_g = 0.01(324) = 3.24in^2$

Design of Tied Column

Solution

Step 2: Determination of Longitudinal Reinforcement

Using #6 bar with $A_b = 0.44 in^2$

Number of bars $= \frac{A_s}{A_b} = \frac{3.24}{0.44} = 7.36 \approx 8$

Hence use 8,#6 bars.

Note:

- To maintain the symmetrical distribution along the perimeter of the cross-section, the number of bars in a square column should be a multiple of 4.
- The configuration may alter for a rectangular or circular column.

Design of Tied Column

Solution

Step 2: Detailing of Lateral / shear Reinforcement

Using #3 bar with $A_b = 0.11in^2$, maximum spacing S_{max} is the least of:

i. $\frac{A_v f_y}{50b}$ = 0.22 x 40,000/ (50x18) = 9.8″

ii. $\frac{A_v f_y}{0.75\sqrt{f_c{}'}b}$ = 0.22 x 40,000 / (0.75$\sqrt{3000}$ x18) = 11.9″

iii. $16d_b$ of longitudinal bar = 16 x 0.75 = 12″

iv. $48d_h$ of hoop/tie bar = 48 x 3/8 = 18″

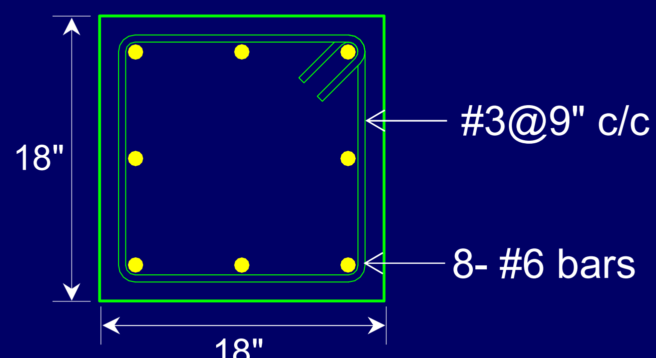
v. Smallest dimension of member = 18″

Therefore, $S_{max} = 9.8$″. Finally provide #3 ties @ 9″ c/c

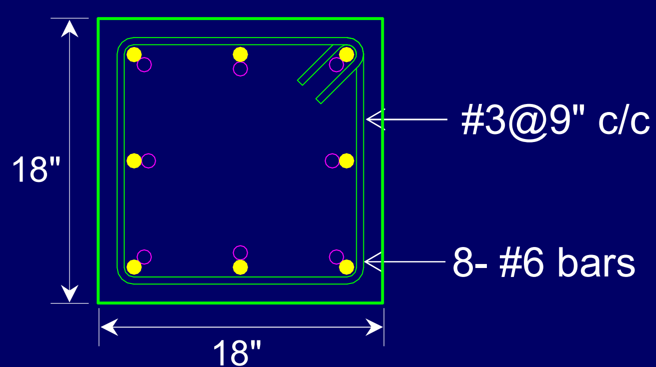
Design of Tied Column

❑ Solution

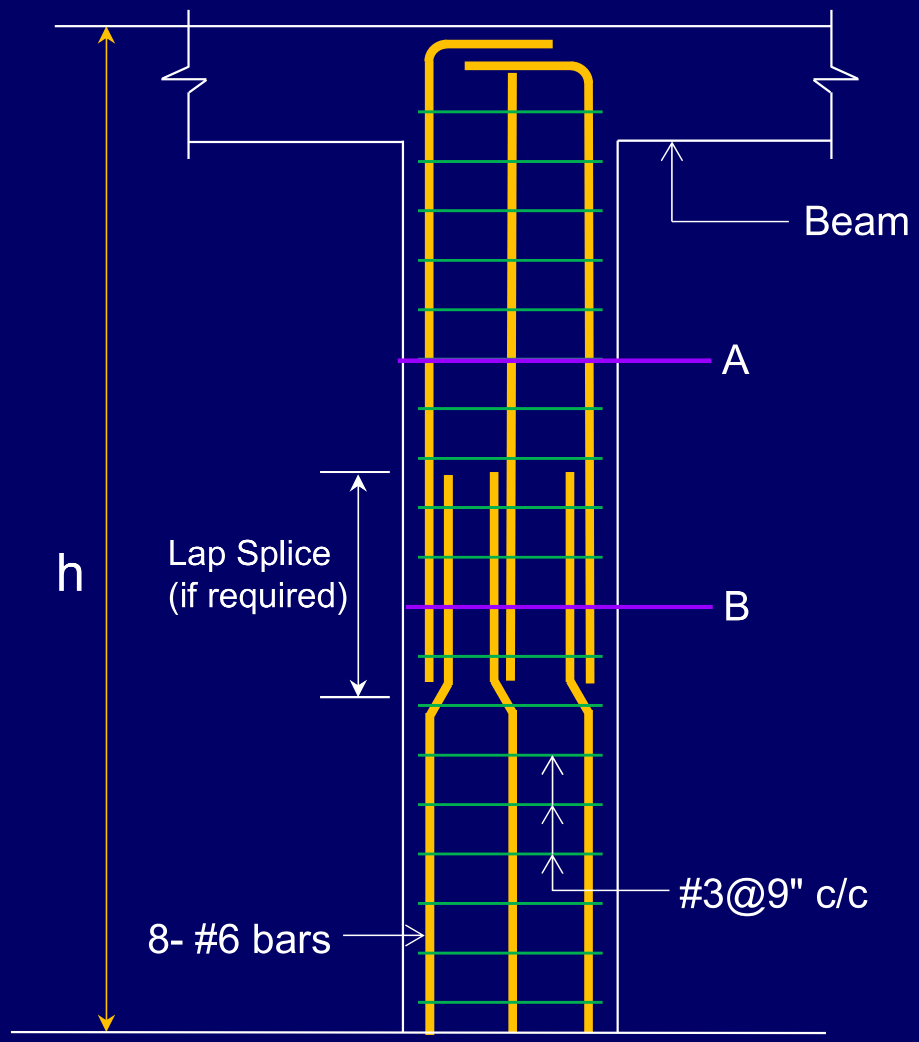
➢ Step 3: Drafting



Section A-A



Section B-B



Design of Spiral Column

Example 8.2

- ***Design*** a circular spiral column having diameter of 24″ to support an axial service dead load of 500 kips and an axial service live load of 230 kips. Take $f_c' = 4ksi$ and $f_y = 60ksi$

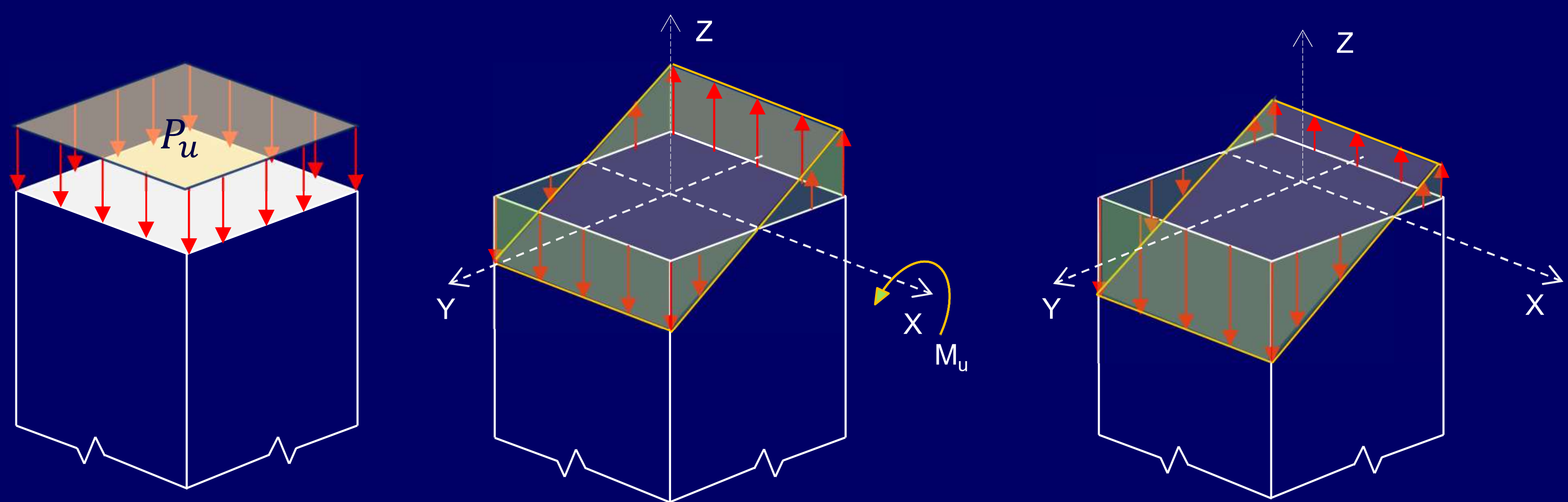
Part - II

Design of Eccentrically Loaded RC Columns

General

❑ Introduction

- An eccentrically loaded column is one that is subjected to both axial load and bending moment simultaneously.
- As a result, combined stresses are induced in the section as shown below.

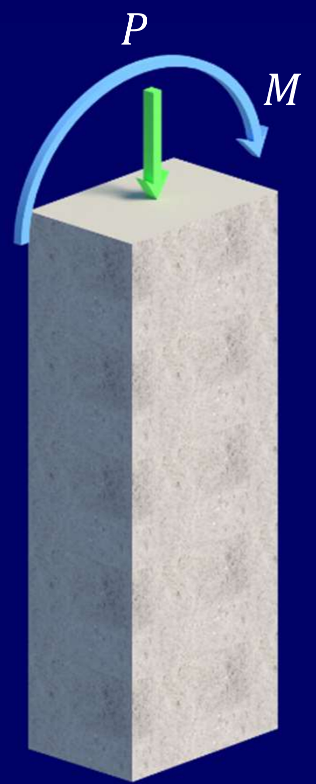


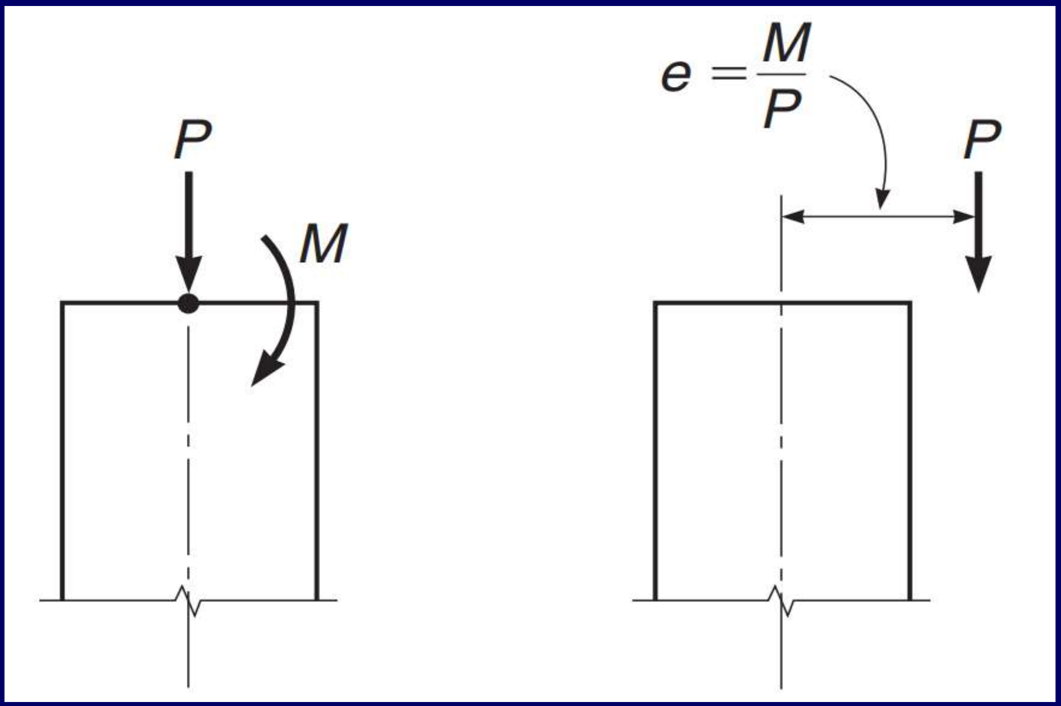
Axial Stress Distribution   Bending Stress Distribution   Combined Stress Distribution

General

Introduction

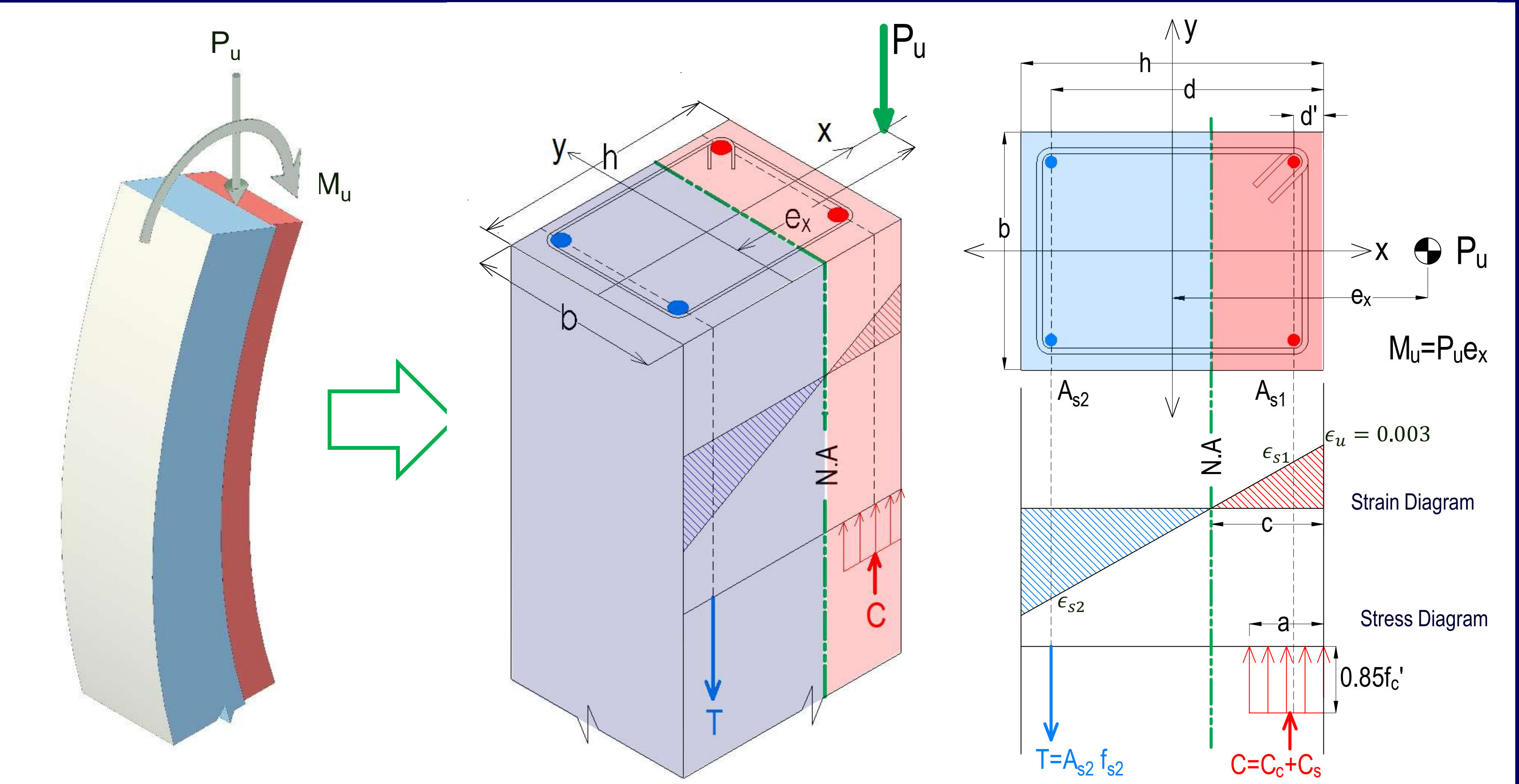
- To simplify the computations, this coupled action can be transformed into P and the equivalent eccentricity e.





General

❑ Introduction



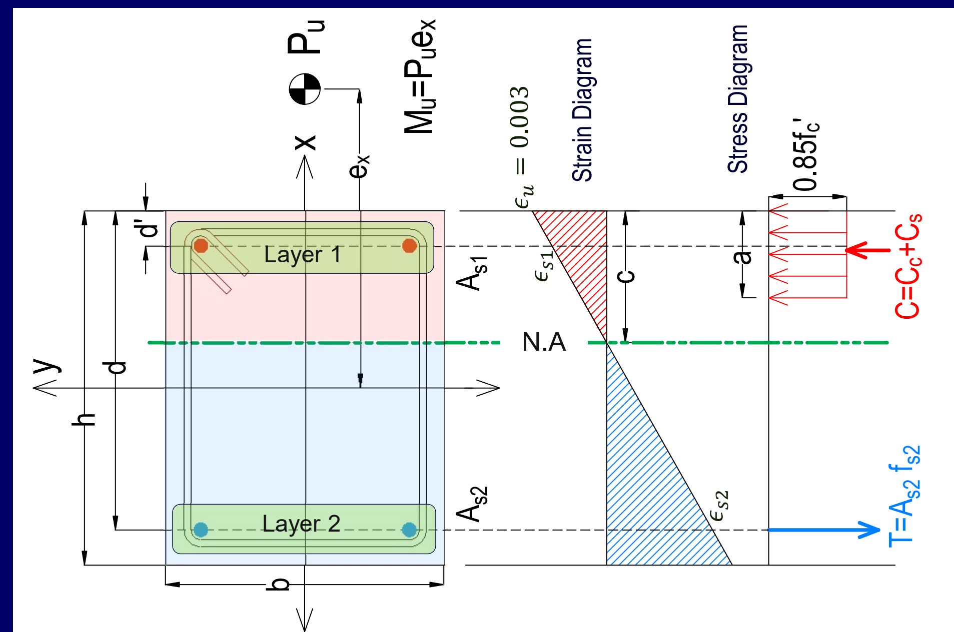
Bending moment can be transformed into eccentricity using $M_u = P_u\, e_x$

Red Region: Compression
Blue Region : Tension

Mechanics

❑ Capacity of Eccentrically loaded Column

a. Axial Capacity



From the Figure;

$$P_n = C_c + C_s - T_s$$

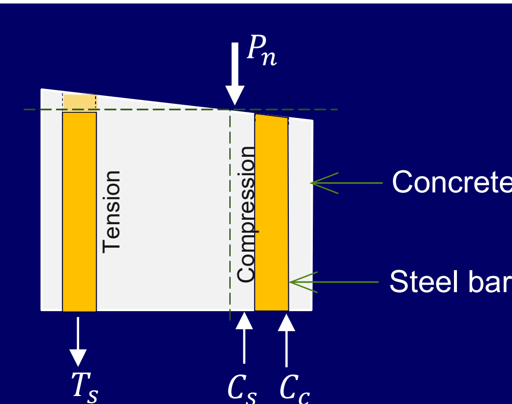
$$P_n = 0.85 f_c' ab + f_{s1} A_{s1} - f_{s2} A_{s2}$$

$$P_n = 0.85 f_c' \beta_1 cb + A_s (f_{s1} - f_{s2})$$

Taking $\beta_1 = 0.85$ gives

$$\emptyset P_n = \emptyset[0.72 f_c' bc + A_s (f_{s1} - f_{s2})] \quad \text{--- (8.3)}$$

(Note that A_s is steel area of a SINGLE layer, not the total steel area)



Mechanics

❑ Capacity of Eccentrically loaded Column

b. Flexural Capacity

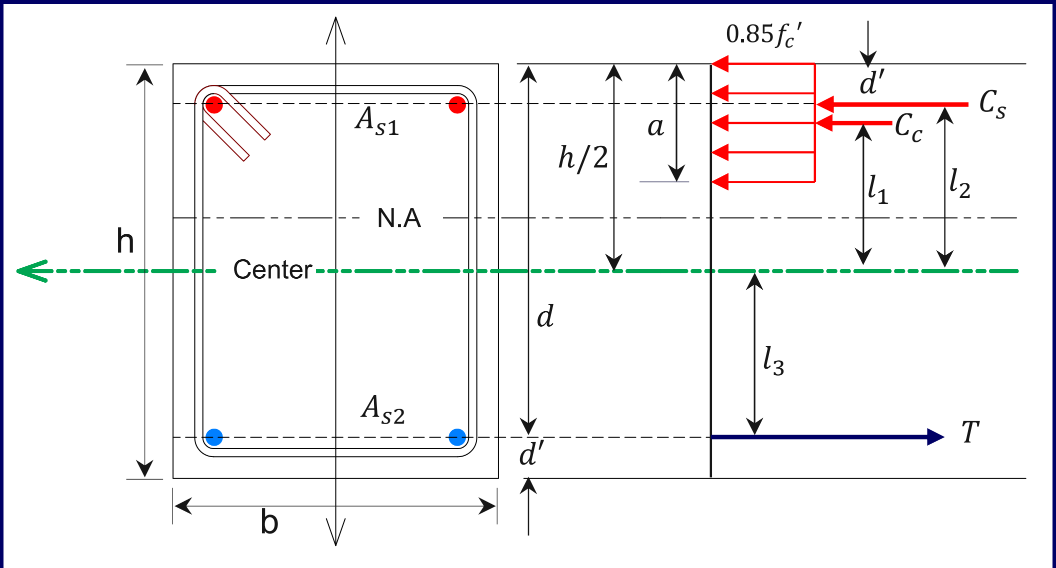
$$M_n = C_c l_1 + C_s l_2 + T_s l_3$$

From figure;

$$l_1 = \frac{h}{2} - \frac{a}{2}$$

$$l_2 = \frac{h}{2} - d'$$

$$l_3 = \frac{h}{2} - d'$$



Now, taking moment about the center of section,

$$M_n = C_c\left(\frac{h}{2} - \frac{a}{2}\right) + C_s\left(\frac{h}{2} - d'\right) + T_s\left(\frac{h}{2} - d'\right)$$

Where;

$C_c = 0.85f_c'ab = 0.85f_c'\beta_1 bc$

$C_s = A_{s1}f_{s1}$

$T_s = A_{s2}f_{s2}$

Mechanics

❑ Capacity of Eccentrically loaded Column

b. Flexural Capacity

$$M_n = 0.85 f_c' \beta_1 bc\left(\frac{h}{2} - \frac{a}{2}\right) + A_{s1} f_{s1}\left(\frac{h}{2} - d'\right) + A_{s2} f_{s2}\left(\frac{h}{2} - d'\right)$$

Since $A_{s1} = A_{s2} = A_s$, therefore

$$M_n = \frac{0.85^2}{2} f_c' bc(h - a) + A_{s1}(h/2 - d')(f_{s1} + f_{s2})$$

$$M_n = 0.36 f_c' bc(h - 0.85c) + A_s(h/2 - d')(f_{s1} + f_{s2})$$

From which the design flexural capacity is determined as,

$$\emptyset M_n = \emptyset[0.36 f_c' bc(h - 0.85c) + A_s(h/2 - d')(f_{s1} + f_{s2})]$$ ---- (8.4)

Mechanics

Capacity of Eccentrically loaded Column

Calculation of Normal Stresses in Steel (f_{s1} and f_{s2})

- Compressive stress f_{s1}

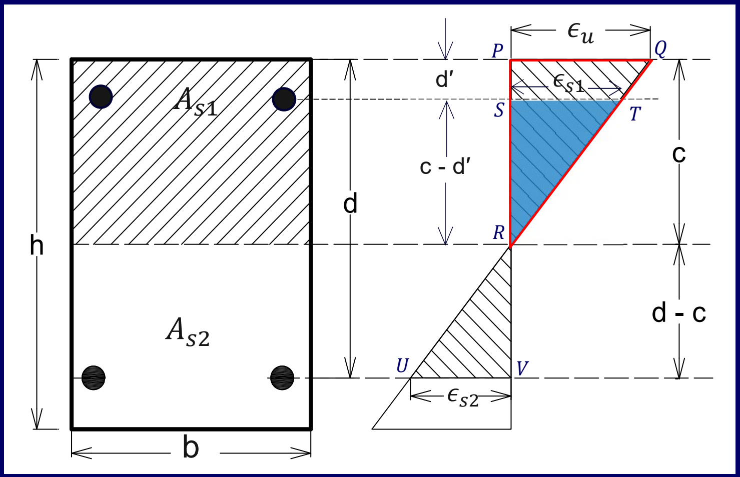
$$f_{s1} = E_s \epsilon_{s1}$$

From $\Delta PQR \leftrightarrow \Delta STR$, we have

$$\frac{\epsilon_{s1}}{c - d'} = \frac{\epsilon_u}{c} \Rightarrow \epsilon_{s1} = \frac{\epsilon_u (c - d')}{c}$$

$$f_{s1} = E_s \frac{\epsilon_u (c - d')}{c}$$

$$f_{s1} = 87\left(1 - \frac{d'}{c}\right) \quad \text{---- (8.5)}$$



Mechanics

❑ Capacity of Eccentrically loaded Column

• Calculation of Normal Stresses in Steel (f_{s1} *and* f_{s2})

- Tensile stress f_{s2}

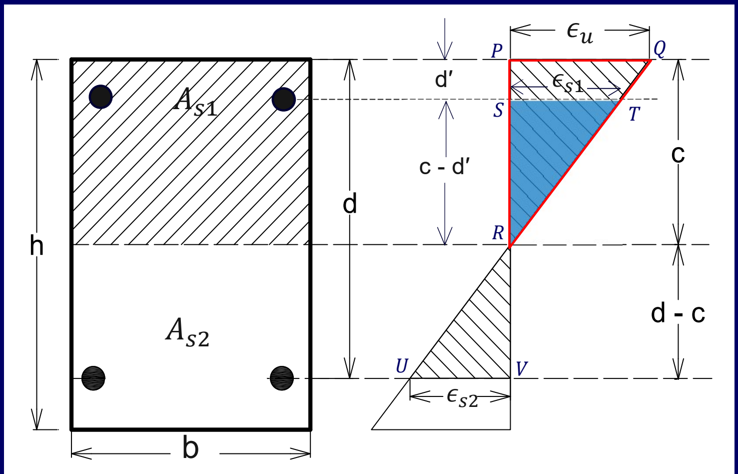
$$f_{s2} = E_s \epsilon_{s2}$$

From $\Delta PQR \leftrightarrow \Delta VUR$, we have

$$\frac{\epsilon_{s2}}{\mathrm{d} - \mathrm{c}} = \frac{\epsilon_u}{c} \Rightarrow \epsilon_{s2} = \frac{\epsilon_u(d - c)}{c}$$

$$f_{s2} = E_s \frac{\epsilon_u(d - c)}{c}$$

$$f_{s2} = 87\left(\frac{d}{c} - 1\right) \quad \text{---- (8.6)}$$



Mechanics

❑ Limitations of Equations 8.3 and 8.4

- It is important to note that equations 8.3 and 8.4 are valid for
  1. Two layers of reinforcement.
  2. $f'_c \leq 4000$ psi (since β was taken 0.85)
- For intermediate layers of reinforcement, the corresponding terms with "A_s" shall be added in the equations.

Mechanics

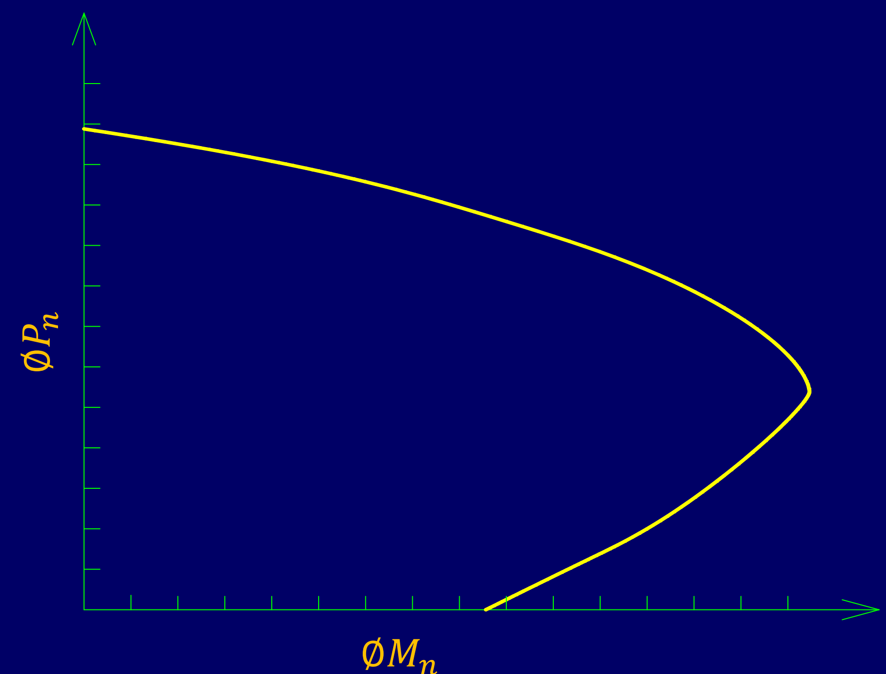
❑ Design Approaches

- Unlike the flexural members, the design of eccentrically loaded columns is relatively complicated due to the coupled action of axial force and bending moment, making it inconvenient to use straightforward equations.
- Two commonly used approaches for designing such columns are
  1. Interaction Diagrams
  2. Design Aids
- Both approaches are discussed in subsequent slides.

Interaction Diagram

❑ Introduction

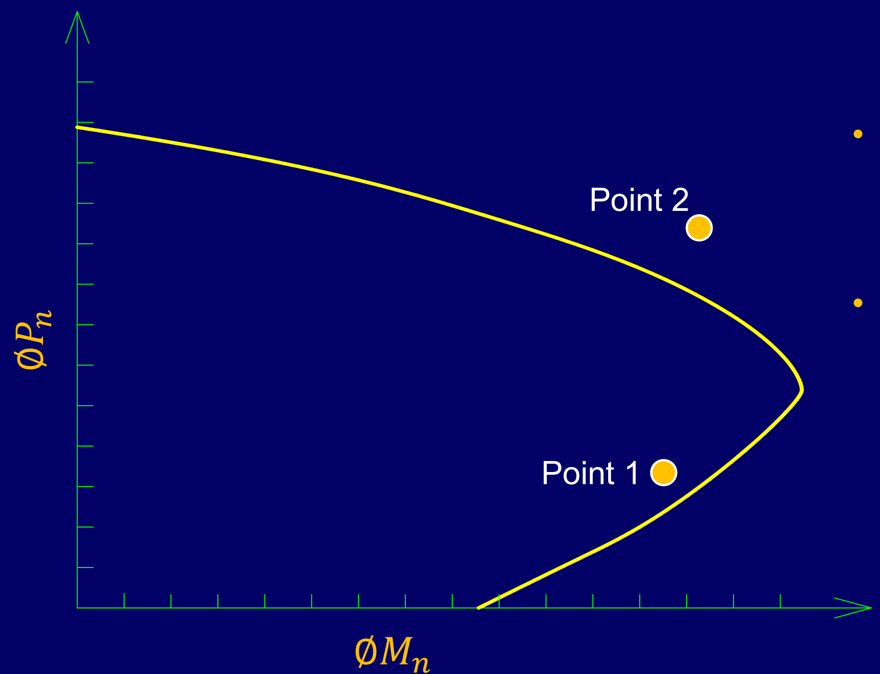
- A graphical representation that shows the interaction/relationship between axial capacity and flexural capacity of a structural member having known material properties, dimensions and reinforcement is called *Interaction diagram* or *Capacity curve*.



Interaction Diagram

❑ Failure Criteria

- If the factored demand in the form of P_u and M_u lies inside or at the border line of the design interaction diagram, the column will be deemed safe against the given demand, otherwise it is failed.

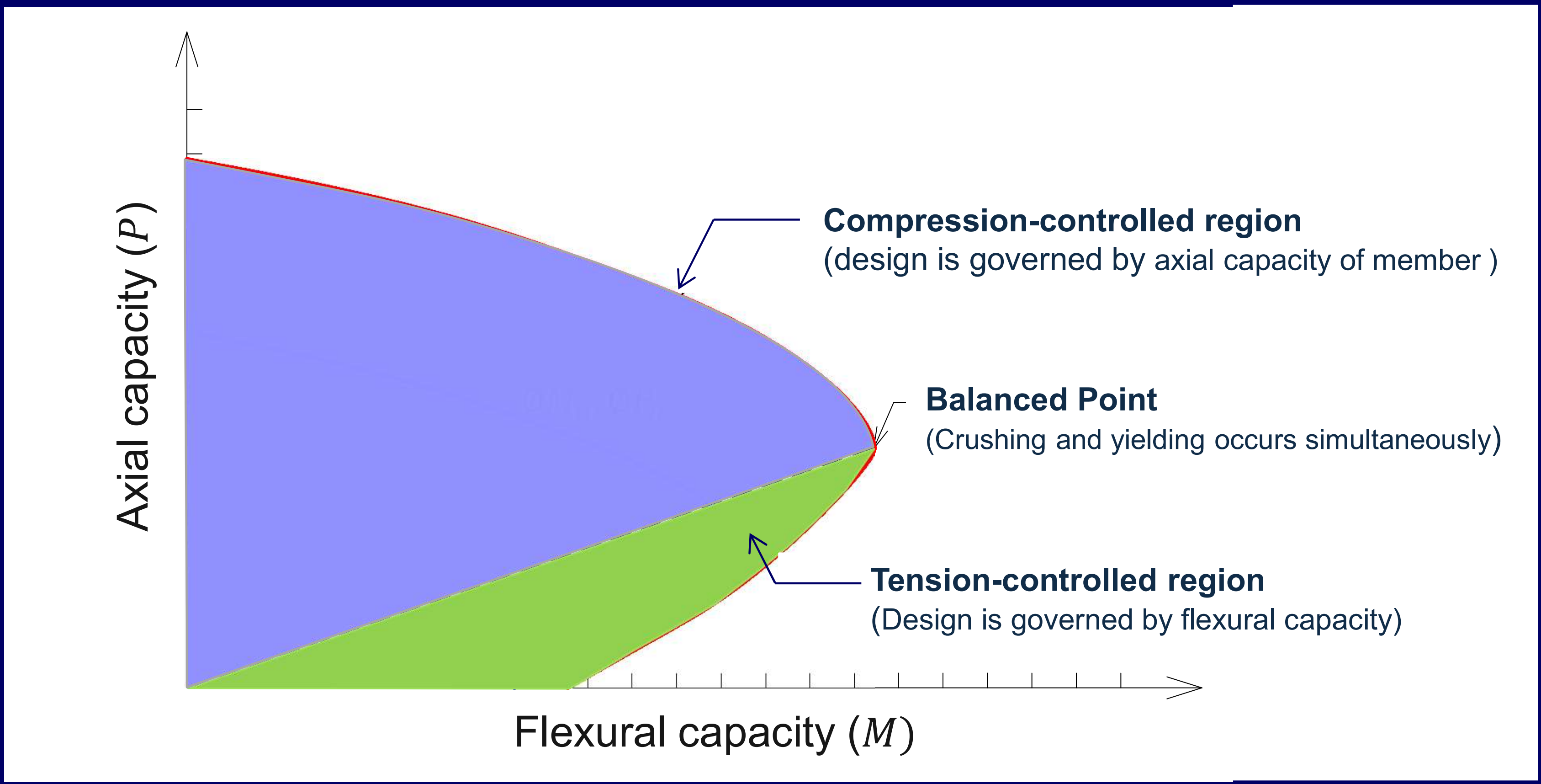


- **Point 1** lies within the curve, indicating that the column is safe against the demand.
- **Point 2** falls outside the curve, showing that the column's capacity is insufficient to carry the given demand.

Interaction Diagram

❑ Important features of Interaction diagram

❖ Control Regions

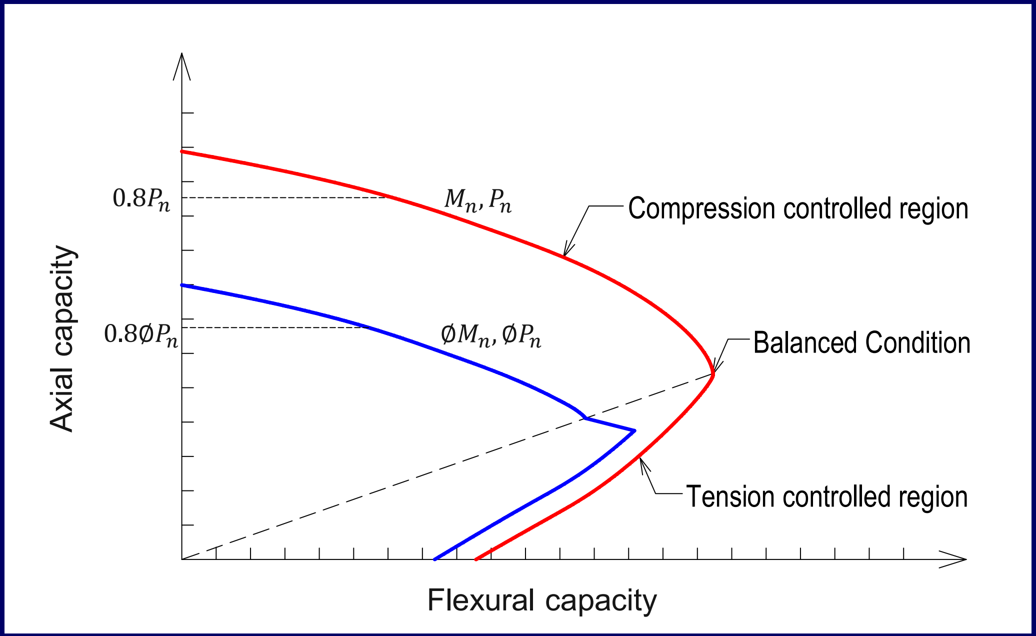


Interaction Diagram

❑ Important features of Interaction diagram

❖ Horizontal Cutoff

- The horizontal cutoff at upper end of the curve at a value of $\alpha \emptyset P_n$ represents the maximum design load specified in the ACI 318-19 10.4.2.1 for small eccentricities i.e., large axial loads.



Interaction Diagram

❑ Important features of Interaction diagram

❖ Linear Variation of Strength Reduction Factor ∅

Variation of Φ from 0.65 to 0.90 is applicable for $\epsilon_t \le f_y/E_s$ to $\epsilon_t = \epsilon_{yt} + 0.003$ respectively.

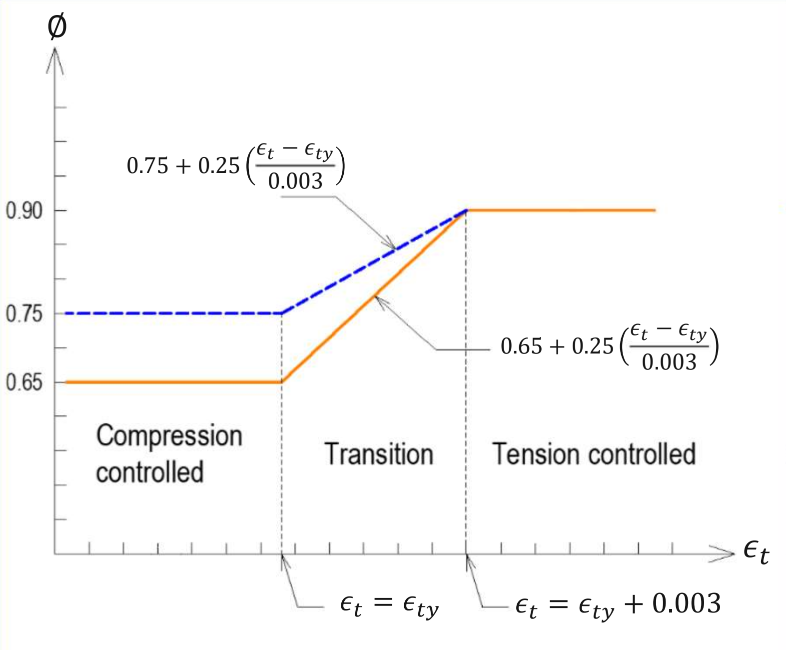
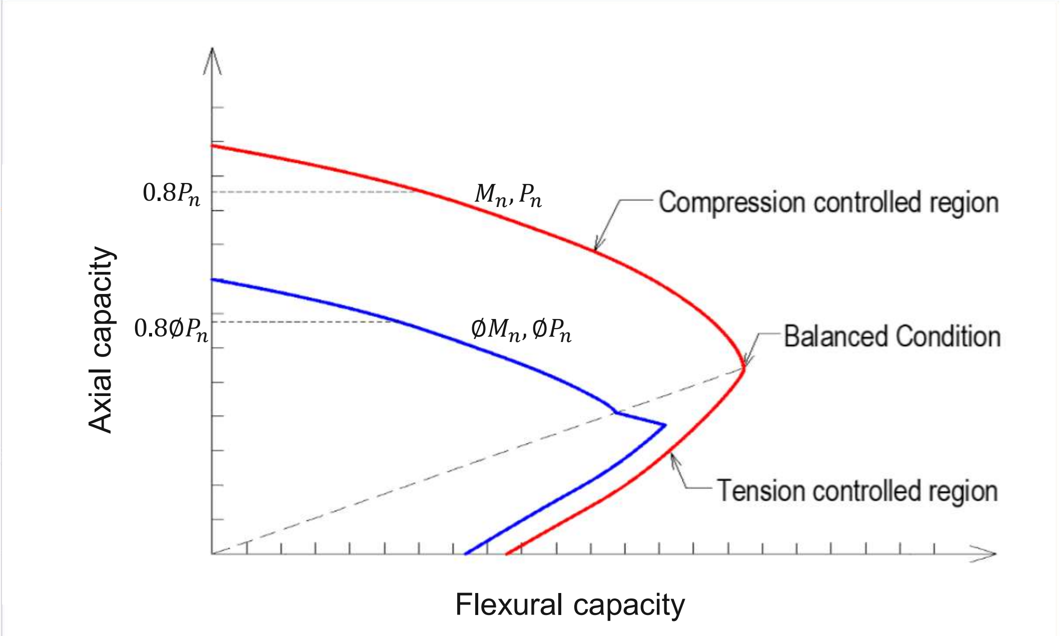


Fig. R21.2.2b



Interaction Diagram

Interaction Diagram

❑ Development of Interaction Diagram

- The interaction diagram can be developed by calculating certain points at key locations, using different values of *c*. These points are obtained from equations 8.3 and 8.4 as described below.

$$\emptyset P_n = \emptyset[0.72 f_c' bc + A_s(f_{s1} - f_{s2})]$$

$$\emptyset M_n = \emptyset[0.36 f_c' bc(h - 0.85c) + A_s(h/2 - d')(f_{s1} + f_{s2})]$$

$$f_{s1} = 87\left(1 - \frac{d'}{c}\right) \le f_y$$

$$f_{s2} = 87\left(\frac{d}{c} - 1\right) \le f_y$$

For a given set of material properties $(f_c{}', f_y)$ dimensions (b, h, d, d') and area of reinforcement (A_s) the only variable that remains unknown is the depth of the neutral axis, c.

Interaction Diagram

Development of Interaction Diagram

- Point 1 is determined using equation of concentrically loaded column ignoring α factor. $\emptyset P_n = \emptyset[0.85 f_c{}'(A_g - A_{st}) + f_y A_{st}]$
- All other control points can be obtained using the following 3 steps.

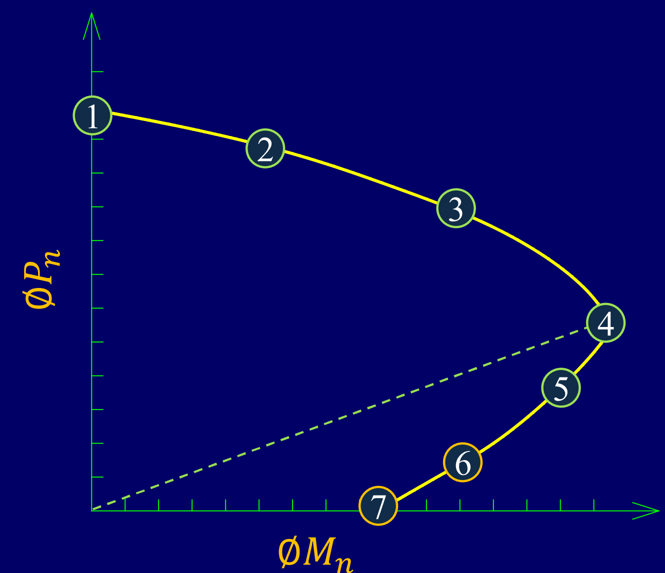
1. Assume reasonable value of c.
2. Compute f_{s1} and f_{s2}

$$f_{s1} = 87\left(1 - \frac{d'}{c}\right) \leq f_y \text{ and } f_{s2} = 87\left(\frac{d}{c} - 1\right) \leq f_y$$

3. Calculate $\emptyset P_n$ and $\emptyset M_n$

$$\emptyset P_n = \emptyset[0.72 f_c' bc + A_s(f_{s1} - f_{s2})]$$

$$\emptyset M_n = \emptyset[0.36 f_c' bc(h - 0.85c) + A_s(h/2 - d')(f_{s1} + f_{s2})]$$

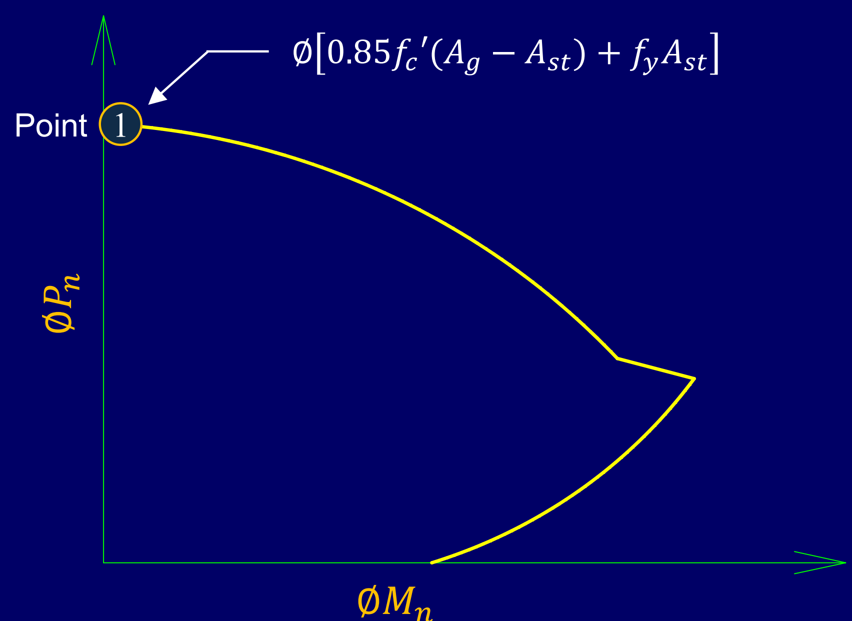


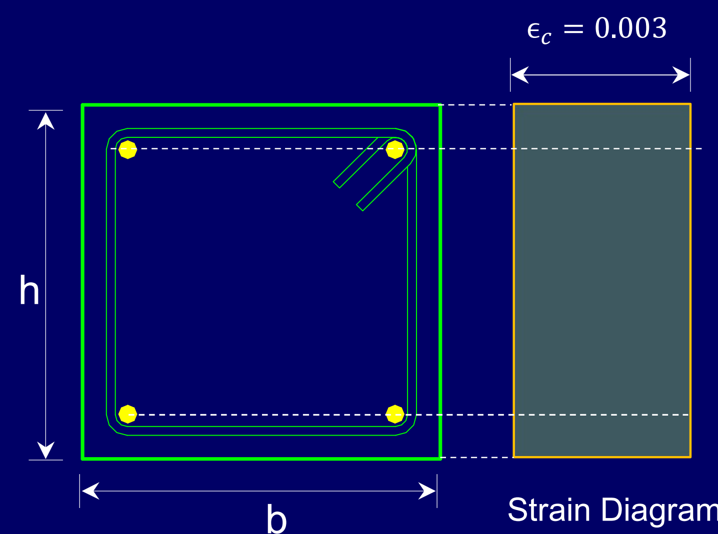
Interaction Diagram

Development of Interaction Diagram

Point 1

- Point representing capacity of column when concentrically loaded.
- This is the point at which M_n = 0.
- Design axial capacity equation of concentric column will be used.



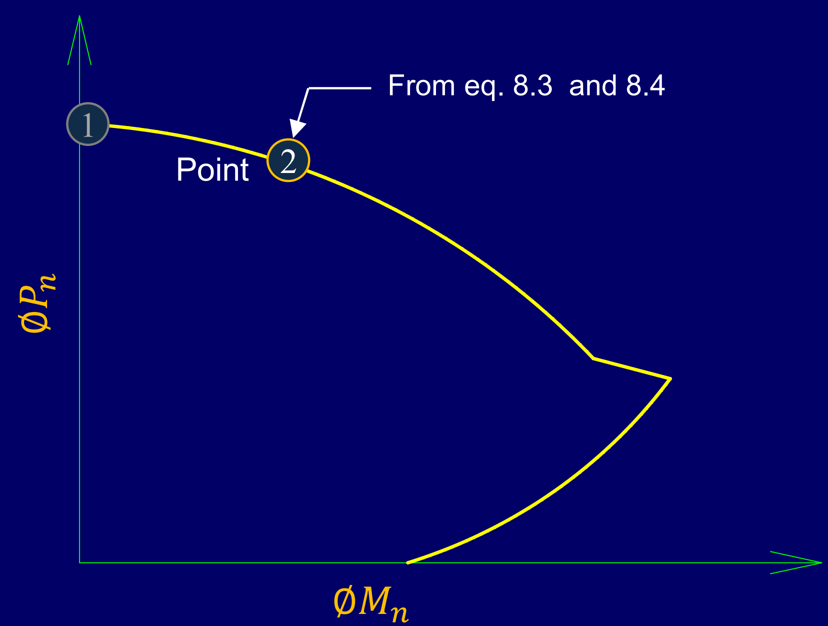


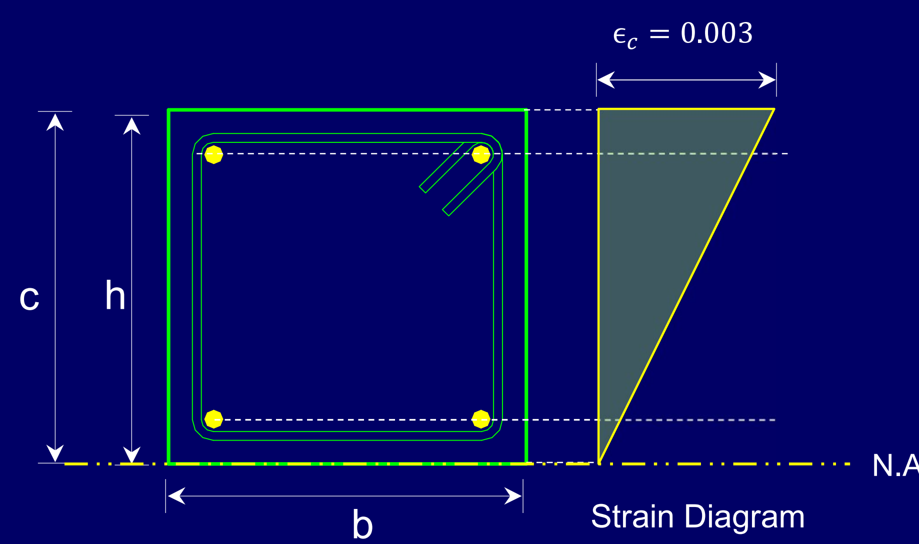
Interaction Diagram

❑ Development of Interaction Diagram

❖ Point 2

- This point corresponds to crushing of the concrete at the compression face of the section and zero stress at the other face.
- $c = h$ and $\emptyset = 0.65$



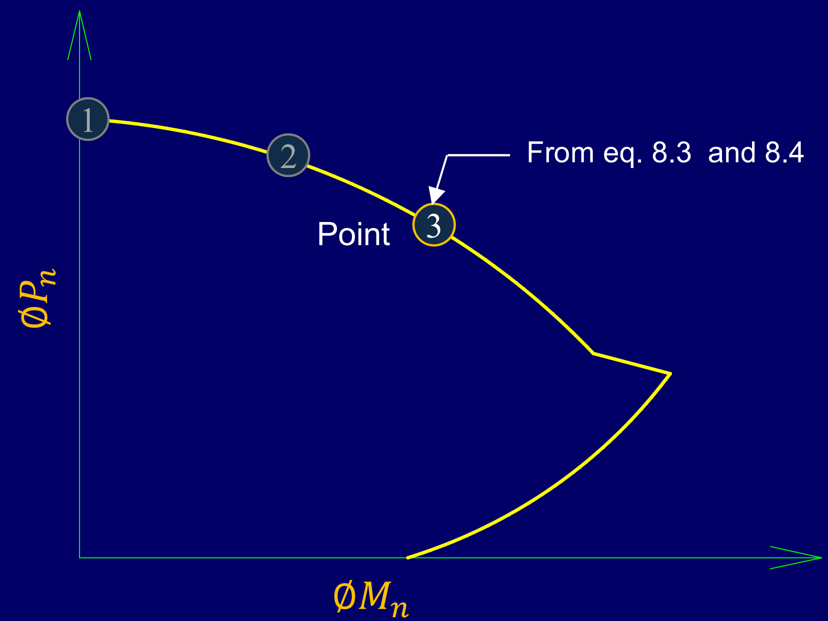


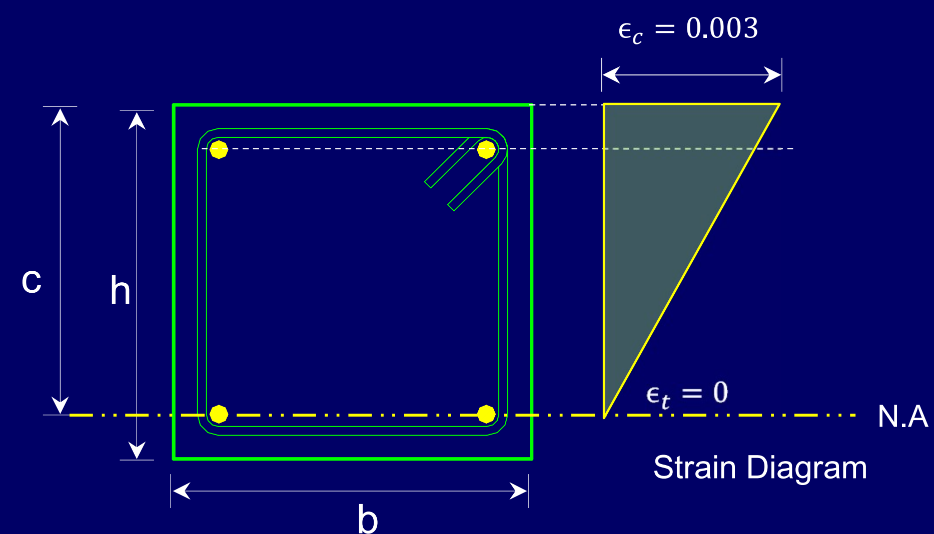
Interaction Diagram

❑ Development of Interaction Diagram

❖ Point 3

- At Point 3, the strain in the reinforcing bars farthest from the compression face is equal to zero.
- $c = h - d'$ and $\emptyset = 0.65$





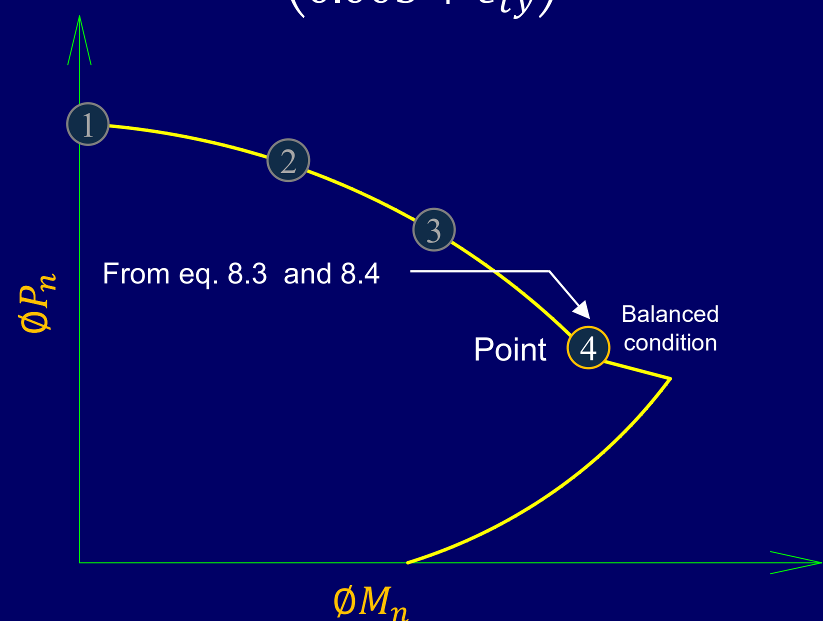
Interaction Diagram

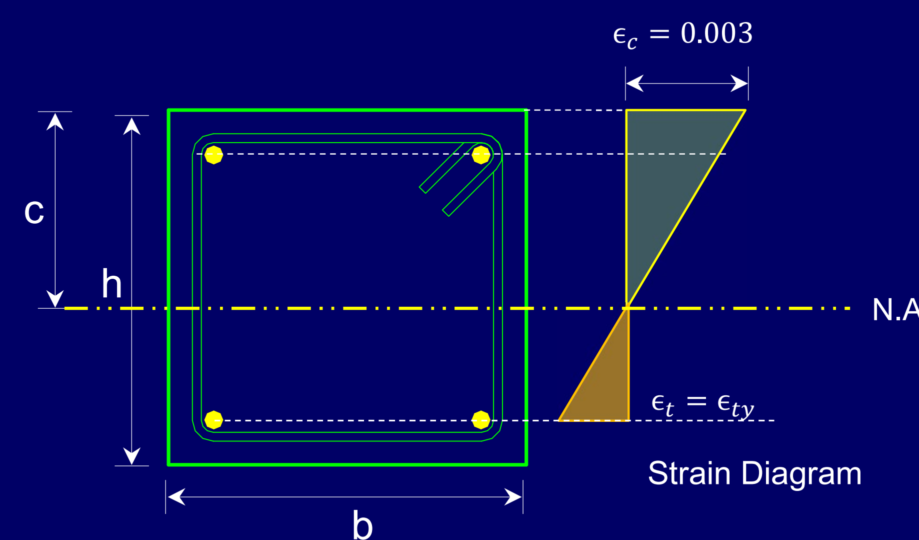
Development of Interaction Diagram

Point 4

- Point representing capacity of column for balance failure condition $\epsilon_t = \epsilon_{ty}$, $\epsilon_c = 0.003$ and $\emptyset = 0.65$

$$c = \left(\frac{0.003}{0.003 + \epsilon_{ty}}\right) d \qquad \Rightarrow c_{40} = 0.69d \quad and \; c_{60} = 0.59d$$





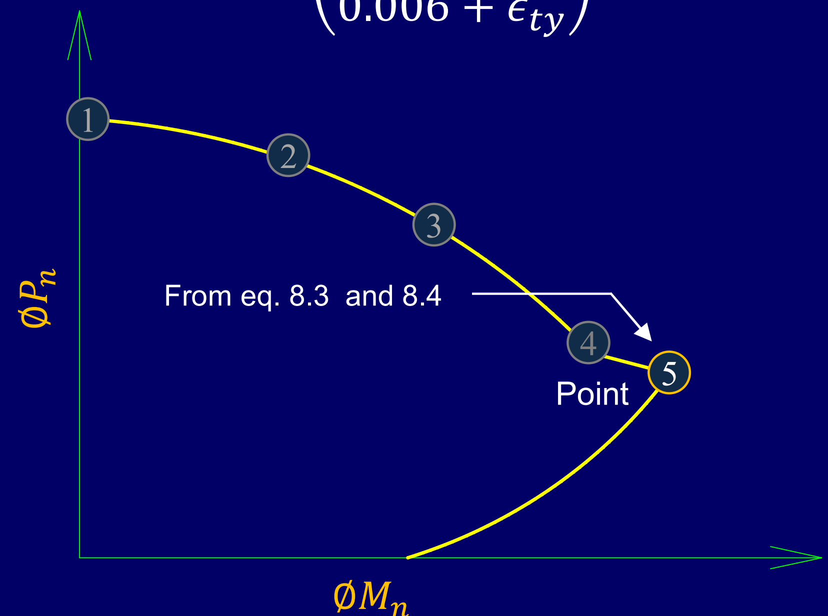
Interaction Diagram

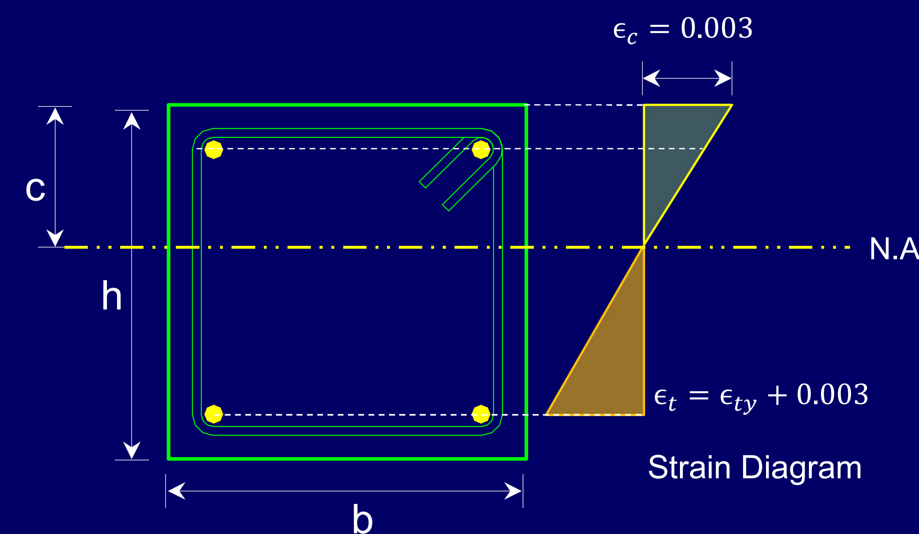
❑ Development of Interaction Diagram

❖ Point 5

- Point on capacity curve for which $\epsilon_t = \epsilon_{ty} + 0.003$, $\epsilon_c = 0.003$
- $\emptyset = 0.90$ or 0.65 (designer's preference)

$$c = \left(\frac{0.003}{0.006 + \epsilon_{ty}}\right)d \quad \Rightarrow c_{40} = 0.41d \quad and\ c_{60} = 0.37d$$



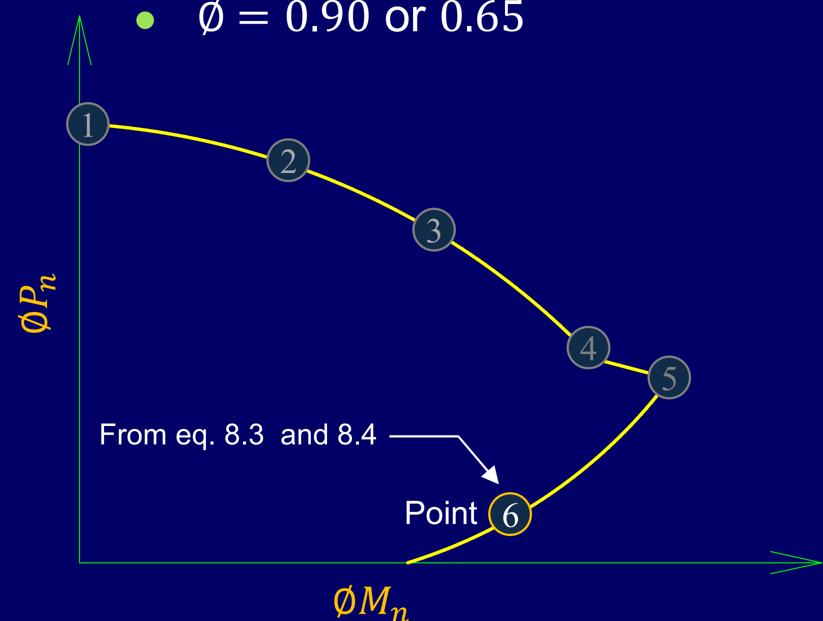


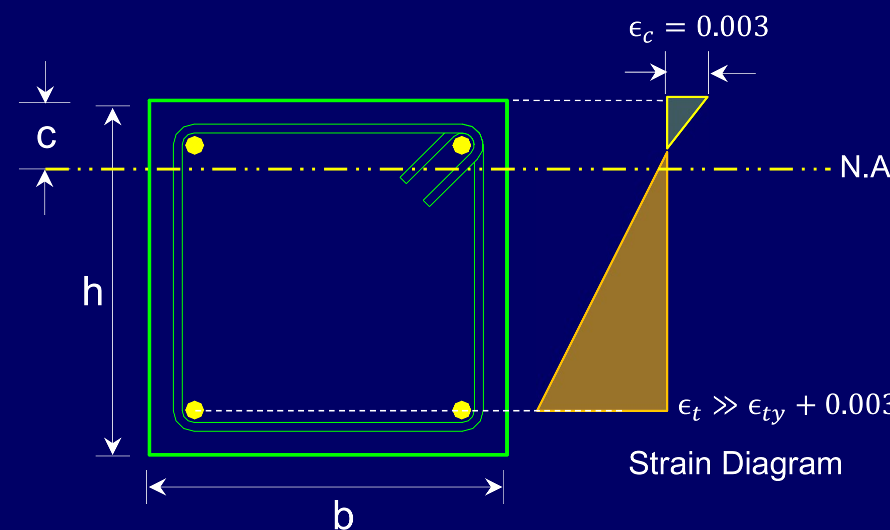
Interaction Diagram

❑ Development of Interaction Diagram

❖ Point 6

- Point on capacity curve at which the strain in tension steel is sufficiently greater than yield. Let consider ϵ_t two times that of point 5, then
- $c_{40} = 0.25d$, $c_{60} = 0.23d$ (for simplicity, assume c = 0.25d for both grades)
- $\emptyset = 0.90$ or 0.65





Interaction Diagram

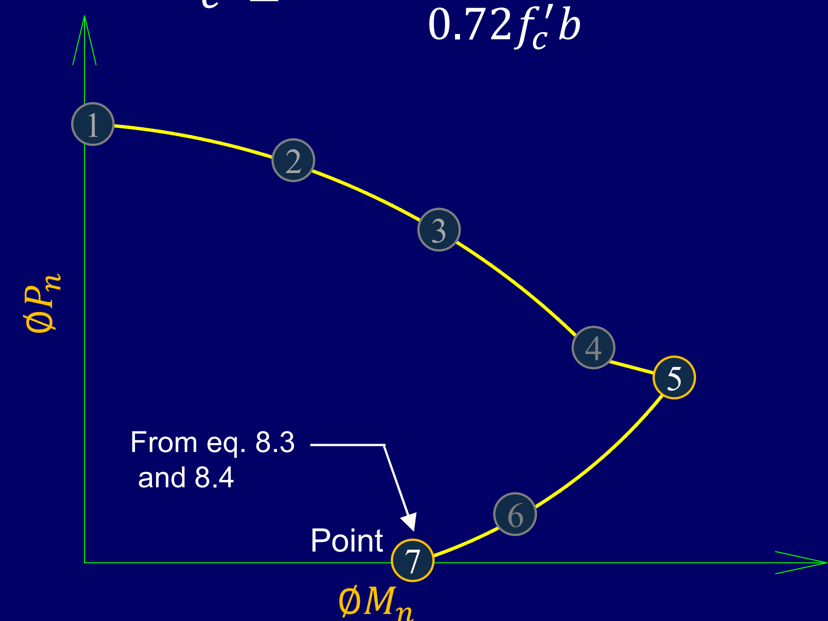
❑ Development of Interaction Diagram

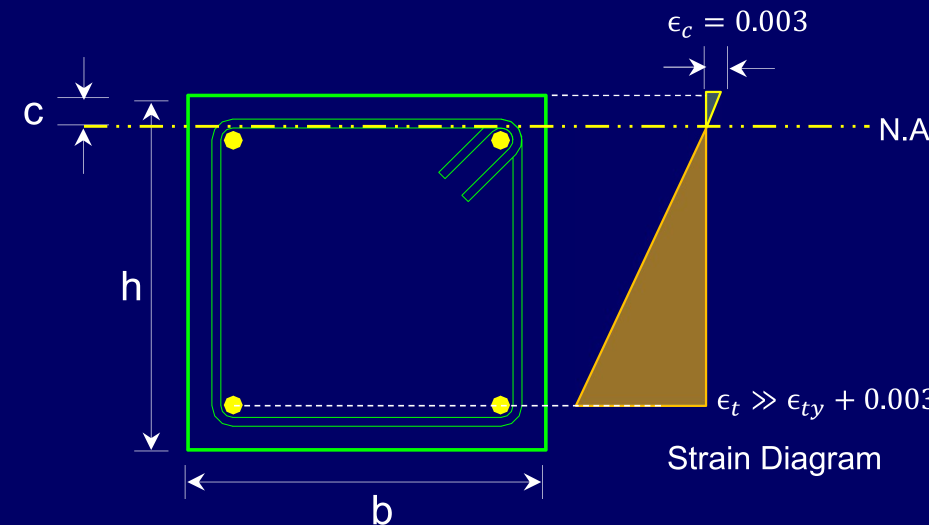
❖ Point 7

- This is the pure bending case on capacity curve at which the axial load is zero and and $\emptyset = 0.90$ or 0.65 and c can be taken as;

$$c = \frac{A_s\left[f_y - 87\left(1 - \frac{d'}{c}\right)\right]}{0.72 f_c' b}$$

(Please refer to the Appendix for the derivation of this equation.)





Interaction Diagram

❑ Development of Interaction Diagram (summary)

<table>
<tr><th>Point</th><th>c
$(in.)$</th><th>f_{s1}
(ksi)</th><th>f_{s2}
(ksi)</th><th>$\emptyset P_n$
(kip)</th><th>$\emptyset M_n$
$(ft.kip)$</th></tr>
<tr><td>1</td><td>Axial capacity</td><td>---</td><td>---</td><td>Eq. (1a)</td><td>0</td></tr>
<tr><td>2</td><td>$c = h$</td><td rowspan="6">$f_{s1} = 87\left(1 - \frac{d'}{c}\right) \leq f_y$</td><td rowspan="6">$f_{s2} = 87\left(\frac{d}{c} - 1\right) \leq f_y$</td><td rowspan="5">Eq. (1b)</td><td rowspan="6">Eq. (2)</td></tr>
<tr><td>3</td><td>$c = h - d'$</td></tr>
<tr><td>4</td><td>$c_{40} = 0.69d \;\; and \;\; c_{60} = 0.59d$</td></tr>
<tr><td>5</td><td>$c_{40} = 0.41d \;\; and \;\; c_{60} = 0.37d$</td></tr>
<tr><td>6</td><td>$c = 0.25d$</td></tr>
<tr><td>7</td><td>$c = \frac{A_s\left[f_y - 87\left(1 - \frac{d'}{c}\right)\right]}{0.72 f_c' b}$</td><td>0</td></tr>
</table>

$.\emptyset P_n = \emptyset[0.85 f_c'(A_g - A_{st}) + f_y A_{st}]$-------- Eq. (1a)

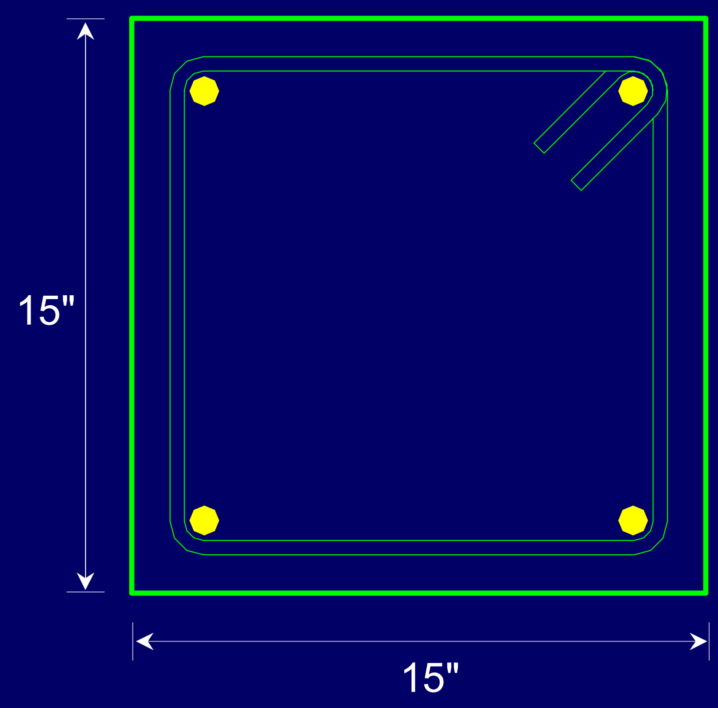
$\emptyset P_n = \emptyset[0.72 f_c' bc + A_s(f_{s1} - f_{s2})]$ -------- Eq. (1b)

$\emptyset M_n = \emptyset[0.36 f_c' bc(h - 0.85c) + A_s(h/2 - d')(f_{s1} + f_{s2})]$ -------- Eq. (2)

Interaction Diagram

❑ Example 3.8

- *Develop* interaction diagram for the given column. The material strengths are $f_c' = 3\ \text{ksi}$ and $f_y = 60\ \text{ksi}$ with 4 - #8 bars.



Interaction Diagram

❑ Solution

- **Given Data**

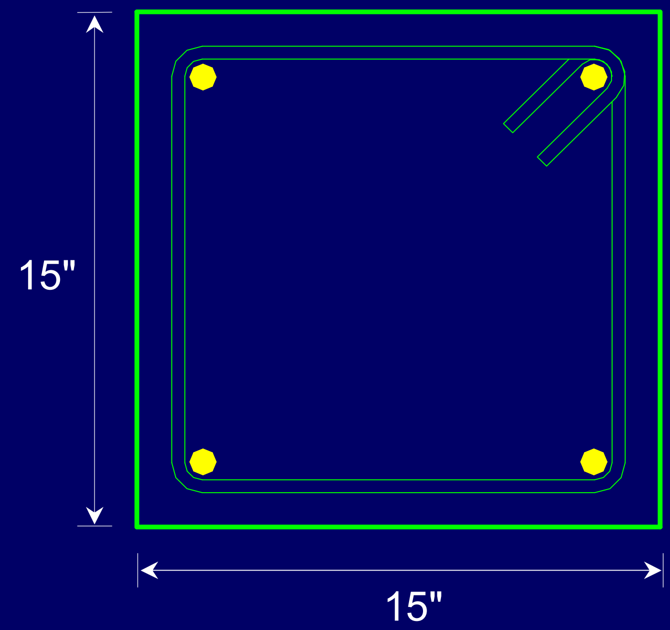
$b = 15''$

$h = 15''$

$A_s = 4 \times 0.79 = 3.16 \text{ in}^2$

$f_c' = 3 \text{ ksi}$

$f_y = 60 \text{ ksi}$



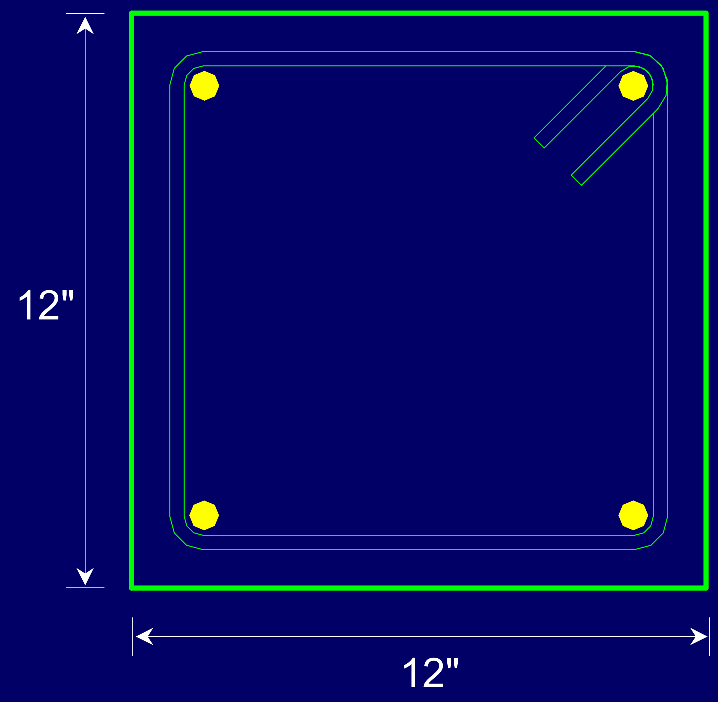
- **Required Data**

Develop Interaction diagram

Interaction Diagram

❑ Example 8.3

- Develop interaction diagram for the given column. The material strengths are $f_c' = 3ksi$ and $f_y = 40ksi$ with 4 #6 bars.



Interaction Diagram

❑ Solution

Point 1: Pure Axial Condition

From eq.(8.1) (ignoring α), we have

$$\emptyset P_n = 0.65\left[0.85 f_c{}'(A_g - A_s) + f_y A_s\right]$$

On substituting values;

$$\emptyset P_n = 0.65[0.85 \times 3\ (144 - 1.76) + 40 \times 1.76]$$

$$\emptyset P_n = 281.5\ kip$$

Now,

$$\emptyset M_n = 0$$

Interaction Diagram

Solution

Point 1: Pure Axial Condition

The pure axial capacity of column (ignoring α) is given by

$$\emptyset P_n = 0.65\left[0.85 f_c{}'(A_g - A_s) + f_y A_s\right]$$

On substituting values;

$$\emptyset P_n = 0.65[0.85 \times 3\ (225 - 3.16) + 60 \times 3.16]$$

$$\emptyset P_n = 490.9 \text{ kip}$$

And

$$\emptyset M_n = 0$$

Interaction Diagram

❑ Solution

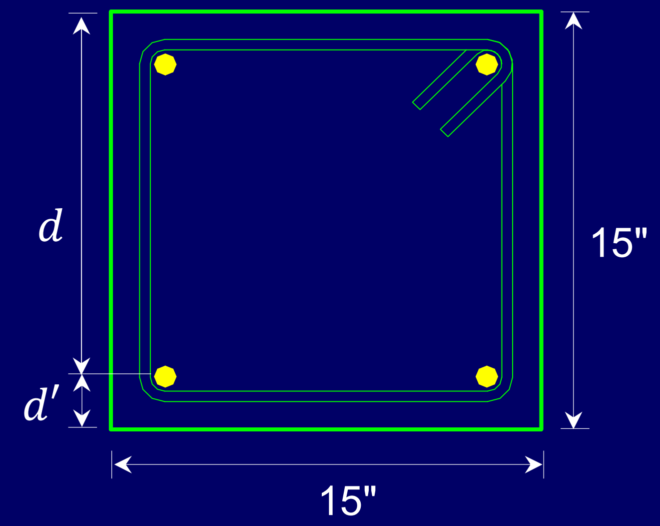
Point 2

d' and d can be calculated as;

$$d' = 1.5 + \frac{3}{8} + \frac{8}{16} = 2.375''$$

and

$$d = 15 - d' = 12.625''$$



Now, with c = h = 15″

$$f_{s1} = 87(1 - d'/c) = 87(1 - 2.375/15) = 73.2 \text{ ksi} > f_y \rightarrow use\ f_{s1} = 60 \text{ ksi}$$

and

$$f_{s2} = 87(d/c - 1) = 87(12.625/15 - 1) = -13.8 \text{ ksi} < f_y \rightarrow use\ f_{s2} = -13.8 \text{ ksi}$$

Interaction Diagram

❑ Solution

❖ Point 2

Now, from eq.(3.3) and (3.4) we have

$$\emptyset P_n = \emptyset[0.72 f_c' bc + A_s(f_{s1} - f_{s2})]$$

⇐ Note that A_s is steel area of single layer.

$$= 0.65[0.72 \times 3 \times 15 \times 15 + 1.58(60 + 13.8)] = \mathbf{391.7\ kip}$$

Similarly,

$$\emptyset M_n = \emptyset[0.36 f_c' bc(h - 0.85c) + A_s(h/2 - d')(f_{s1} + f_{s2})]$$

$$= 0.65[0.36 \times 3 \times 15 \times 15(15 - 0.85 \times 15) + 1.58(7.5 - 2.375)(60 - 13.8)]$$

$$= 598.56\ \text{in.kip} \quad \text{or} \quad \mathbf{49.9\ ft.kip}$$

Interaction Diagram

Solution

Point 3

with $c = h - d' = 15 - 2.375 = 12.625''$

$$f_{s1} = 87(1 - 2.375/12.625) = 70.6 \text{ ksi} > f_y \rightarrow use\ f_{s1} = 60 \text{ ksi}$$

$$f_{s2} = 87(12.625/12.625 - 1) = 0$$

Now,

$$\emptyset P_n = 0.65[0.72 \times 3 \times 15 \times 12.625 + 1.58(60 - 0)] = \mathbf{327.5\ kip}$$

$$\emptyset M_n = 0.65[0.36 \times 3 \times 15 \times 12.625(15 - 0.85 \times 12.625) + 1.58(5.125)(60 + 0)]$$

$$= 883.29 \text{ in. kip} \quad \text{or} \quad \mathbf{73.6\ ft.kip}$$

Interaction Diagram

❑ Solution

❖ Point 4: Balanced Condition

with $c_{60} = 0.59d = 0.59 \text{ x } 12.625 = 7.45''$

$f_{s1} = 87(1 - 2.375/7.45) = 59.3\ ksi < f_y \rightarrow use\ f_{s1} = 59.3 \text{ ksi}$

$f_{s2} = 87(12.625/7.45 - 1) = 60.43 > f_y \rightarrow use\ f_{s2} = 60 \text{ ksi}$

Now,

$\emptyset P_n = 0.65[0.72 \times 3 \times 15 \times 7.45 + 1.58(59.3 - 60)] = \mathbf{156.2\ kip}$

$\emptyset M_n = 0.65[0.36 \times 3 \times 15 \times 7.45(15 - 0.85 \times 7.45) + 1.58(5.125)(119.30)]$

$= 1307.87 \text{ in. kip} \quad \text{or} \quad \mathbf{109.0\ ft.kip}$

Interaction Diagram

❑ Solution

❖ Point 5

with $c_{60} = 0.37d = 0.37 \text{ x } 12.625 = 4.67''$

$f_{s1} = 87(1 - 2.375/4.67) = 42.8\ ksi < f_y \rightarrow use\ f_{s1} = 42.8 \text{ ksi}$

$f_{s2} = 87(12.625/4.67 - 1) = 148.3 > f_y \rightarrow use\ f_{s2} = 60 \text{ ksi}$

Now,

$\emptyset P_n = 0.90[0.72 \times 3 \times 15 \times 4.67 + 1.56(42.8 - 60)] = \mathbf{111.8\ kip}$

$\emptyset M_n = 0.90[0.36 \times 3 \times 15 \times 4.67(15 - 0.85 \times 4.67) + 1.58(5.125)(42.8 + 60)]$

$= 1500.23 \text{ in.kip } or\ \mathbf{125.0\ ft.kip}$

Interaction Diagram

Solution

Point 6

with $c = 0.25d = 0.25 \text{ x } 12.625 = 3.16''$

$f_{s1} = 87(1 - 2.375/3.16) = 21.6 \text{ ksi} < f_y \rightarrow use\ f_{s1} = 21.6 \text{ ksi}$

$f_{s2} = 87(12.625/3.16 - 1) = 260.6 > f_y \rightarrow use\ f_{s2} = 60 \text{ ksi}$

Now,

$\emptyset P_n = 0.90[0.72 \times 3 \times 15 \times 3.16 + 1.58(21.6 - 60)] = \mathbf{37.5\ kip}$

$\emptyset M_n = 0.90[0.36 \times 3 \times 15 \times 3.16(15 - 0.85 \times 3.16) + 1.58(5.125)(21.6 + 60)]$

$= 1162.02 \text{ in.kip} \quad \text{or} \quad \mathbf{96.8\ ft.kip}$

Interaction Diagram

Solution

Point 7: Pure Bending Condition

$$c = \frac{A_s\left[f_y - 87\left(1 - \frac{d'}{c}\right)\right]}{0.72 f_c' b} \Rightarrow \text{on solving and neglecting negative root}, \text{c} = 2.58''$$

$$f_{s1} = 87(1 - 2.375/2.58) = 6.9 \text{ ksi} < f_y \rightarrow use\ f_{s1} = 6.9 \text{ ksi}$$

$$f_{s2} = 87(12.625/2.58 - 1) = 338.7 \text{ ksi} > f_y \rightarrow use\ f_{s2} = 60\ ksi$$

Now,

$$\emptyset P_n = 0$$

$$\emptyset M_n = 0.90[0.36 \times 3 \times 15 \times 2.58(15 - 0.85 \times 2.58) + 1.58(5.125)(66.9)]$$

$$= 969.30 \text{ in. kip} \quad \text{or} \quad \mathbf{80.8\ ft.kip}$$

Interaction Diagram

Solution

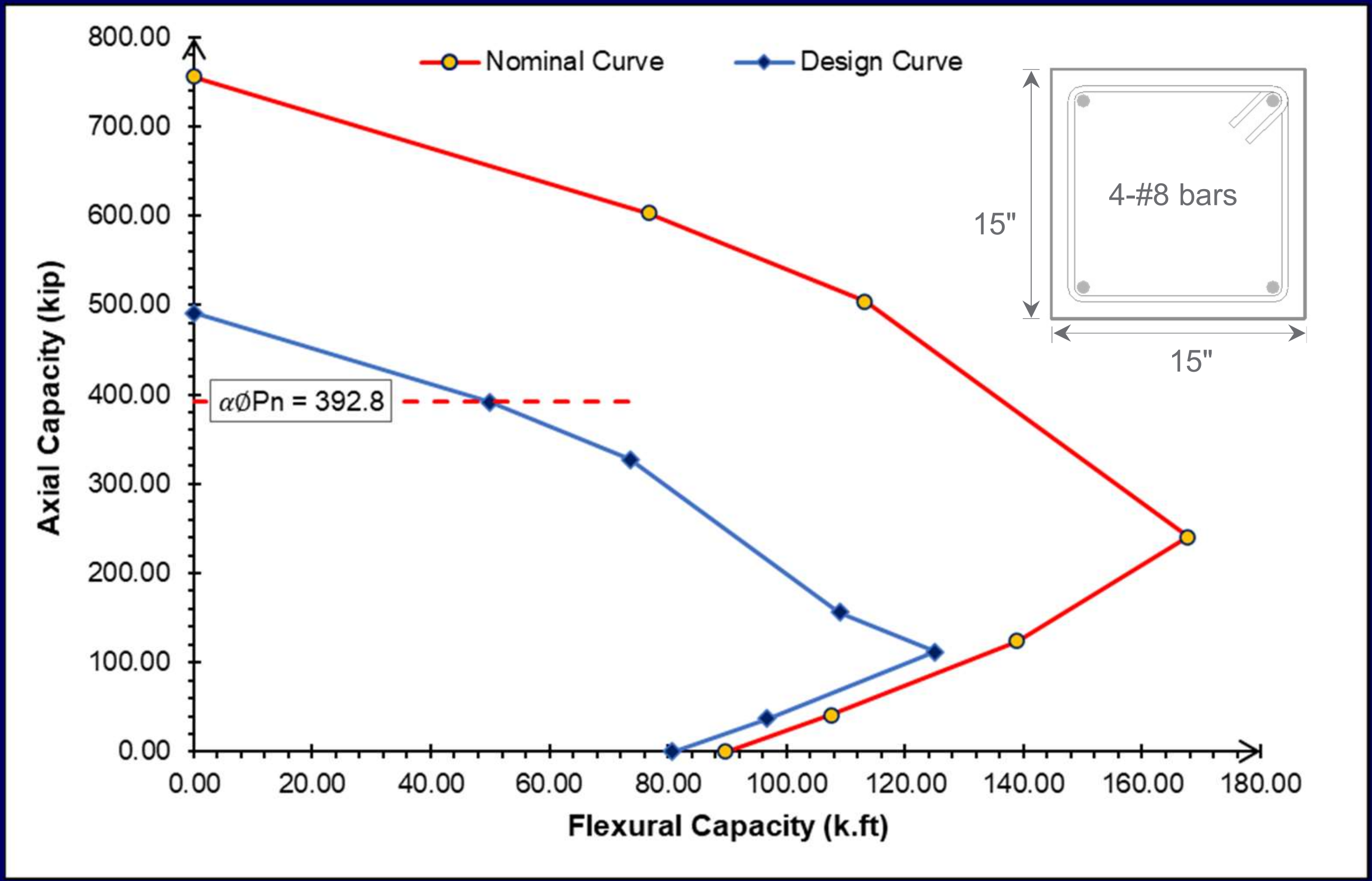
Summary of Calculations

Point	c $(in.)$	f_{s1} (ksi)	f_{s2} (ksi)	$\emptyset P_n$ (kip)	$\emptyset M_n$ $(kip.ft)$	Remarks
1	---	---	---	281.5	0	Compression controlled region
2	15.00	60.0	-13.8	391.7	49.9	
3	12.625	60.0	0.0	327.5	73.6	
4	7.45	59.3	60.0	156.2	109.0	Balanced condition
5	4.67	42.8	60.0	111.8	125.0	Tension controlled region
6	3.16	21.6	60.0	37.5	96.8	
7	2.58	6.9	60.0	0.0	80.8	

Interaction Diagram

❑ Solution

❖ Plot of Interaction Curve



Interaction Diagram

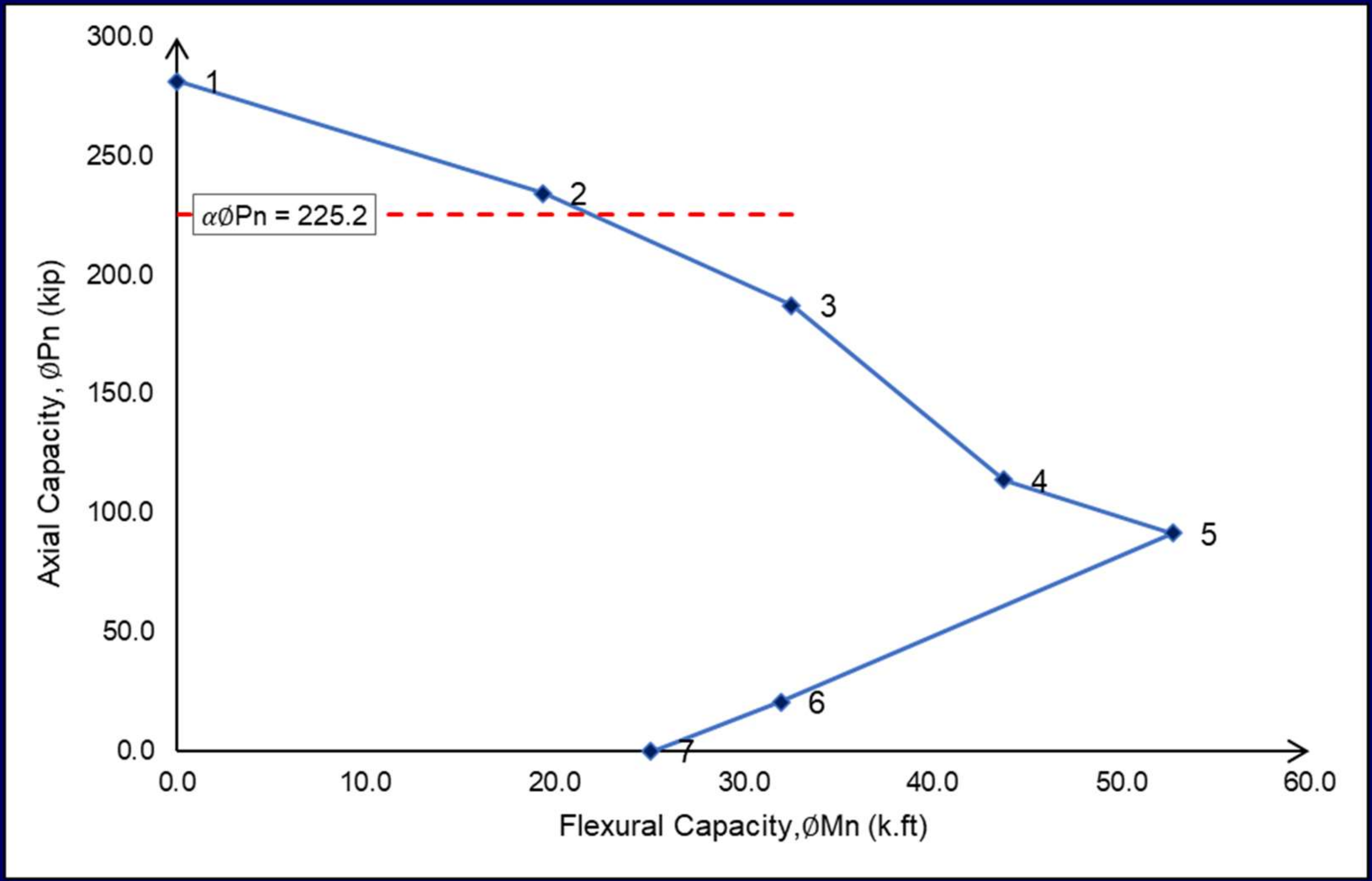
❑ Solution

❖ Summary of calculations

Point	c $(in.)$	f_{s1} (ksi)	f_{s2} (ksi)	$\emptyset P_n$ (kip)	$\emptyset M_n$ $(kip.ft)$	Remarks
1	---	---	---	281.5	0	Compression controlled region
2	12.00	40.0	-16.3	234.4	19.4	
3	9.75	40.0	0.0	187.1	32.6	
4	6.73	40.0	39.1	113.9	43.8	Balanced condition
5	4.00	38.0	40.0	91.8	52.8	Tension controlled region
6	2.25	9.8	40.0	20.8	32.0	
7	1.90	0	-16.0	0	25.1	

Interaction Diagram

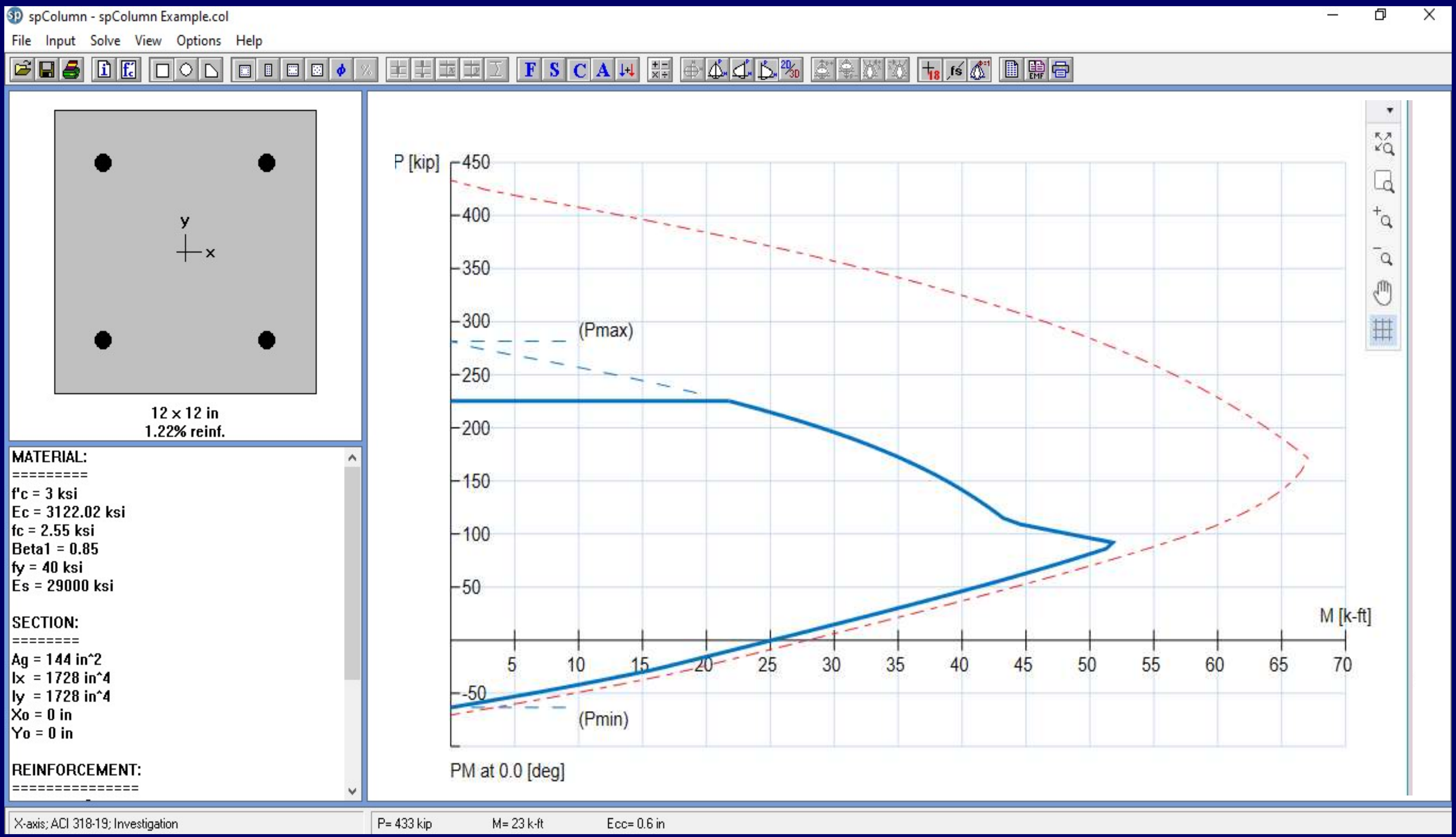
- Solution
  - Plot of Interaction Curve



Interaction Diagram

Solution

- Plot of Interaction curve (in sPCOLUMN)



Design Aids

Introduction

- In practice, Design Aids are used for the design of eccentrically loaded RC columns.
- They can be found in handbooks and special volumes published by the American Concrete Institute (ACI).
- They cover the most frequent practical cases, such as symmetrically reinforced rectangular and square columns and circular spirally reinforced columns.
- Design Aids for different ranges of f_c' and f_y are provided in Appendix. (at the end of this lecture).

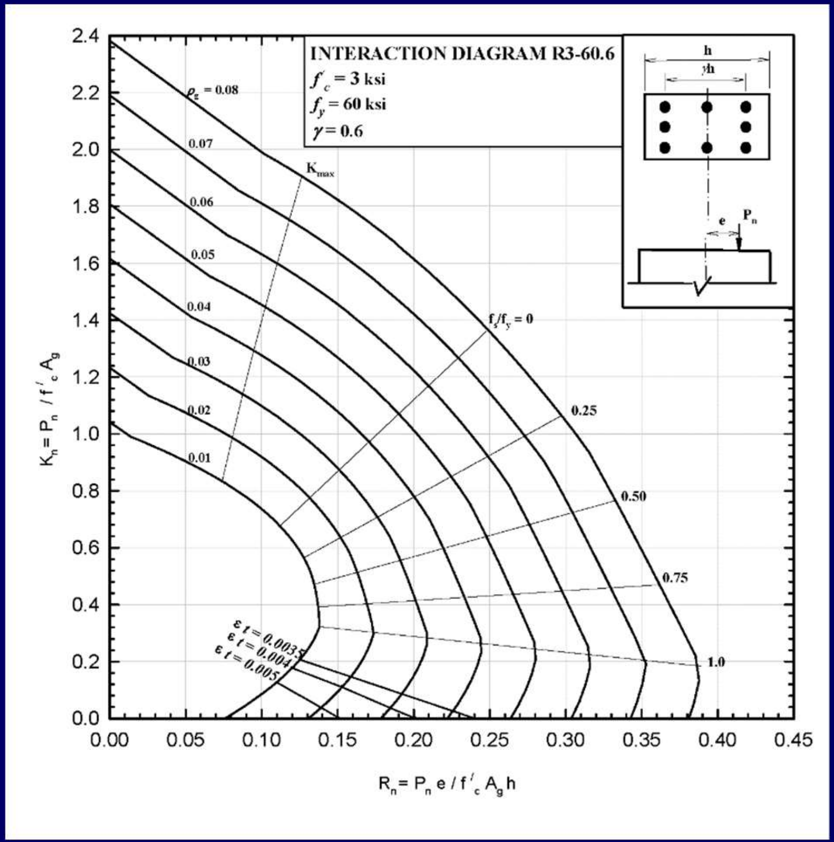
Design Aids

❑ Procedure of using Design Aids

1. Select a trial cross-sectional dimensions b and h

2. Calculate the ratio γ based on required cover distances to the bar centroids and select the corresponding column design chart.

$$\gamma = \frac{h - 2\,d'}{h}$$



Design Aids

❑ Procedure of using Design Aids

4. Calculate K_n and R_n factor

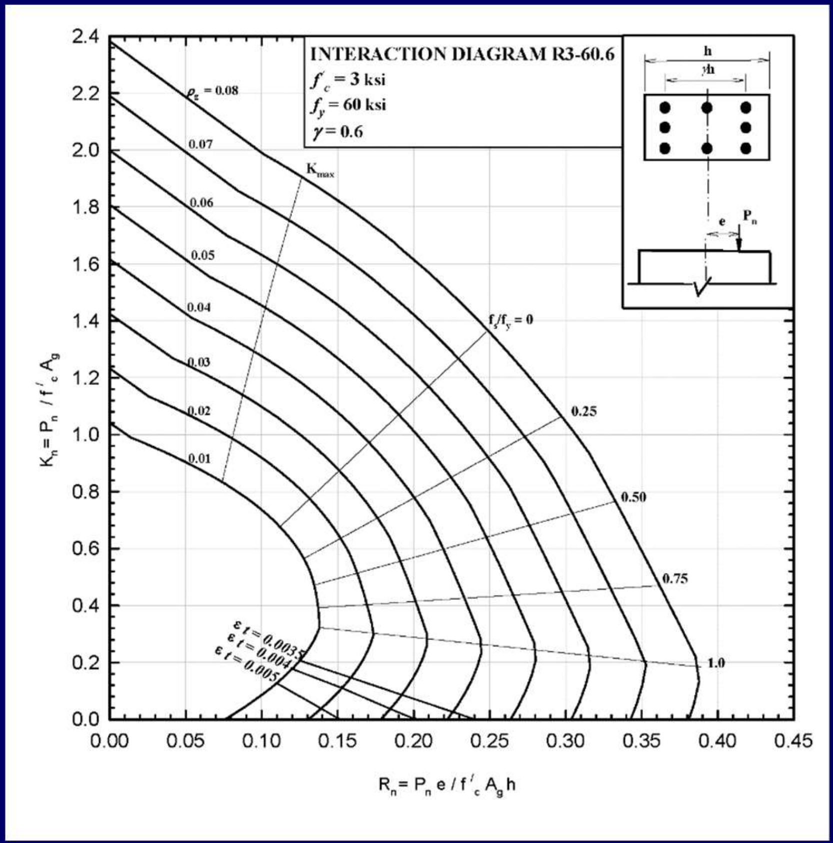
$$K_n = \frac{P_u}{\emptyset f_c' bh}$$

$$R_n = \frac{M_u}{\emptyset f_c' bh^2}$$

5. Using values of K_n and R_n, read the required reinforcement ratio ρ_g from the graph.

6. Calculate the total steel area

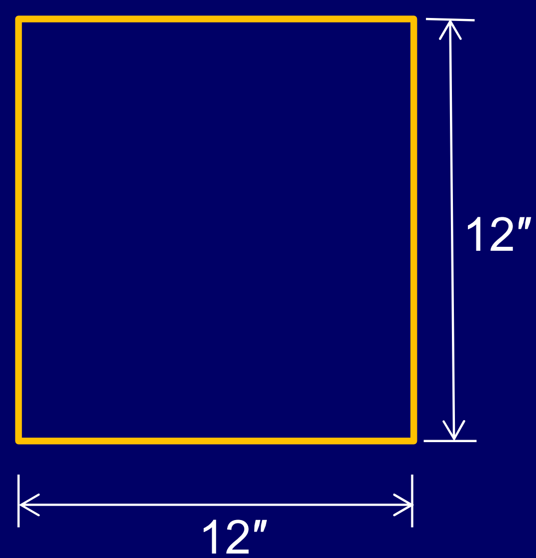
$$A_{st} = \rho_g bh$$



Design Aids

Example 8.4

- Using design aids, design a 12″ square column section to support a factored load of 145 kip and a factored moment of 40 kip-ft. The material strengths are $f_c{}' = 4\ ksi$ and $f_y = 60\ ksi$.



Design Aids

Solution

1. Dimensions are already given to us;

$b = h = 12"$

2. Calculate ratio γ

$$\gamma = \frac{h - 2d'}{h}$$

Assuming $d' = 2.5in$

$$\gamma = \frac{12 - 2(2.5)}{12} = 0.583$$

$$\gamma \approx 0.60$$

Design Aids

Solution

3. Calculate K_n and R_n factor

$$K_n = \frac{P_u}{\emptyset f_c' bh} = \frac{145}{0.65 \times 4 \times 12 \times 12}$$

$$K_n = 0.40$$

$$R_n = \frac{M_u}{\emptyset f_c' bh^2} = \frac{40 \times 12}{0.65 \times 4 \times 12 \times 12^2}$$

$$R_n = 0.11$$

For $\gamma = 0.60$, $f_c' = 4ksi$ and $f_y = 60ksi$, The relevant Design Aid is DA-5 (from Appendix)

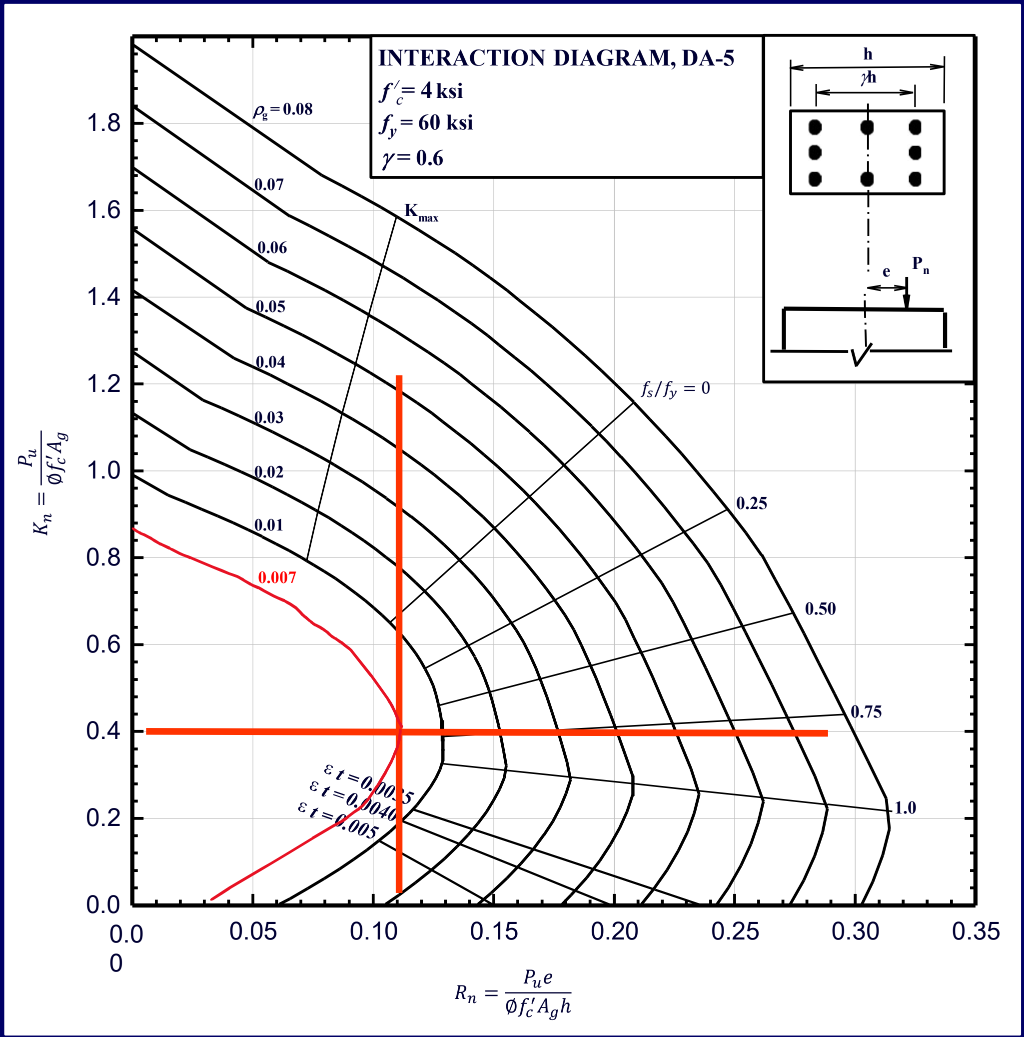
Design Aids

❑ Solution

4. Read ρ_g from the graph

$$\rho_g = 0.007 < 0.01$$

$$\rightarrow Take\ \ \rho_g = 0.01$$



5. Calculate Area of steel

$$A_{st} = 0.01A_g = 1.44in^2$$

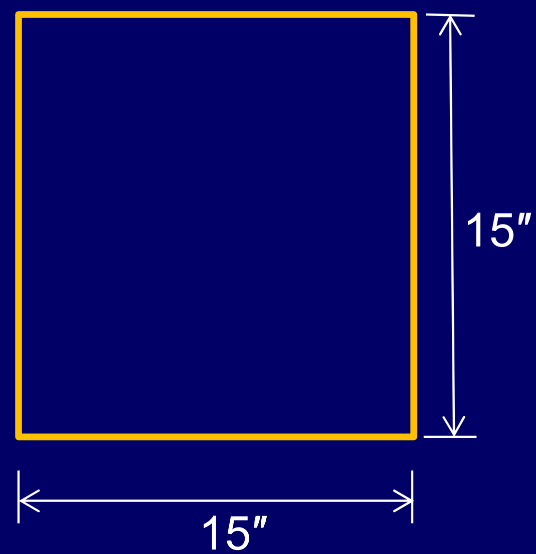
Using #6 bar

$$\text{No. of bars } = \frac{1.44}{0.44} \approx 4$$

Design Aids

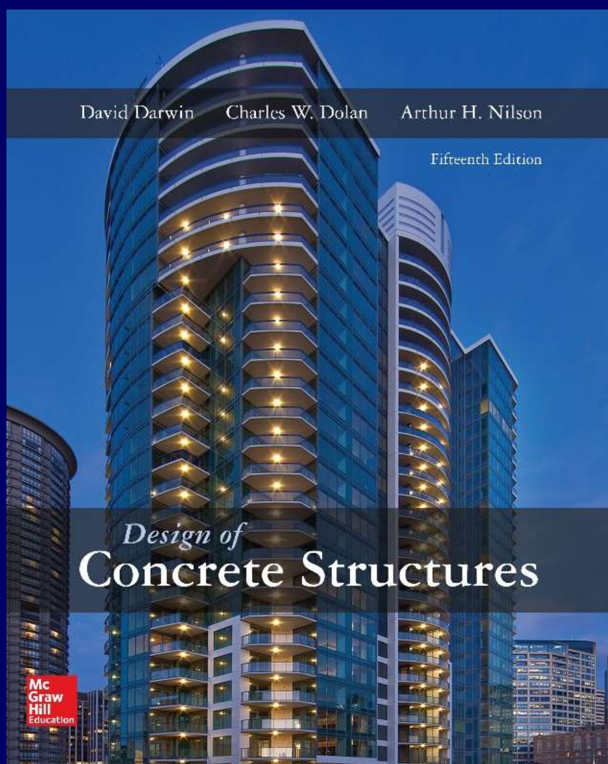
❑ Example 8.5 (Class Activity)

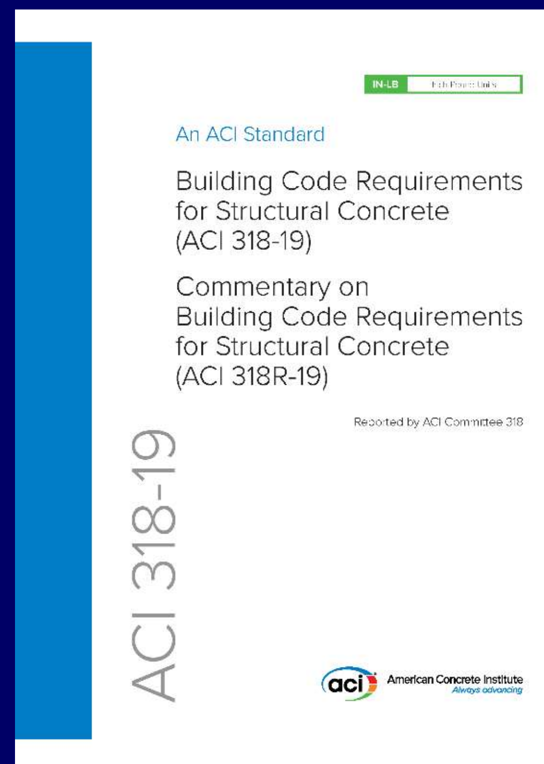
- Using design aids, design a 15″ square column section to support a factored load of 200 kip and a factored moment of 80 kip-ft. The material strengths are $f_c{}' = 4\ ksi$ and $f_y = 60\ ksi$.



References

- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-19)





Appendix

Derivation of c for Pure Bending Condition

As

$P = C_c + C_s - T_s$

For pure bending case, $P = 0$

$T_s = C_c + C_s$

$A_{s2}f_2 = 0.85f_c'ab + A_{s1}f_{s1} \quad \Rightarrow a = \dfrac{A_{s2}f_2 - A_{s1}f_{s1}}{0.85f_c'b}$

Here $A_{s1} = A_{s2} = A_s, \;\; f_{s1} = 87(1 - d'/c), \; f_{s2} = f_y$ and $a = 0.85c$

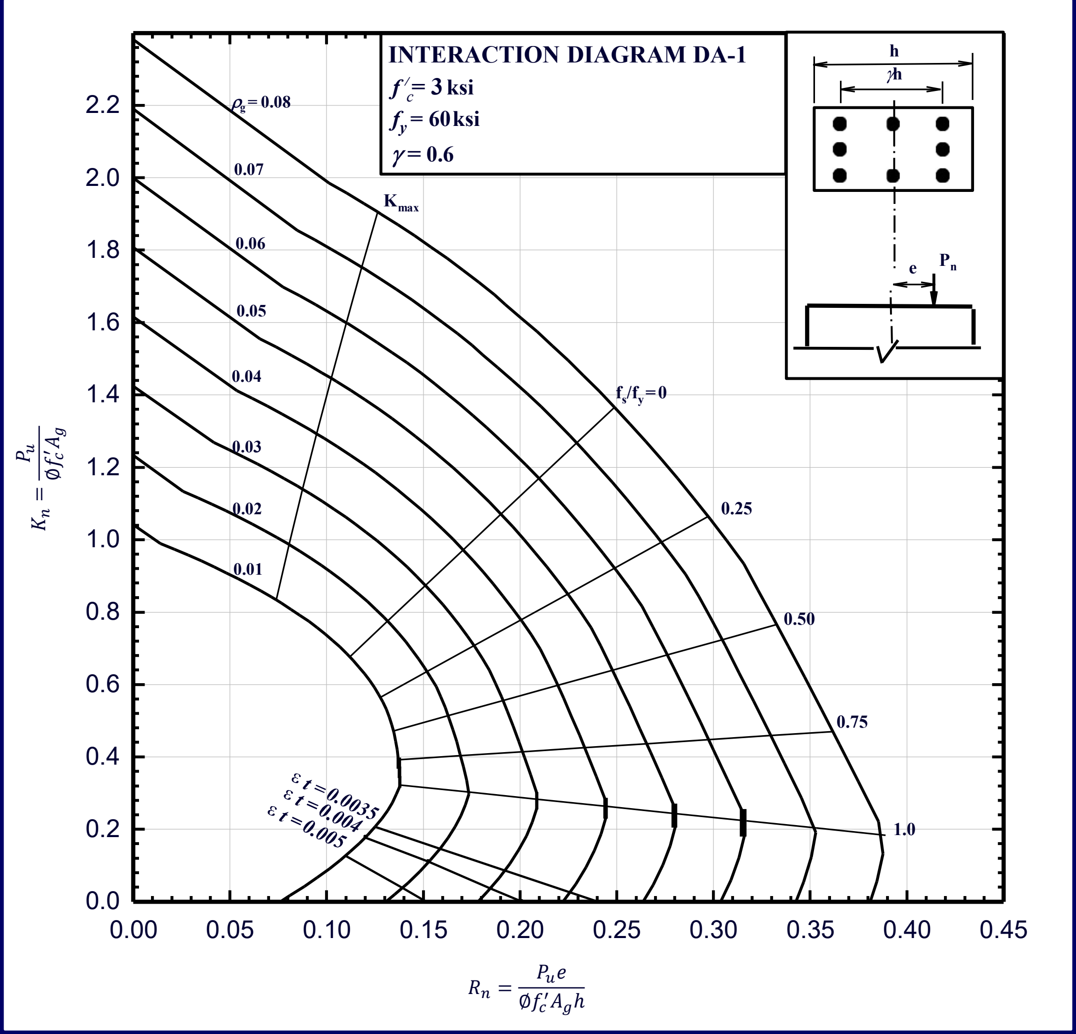
Substituting the above values, we get

$$c = \frac{A_s[f_y - 87\left(1 - \frac{d'}{c}\right)}{0.72f_c'b}$$

(This is an implicit equation, hence shall be solved by Equation Solver)

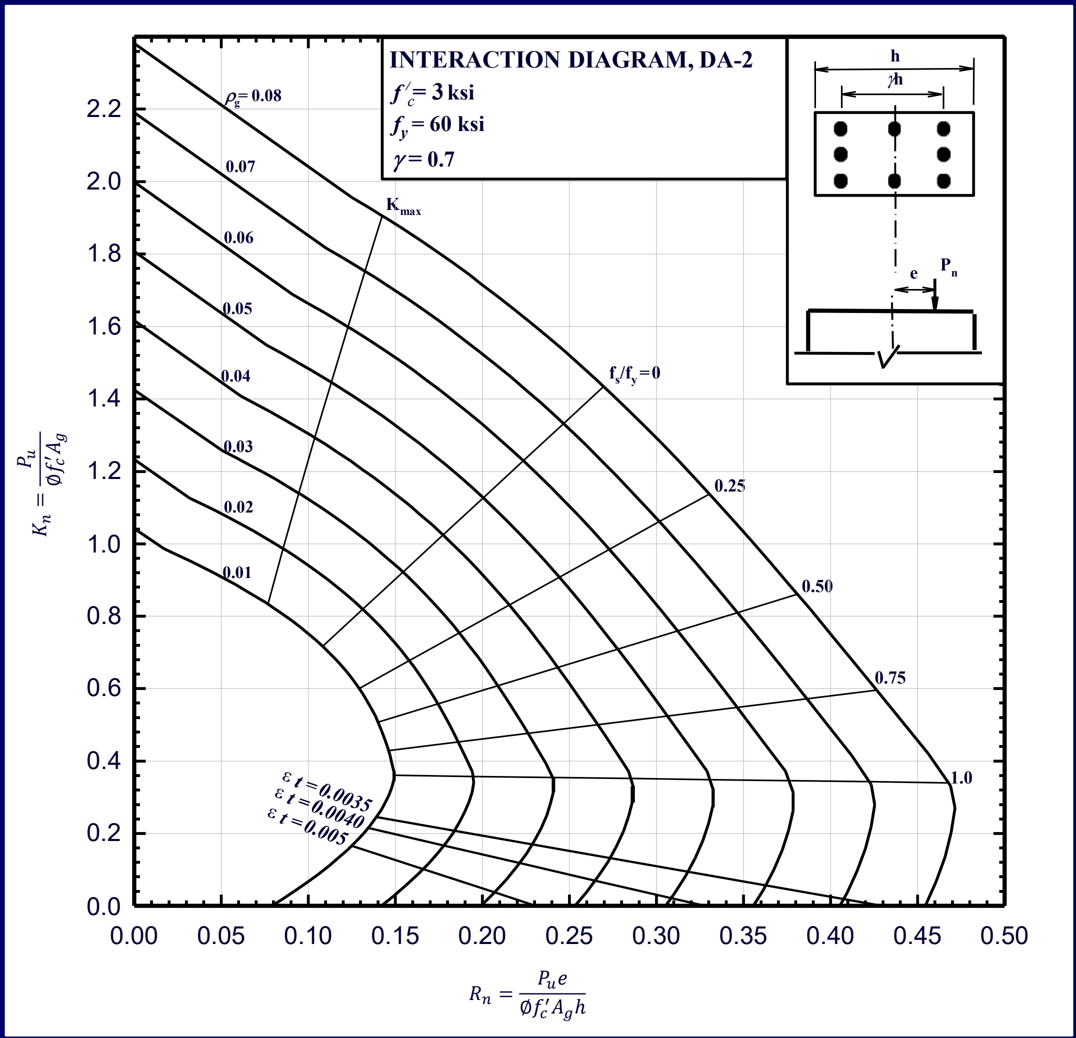
Appendix

❑ DESIGN AIDS (DA-1)



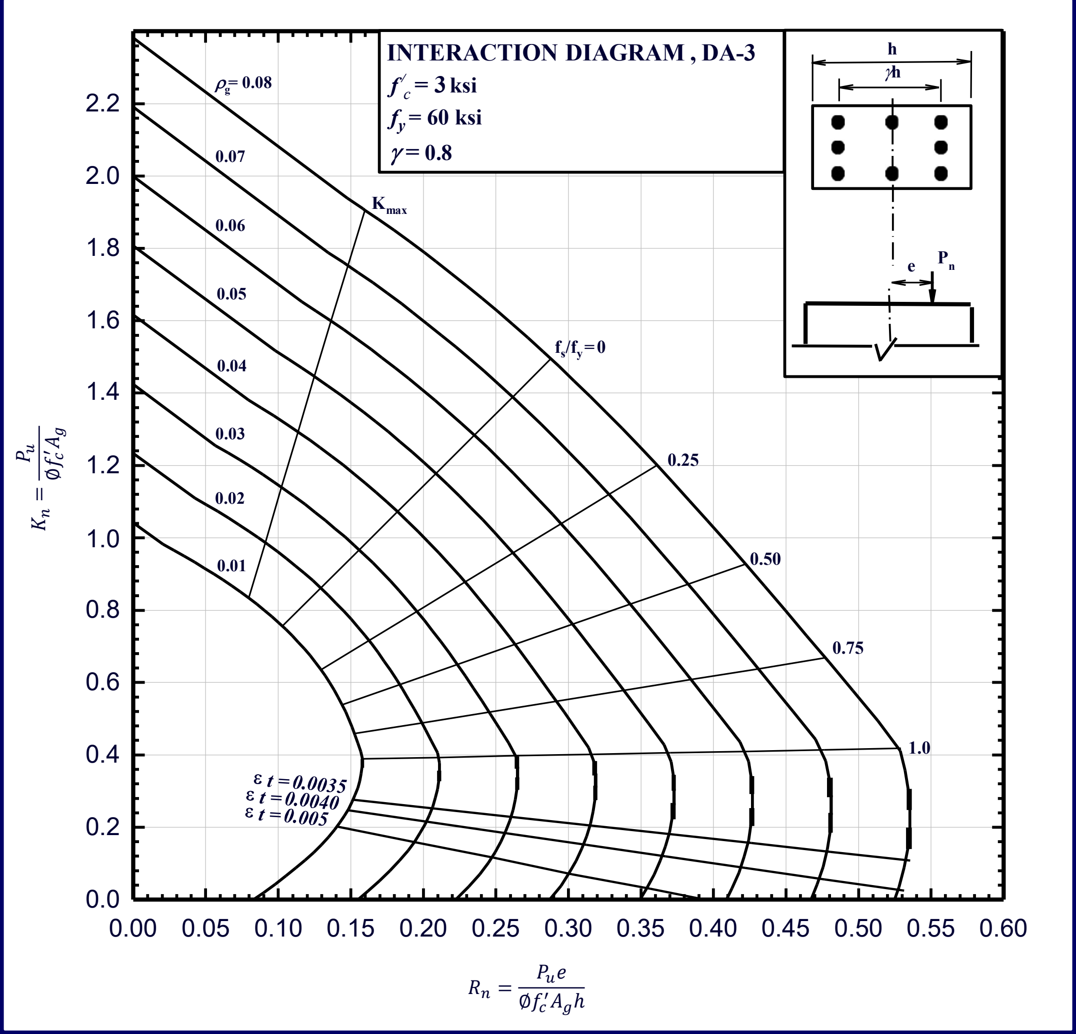
Appendix

❑ DESIGN AIDS (DA-2)



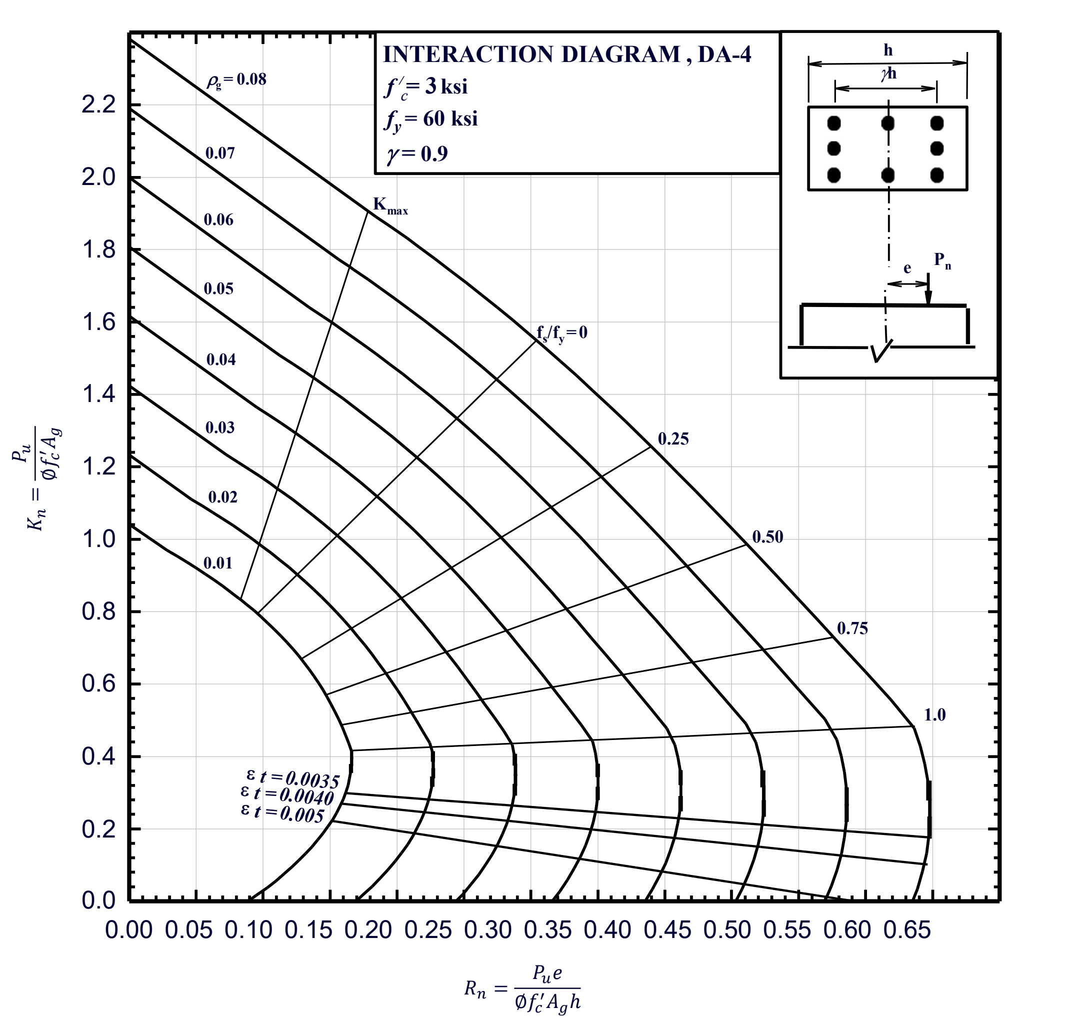
Appendix

❑ DESIGN AIDS (DA-3)



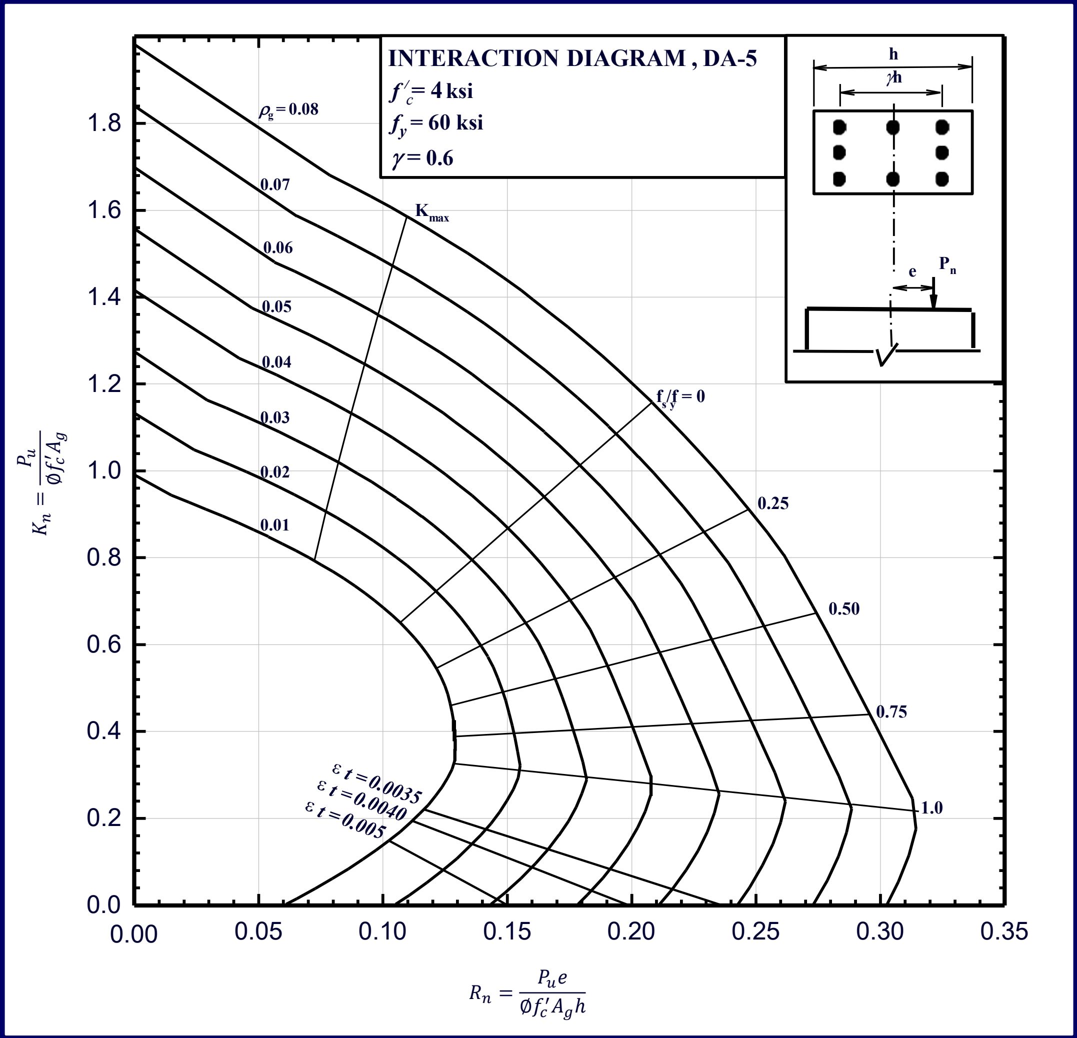
Appendix

❑ DESIGN AIDS (DA-4)



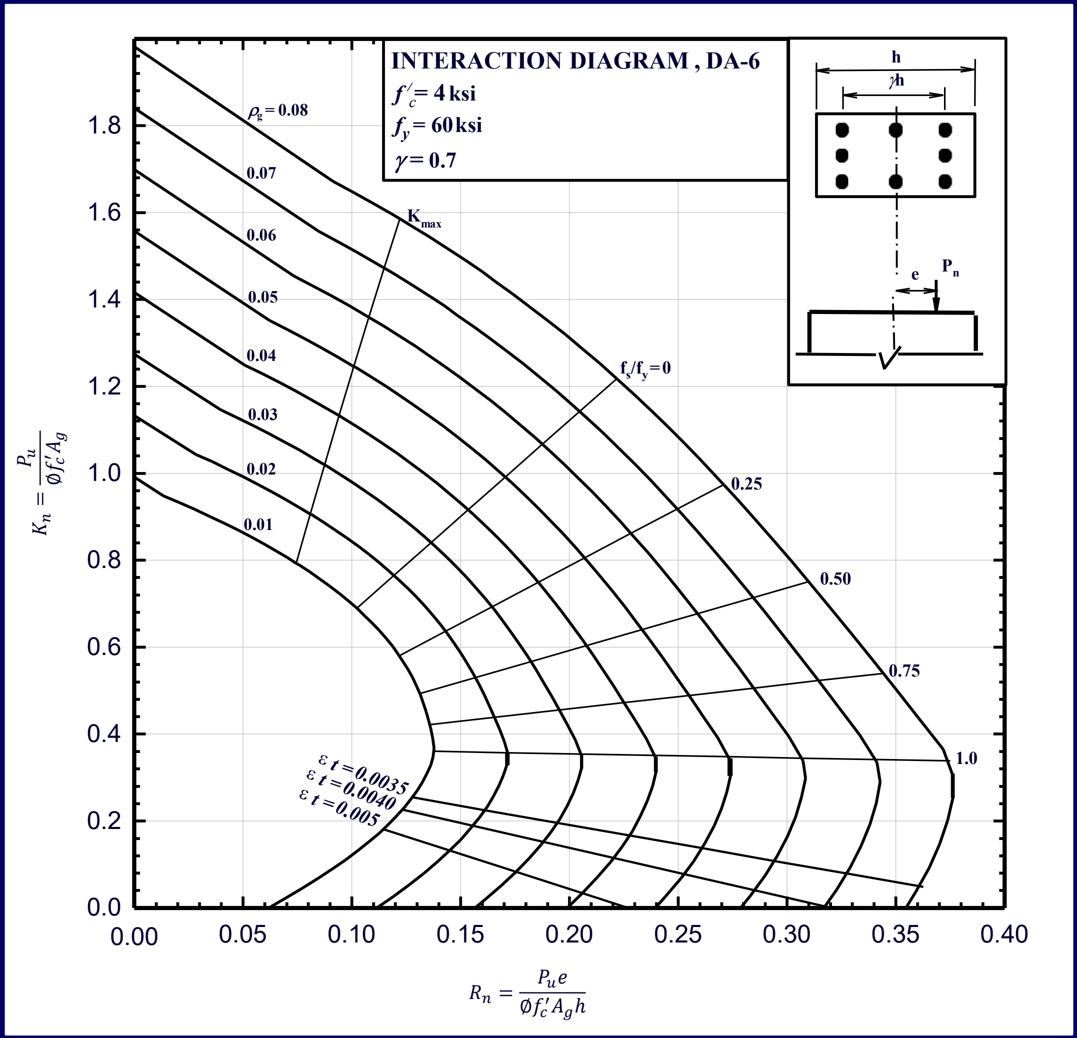
Appendix

❑ DESIGN AIDS (DA-5)



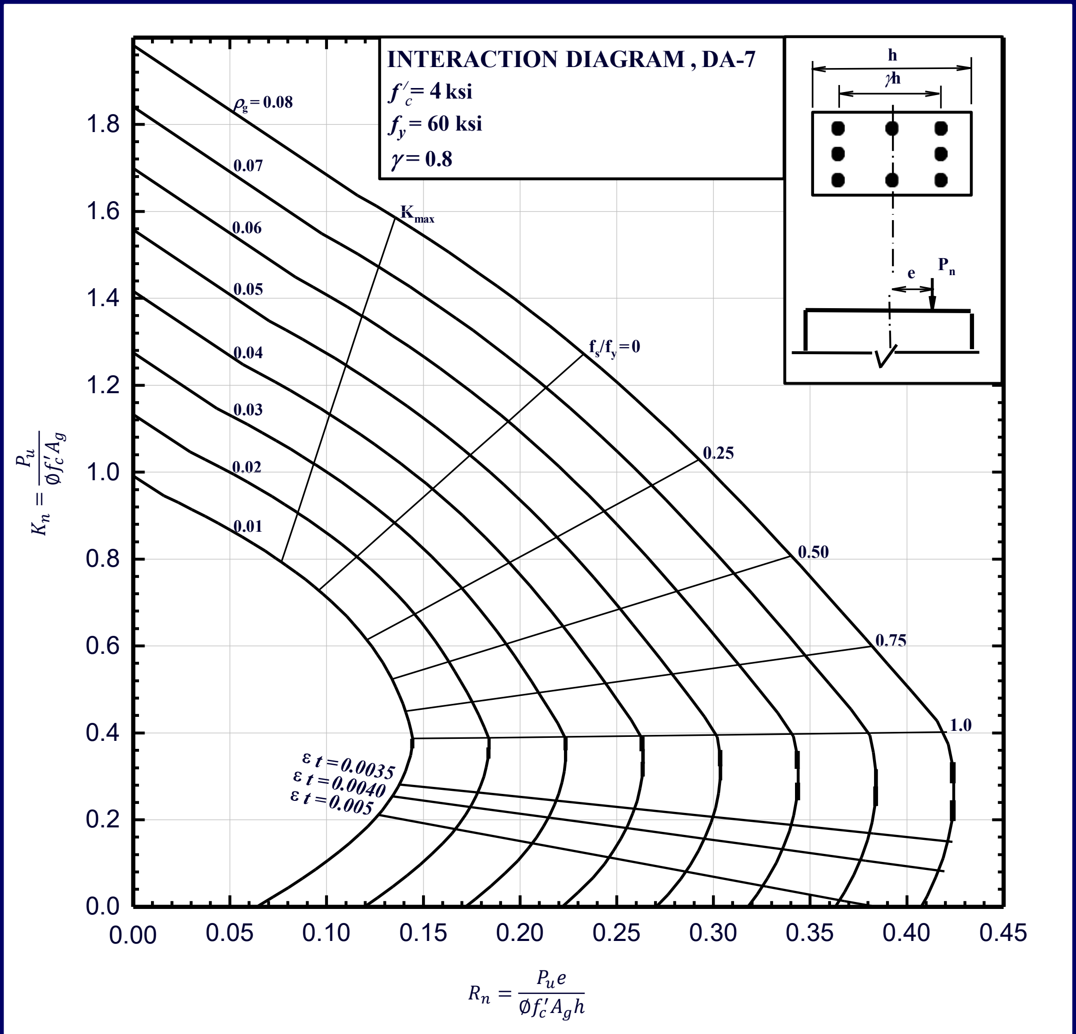
Appendix

❑ DESIGN AIDS (DA-6)



Appendix

❑ DESIGN AIDS (DA-7)



Appendix

❑ DESIGN AIDS (DA-8)