Lecture 06

Design of RC Slab Systems

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Organization of Lecture

Organization of Lecture

❑ Design of One-way Slab System

- The ACI code approximate method of analysis called as strip method is used for the analysis of one-way slabs.
- This topic has already been covered at BSc level. Please download **Lecture 2 of RCD – II** from the website.

❑ Design of One-way Joist System

- This topic will be covered in this lecture.

Organization of Lecture

❑ Design of Two-way Slab System with Beams

- The ACI code approximate method of analysis called as moment coefficient method is generally used for the analysis of two-way slabs supported on stiff beams or walls.
- This topic has already been covered at BSc level. Please download **Lecture 3 of RCD – II** from the website.

Organization of Lecture

❑ Design of Two-way Slab System without Beams

- The ACI code approximate method of analysis called as Direct Design Method (DDM) is used for the analysis of flat plates and flat slabs.

- This topic has been covered at BSc Level. Students can download **Lecture 04 of RCD – II** from the website. However, this topic will also be quickly discussed in this lecture.

- Please note that DDM can also be used for the analysis of two-way slabs with beams. However, as the application of this method to such systems is relatively difficult, therefore, these systems are generally analyzed using moment coefficient method instead of DDM, if beams are relatively stiff.

Organization of Lecture

- ❑ **Design of Two-way Joist System**
  - This topic will be discussed in this lecture.
- ❑ **Summary**

In short, only the following topics will be discussed.

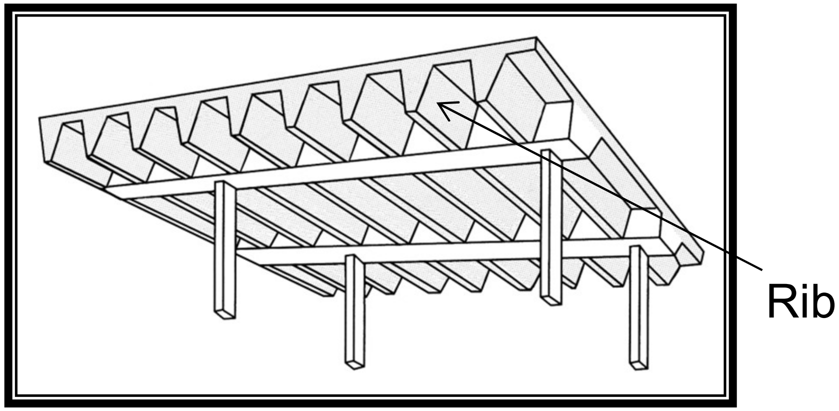
- Analysis & design of one-way joist system
- Analysis & design of two-way slab system without beams (flat plate & flat slabs)
- Analysis & design of two-way joist system

Design of One-way Joist System

General

❑ Introduction

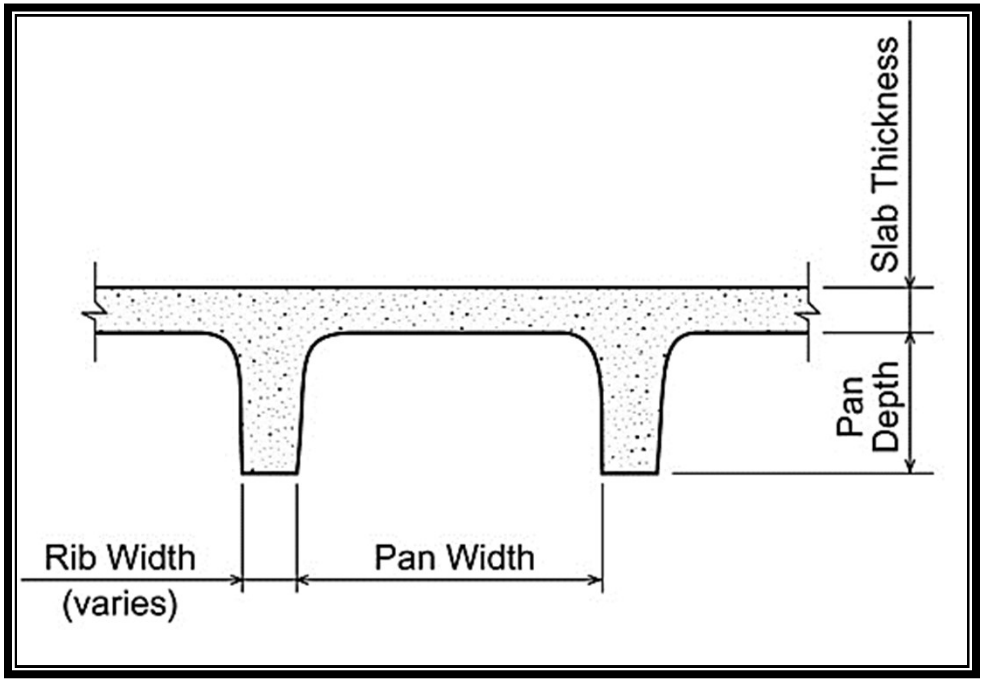
- **Joist:** T-beams called as joists are formed by creating void spaces in what otherwise would be a solid slab.
- **Joist Construction:** Joist construction consists of a monolithic combination of regularly spaced ribs and a top slab (T beam or Joist) arranged to span in one direction or two orthogonal directions.



General

❑ Introduction

- ACI 9.8 contains provisions for one-way joist construction.
- A structural system will be called as joist system if the pan width (clear spacing between ribs) is less than or equal to 30 inches (ACI 9.8.1.4).



General

❑ Introduction

- If the system does not fulfill the requirements of joist system, then it shall be designed as regular slab beam system which means that slab shall be designed for flexure and beam shall be designed for flexure and shear.
- One-way joist systems are suitable for longer spans, typically falling within the range of 30 to 50 feet.

General

❑ Characteristics

- Reduced dead load through pan voids.
- Ability to place electrical/mechanical equipment between joists.
- Clear, unobstructed tenant space.
- Decreased susceptibility to vibrations due to a stiffer system.
- Standardized void space forms: 20 or 30 inches wide and 8, 10, 12, 16, or 20 inches deep.
- Cost-effectiveness for buildings like apartments, hotels, and hospitals with small live loads and longer spans.
- Forms tapered in cross-section at a 1 to 12 slope for easier removal.

Basic Design Steps

❑ Step 1: Selection of Sizes

❖ Depth of Joist

- Same tables as used for one-way slabs and beams are used to assume the initial depth of joist.

Table 7.3.1.1 & Table 9.3.1.1– Minimum Thickness of Non-Prestressed Beams or One-way Slabs Unless Deflections are Computed

Member	Minimum thickness, h [1]			
	Simply supported	*One end continuous*	*Both ends continuous*	*Cantilever*
Solid one-way slabs	$l/20$	$l/24$	$l/28$	$l/10$
Beams or ribbed one-way slabs	$l/16$	$l/18.5$	$l/21$	$l/8$

- $l =$ Span length (center to center length in case of interior spans and clear projection in case of cantilevers) (Section 2.2).
- [1] For f_y other than 60,000 psi, the expressions in the table shall be multiplied by (0.4 + f_y /100,000)

Basic Design Steps

❑ Step 1: Selection of Sizes

❖ Width

- Minimum width of rib = depth of rib/3.5 but not less than 4″ (ACI 9.8.1.2).

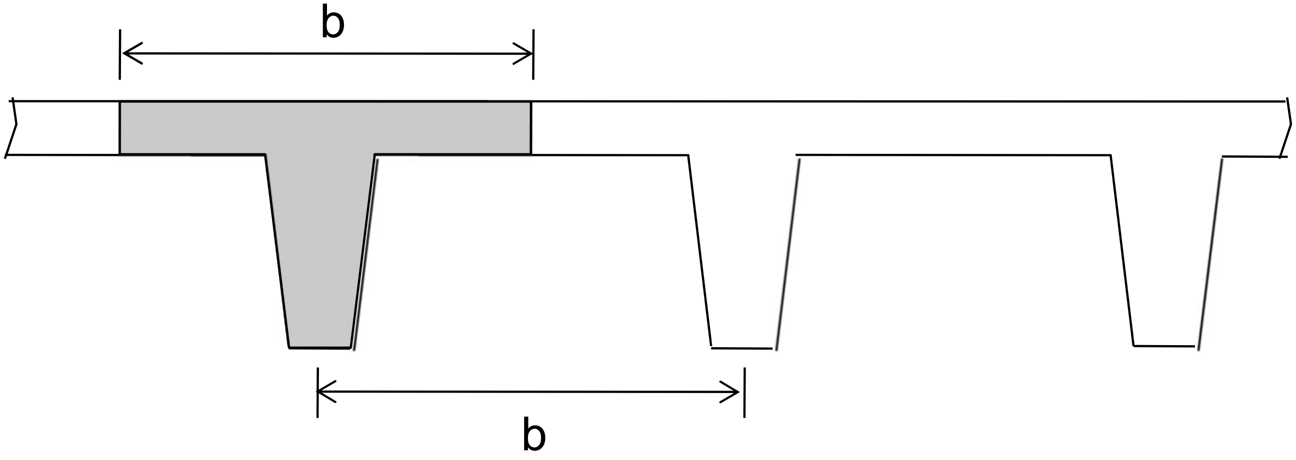
❖ Thickness of Slab

- Minimum slab thickness = clear distance between ribs/12 but not less than 2″ (ACI 9.8.2.1.1).
- Note: the depth of joist includes the slab thickness.

Basic Design Steps

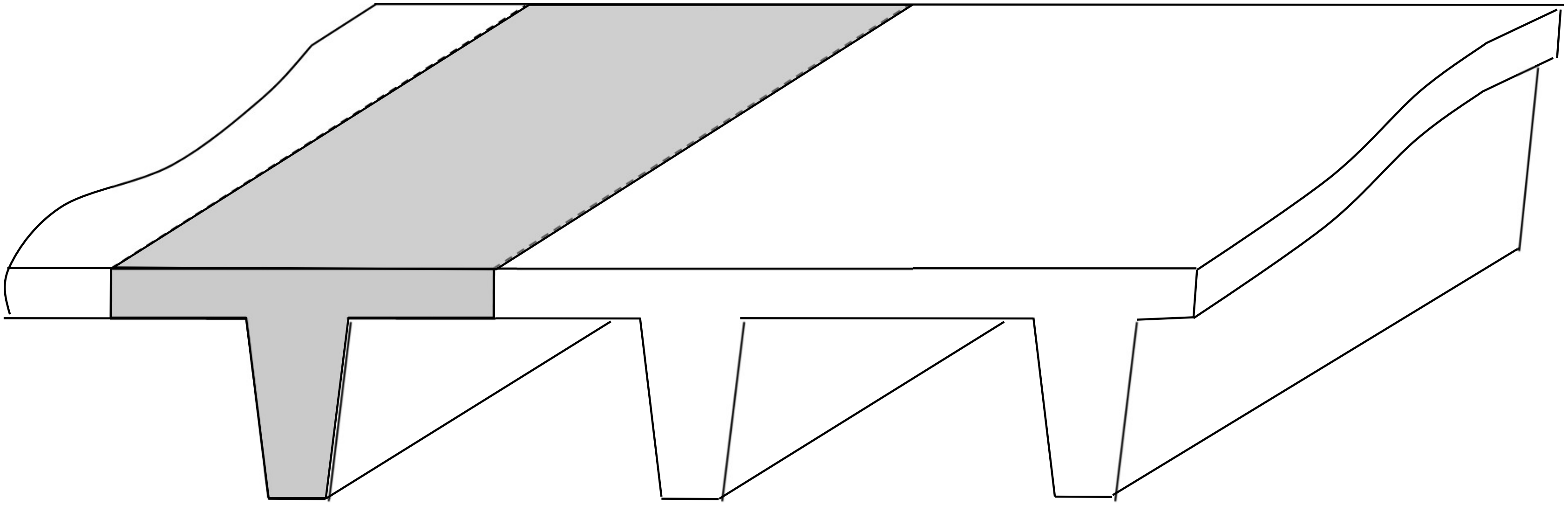
❑ Step 2: Calculation of Loads

- Loads on structure are determined based on occupational characteristics and functionality.
- One way Joist systems are usually designed for gravity loading (U = 1.2D + 1.6L).
- The joist is analyzed for load (per running foot) over a width equal to the center to center spacing between the joists as shown below.



Basic Design Steps

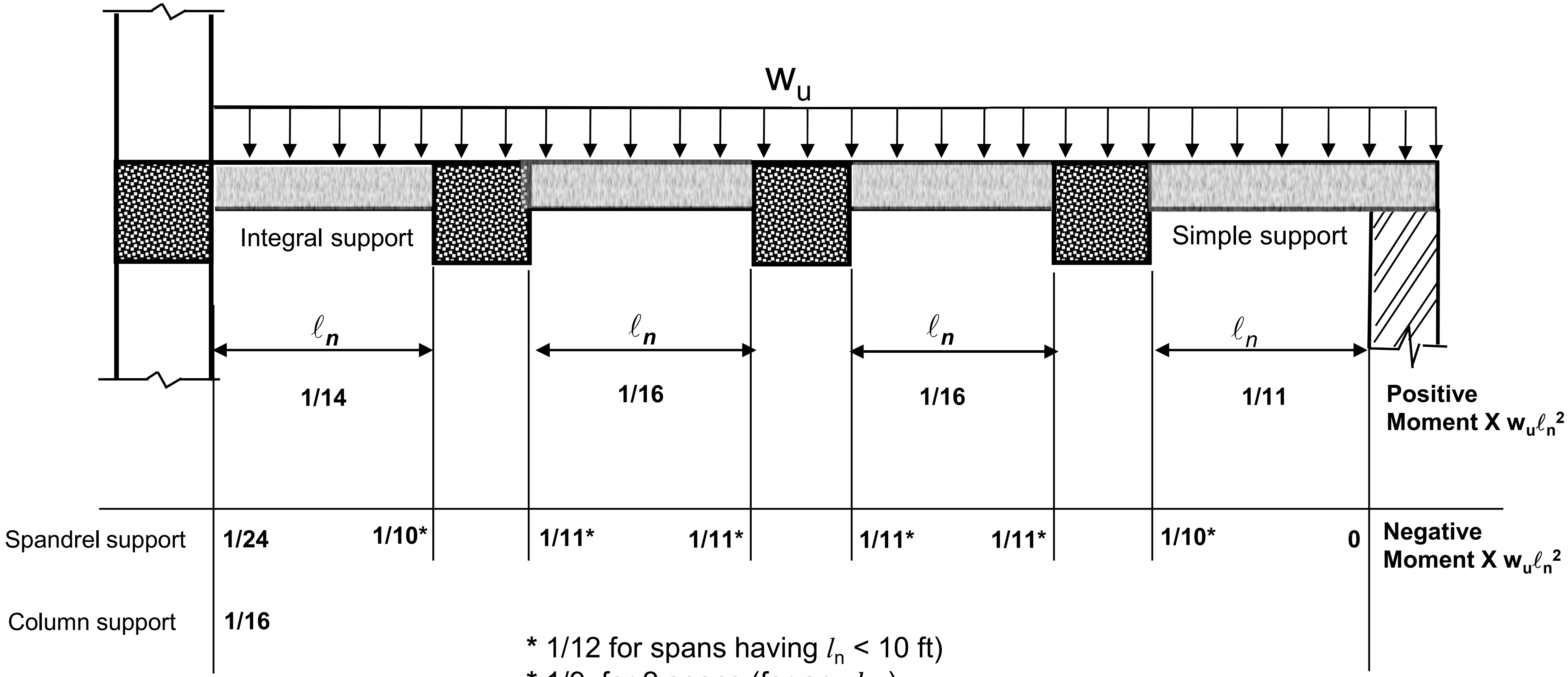
- **Step 3: Analysis**
  - Effect of loads are calculated on all structural elements.
  - For the purpose of analysis, a single T shaped joist is considered.



Basic Design Steps

❑ Step 3: Analysis

- ACI approximate analysis is applicable.



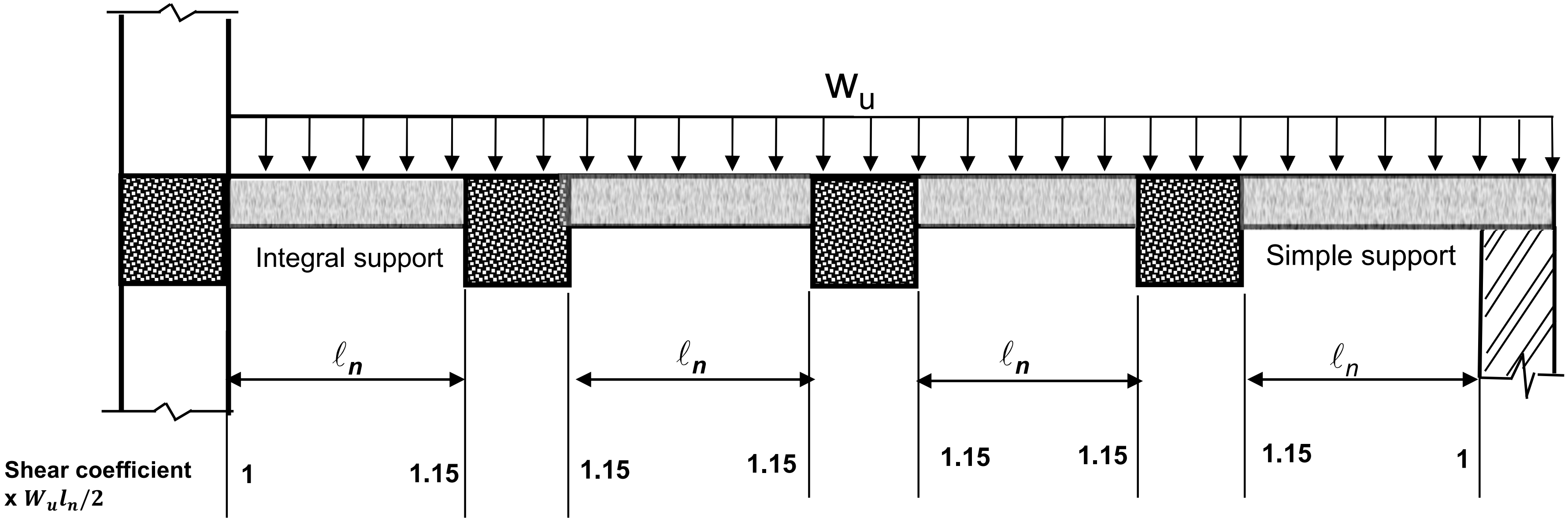
* 1/12 for spans having l_n < 10 ft)
* 1/9 for 2 spans (for any l_n)

Note: (i) For simply supported members, M = $w_u l^2/8$, where l = center to center distance.
(ii) To calculate negative moments, ℓ_n shall be the average of the adjacent clear span lengths.

Basic Design Steps

❑ Step 3: Analysis

- ACI approximate analysis is applicable.



Basic Design Steps

- ☐ **Step 3: Analysis**
  - **Limitations of ACI Approximate Coefficients Method**

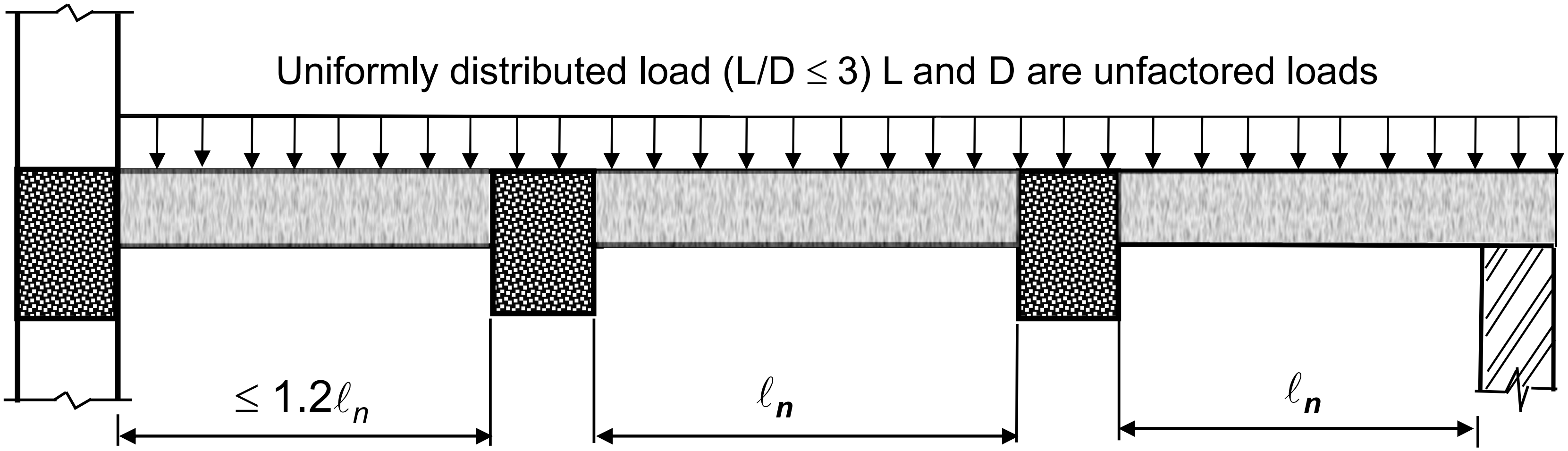
1. Applicable to one-way slabs, one-way joists and beams.
2. Loads must be uniformly distributed (not applicable to point loads).
3. The ratio of service live load to service dead load shall not exceed 3 (L/D ≤ 3).
4. Suitable for two or more spans.
5. Members must be prismatic.

Basic Design Steps

❑ Step 3: Analysis

❖ Limitations ACI Approximate Coefficients Method

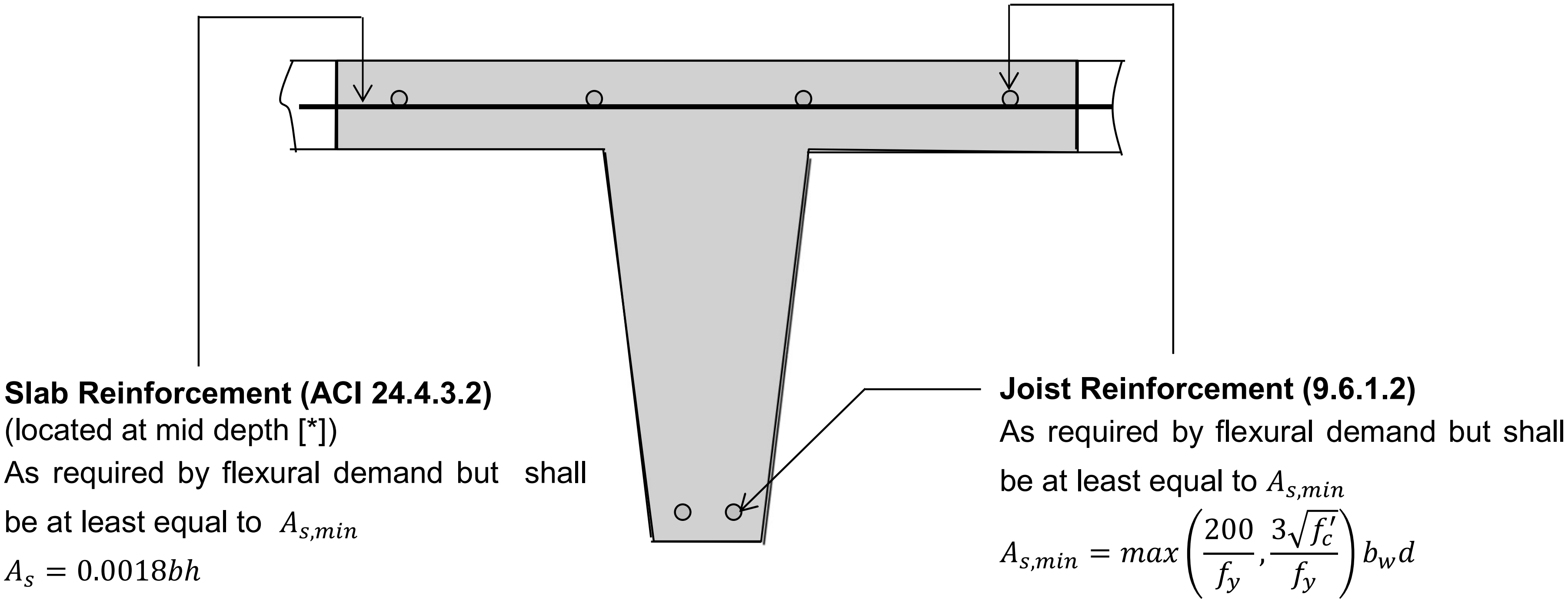
6. The longer of two adjacent spans does not exceed the shorter by more than 20 percent.



Basic Design Steps

❑ Step 4: Determination of Reinforcement

❖ Flexural Reinforcement



Slab Reinforcement (ACI 24.4.3.2)
(located at mid depth [*])
As required by flexural demand but shall be at least equal to $A_{s,min}$
$A_s = 0.0018bh$

Joist Reinforcement (9.6.1.2)
As required by flexural demand but shall be at least equal to $A_{s,min}$
$$A_{s,min} = max\left(\frac{200}{f_y}, \frac{3\sqrt{f_c'}}{f_y}\right) b_w d$$

[*] The slab reinforcement normal to the ribs is often located at mid-depth of the slab to resist both positive and negative moments.

Basic Design Steps

❑ Step 4: Determination of Reinforcement

❖ Skin Reinforcement

- ACI 9.7.2.3 states that if the depth d of a beam or joist exceeds 36 inches, longitudinal skin reinforcement shall be provided as per ACI section 24.3.2.

❖ Maximum Spacing for Flexural Reinforcement in Slab

- If spacing is obtained by flexural demand, then compare spacing with:
  - Least of 3h or 18” (ACI 7.7.2.3)
- If shrinkage reinforcement is governing, then compare spacing with:
  - Least of 5h or 18” (ACI 7.7.2.4)

Basic Design Steps

❑ Step 4: Determination of Reinforcement

❖ Shear Reinforcement

- If $V_u \geq \Phi V_c$, then shear capacity shall be increased either by increasing rib depth or width or by providing single legged shear reinforcement.
- If $V_u < \Phi V_c$ the section is safe against shear. Even minimum reinforcement as required for the beams with $[\Phi V_c/2 < V_u < \Phi V_c]$ is not required as per ACI 22.5.
- For joist construction, contribution of concrete to shear strength V_c shall be permitted to be 10 percent more than that specified in Chapter 22.
- Critical shear demand section shall be at a distance “d” from the face of the support.

Example 6.1

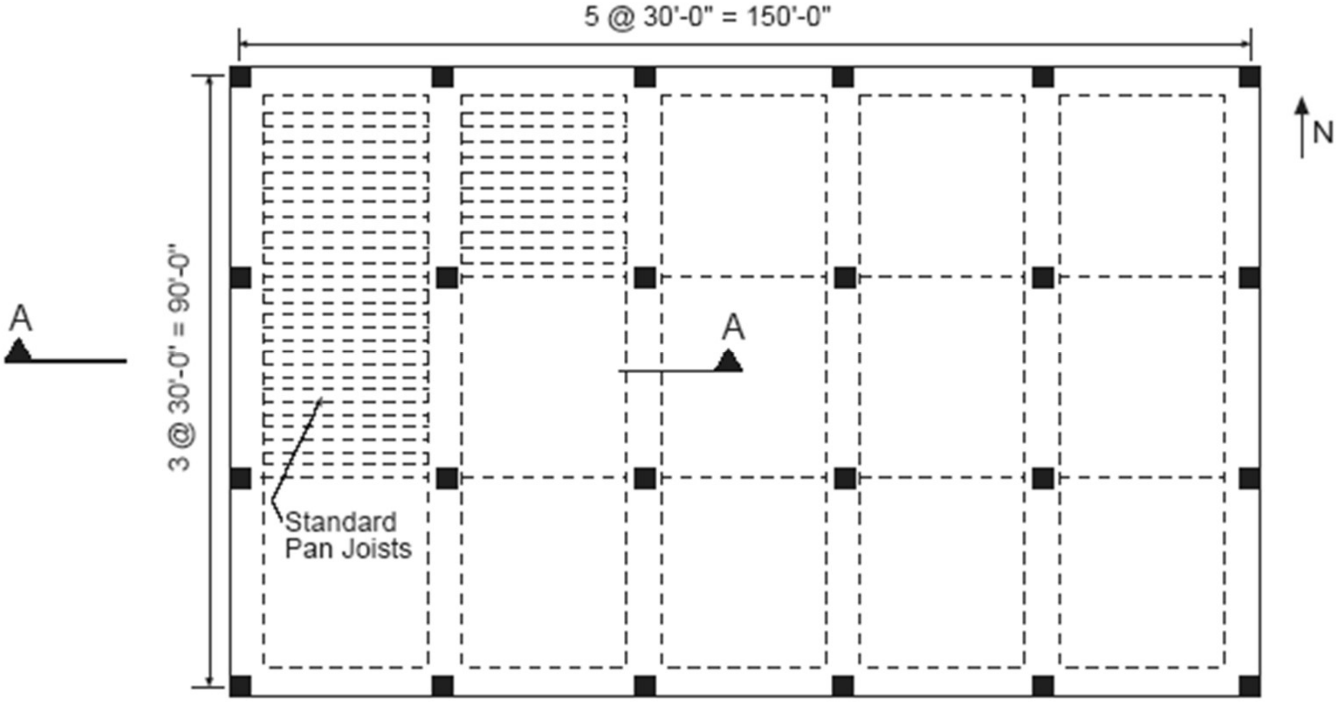
❑ Problem Statement

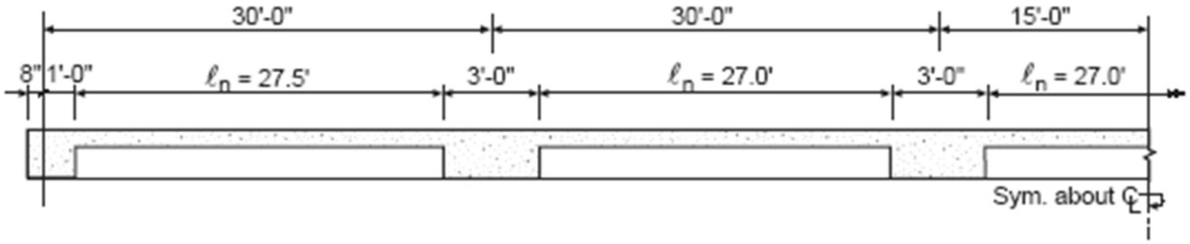
Determine the required depth and reinforcement for the one-way joist system shown in next slide. The joists are 6 inches wide and are spaced 36 inches c/c. The slab is 3.5 inches thick.

Use the following Data

- Superimposed Dead Load (SDL) = 64 psf
- Service Live Load (LL) = 60 psf
- Width of spandrel beams = 20 inches ; Width of interior beams = 36″
- Columns: interior = 18″ × 18″ ; exterior = 16″ × 16″
- Story height (typical) = 13 ft
- $f_c' = 4000 \text{ psi}$ & $f_y = 60{,}000 \text{ psi}$

Example 6.1



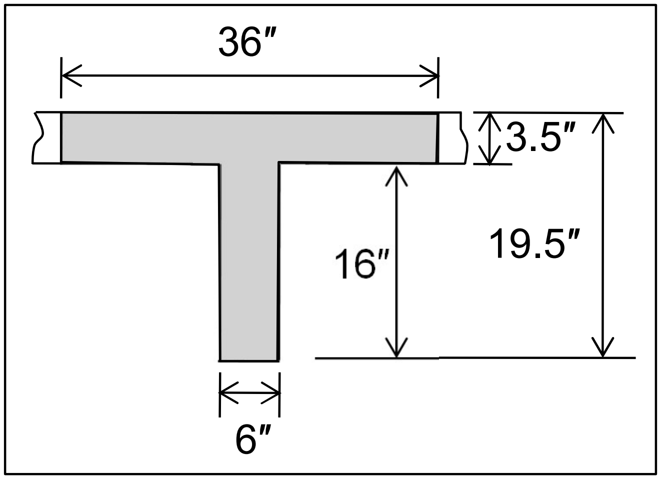


Section A-A

Example 6.1

❑ Solution

➢ Step 1: Selection of Sizes



$$h_{min} = \frac{l}{18.5} = \frac{30}{18.5} \times 12 = 19.5''$$

Take 19.5″ deep joist (16″ + 3.5″)

Assuming #8 bar,

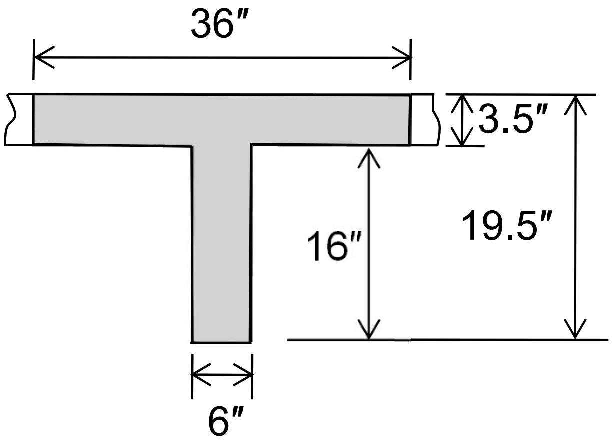
$$d = h - C_c - \frac{d_b}{2} = 19.5 - 0.75 - \frac{1}{2} = 18.25''$$

Width of the joist = 6″ > (rib depth/ 3.5 = 16/3.5 = 4.57″), OK.

Example 6.1

❑ Solution

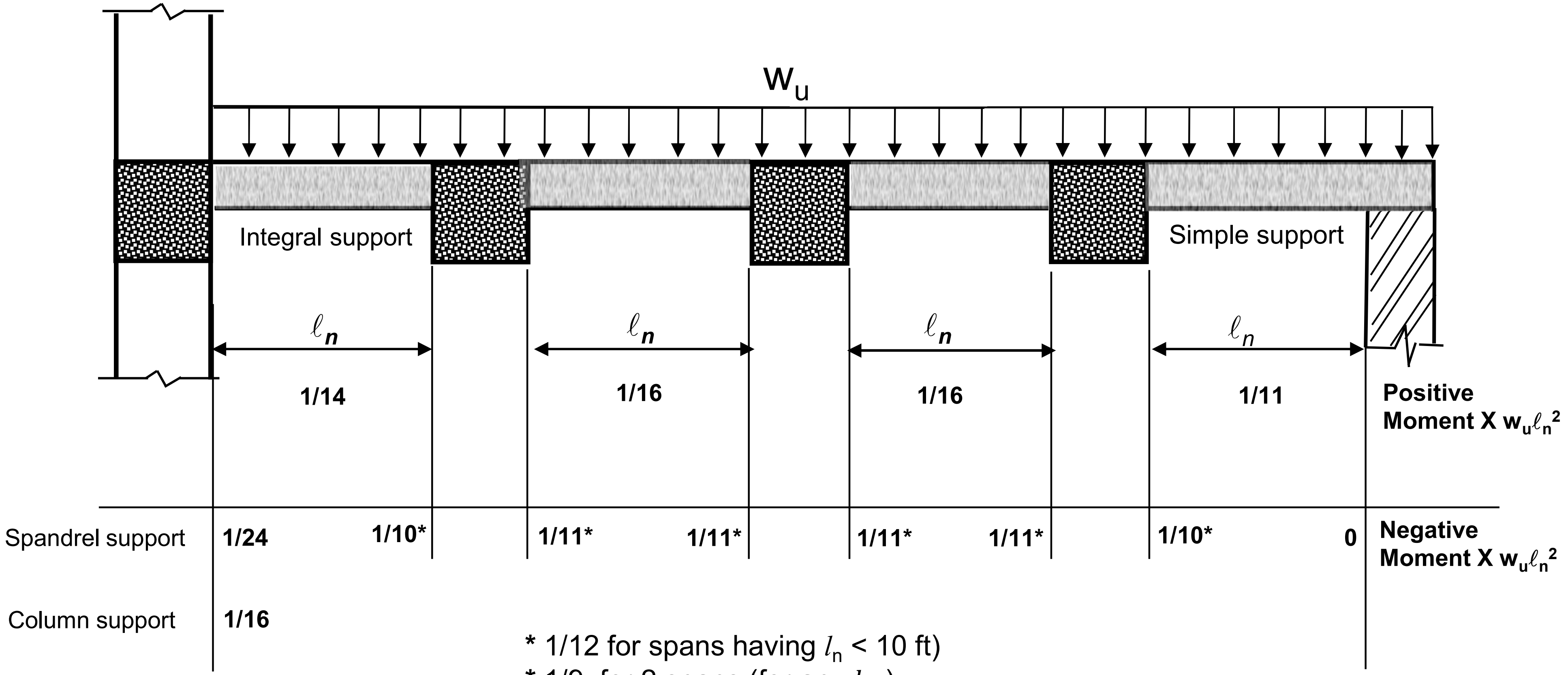
➢ Step 2: Calculation of Loads

Load	Calculation	
Rib weight	$(6 \times 16/144) \times 0.150 = 0.10\ \text{kip/ft}$	
Slab weight	$(3.5 \times 36/144) \times 0.150 = 0.131\ \text{kip/ft}$	
SDL on slab	$(0.064 \times 36/12) = 0.192\ \text{kip/ft}$	
Live load on slab	$(0.060 \times 36/12) = 0.180\ \text{kip/ft}$	
Total	$W_u = 1.2D + 1.6L$ $= 1.2 \times (0.10+0.131+0.192)+1.6 \times 0.18$ = $\mathbf{0.795\ kip/ft}$	

Example 6.1

❑ Solution

➢ Step 3: Analysis



w_u

Integral support

Simple support

ℓ_n ℓ_n ℓ_n ℓ_n

1/14 **1/16** **1/16** **1/11** **Positive Moment X $w_u \ell_n^2$**

Spandrel support **1/24** **1/10*** **1/11*** **1/11*** **1/11*** **1/11*** **1/10*** **0** **Negative Moment X $w_u \ell_n^2$**

Column support **1/16**

* 1/12 for spans having l_n < 10 ft)

* 1/9 for 2 spans (for any l_n)

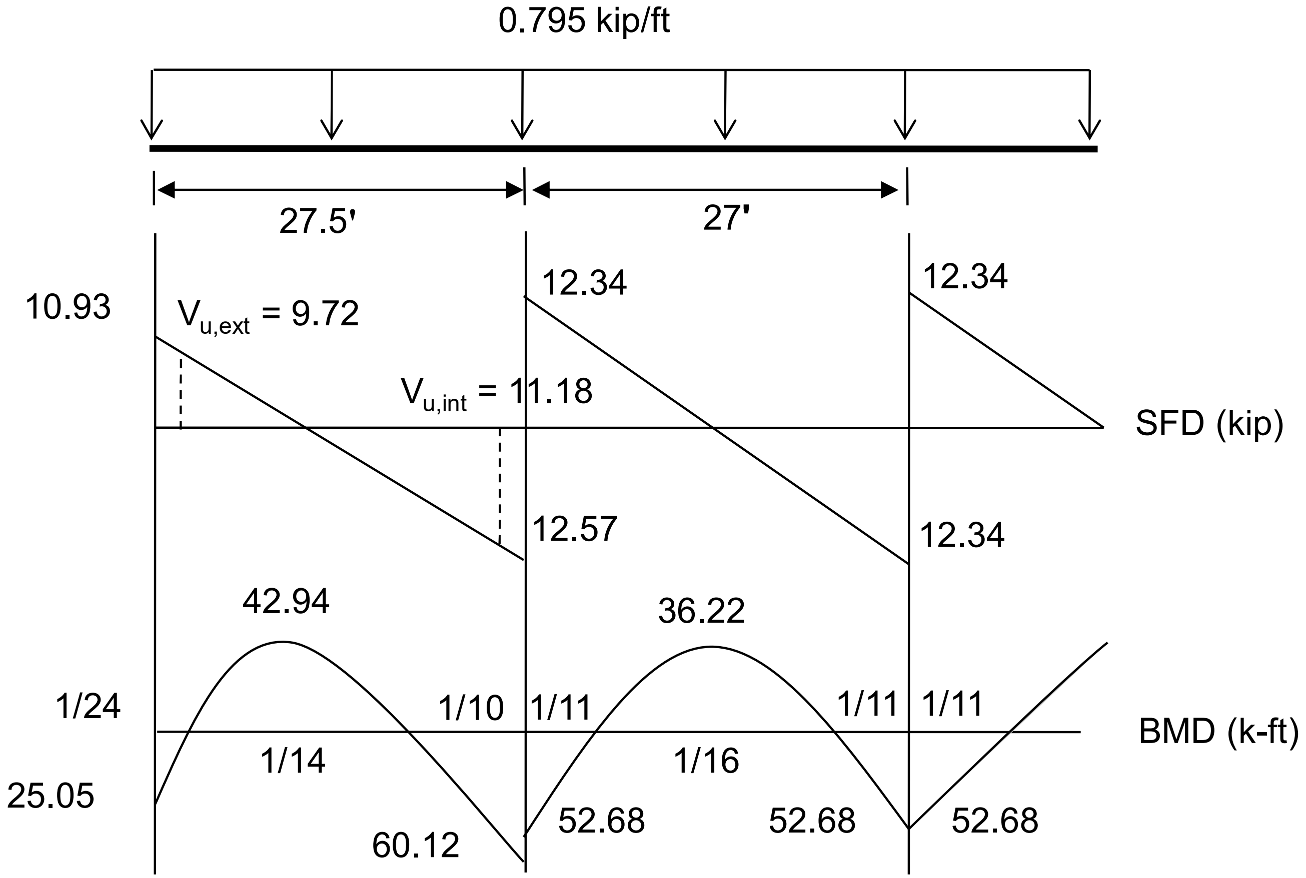
Note: (i) For simply supported members, M = $w_u l^2/8$, where l = center to center distance.

(ii) To calculate negative moments, ℓ_n shall be the average of the adjacent clear span lengths.

Example 6.1

Solution

- **Step 3: Analysis**



Example 6.1

❑ Solution

➢ Step 4: Determination of Reinforcement

- **Shear Reinforcement for Joist**

$V_{u,ext} = 9.72\ kip$

$V_{u,int} = 11.18\ kip$

$\emptyset V_c = 1.10\emptyset 2\sqrt{f_c'}bd$

$$= 1.10 \times 0.75 \times 2\sqrt{4000} \times 6 \times 18.25 \times \frac{1}{1000} = 11.42\ kip$$

$\emptyset V_c > V_{u,ext}\ \ and\ \ V_{u,int} \rightarrow OK$

Example 6.1

❑ Solution

➢ Step 4: Determination of Reinforcement

- **Flexural Reinforcement for Joist**

Moment (in.kip)	A_s (in^2)	$A_{s,min}$ (in^2)	**Detailing**
$M_{u+} = 515.28$	0.53	0.365	2- #5 bars
$M_{u-,ext} = 300.6$	0.31	0.365 (governs)	4 - #3 bars
$M_{u-,int} = 721.44$	0.78	0.365	8 - #3 bars

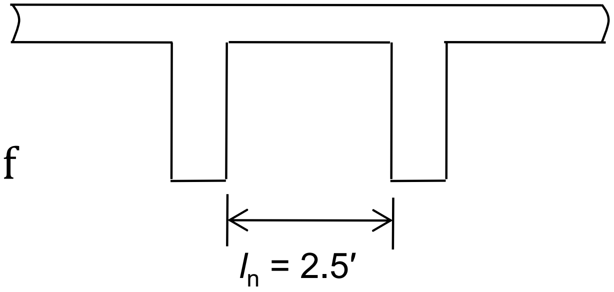
$$A_s = \frac{M_u}{\emptyset f_y\left(d - \frac{a}{2}\right)} \;\; ; \; a = d - \sqrt{d^2 - \frac{2.614 M_u}{f_c' b}} \;\; and \;\; A_{s,min} = max\left(3\sqrt{f_c'}, 200\right) \times \frac{bd}{f_y}$$

Example 6.1

❑ Solution

➢ Step 4: Determination of Reinforcement

❖ **Flexural Reinforcement for Slab**



$$w_u = 1.2\ (0.044 + 0.064) + 1.6 \times 0.06 = 0.23\ \text{ksf}$$

Over the support, we have

$$M_u = \frac{w_u {l_n}^2}{12} = 0.23 \times \frac{2.5^2}{12} = 0.12\ \ \text{ft.kip}$$

This moment requires $A_s = 0.015\ \text{in}^2/\text{ft}$

$$A_{s,\,min} = 0.0018 \times 12 \times 3.5 = 0.08\ \text{in}^2/\text{ft, governs}\ (\#3\ @\ 16.5''\ c/c)$$

Example 6.1

- **Solution**

Step 4: Determination of Reinforcement

Flexural Reinforcement for Slab

Maximum spacing allowed for temperature steel reinforcement

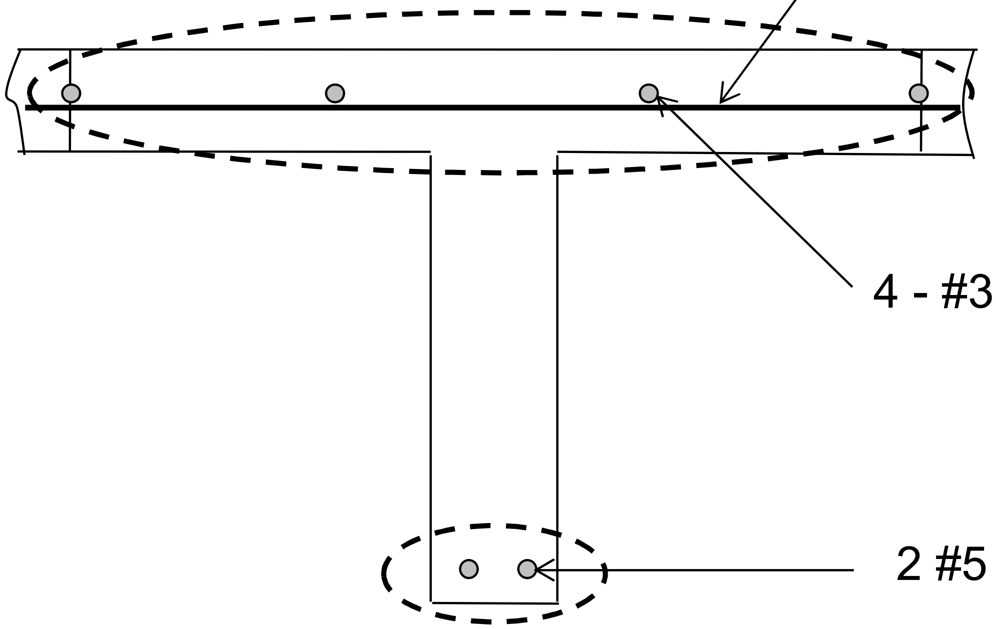
$5h_f = 5 \times 3.5 = 17.5''$ or $18''$

Finally provide #3 @ 16″ c/c.

Example 6.1

- ☐ **Solution**

  - ➢ **Step 5: Drafting**



#3 @ 16″ c/c, at mid depth
(shrinkage & temperature)

4 - #3 equally spaced (Main –ve)

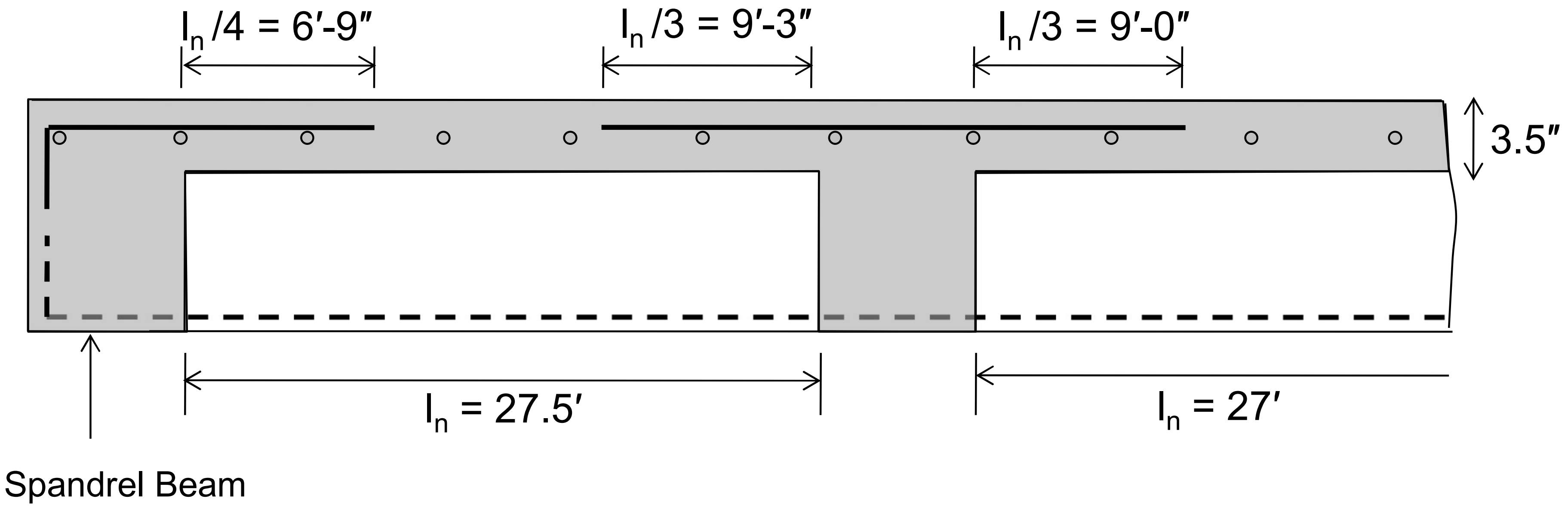
2 #5 bars (Main +ve)

<u>Cross Section</u>

Example 6.1

❑ Solution

➢ Step 5: Drafting



Long Section

Design of Two-way Slab System without Beams (Flat Plates and Flat Slabs)

Section – I

Flexural Design of Two-Way Slab System without Beams

[Direct Design Method]

Direct Design Method

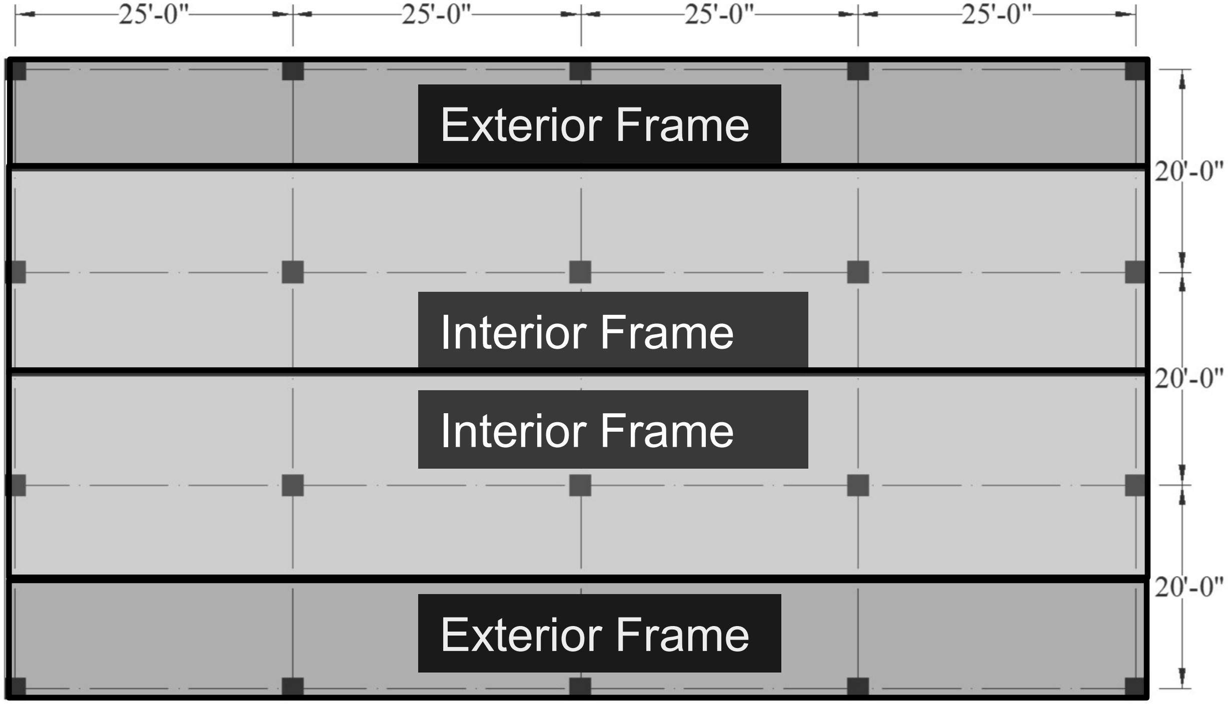
❑ Introduction

- The Direct Design Method (DDM) consists of a set of rules for distributing moments to slab and beam sections to satisfy safety requirements and most serviceability requirements simultaneously.
- The DDM is applicable to two-way slabs both with and without beams. However, it becomes comparatively challenging when dealing with slabs that have beams or walls. Hence, it will be only used for slabs without beams.
- Although the provisions for DDM present in ACI 318-14 Section 8.10 have been eliminated from the latest version (ACI 318-19), it can still be used as per ACI R6.2.4.1.

Direct Design Method

❑ Introduction

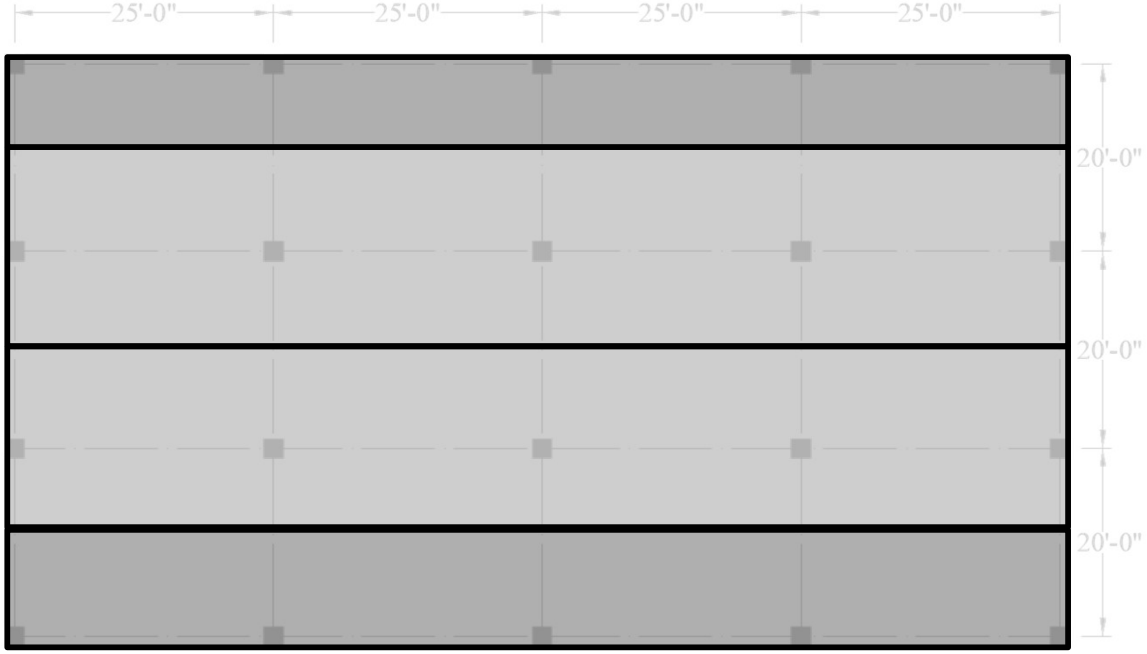
- In DDM, frames rather than panels are analyzed.



Direct Design Method

❑ Introduction

- For complete analysis of slab system, frames are analyzed in E-W and N-S directions.

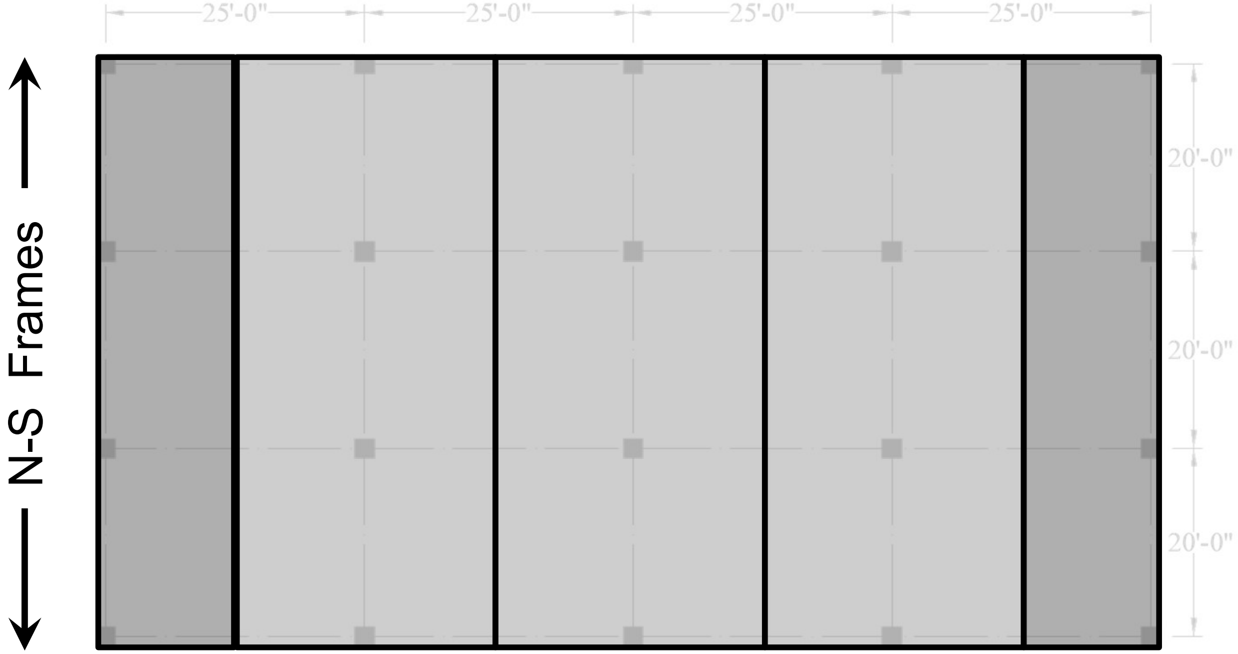


E-W Frames

Direct Design Method

❑ Introduction

- For complete analysis of slab system, frames are analyzed in E-W and N-S directions.



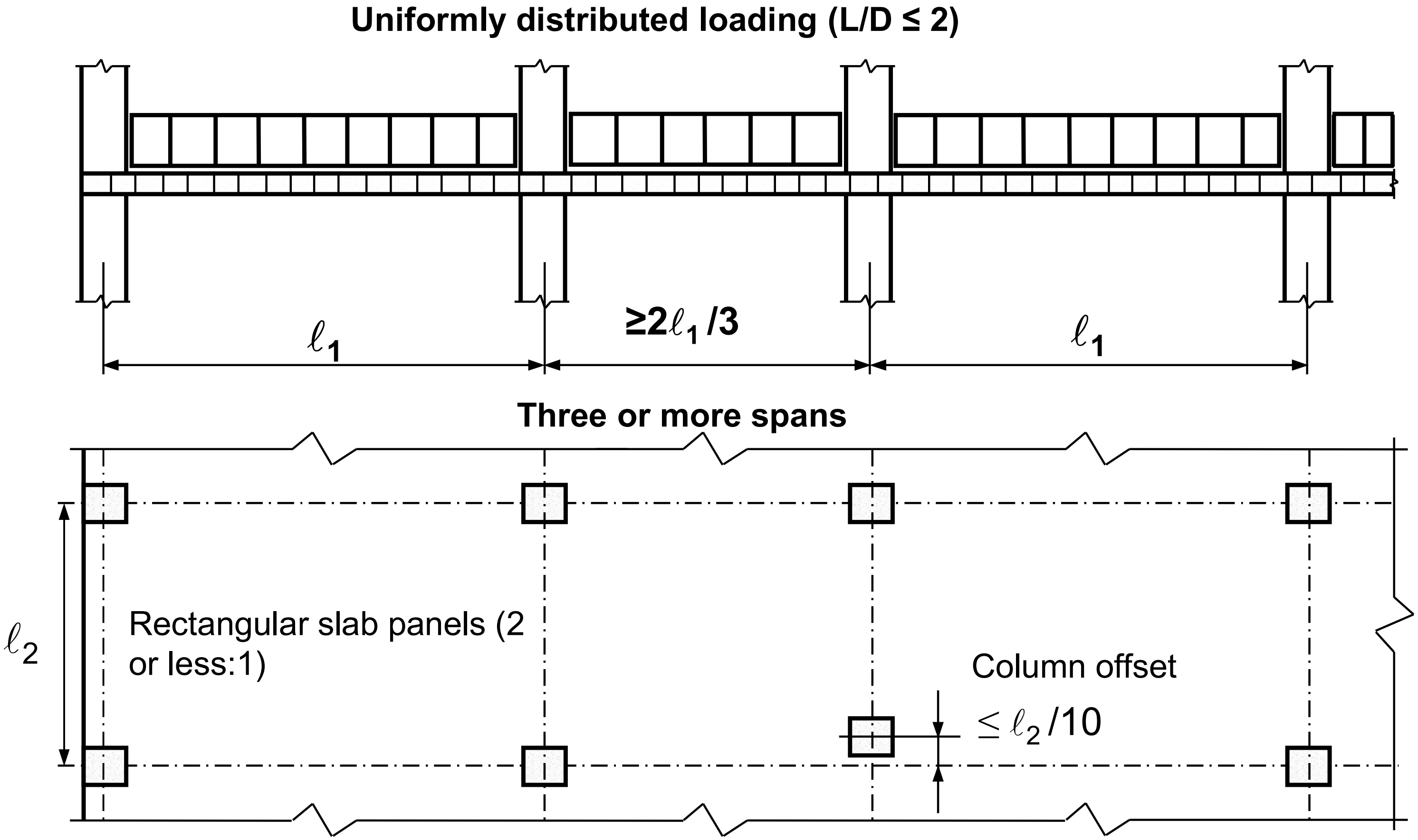
Direct Design Method

❑ Limitations

- Though DDM is useful for analysis of slabs, specially without beams, the method is applicable with some limitations as discussed next.

Direct Design Method

❑ Limitations (ACI 8.10.2)



Analysis Procedure using DDM

Step 1: Selection of Sizes

- ACI table 8.3.1.1 is used for finding the slab thickness.

Table 8.3.1.1 —Minimum thickness of nonprestressed two-way slabs without interior beams (in.)

f_y (psi)	Without drop panels			With drop panels		
	Exterior Panels		Interior panels	Exterior Panels		Interior panels
	Without edge beams	With edge beams		Without edge beams	With edge beams	
40,000	$l_n/33$	$l_n/36$	$l_n/36$	$l_n/36$	$l_n/40$	$l_n/40$
60,000	$l_n/30$	$l_n/33$	$l_n/33$	$l_n/33$	$l_n/36$	$l_n/36$
80,000	$l_n/27$	$l_n/30$	$l_n/30$	$l_n/30$	$l_n/33$	$l_n/33$

- l_n is the clear span in the long direction, measured face-to-face of supports (in.).
- $h_{min} = 5''$ for slabs without drop panels
- $h_{min} = 4''$ for slabs with drop panels

Analysis Procedure using DDM

❑ Step 2: Calculation of Loads

- The slab load is calculated in usual manner.

$$W_u = 1.2D + 1.6L$$

Where;

W_u = ultimate/ factored load

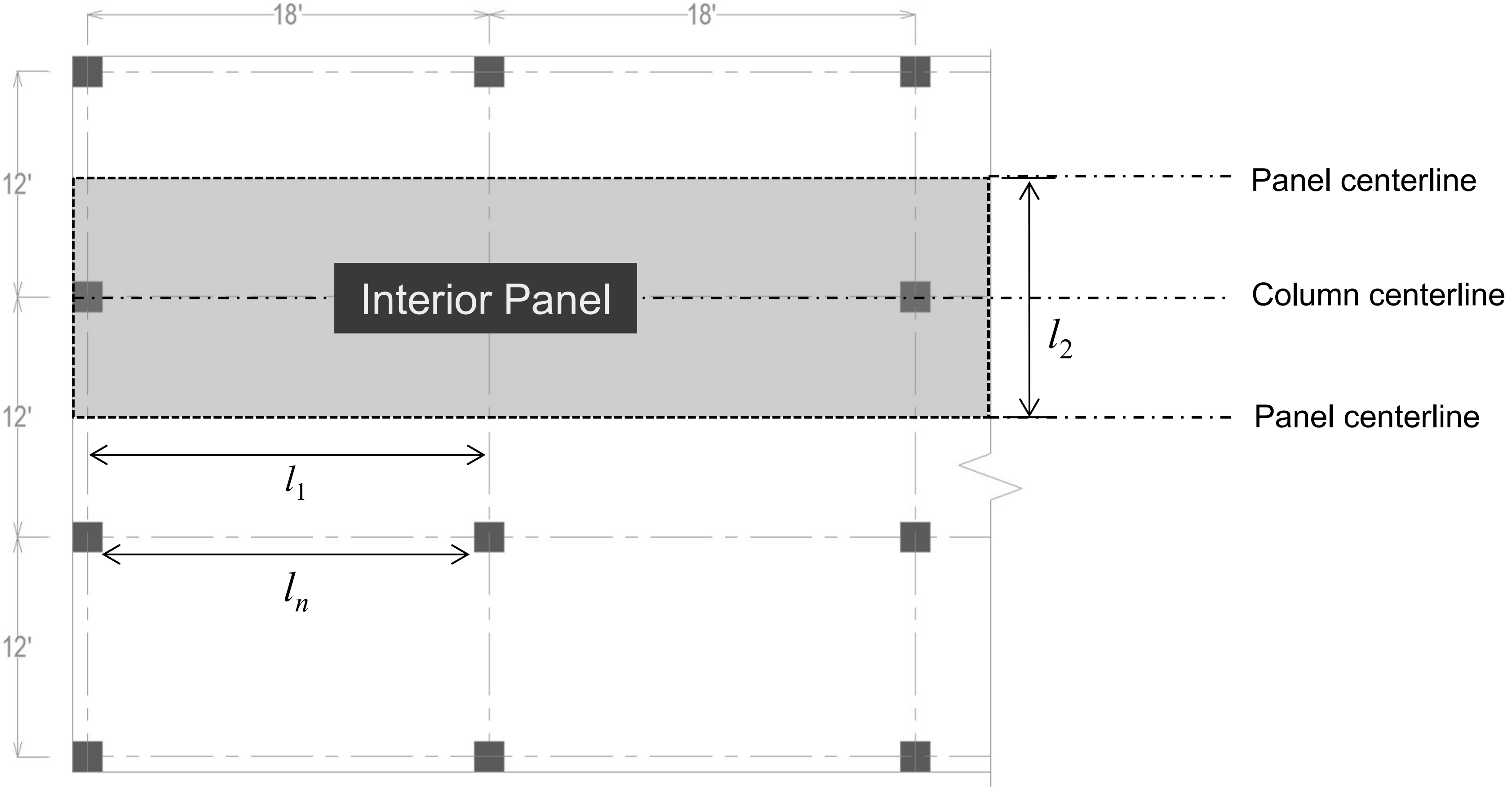
D = service dead load

L = service live load

Analysis Procedure using DDM

❑ Step 3: Analysis

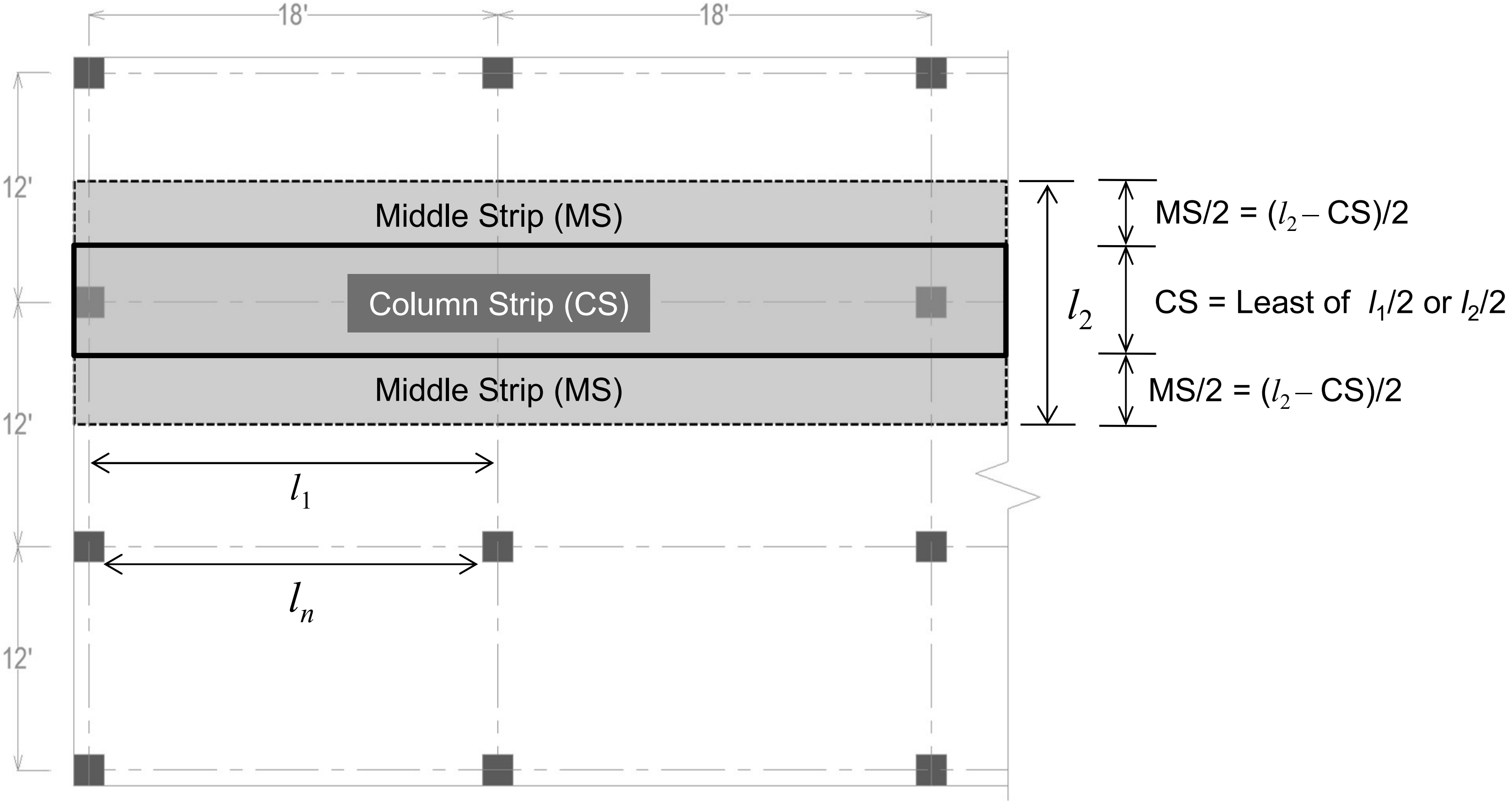
❖ Marking Frames in Each Direction



Analysis Procedure using DDM

❑ Step 3: Analysis

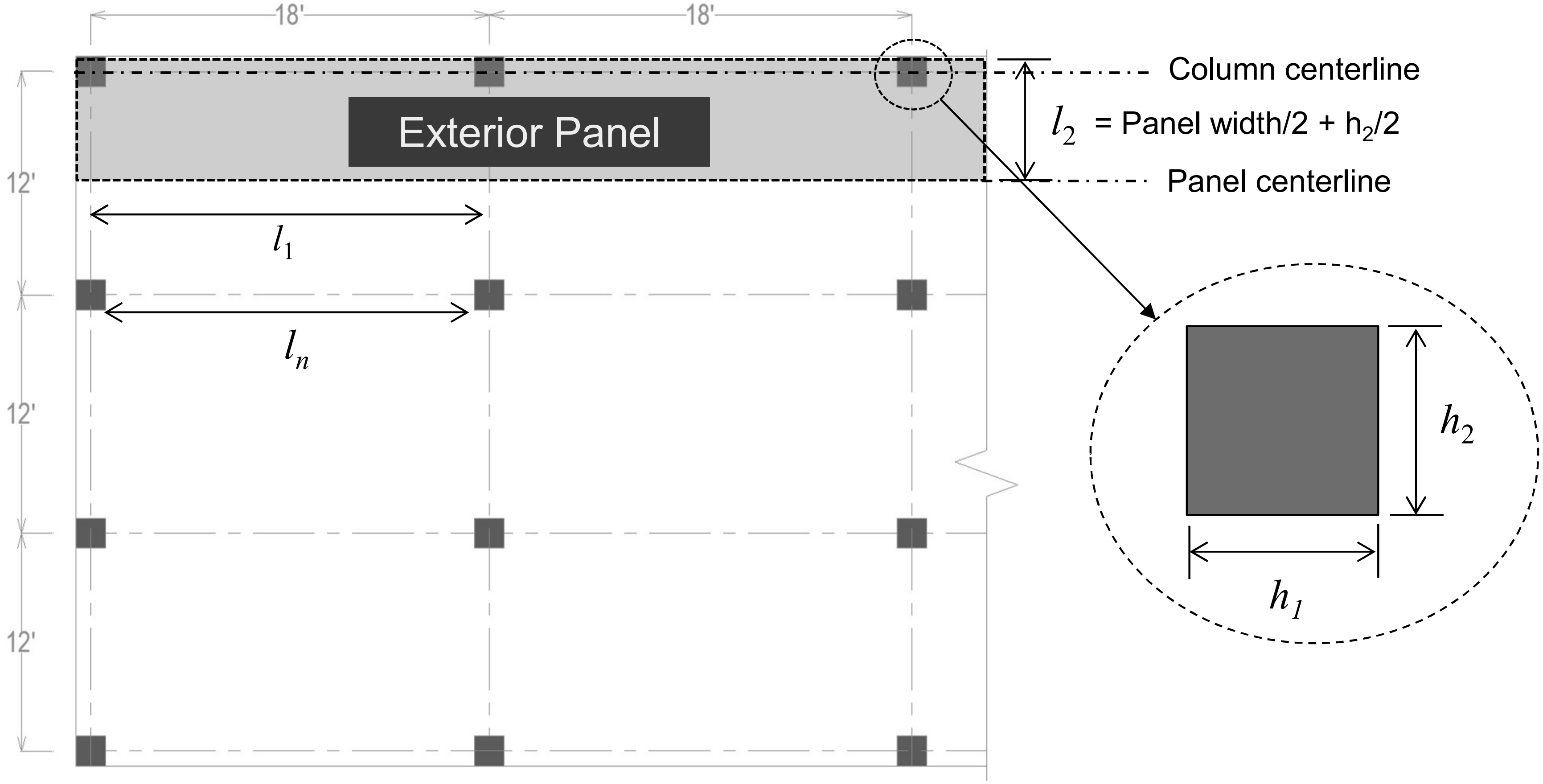
- **Marking Frames in Each Direction**



Analysis Procedure using DDM

❑ Step 3: Analysis

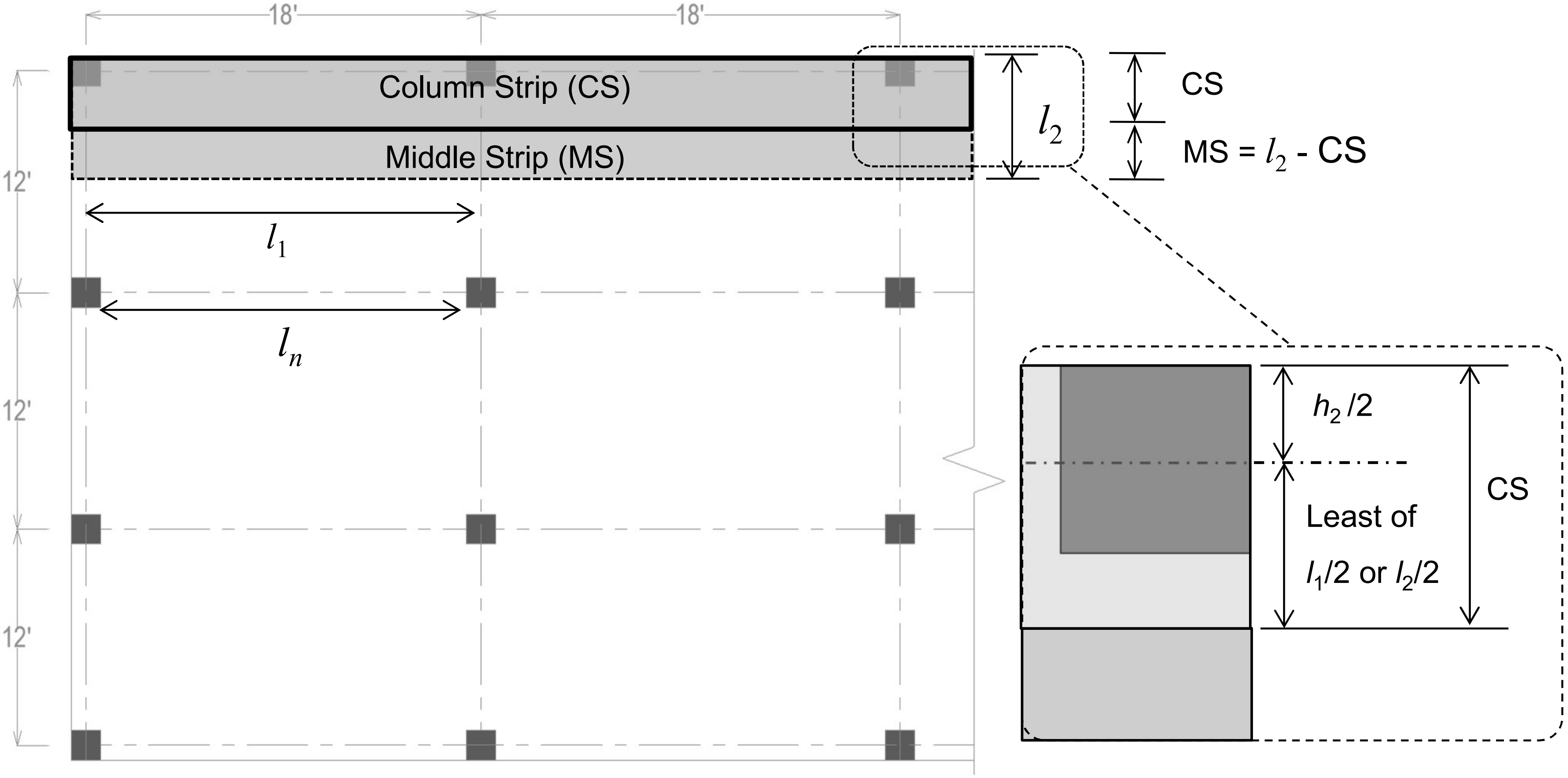
❖ Marking Frames in Each Direction



Analysis Procedure using DDM

Step 3: Analysis

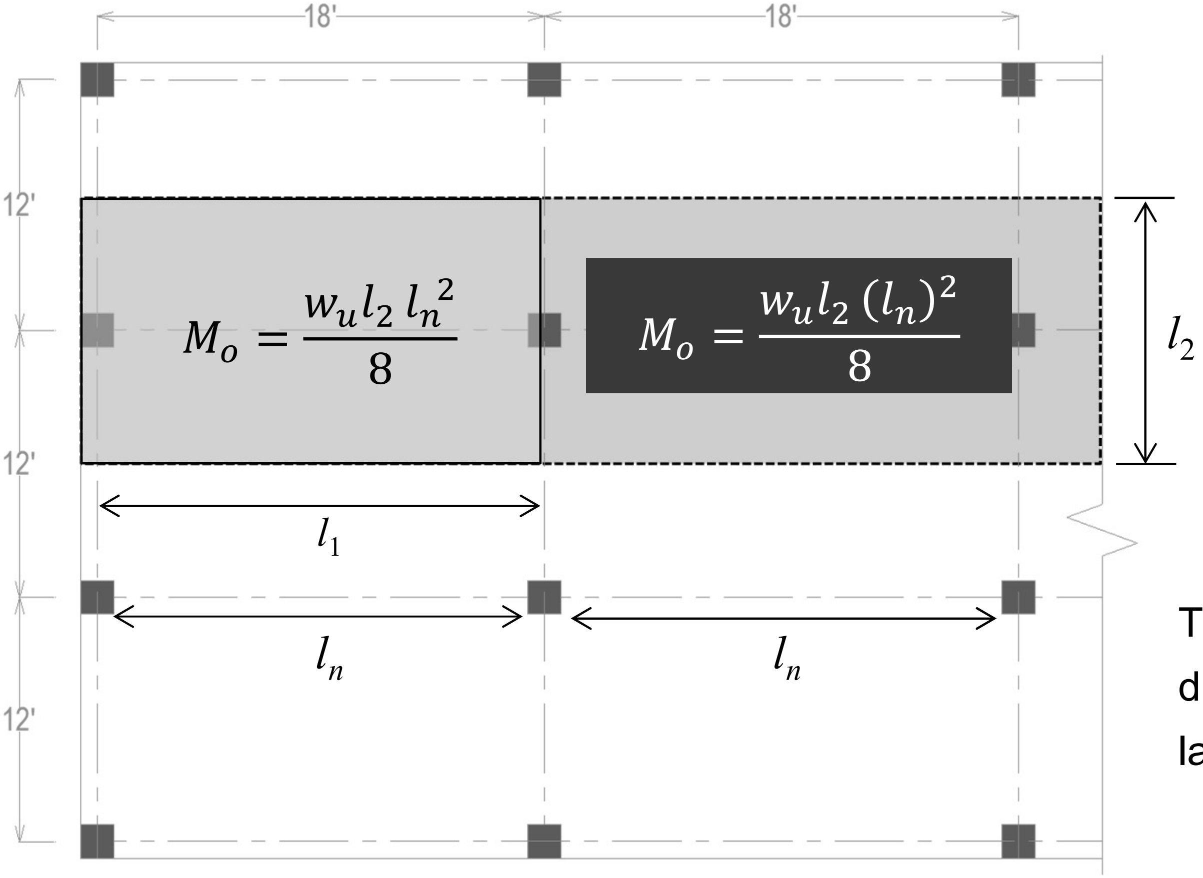
Marking Frames in Each Direction



Analysis Procedure using DDM

❑ Step 3: Analysis

❖ Total Static Moment



$$M_o = \frac{w_u l_2 \, l_n{}^2}{8}$$

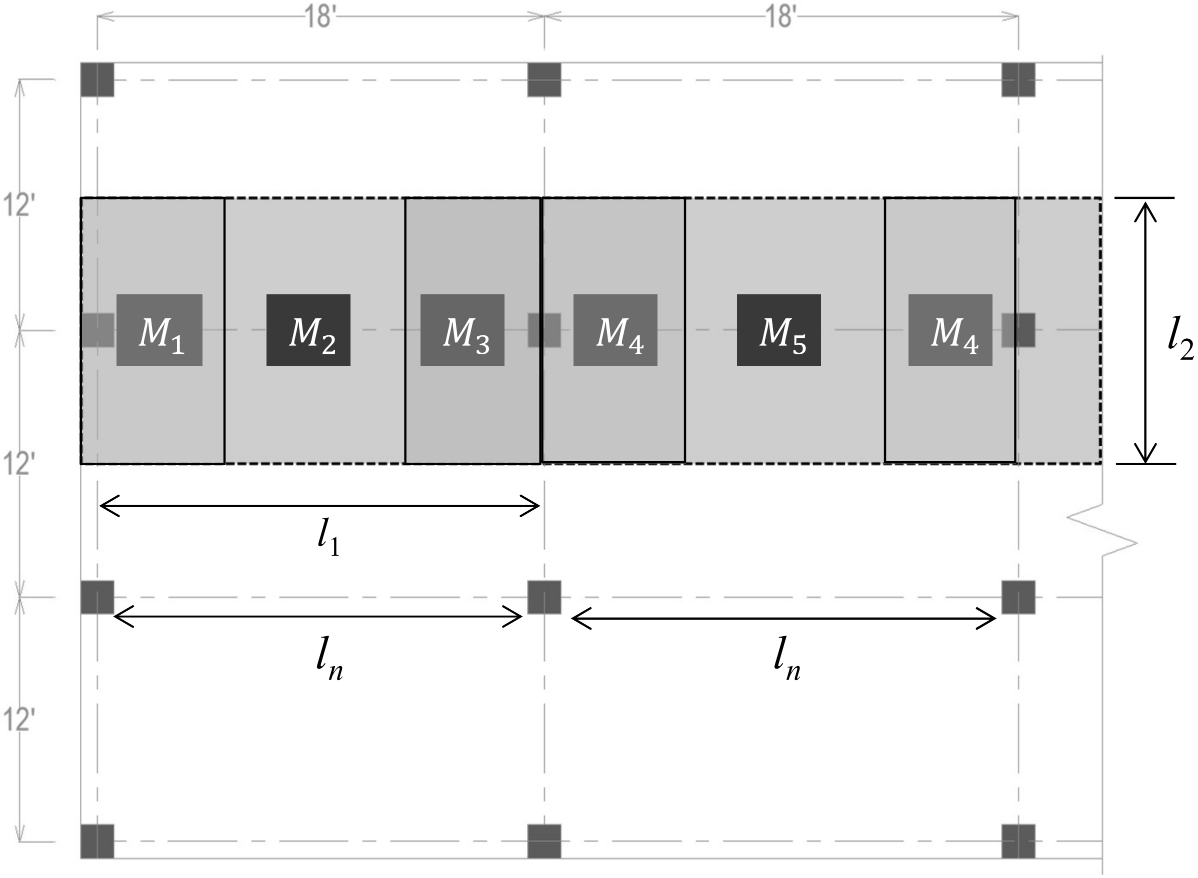
$$M_o = \frac{w_u l_2 \,(l_n)^2}{8}$$

This total moment is then distributed longitudinally and laterally as illustrated next.

Analysis Procedure using DDM

❑ Step 3: Analysis

❖ Longitudinal Distribution of Total Static Moment



$$M_o = \frac{w_u l_2 \, l_n^2}{8}$$

$M_1 = 0.26M_o$

$M_2 = 0.52M_o$

$M_3 = 0.70M_o$

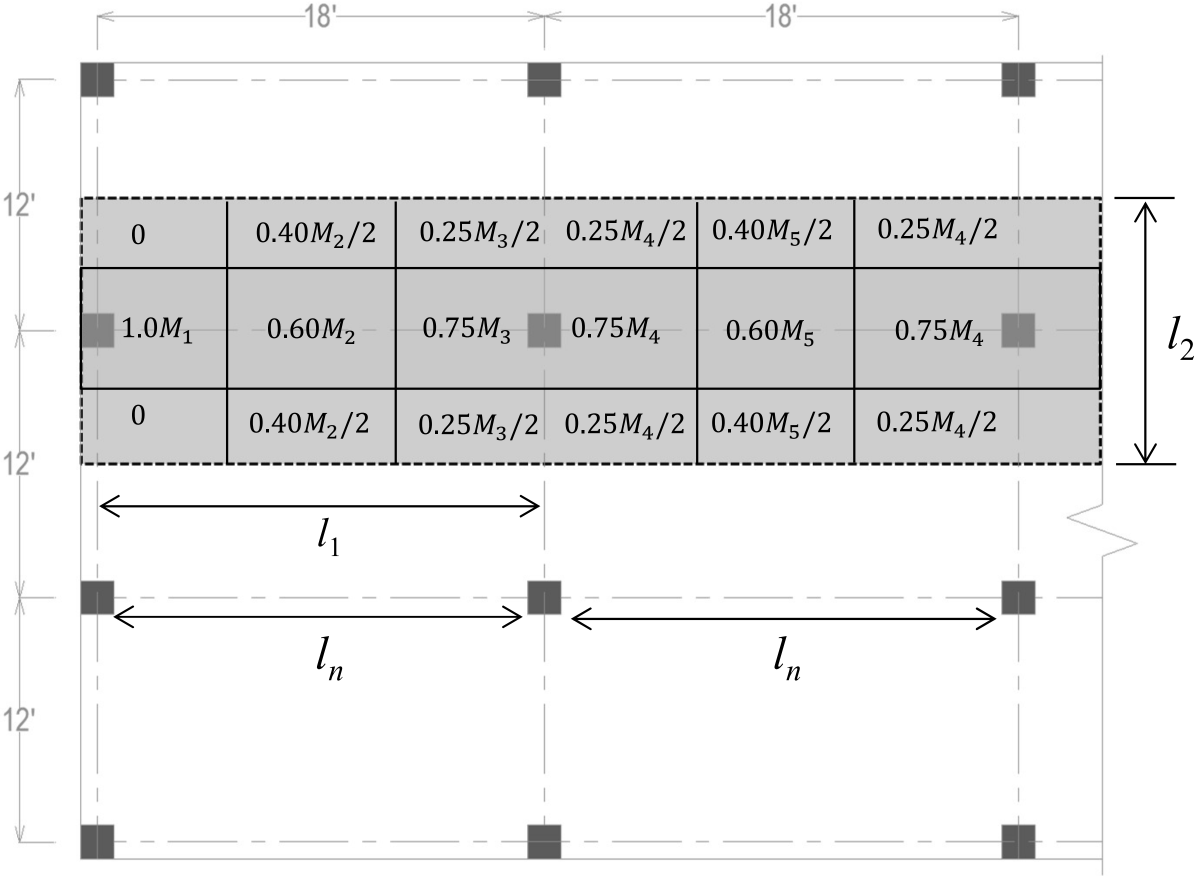
$M_4 = 0.65M_o$

$M_5 = 0.35M_o$

Analysis Procedure using DDM

❑ Step 3: Analysis

❖ Lateral Distribution of Calculated Moments



0	$0.40M_2/2$	$0.25M_3/2$	$0.25M_4/2$	$0.40M_5/2$	$0.25M_4/2$
$1.0M_1$	$0.60M_2$	$0.75M_3$	$0.75M_4$	$0.60M_5$	$0.75M_4$
0	$0.40M_2/2$	$0.25M_3/2$	$0.25M_4/2$	$0.40M_5/2$	$0.25M_4/2$

$$M_o = \frac{w_u l_2 \, l_n^2}{8}$$

$M_1 = 0.26M_o$

$M_2 = 0.52M_o$

$M_3 = 0.70M_o$

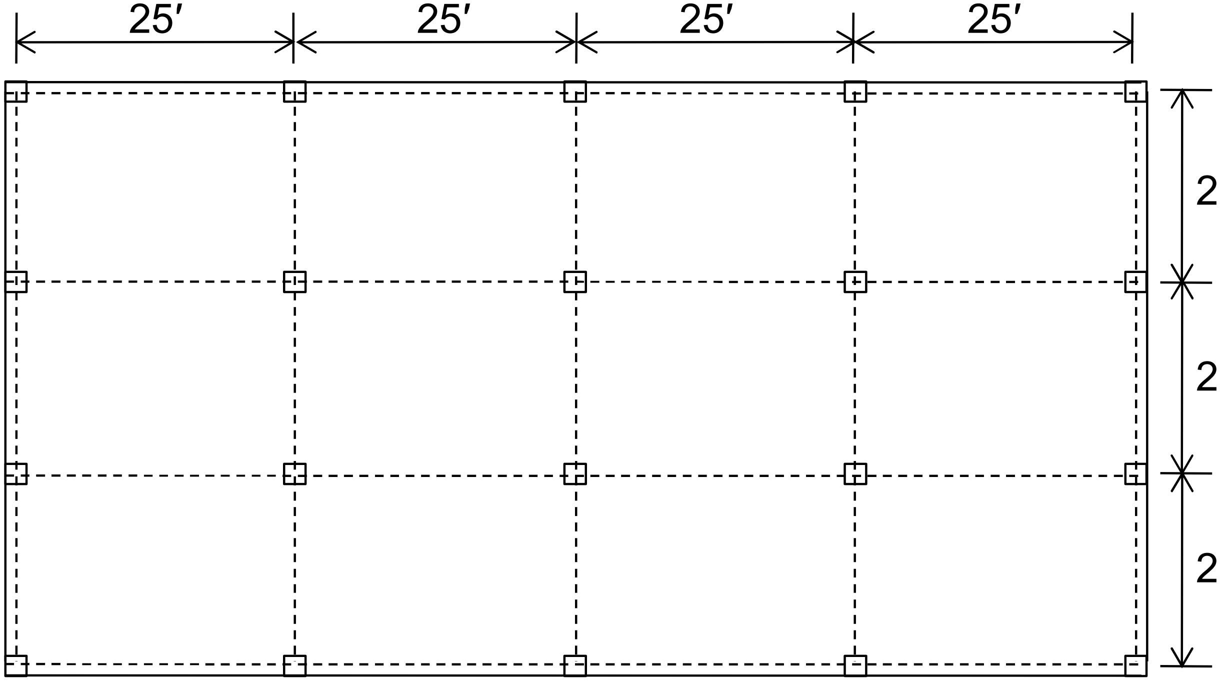
$M_4 = 0.65M_o$

$M_5 = 0.35M_o$

Example 6.2

❑ Problem Statement

- ***Analyze*** the flat plate shown below using DDM. The slab supports a uniformly distributed live load of 144 psf. All columns are 14″ square. Take $f_c' = 3$ ksi and $f_y = 60$ ksi



Example 6.2

❑ Solution

- A summarized solution of this example has been provided in subsequent slides.
- Please refer **Lecture 4 of RCD – II** for the compete step-by-step solution.

Example 6.2

❑ Solution

➢ Step 1: Selection of Sizes

- ACI table 8.3.1.1 is used for finding flat plate and flat slab thickness

Table 8.3.1.1 —Minimum thickness of nonprestressed two-way slabs without interior beams (in.)

f_y (psi)	Without drop panels			With drop panels		
	Exterior Panels		Interior panels	Exterior Panels		Interior panels
	Without edge beams	With edge beams		Without edge beams	With edge beams	
40,000	$l_n/33$	$l_n/36$	$l_n/36$	$l_n/36$	$l_n/40$	$l_n/40$
60,000	$l_n/30$	$l_n/33$	$l_n/33$	$l_n/33$	$l_n/36$	$l_n/36$
80,000	$l_n/27$	$l_n/30$	$l_n/30$	$l_n/30$	$l_n/33$	$l_n/33$

Example 6.2

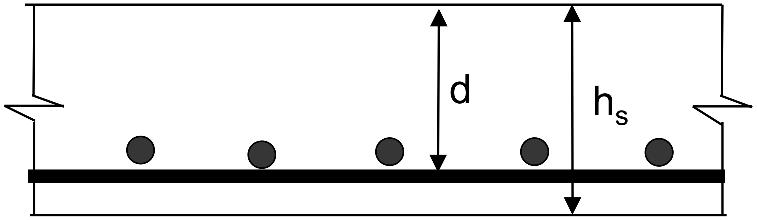
❑ Solution

➢ Step 1: Selection of Sizes

Exterior panel governs. Therefore;

$$h_{min} = \frac{l_n}{30} = \frac{25 - 2(14/12)}{30} = 0.76' \text{ or } 9.12''$$

Take $h_s = 10''$



Slab

Assuming #6 bar ;

$d = h_s - 0.75 - 0.75 = 8.5''$

Step 2: Calculation of Loads

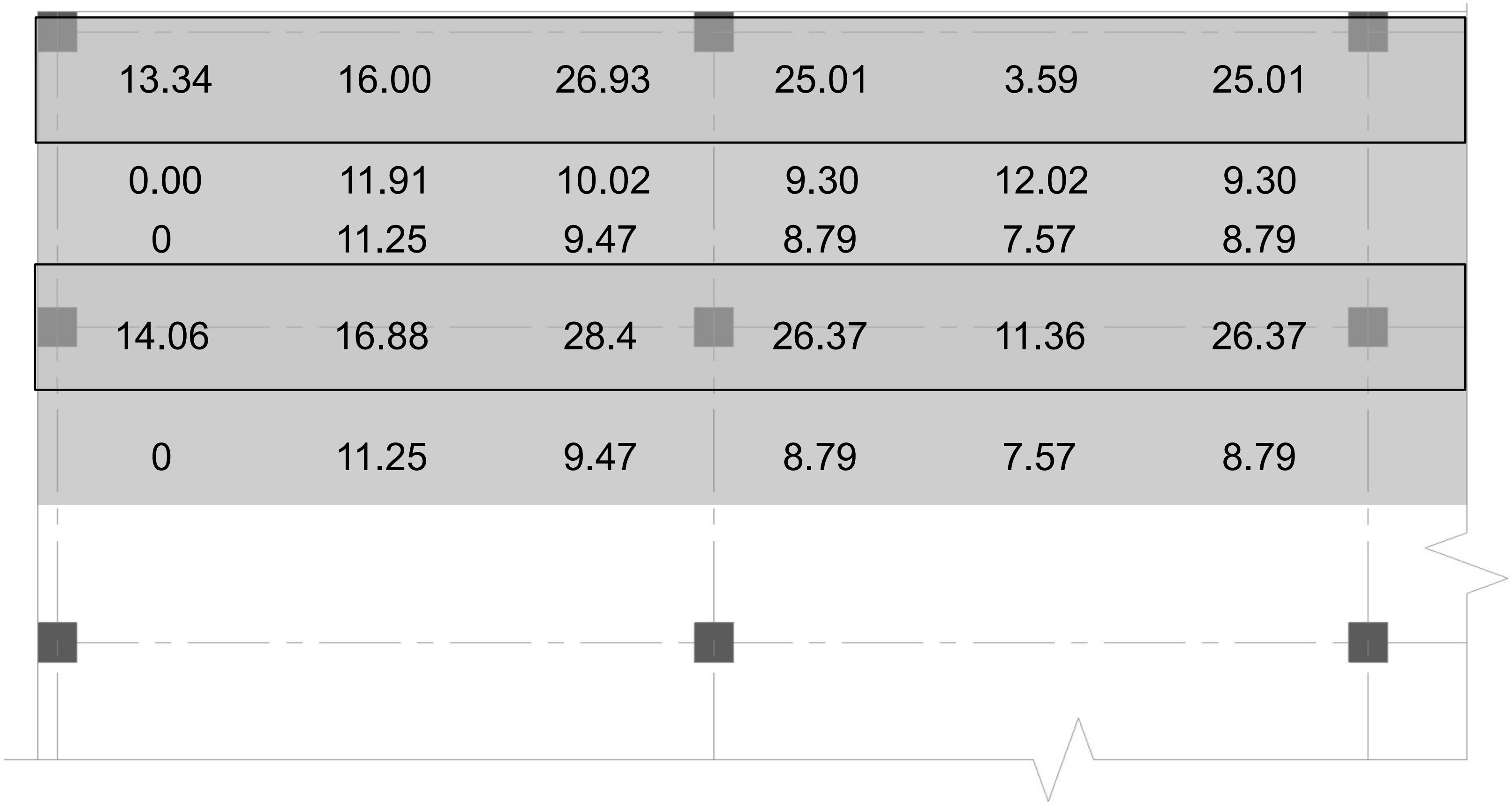
$W_u = 1.2D + 1.6L = 0.381\ ksf$

Example 6.2

❑ Solution

➢ Step 3: Analysis (E-W Direction)

❖ **Summary of Moments for Interior Frame (per unit strip)**



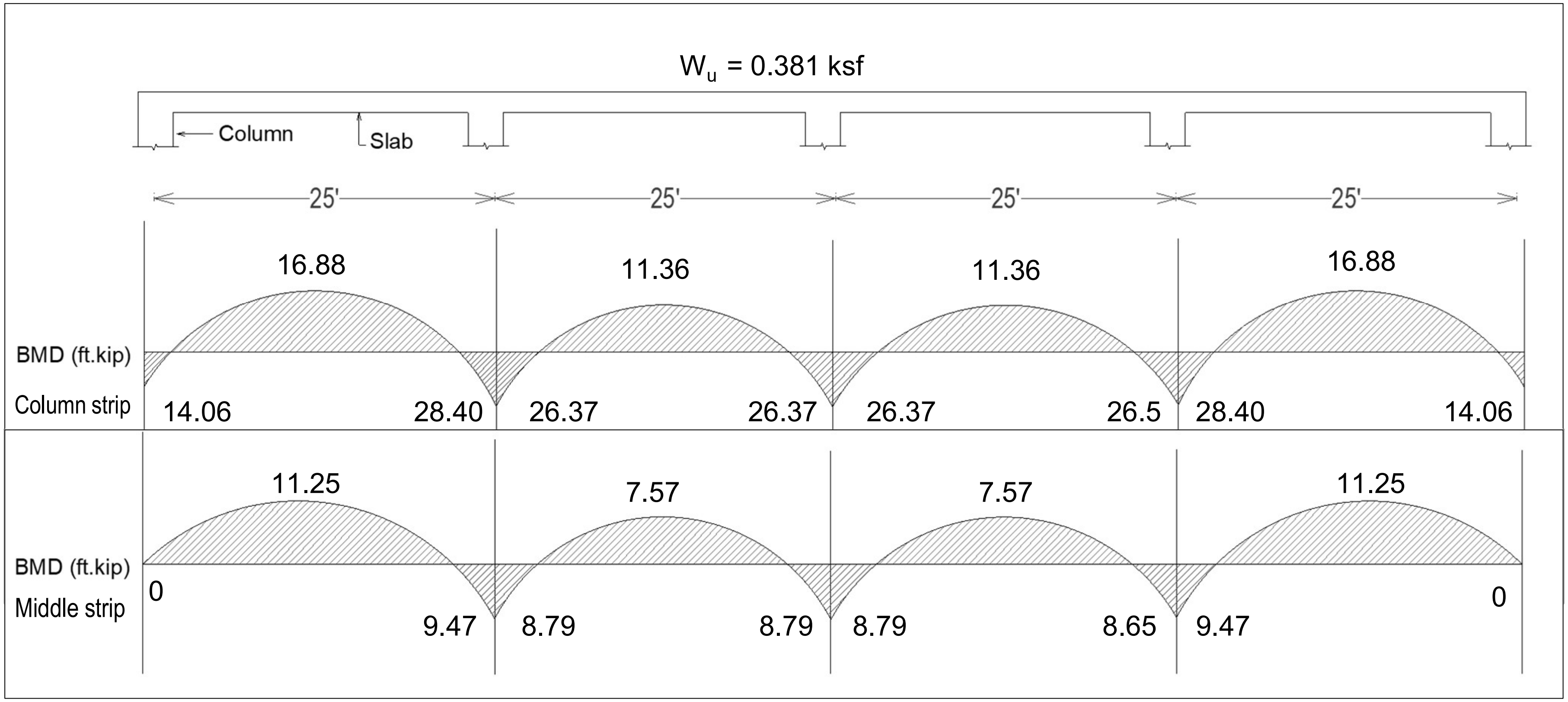
13.34	16.00	26.93	25.01	3.59	25.01
0.00	11.91	10.02	9.30	12.02	9.30
0	11.25	9.47	8.79	7.57	8.79
14.06	16.88	28.4	26.37	11.36	26.37
0	11.25	9.47	8.79	7.57	8.79

Example 6.2

❑ Solution

➢ Step 3: Analysis (E-W Direction)

- ❖ **Bending Moment Diagrams**

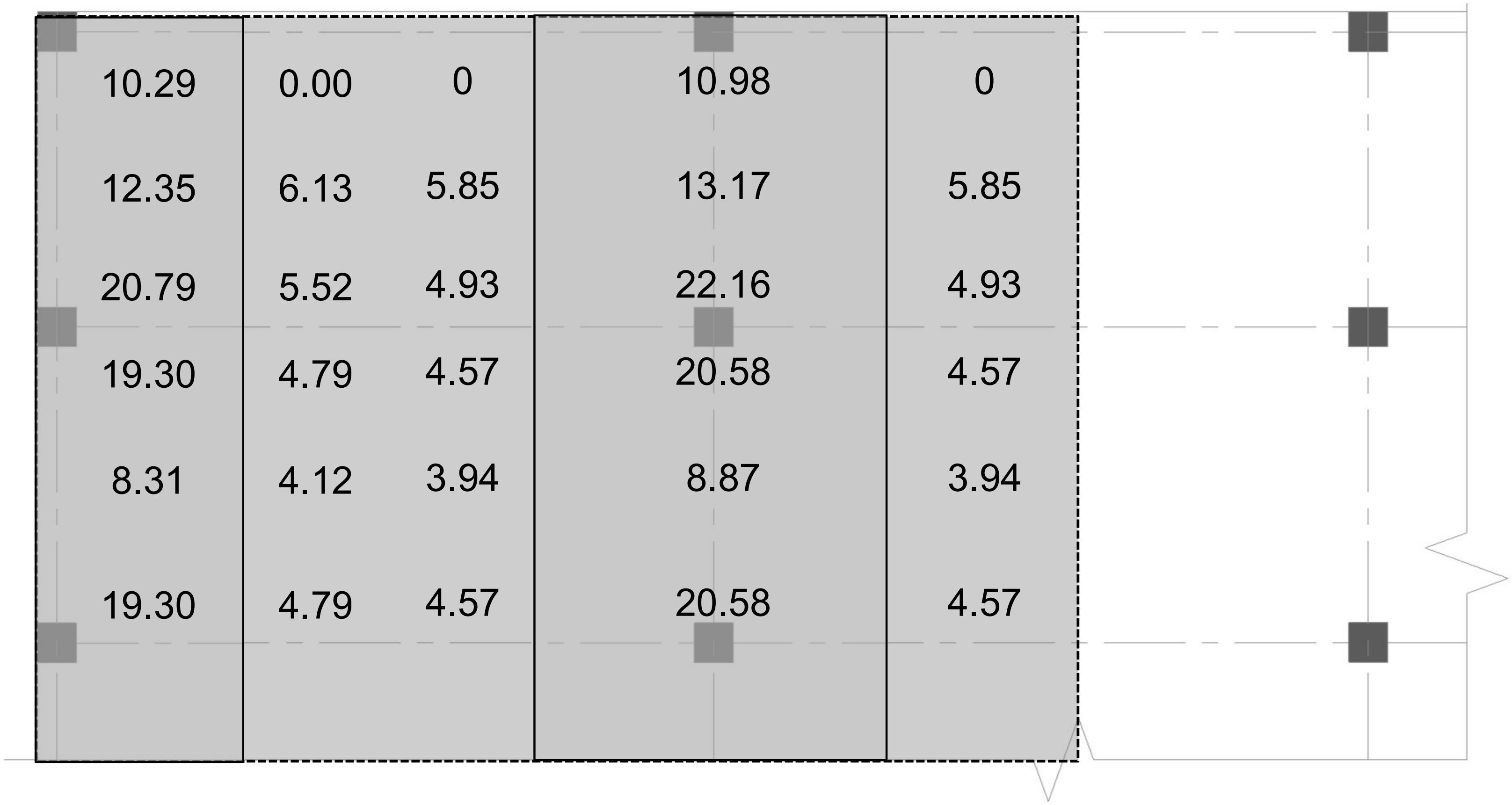


Example 6.2

❑ Solution

➢ Step 3: Analysis (N-S Direction)

- ❖ **Summary of Moments for Exterior Frame (per unit strip)**



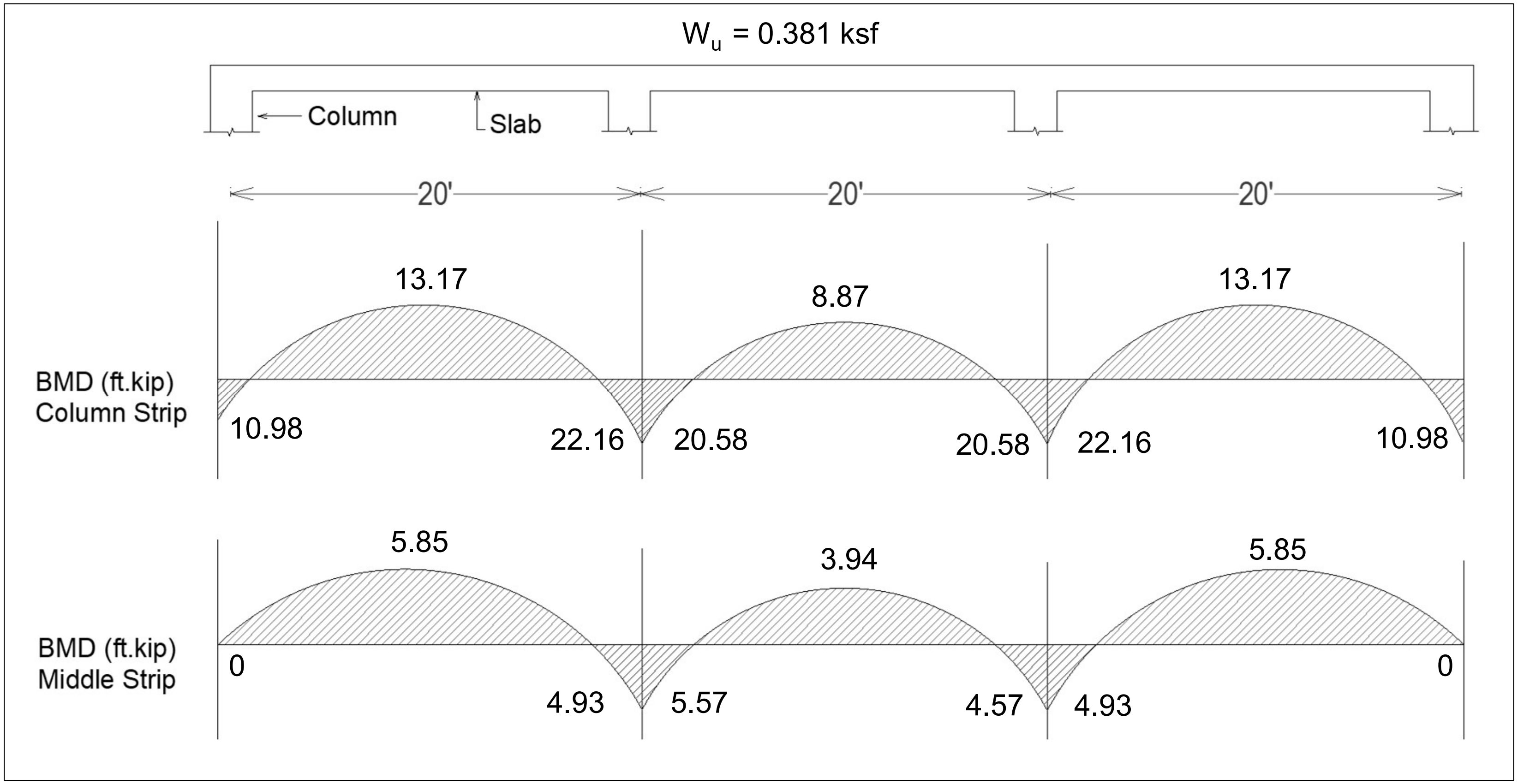
10.29	0.00	0	10.98	0
12.35	6.13	5.85	13.17	5.85
20.79	5.52	4.93	22.16	4.93
19.30	4.79	4.57	20.58	4.57
8.31	4.12	3.94	8.87	3.94
19.30	4.79	4.57	20.58	4.57

Example 6.2

❑ Solution

➢ Step 3: Analysis (N-S Direction)

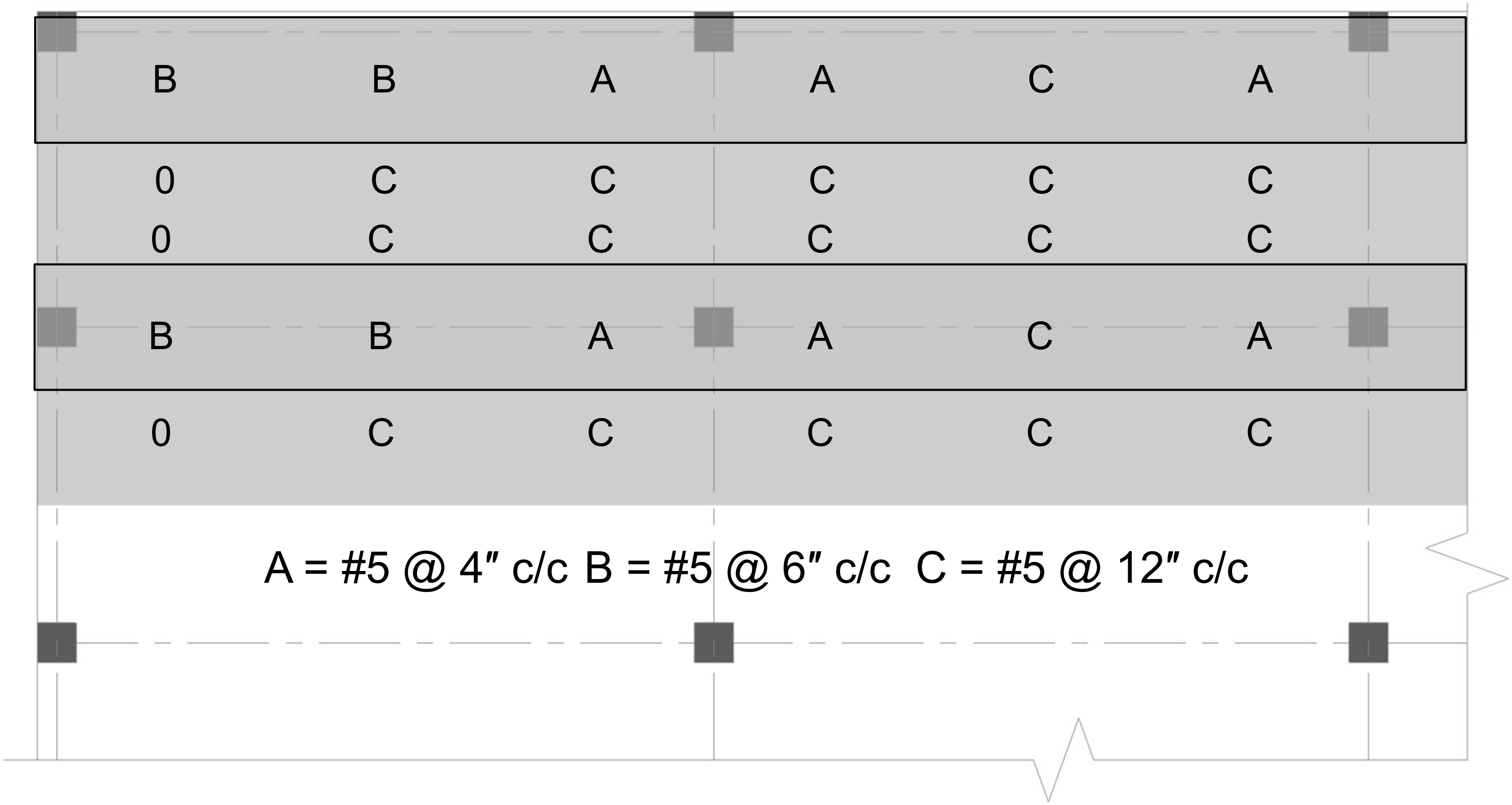
- Bending Moment Diagrams



Example 6.2

❑ Solution

- **Step 4: Reinforcement Detailing (E-W Direction)**

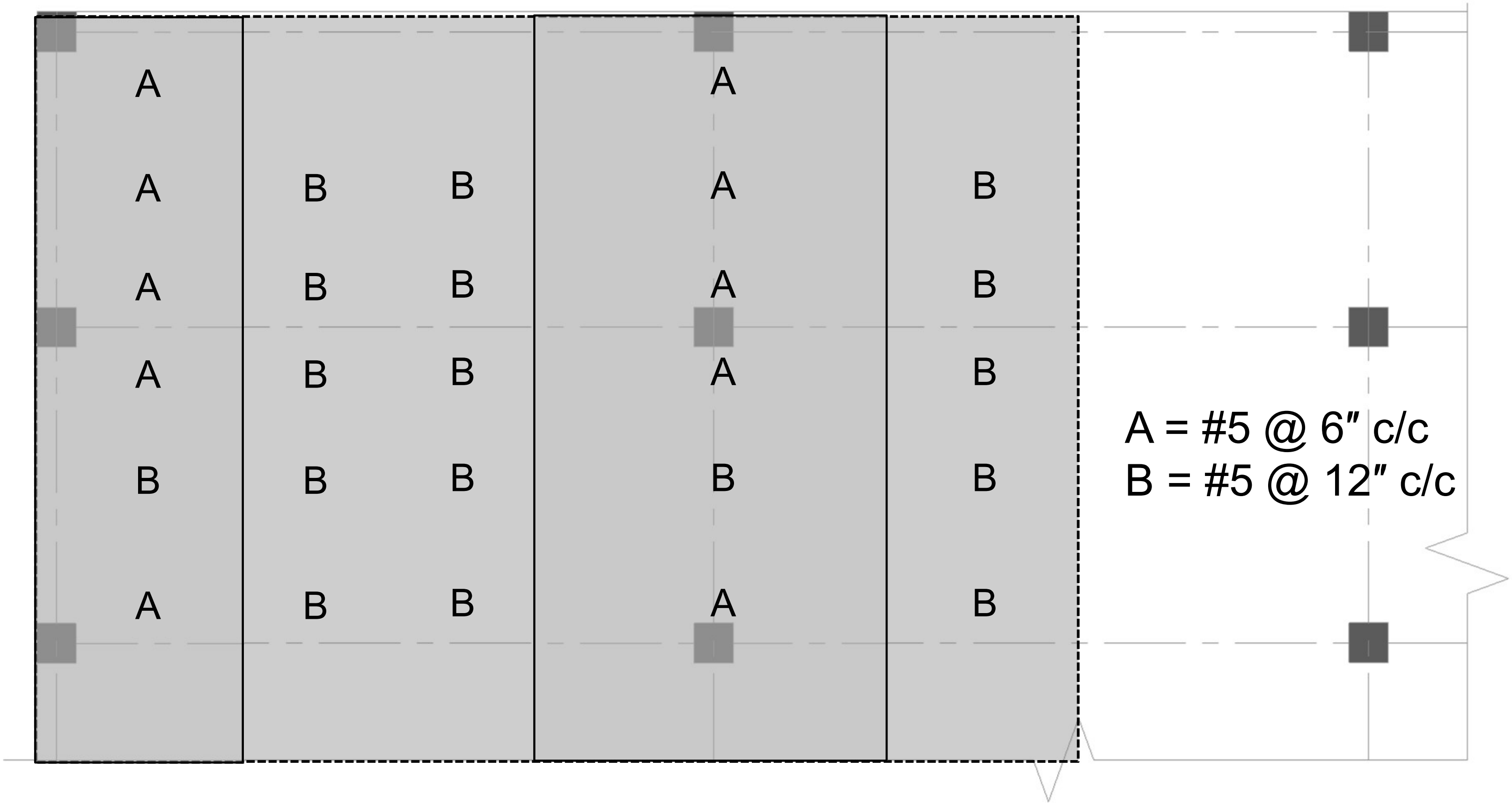


A = #5 @ 4″ c/c B = #5 @ 6″ c/c C = #5 @ 12″ c/c

Example 6.2

❑ Solution

- **Step 4: Reinforcement Detailing (N-S Direction)**



A = #5 @ 6″ c/c
B = #5 @ 12″ c/c

Other ACI Provisions for Flat Slabs

❑ Reinforcement and Spacing Limits

- **Minimum Reinforcement (ACI 24.4.3.2)**

$$A_{s,min} = 0.0018\, bh_s$$

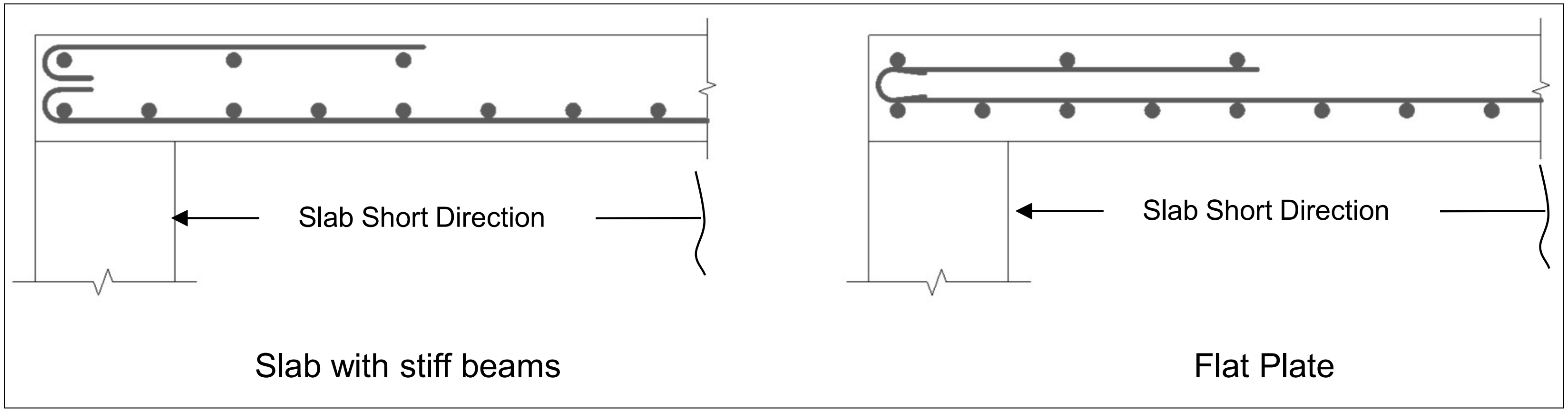
- **Maximum spacing (ACI 8.7.2.2)**

$S_{max} = 2h_s$ in each direction

Other ACI Provisions for Flat Slabs

❑ Detailing of Flexural Reinforcement

- In two-way slabs with beam supports, bars in shorter direction are placed closer to the surface due to greater moments in the shorter direction.
- However, for flat plates, long-direction bars in middle and column strips are placed nearer the surface due to larger moments in the longer direction.



Other ACI Provisions for Flat Slabs

❑ Detailing of Flexural Reinforcement

❖ Splicing

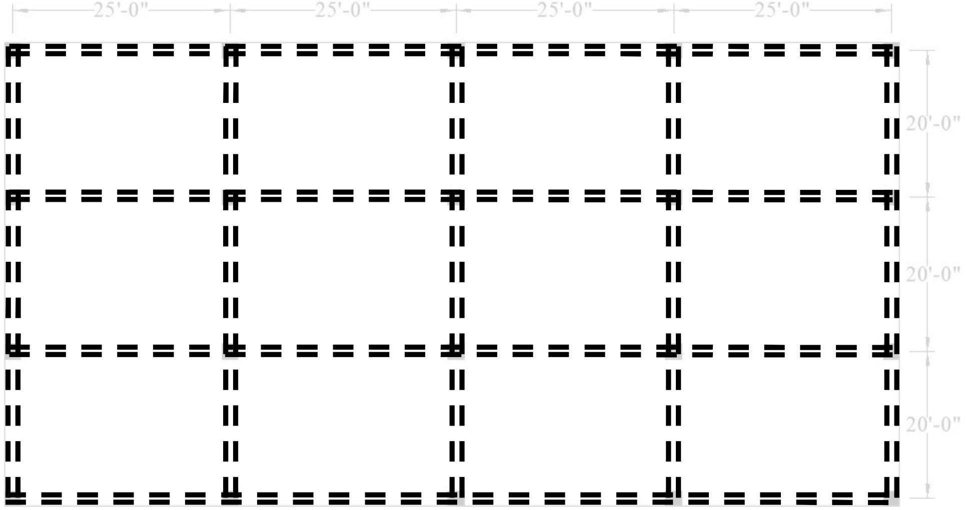
- ACI 8.7.4.2.1 requires that all bottom bars within the column strip in each direction be continuous or spliced with length equal to 1.0 l_d, (For development length see ACI 25.4.2.3 or Nelson 13th Ed, page 172 chapter 5 or mechanical or welded splices).

Other ACI Provisions for Flat Slabs

❑ Detailing of Flexural Reinforcement

❖ Continuity of Bars

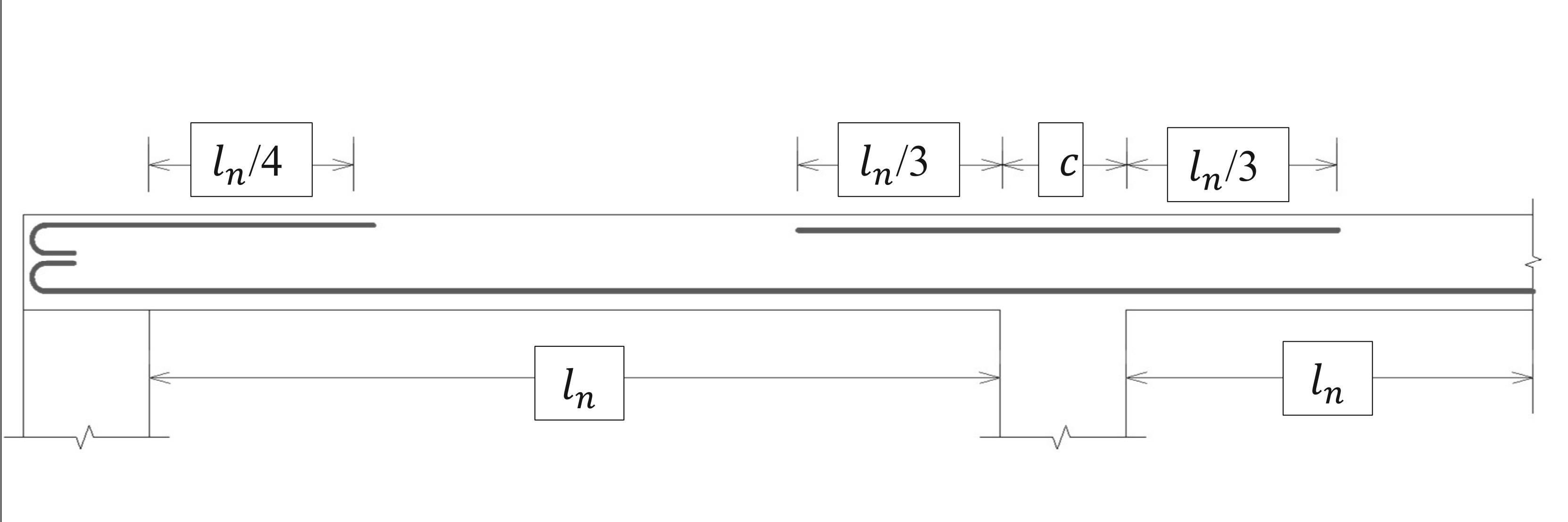
- ACI 8.7.4.2.2 requires that at least two of the column strip bottom bars in each direction must pass within the column core and must be anchored at exterior supports.



Other ACI Provisions for Flat Slabs

❑ Detailing of Flexural Reinforcement

- Standard Bar Cut off Points (Practical Recommendation) for column and middle strips both are shown below.

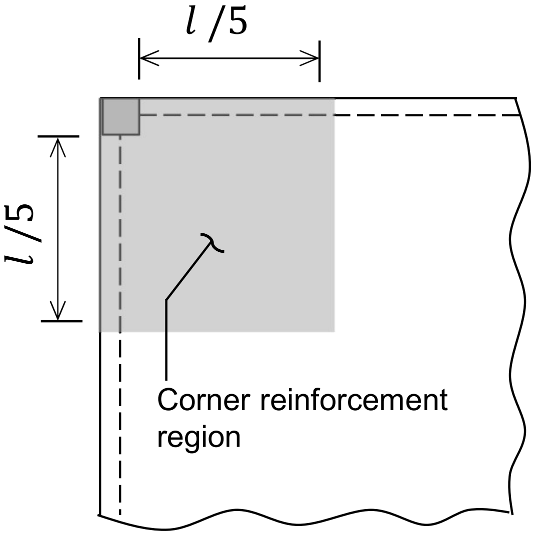


Other ACI Provisions for Flat Slabs

❑ Detailing of Flexural Reinforcement

❖ Reinforcement at Exterior Corners

- ACI 8.7.3.1.2 mandates reinforcement at exterior corners on the top and bottom of the slab, extending one-fifth the longer span of the corner panel in both directions as shown in the figure.
- The positive and negative reinforcement should be of size and spacing equivalent to that required for maximum positive moments (per foot of width) in the panel.



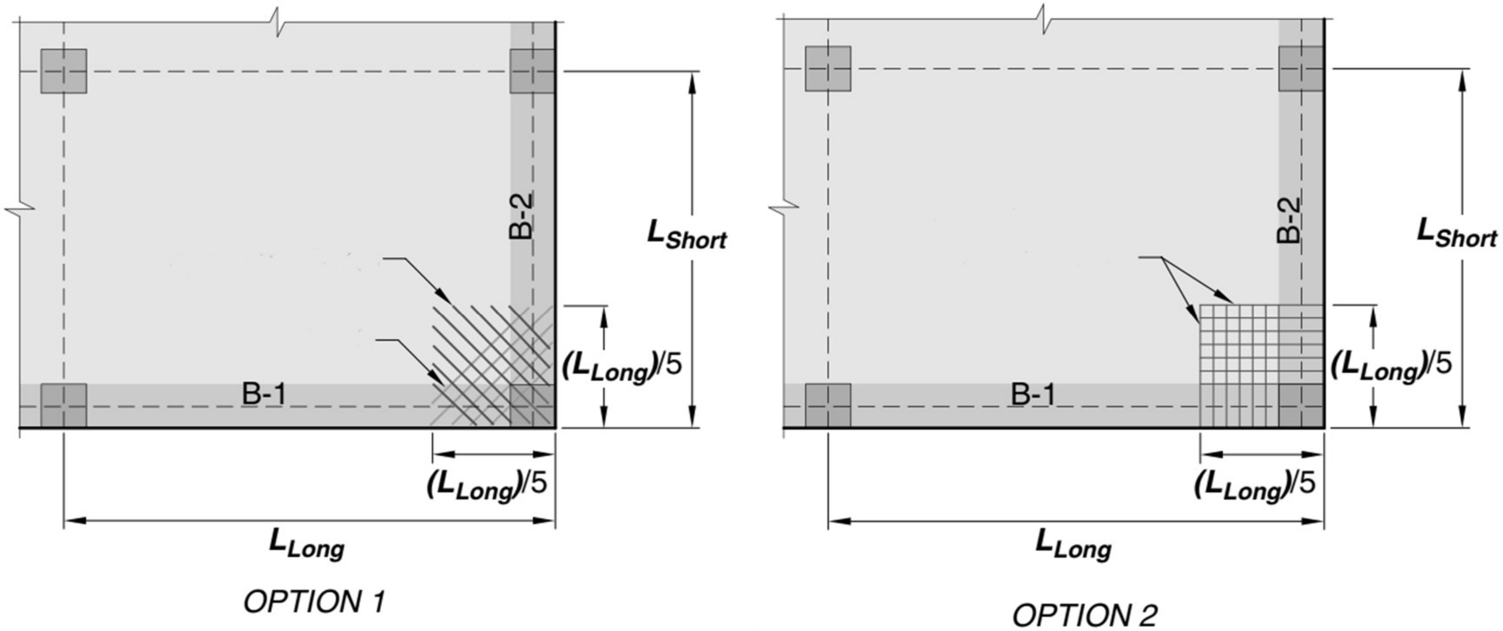
l = longer clear span

Other ACI Provisions for Flat Slabs

❑ Detailing of Flexural Reinforcement

❖ Reinforcement at Exterior Corners

- There are two options available for providing corner reinforcement.



Slab Corner Reinforcement (ACI Fig. R8.7.3.1)

Section – II

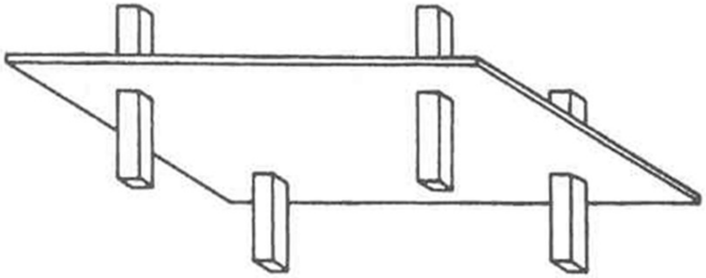
Shear Design of Two-Way Slab System without Beams

(Flat Plates and Flat Slabs)

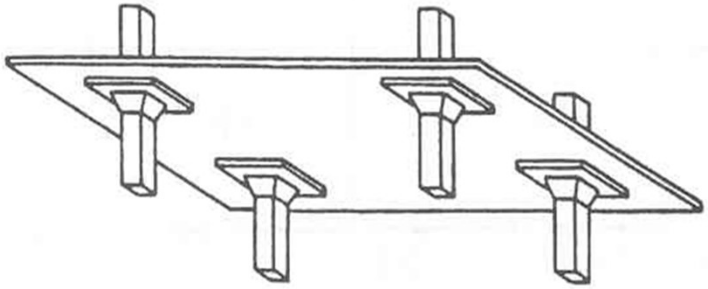
General

☐ Flat Plates and Flat Slabs

- These are the most common types of two-way slab system, which are commonly used in multi-story construction.
- They render low story heights and have easy construction and formwork.



Flat Plate



Flat Slab

General

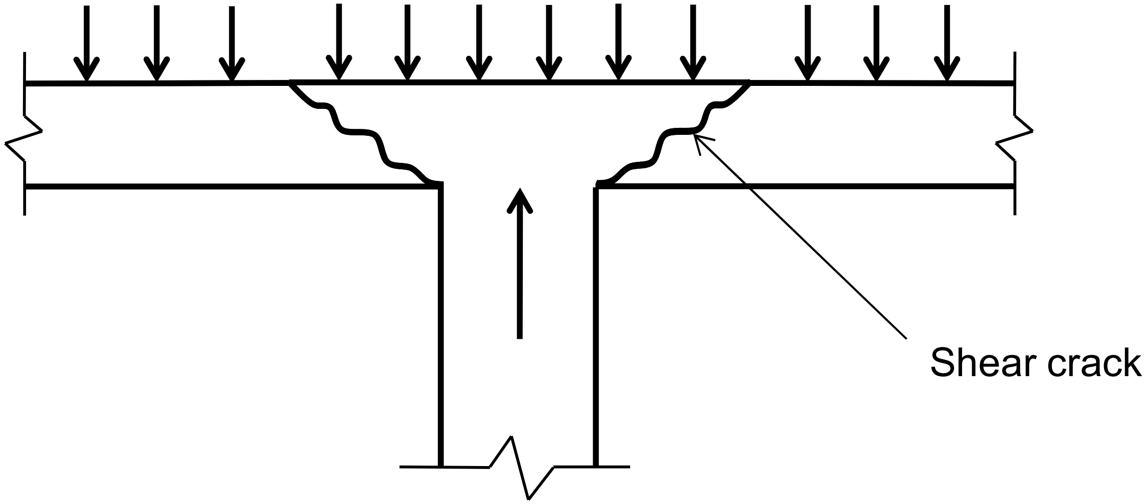
❑ Behavior

- These two-way slabs are directly supported by columns, shear near the column (punching shear) is of critical importance.
- Therefore, in addition to flexure, flat plates shall also be designed for two-way shear (punch out shear) stresses.

General

❑ Punching Shear in Flat Plates

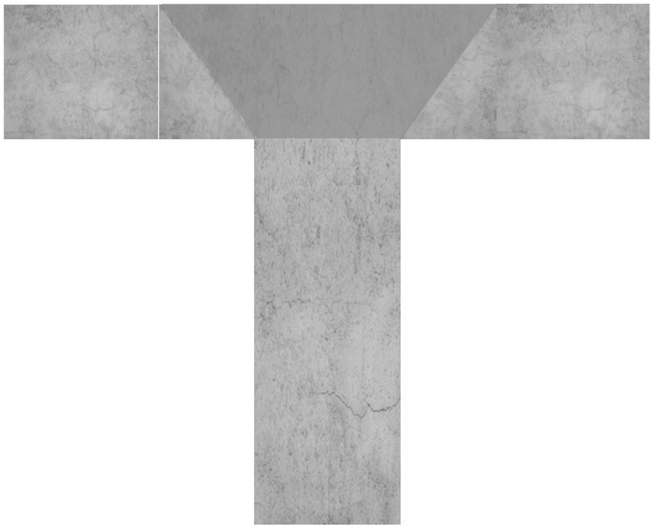
- Punching shear occurs at column support points in flat plates and flat slabs.

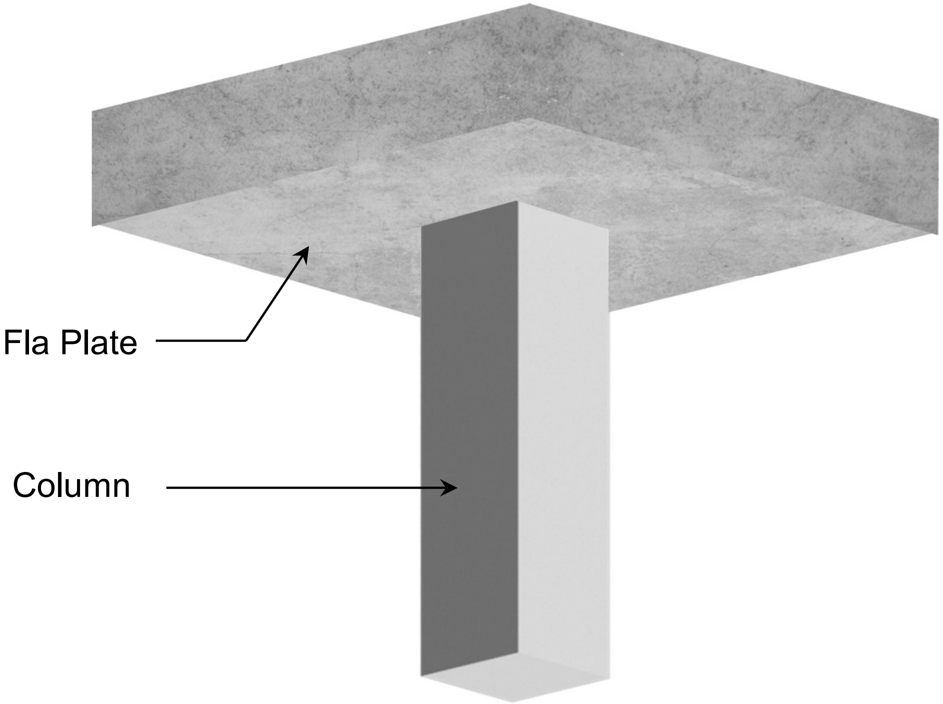


Punch Out

General

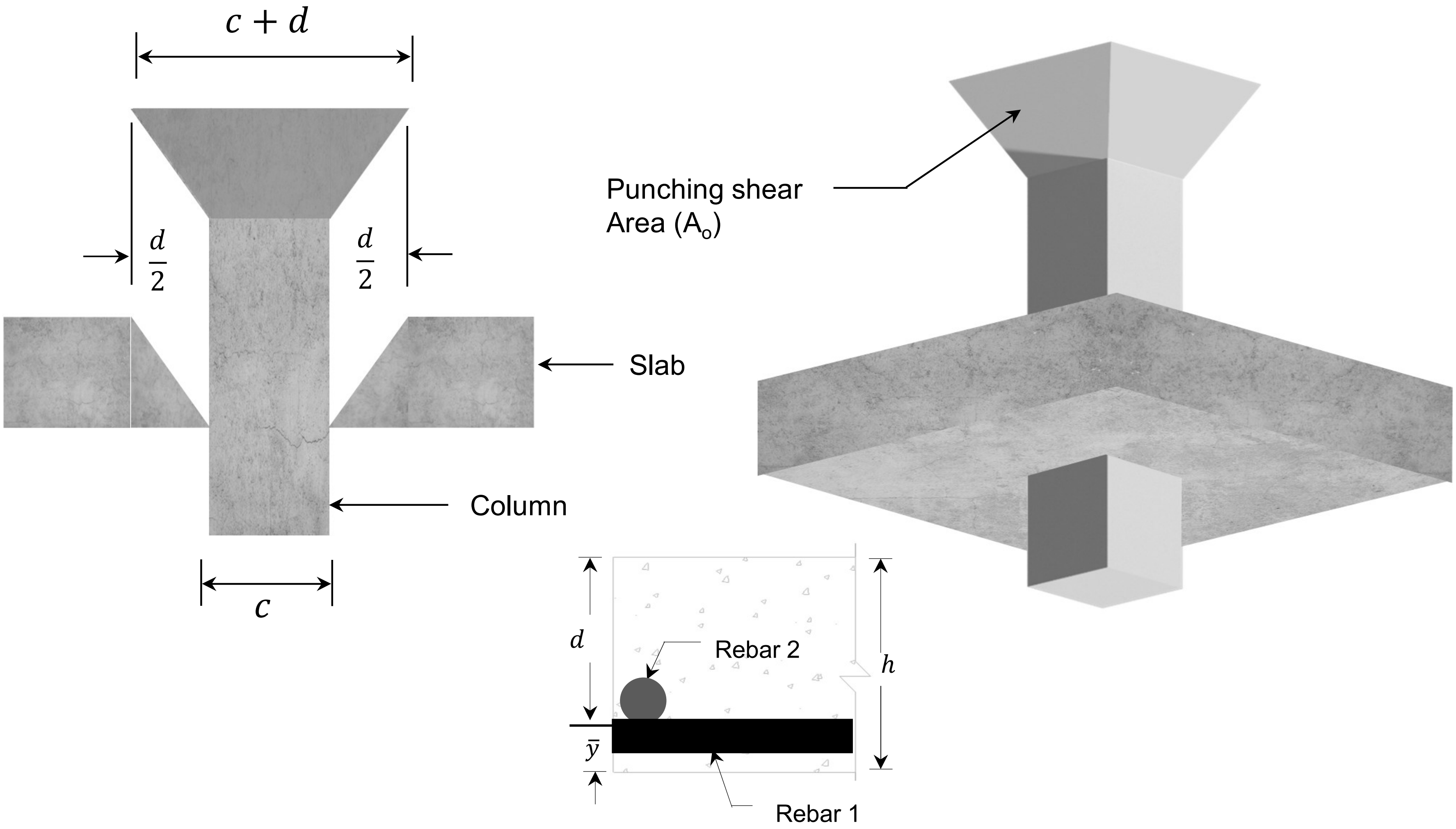
❑ Critical Section for Punching Shear





General

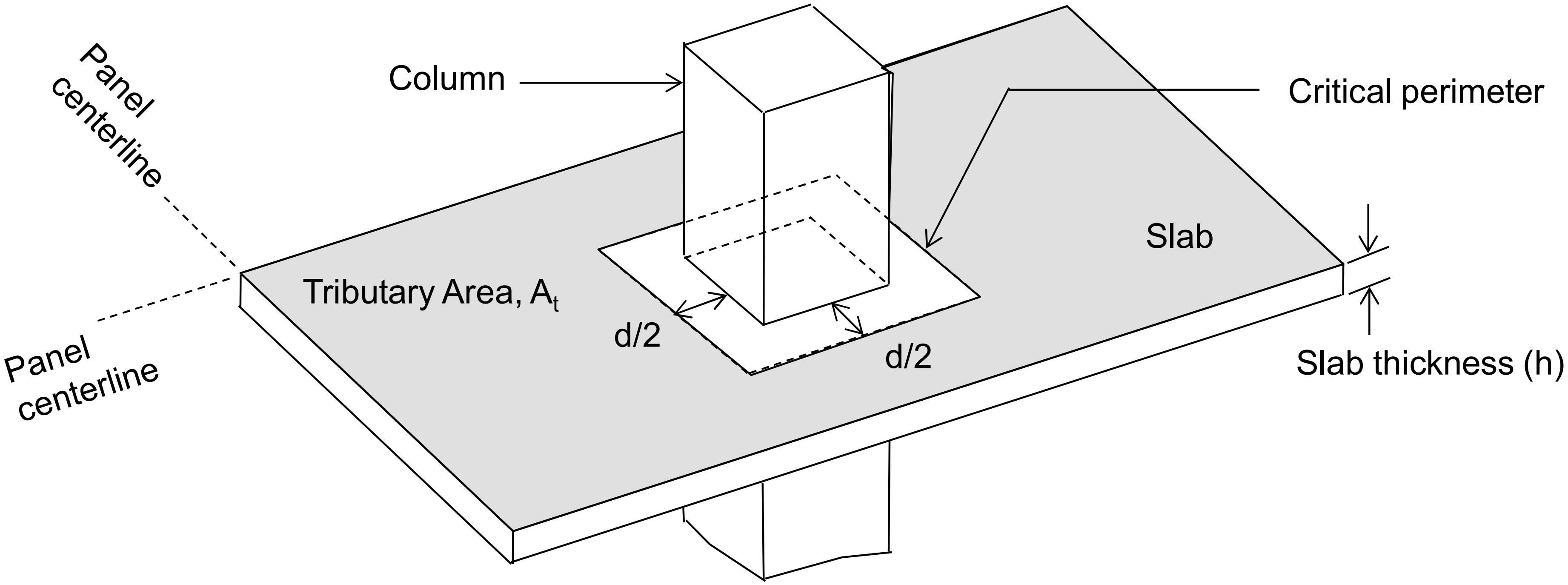
❑ Critical Section for Punching Shear



General

❑ Critical Section for Punching Shear

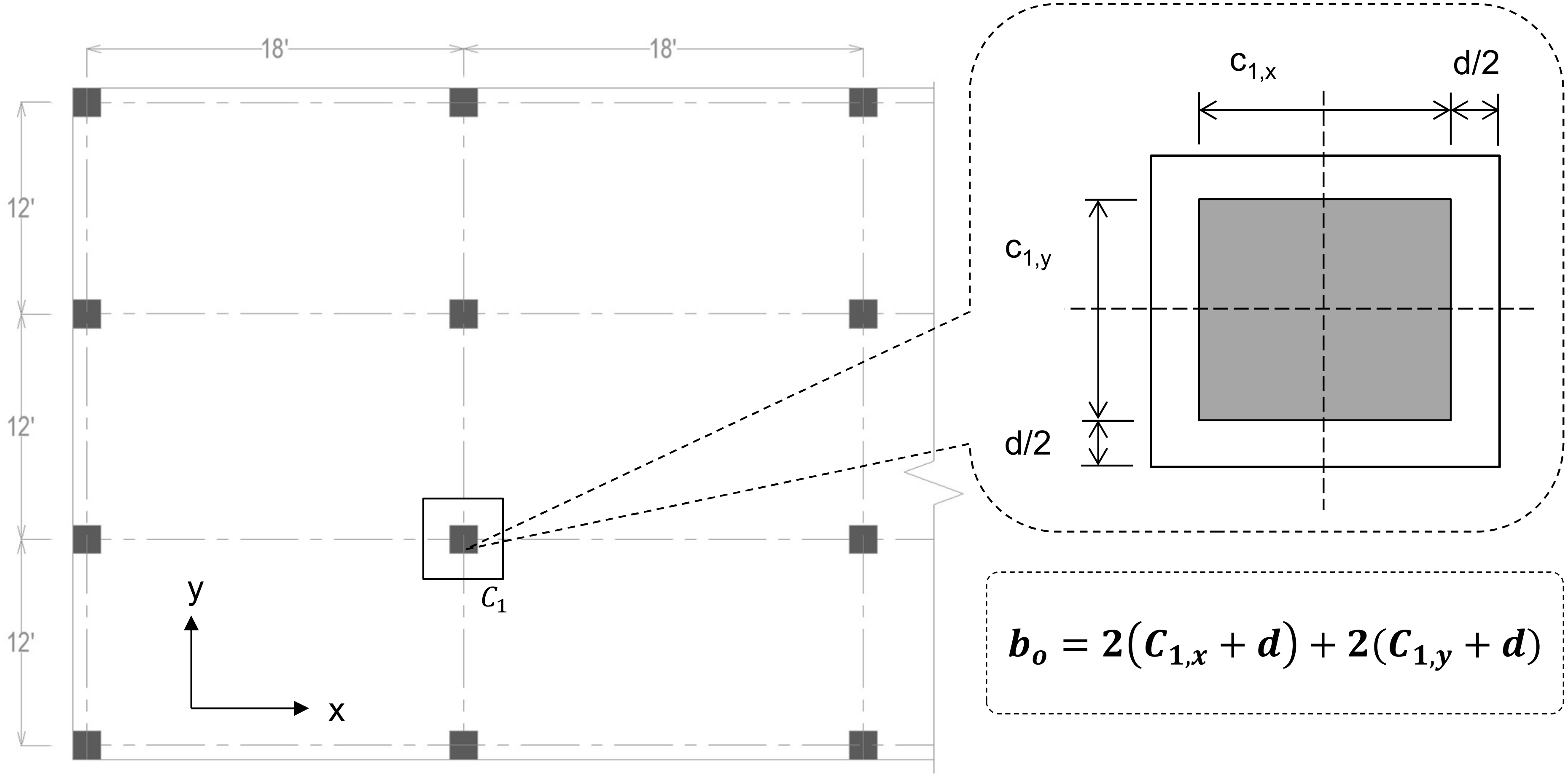
- In shear design of flat plates, the critical section is an area taken at a distance “d/2” from all face of the support.



General

❑ Calculation of Critical Perimeter b_o

❖ Interior Columns

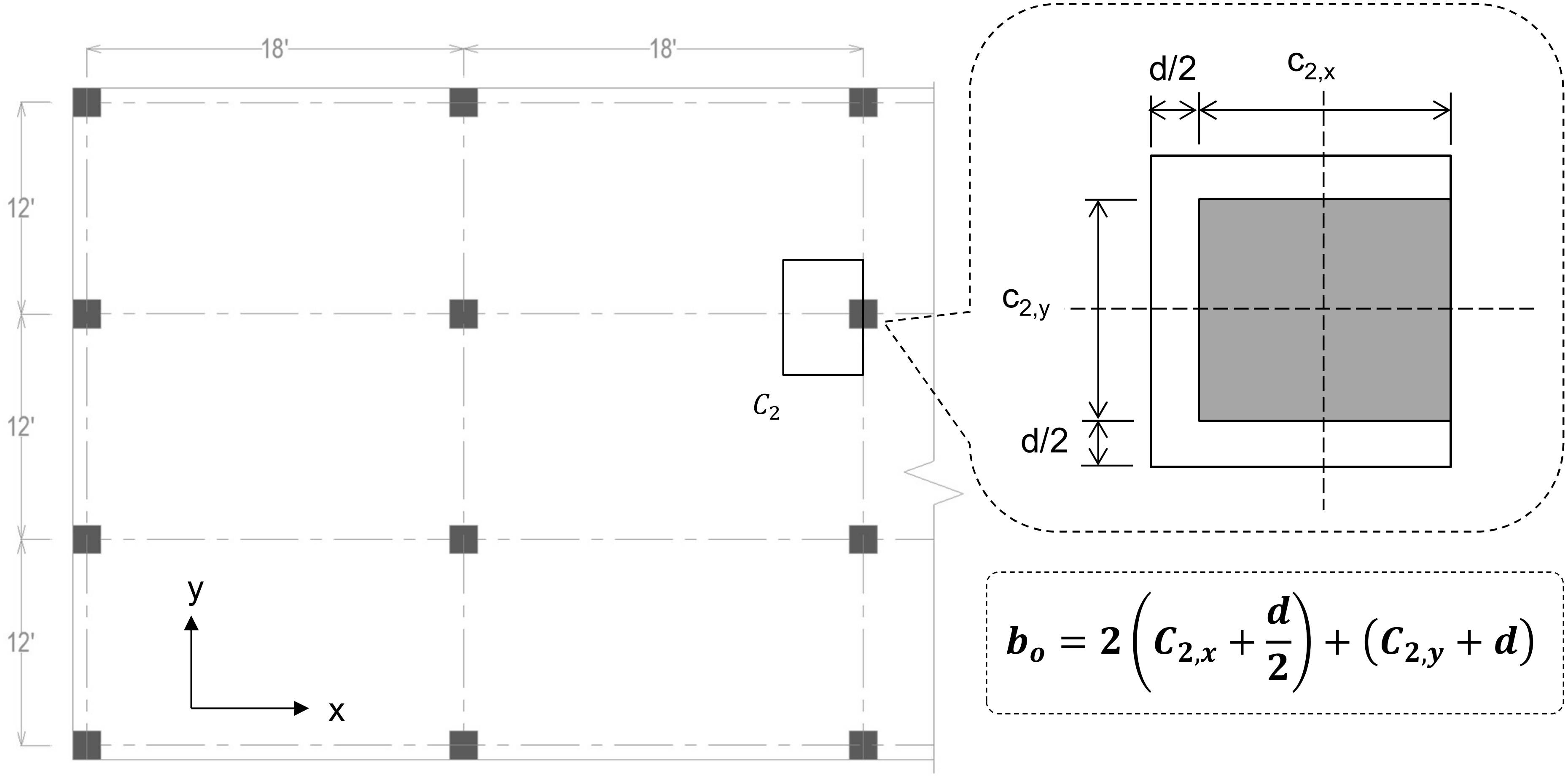


$$b_o = 2\left(C_{1,x} + d\right) + 2\left(C_{1,y} + d\right)$$

General

❑ Calculation of Critical Perimeter b_o

❖ Edge Columns

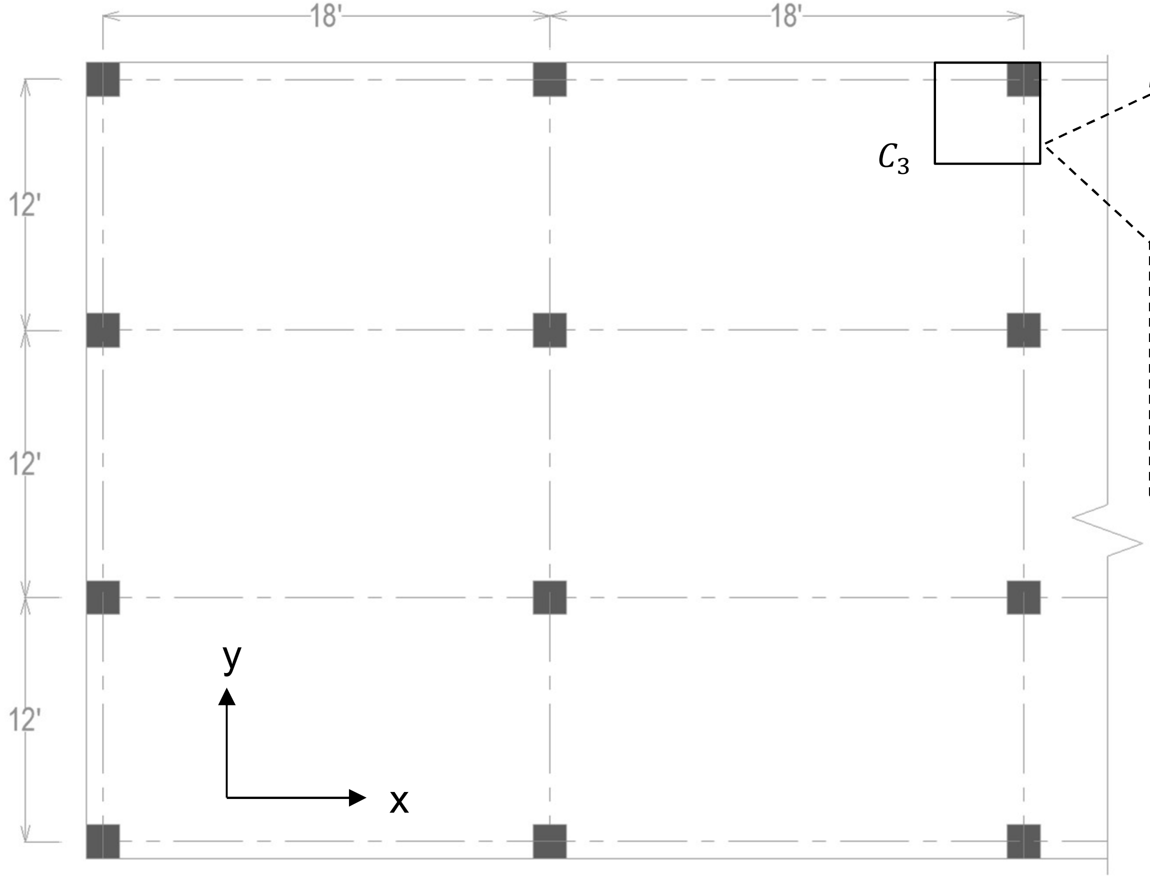


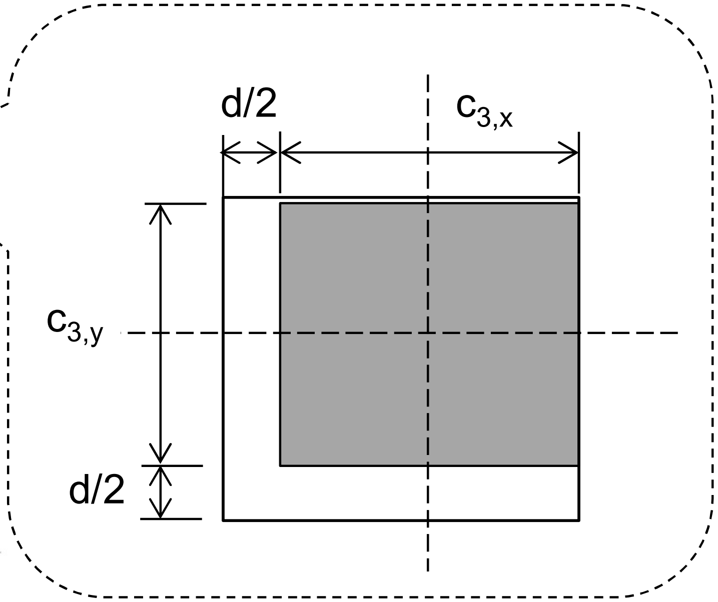
$$b_o = 2\left(C_{2,x} + \frac{d}{2}\right) + \left(C_{2,y} + d\right)$$

General

❑ Calculation of Critical Perimeter b_o

❖ Corner Columns





$$b_o = \left(C_{3,x} + \frac{d}{2}\right) + \left(C_{3,y} + \frac{d}{2}\right)$$

General

❑ Punching Shear Demand V_u for Square Columns

- Punching shear demand for square columns can be determined using the following steps.

1. **Calculate Tributary Area**

$$A_t = l_1 \times l_2 - A_o$$

where;

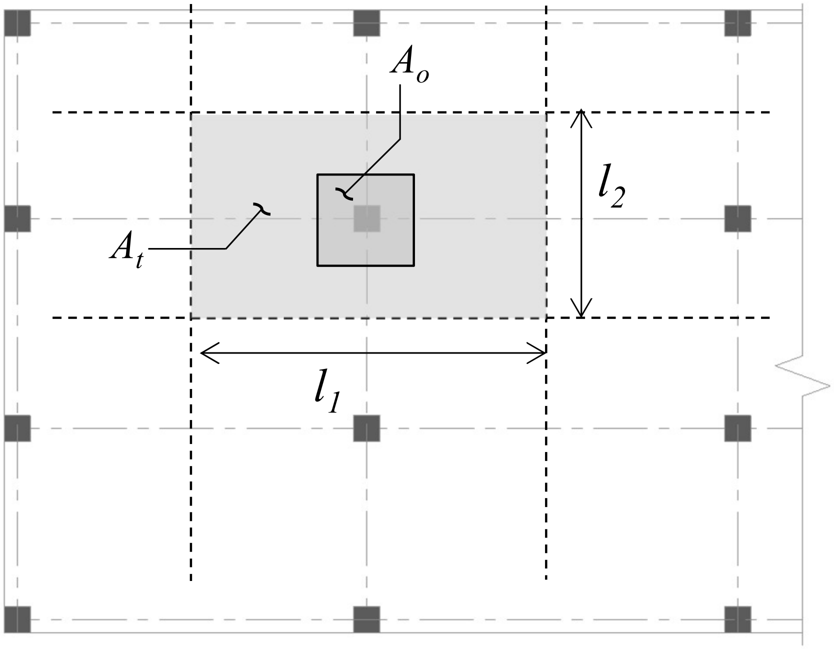
$$A_o = (c + d)^2 \rightarrow \text{for square columns}$$

$$A_t = l_1 l_2 - (c + d)^2/144$$

[l_1 & l_2 are in ft. and c & d are in inches]

2. **Calculate Demand V_u**

$$V_u = w_u A_t$$



General

❑ Capacity of Slab in Punching Shear

The total punching shear capacity is given by

$$\emptyset V_n = \emptyset V_c + \emptyset V_s$$

$\emptyset V_c$ calculated from ACI Table 22.6.5.2 , is the least of:

$$V_c = min\left(4, 2 + \frac{4}{\beta_c}, \frac{\alpha_s d}{b_o} + 2\right)\emptyset \lambda_s \sqrt{f_c'} b_o d$$

Where;

β_c = longer side of column/shorter side of column

α_s = 40 for interior column, 30 for edge column, 20 for corner columns

λ_s = size effect factor that can be determined as per 22.5.5.1.3

$$\lambda_s = \sqrt{\frac{2}{1 + d/10}} \leq 1$$

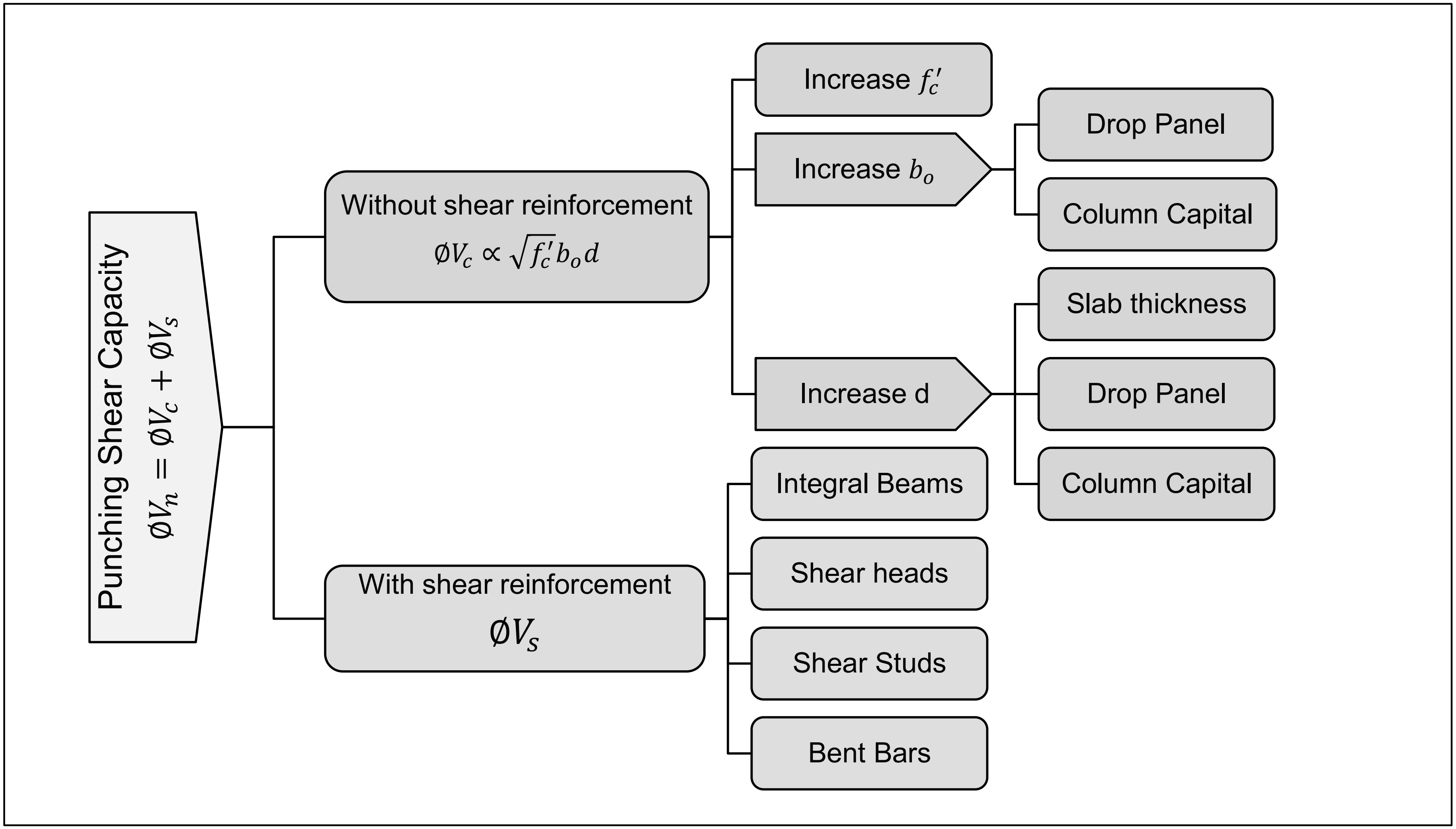
Design of Slabs for Punching Shear

❑ Various Design Options

- When $\Phi V_c \geq V_u$ ($\Phi = 0.75$) → nothing is required.
- When $\Phi V_c < V_u$, → increase the punching shear capacity of the slab.
- Various options are available for increasing the punching shear capacity as described next.

Design of Slabs for Punching Shear

❑ Various Design Options



Punching Shear Capacity

$\varnothing V_n = \varnothing V_c + \varnothing V_s$

Without shear reinforcement

$\varnothing V_c \propto \sqrt{f_c'}\, b_o d$

Increase f_c'

Increase b_o

Drop Panel

Column Capital

Increase d

Slab thickness

Drop Panel

Column Capital

With shear reinforcement

$\varnothing V_s$

Integral Beams

Shear heads

Shear Studs

Bent Bars

Design of Slabs for Punching Shear

❑ Punching Shear Design without Shear Reinforcement

❖ Drop Panel

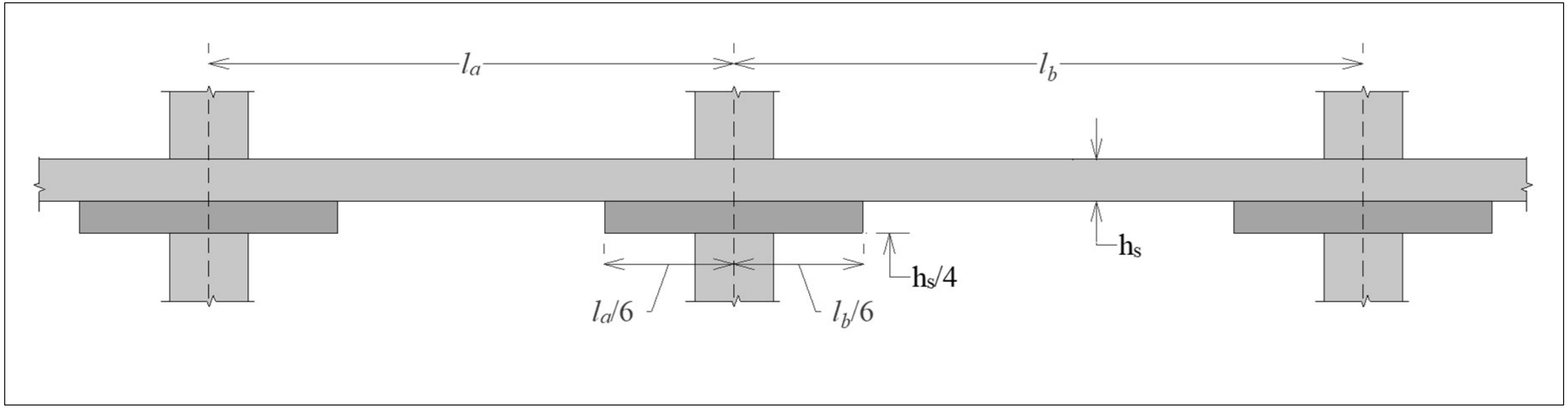
- A drop panel is the projection of slab provided at the vicinity of column to minimize punching shear demand.

Design of Slabs for Punching Shear

❑ Punching Shear Design without Shear Reinforcement

❖ Drop Panel

- ACI Section 8.2.4 requires that drop panel shall:
  a) Project below the slab at least one-fourth of the adjacent slab thickness.
  b) Extend in each direction from the centerline of support a distance not less than one-sixth the span length measured from center-to-center of supports in that direction.



Design of Slabs for Punching Shear

❑ Punching Shear Design without Shear Reinforcement

❖ Column Capital

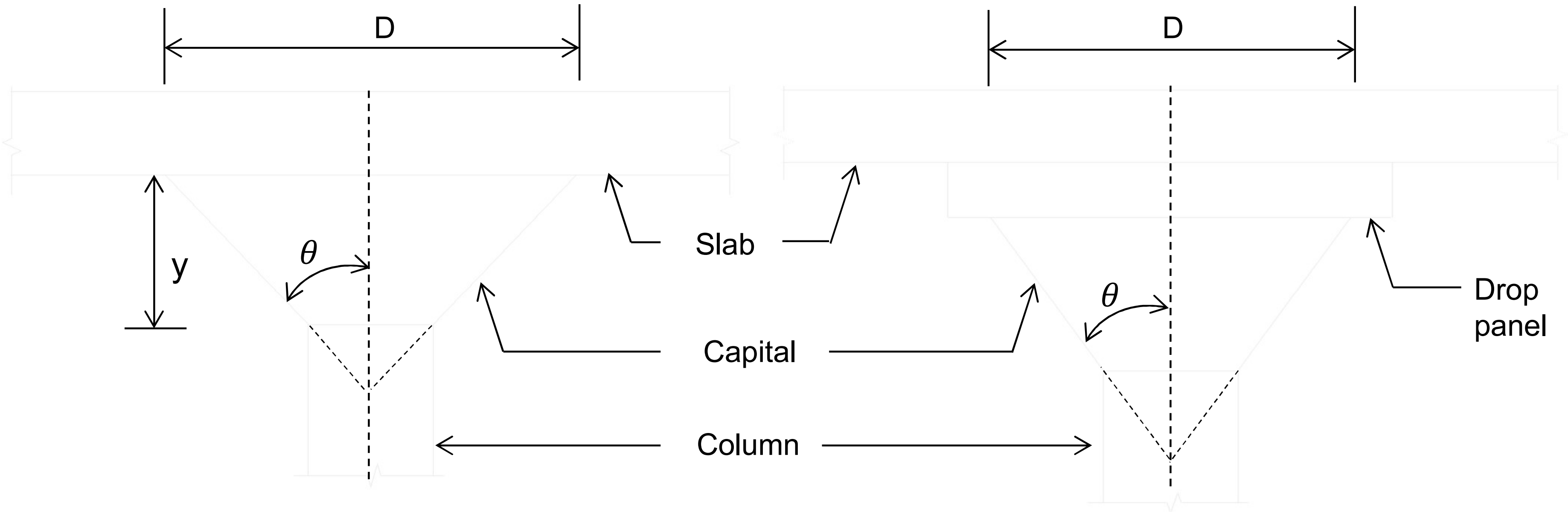
- Column capital is the enlargement of the top of a concrete column located directly below the slab or drop panel that is cast monolithically with the column (ACI 2.2).

Design of Slabs for Punching Shear

❑ Punching Shear Design without Shear Reinforcement

❖ Column Capital

- ACI Section 8.4.1.4 requires the column capital should be oriented no greater than 45° to the axis of the column ($\theta \leq 45^o$).



Design of Slabs for Punching Shear

❑ Punching Shear Design without Shear Reinforcement

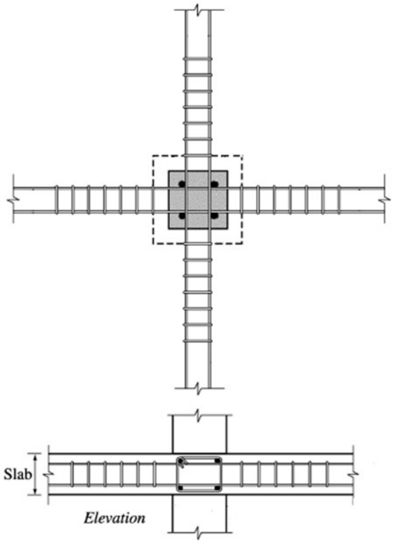
❖ Column Capital

- ACI Section 26.5.7 (d) requires that the concrete be placed at the same time as the slab concrete. As a result, the floor forming becomes considerably more complicated and expensive.
- The increased perimeter can be computed by equating V_u to $\emptyset V_c$ and simplifying the resulting equation for b_o.

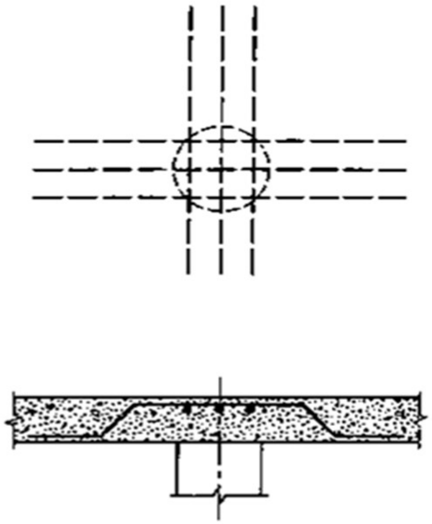
Design of Slabs for Punching Shear

❑ Punching Shear Design with Shear Reinforcement

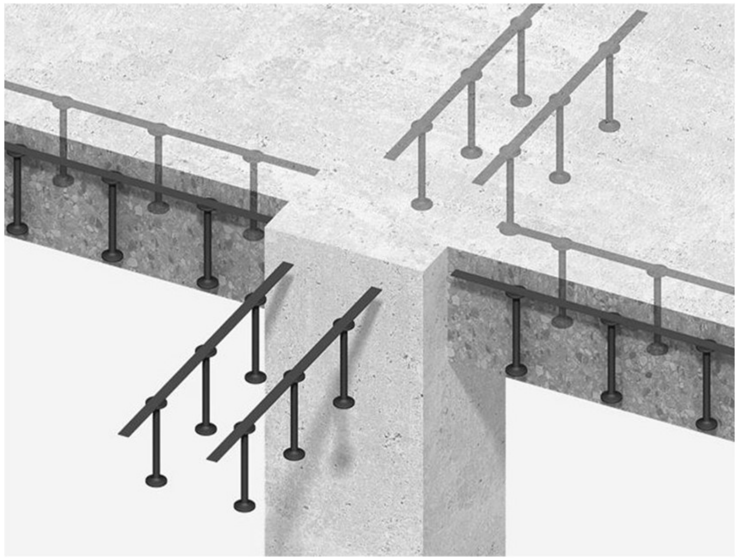
- The shear reinforcement can be provided by any of the following methods.



Integral Beams



Integral Beams



Shear Studs

- Only the design of Integral beams will be discussed next.

Design of Slabs for Punching Shear

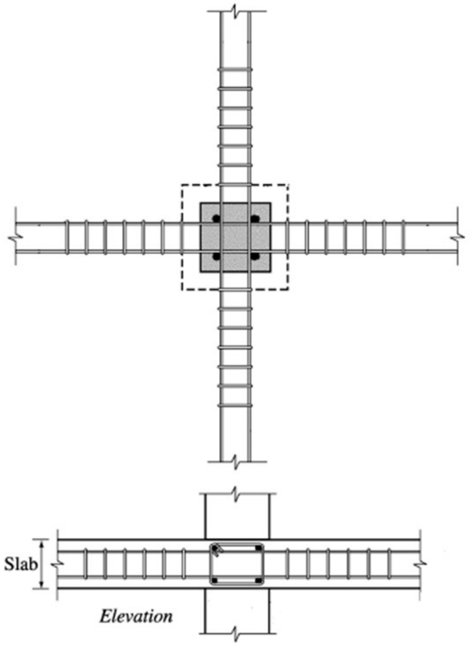
❑ Punching Shear Design with Shear Reinforcement

❖ Integral Beams

- For integral beams or bent bar reinforcement following must be satisfied.

  i. The slab effective depth d to be at least 6 in., but not less than 16 times the diameter of the shear reinforcement (ACI 26.6.7.1).

  ii. When bent bars and integral beams are to be used, reduce ΦV_c by 2 (ACI R22.6.6.1).

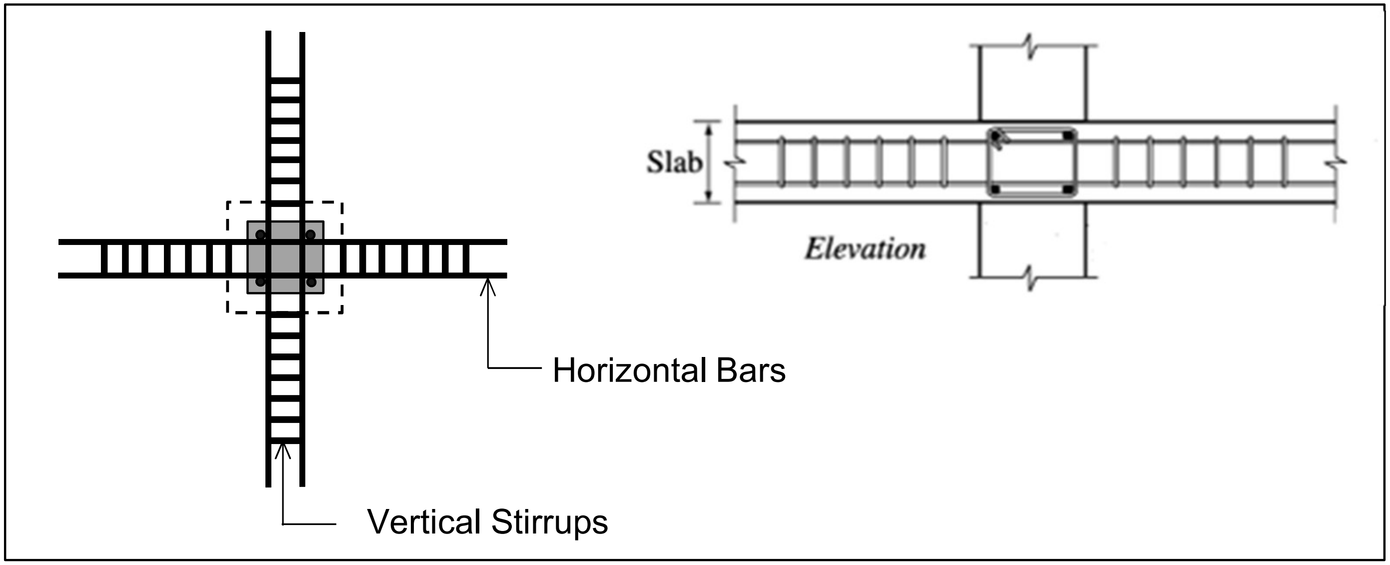


Design of Slabs for Punching Shear

❑ Punching Shear Design with Shear Reinforcement

❖ Integral Beams

- Integral Beams require the design of two main components:
  1. Vertical stirrups
  2. Horizontal bars radiating outward from column faces.



Design of Slabs for Punching Shear

❑ Punching Shear Design with Shear Reinforcement

❖ Integral Beams

1. **Vertical Stirrups**

- Vertical stirrups are used in conjunction with supplementary horizontal bars radiating outward in two perpendicular directions from the support to form what are termed integral beams contained entirely within the slab thickness.
- In such a way, critical perimeter is increased as illustrated next.

Design of Slabs for Punching Shear

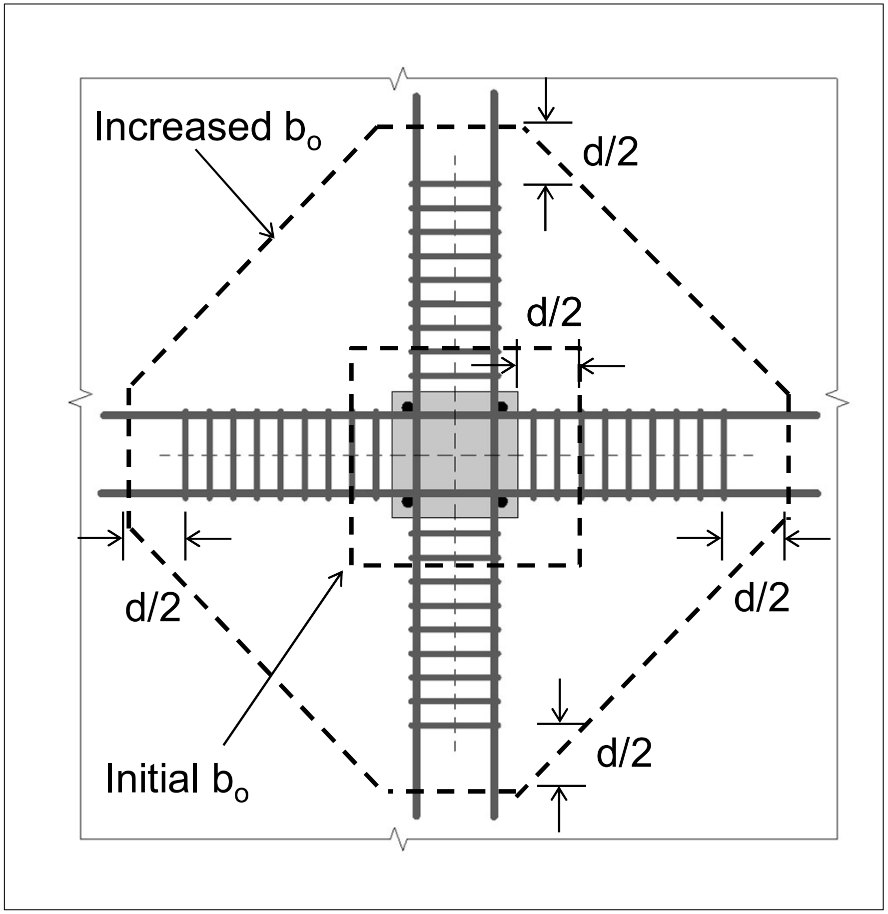
❑ Punching Shear Design with Shear Reinforcement

❖ Integral Beams

1. **Vertical Stirrups**

- Stirrups are placed vertically on all four sides, resulting in a total stirrup area that equals four times the area of each individual two-legged stirrup.

$$A_v = 4(2A_b) = 8A_b$$



Design of Slabs for Punching Shear

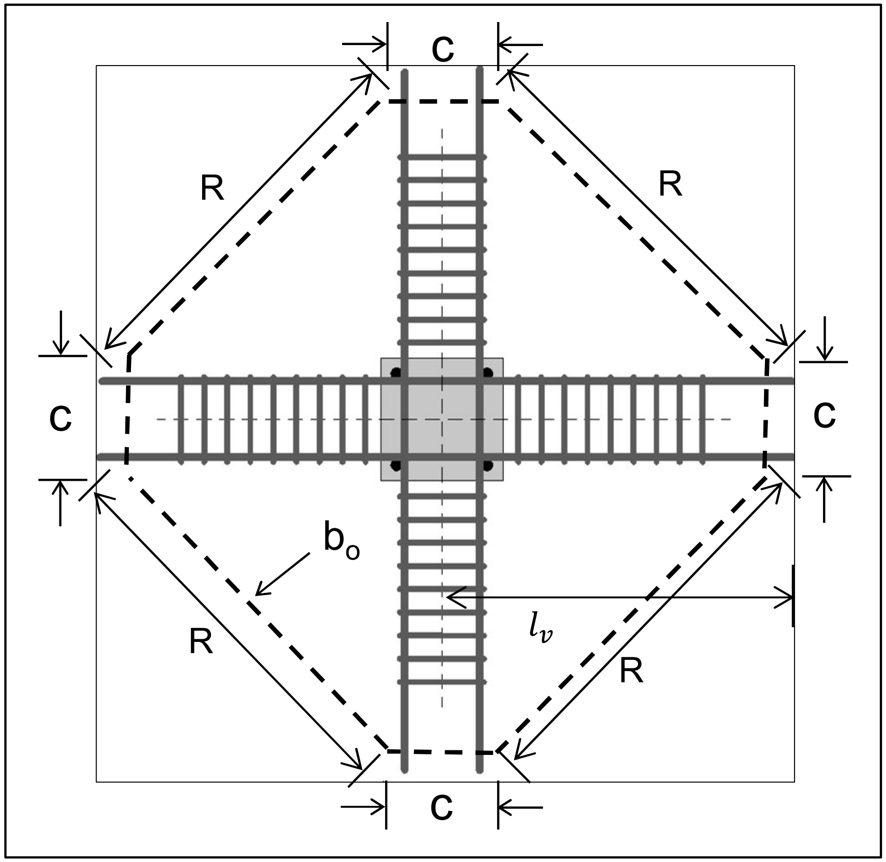
❑ Punching Shear Design with Shear Reinforcement

❖ Integral Beams

2. Horizontal Bars

- The length of horizontal bars l_v from the center of column can be determined using critical perimeter b_o.
- For square column of size "c", we have

$b_o = 4R + 4c \;--(A)$



Design of Slabs for Punching Shear

❑ Punching Shear Design with Shear Reinforcement

❖ Integral Beams

2. Horizontal Bars

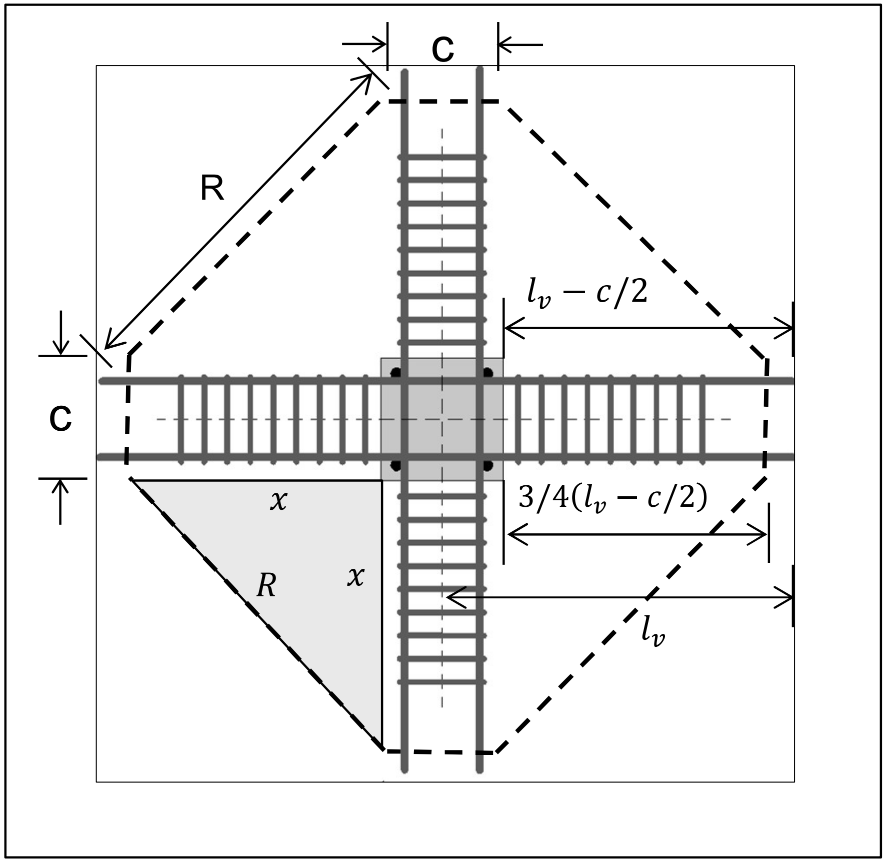
- The projection from the face of column to the boundary of critical perimeter x is:

$$x = \frac{3}{4}\left(l_v - \frac{c}{2}\right)$$

From the shaded triangle

$$R = \sqrt{x^2 + x^2} = \sqrt{2}\,x$$

$$R = \sqrt{2}\left[\frac{3}{4}\left(l_v - \frac{c}{2}\right)\right]$$



Design of Slabs for Punching Shear

❑ Punching Shear Design with Shear Reinforcement

❖ Integral Beams

2. Horizontal Bars

Substituting the value in equation (A), we get

$$b_o = 4R + 4c = 4\left[\sqrt{2} \times \frac{3}{4}\left(l_v - \frac{c}{2}\right)\right] + 4c = 4.24l_v + 1.88c$$

Solving for l_v, we get

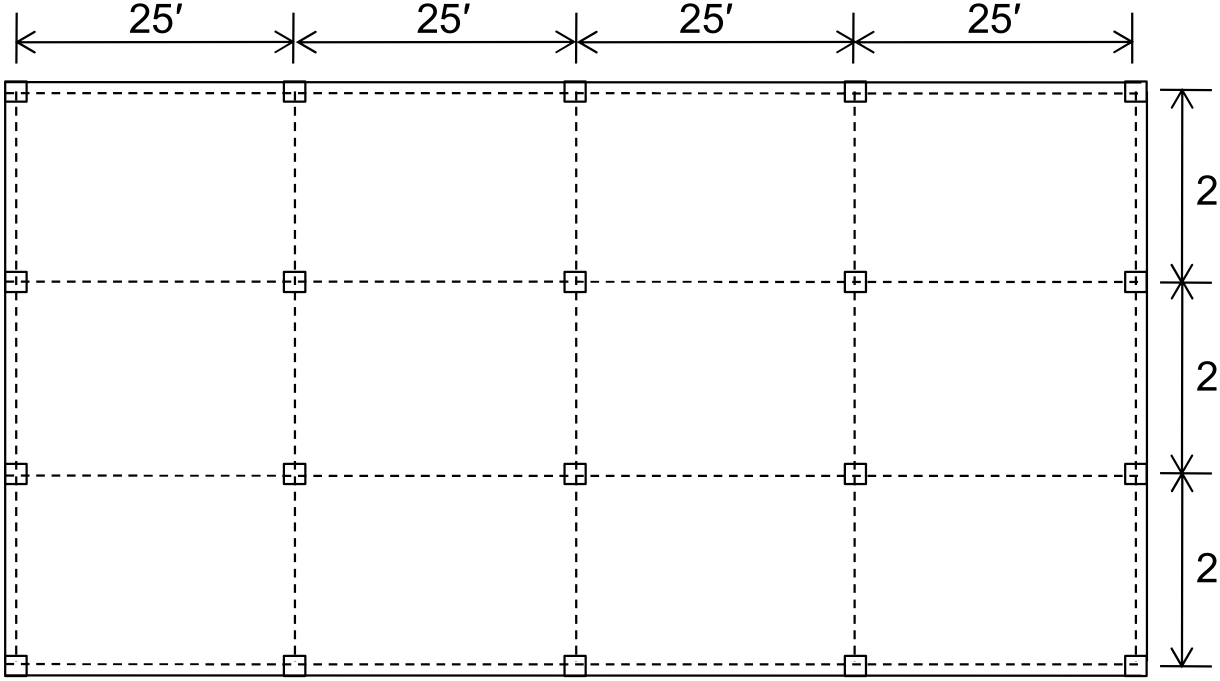
$$l_v = \frac{b_o - 1.88c}{4.24}$$

This equation can be used for determining the length up to which the horizontal bars should be extended beyond the face of column. The b_o can be obtained by equating V_u with $\emptyset V_c$ and solving the resulting equation for b_o.

Example 6.3

❑ Problem Statement

- A 10-inch-thick flat plate shown below supports a uniformly distributed factored load (including self weight of plate) of 0.381 ksf. ***Design*** the plate for punching shear using various options. Take $f_c' = 4$ ksi and $f_y = 60$ ksi.



All columns are 14″ square

Example 6.3

❑ **Solution**

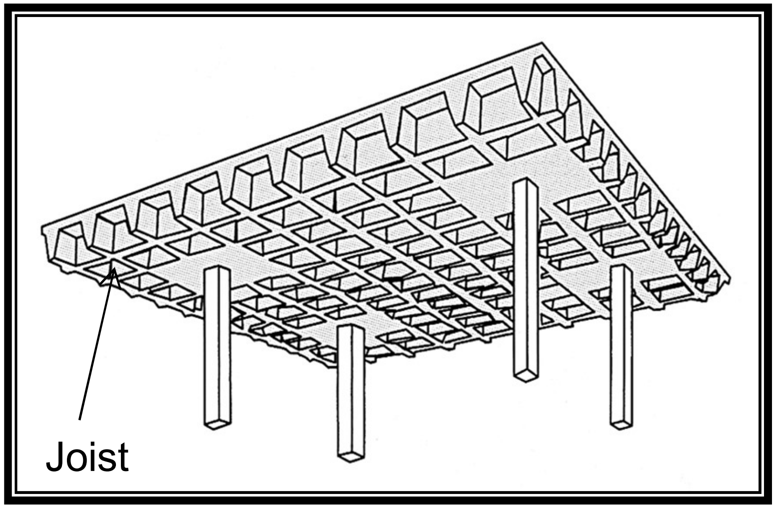
- Please refer to **Lecture 4 of RCD – II** for solution of this Example

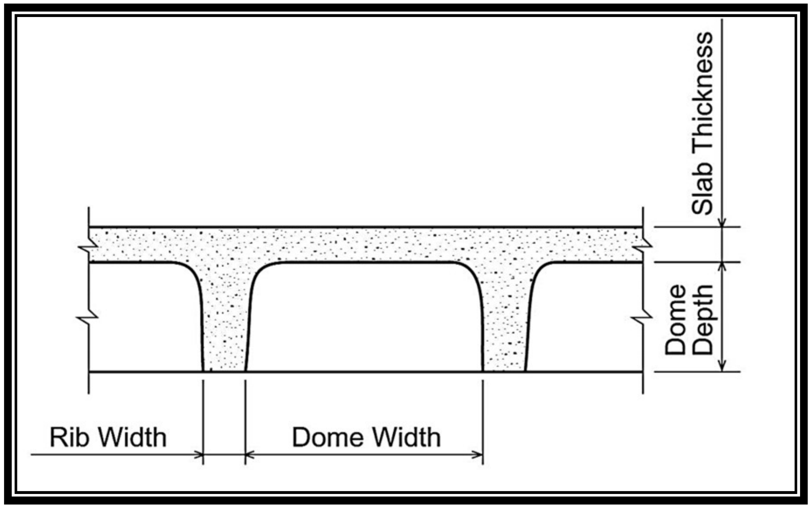
Two-Way Joist System

General

❑ Introduction

- A two-way joist system, or waffle slab, comprises evenly spaced concrete joists spanning in both directions and a reinforced concrete slab cast integrally with the joists.
- Like one-way joist system, a two-way system will be called as two-way joist system if clear spacing between ribs (dome width) does not exceed 30 inches.

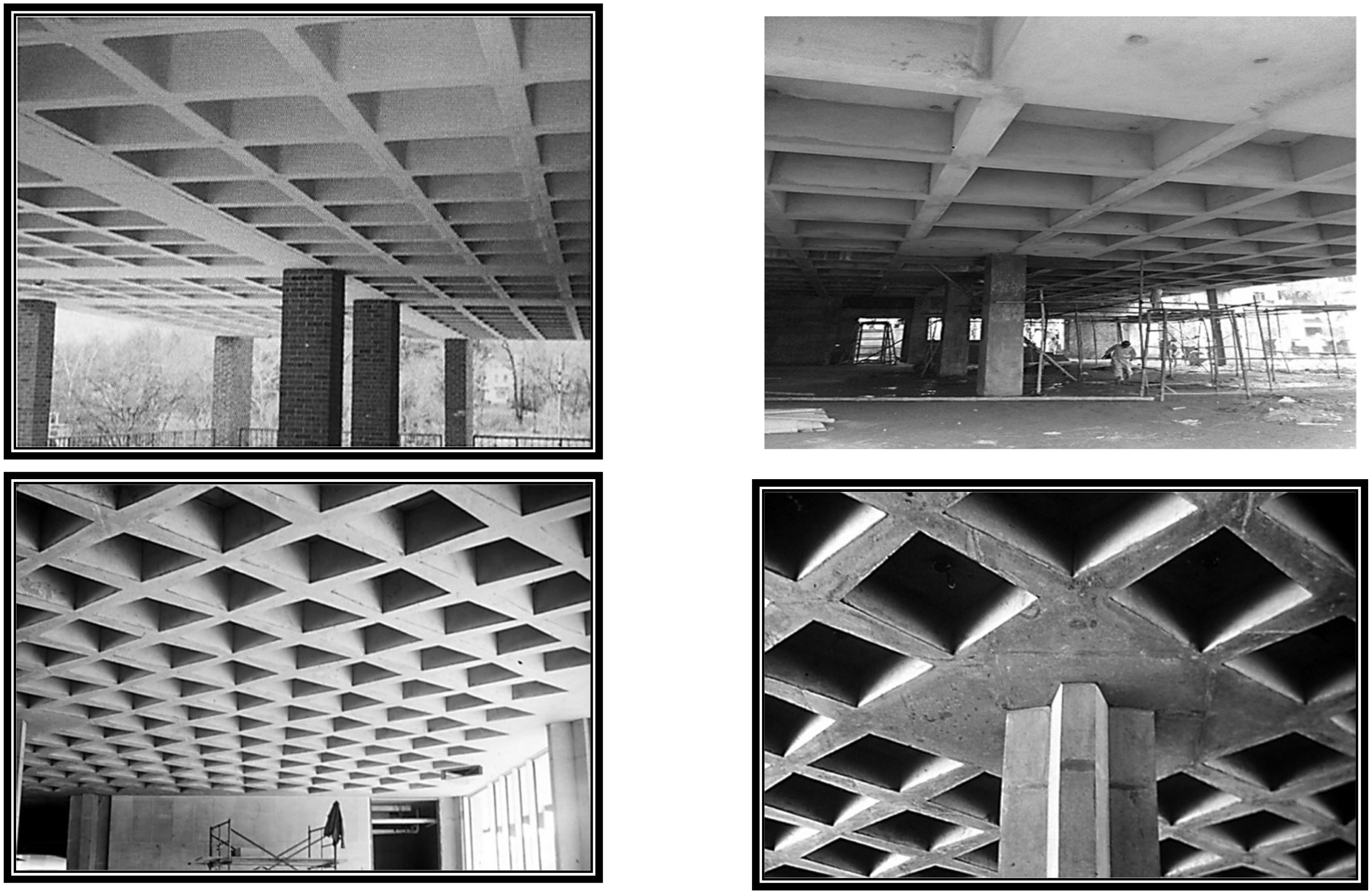




General

❑ Introduction

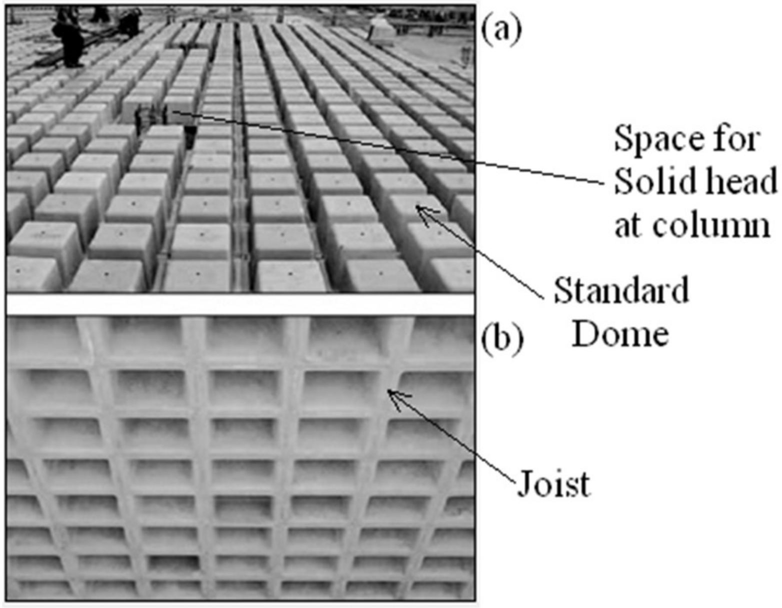
- Some pictures of two-way joist systems are shown below

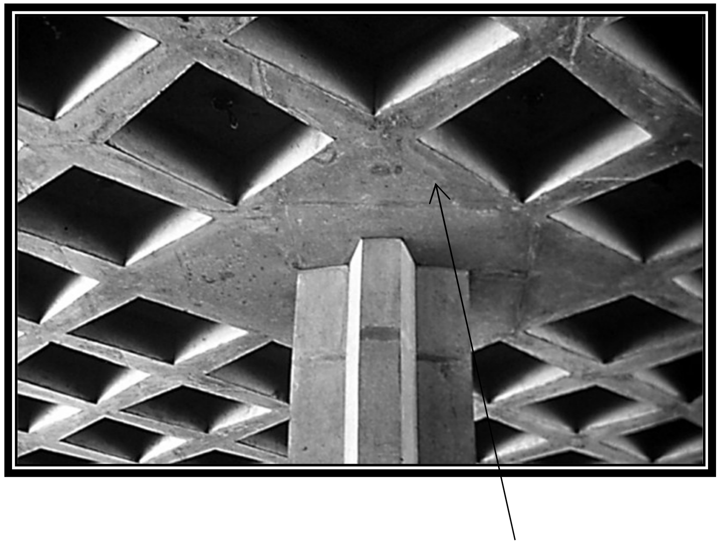


General

❑ Introduction

- The joists are commonly formed by using standard square “dome” forms and the domes are omitted around the columns to form the solid heads.





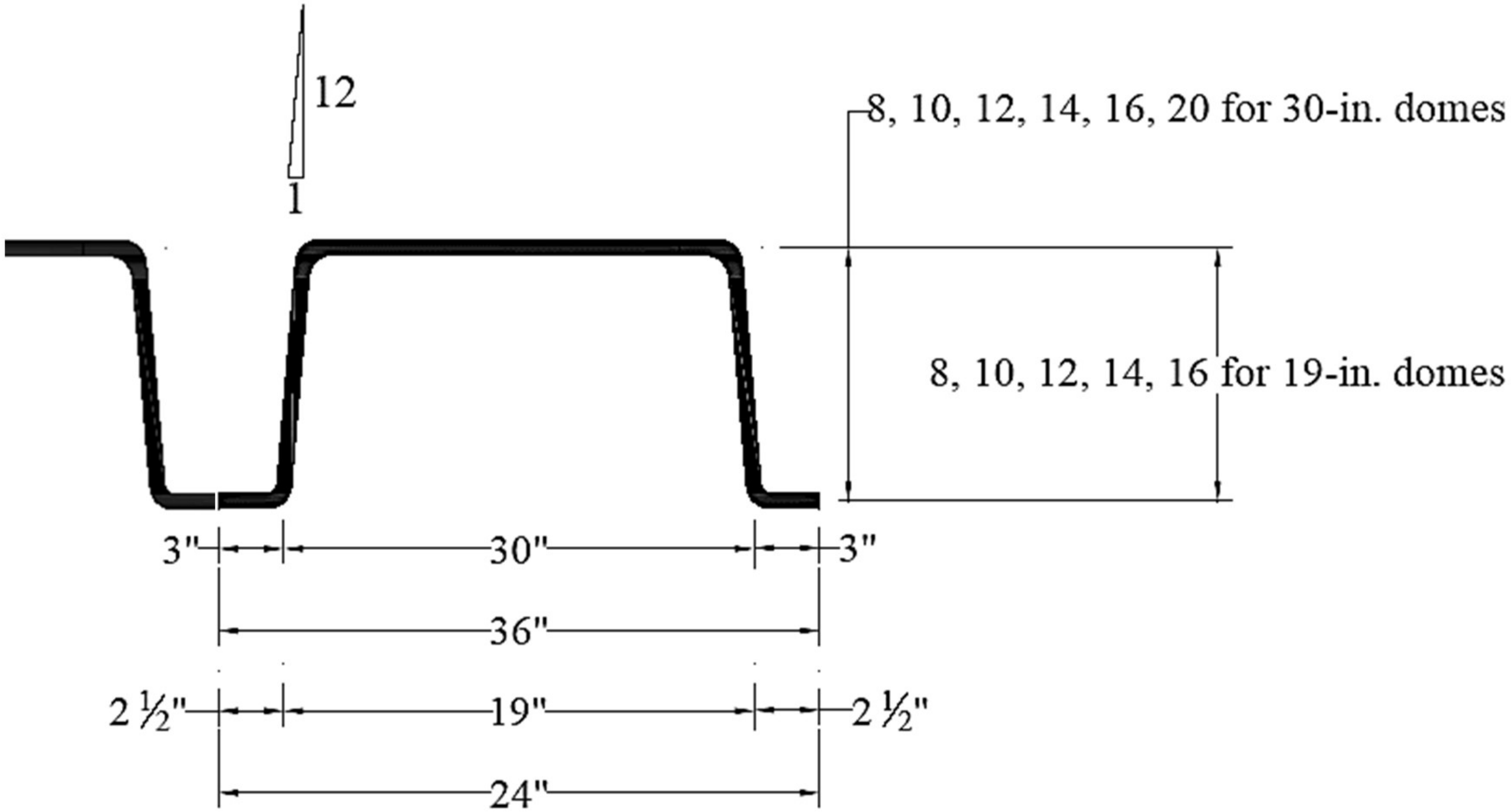
General

❑ Standard Dome Data

- The dome for waffle slab can be of any size. However, the commonly used standard domes are discussed as follows:
    - 30-inch × 30-inch square domes with 3-inch flanges; from which 6-inch-wide joist ribs at 36-inch centers are formed: these are available in standard depths of 8, 10, 12, 14, 16 and 20 inches.
    - 19-inch × 19-inch square domes with 2 ½-inch flanges, from which 5-inch-wide joist ribs at 24-inch centers are formed. These are available in standard depths of 8, 10, 12, 14 and 16 inches.

General

- **Standard Dome Data**



General

❑ Behavior

- The behavior of two-way joist slab is similar to a two-way flat Slab system.

General

❑ Characteristics

- Dome voids reduce dead load.
- Attractive ceiling (waffle like appearance).
- Electrical fixtures can be placed in the voids.
- Particularly advantageous where the use of longer spans and/or heavier loads are desired without the use of deepened drop panels or supported beams.

Basic Design Steps

☐ Step 1: Selection of Sizes

❖ Minimum Joist Depth

- For joist depth determination, waffle slabs are considered as flat slab.
- The thickness of equivalent flat slab is taken from table 8.3.1.1.
- The thickness of slab and depth of rib of waffle slab can be then computed by equalizing the moment of inertia of equivalent flat slab to that of waffle slab.
- However, since this practice is time consuming, tables have been developed to determine the size of waffle slab from equivalent flat slab thickness.

Basic Design Steps

❑ Step 1: Selection of Sizes

❖ Minimum Joist Depth

- Equivalent Flat Slab Thickness

Table 8.3.1.1 —Minimum thickness of nonprestressed two-way slabs without interior beams (in.)

f_y (psi)	Without drop panels			With drop panels		
	Exterior Panels		Interior panels	Exterior Panels		Interior panels
	Without edge beams	With edge beams		Without edge beams	With edge beams	
40,000	$l_n/33$	$l_n/36$	$l_n/36$	$l_n/36$	$l_n/40$	$l_n/40$
60,000	$l_n/30$	$l_n/33$	$l_n/33$	$l_n/33$	$l_n/36$	$l_n/36$
80,000	$l_n/27$	$l_n/30$	$l_n/30$	$l_n/30$	$l_n/33$	$l_n/33$

- l_n is the clear span in the long direction, measured face-to-face of supports (in.).
- $h_{min} = 5''$ for slabs without drop panels
- $h_{min} = 4''$ for slabs with drop panels

Basic Design Steps

Step 1: Selection of Sizes

Minimum Joist Depth

- Slab and rib depth from equivalent flat slab thickness

Waffle flat slabs (19" × 19" voids at 2'-0")-Equivalent thickness	
Rib + Slab Depths (in.)	**Equivalent Thickness t_e (in.)**
8 + 3	8.89
8 + 4 ½	10.11
10 + 3	10.51
10 + 4 ½	11.75
12 + 3	12.12
12 + 4 ½	13.38
14 + 3	13.72
14 + 4 ½	15.02
16 + 3	15.31
16 + 4 ½	16.64

Reference: Table 11-2 of CRSI Design Handbook.
Note: Only first two columns of the table are reproduced here.

Basic Design Steps

❑ Step 1: Selection of Sizes

❖ Minimum Joist Depth

- Slab and rib depth from equivalent flat slab thickness

Waffle flat slabs (30" × 30" voids at 3'-0")-Equivalent thickness	
Rib + Slab Depths (in.)	**Equivalent Thickness t_e (in.)**
8 + 3	8.61
8 + 4 ½	9.79
10 + 3	10.18
10 + 4 ½	11.37
12 + 3	11.74
12 + 4 ½	12.95
14 + 3	13.3
14 + 4 ½	14.54
16 + 3	14.85
16 + 4 ½	16.12

Reference: Table 11-2 of CRSI Design Handbook.
Note: Only first two columns of the table are reproduced here.

Basic Design Steps

❑ Step 1: Selection of Sizes

❖ **Minimum Width of Rib**

- ACI 8.8.1.2 states that ribs shall be not less than 4 inches in width.

❖ **Maximum Depth of Rib**

- ACI 8.8.1.3 also states that ribs shall have a depth of not more than 3 ½ times the minimum width of rib.

❖ **Minimum Slab Thickness**

- ACI 8.8.3.1 states that slab thickness shall be not less than one-twelfth the clear distance between ribs, nor less than 2 inch.

Basic Design Steps

❑ Step 1: Selection of Sizes

❖ Solid Head

- Dimension of solid head on either side of column centerline is equal to $l/6$.
- The depth of the solid head is equal to the depth of the combined depth of ribs and top slab.

Basic Design Steps

❑ Step 2: Calculations of Loads

- Floor dead load for two-way joist with certain dome size, dome depth can be calculated from the table shown for two options of slab thicknesses (3 inches and 4 ½ inches).

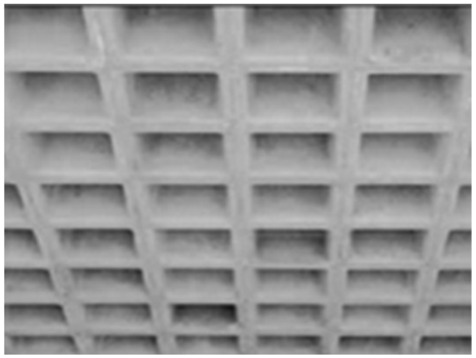
Standard Dome Dimensions and other Data (*Table 11-1, CRSI Design Handbook)*

Dome Size	Dome Depth (inches)	Volume of Void (ft^3)	Floor Dead Load (psf) per slab thickness	
			3 inches	4 ½ inches
30 inches	8	3.98	71	90
	10	4.92	80	99
	12	5.84	90	109
	14	6.74	100	119
	16	7.61	111	129
	20	9.3	132	151
19 inches	8	1.56	79	98
	10	1.91	91	110
	12	2.25	103	122
	14	2.58	116	134
	16	2.9	129	148

Basic Design Steps

❑ Step 2: Calculations of Loads

- Floor dead load (wdj) for two-way joist can also be calculated as follows:

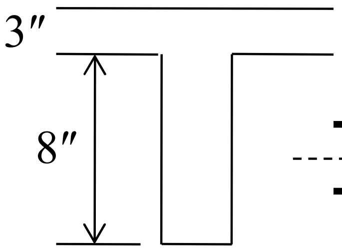


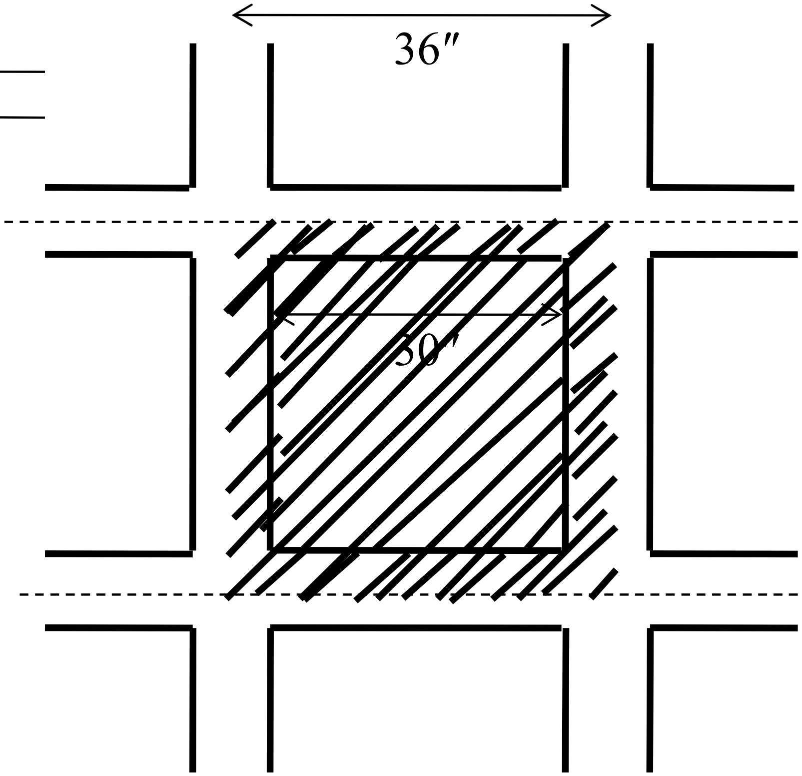
Volume of solid:
$V_{solid} = (36 \times 36 \times 11)/1728 = 8.24 \text{ ft}^3$

Volume of void:
$V_{void} = (30 \times 30 \times 8)/1728 = 4.166 \text{ ft}^3$

Total Load of joists per dome:
$w_{dj} = (V_{solid} - V_{void}) \times \gamma_{conc}$
$= (8.24 - 4.166) \times 0.15 = 0.61$ kips/ dome

Total Load of joists per sq. ft:
$w_{dj}/$ (dome area) $= 0.61/(3 \times 3) = 0.0679$ ksf
$= 68 \text{ psf} \approx 71$ psf (from table 03)
The difference is because sloped ribs are not considered.



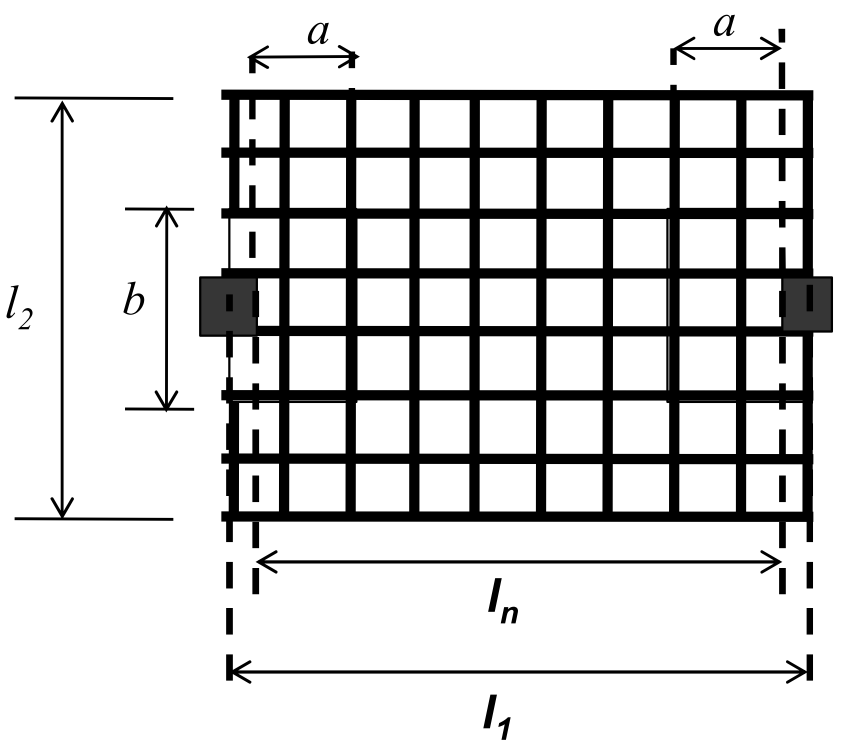


Plan

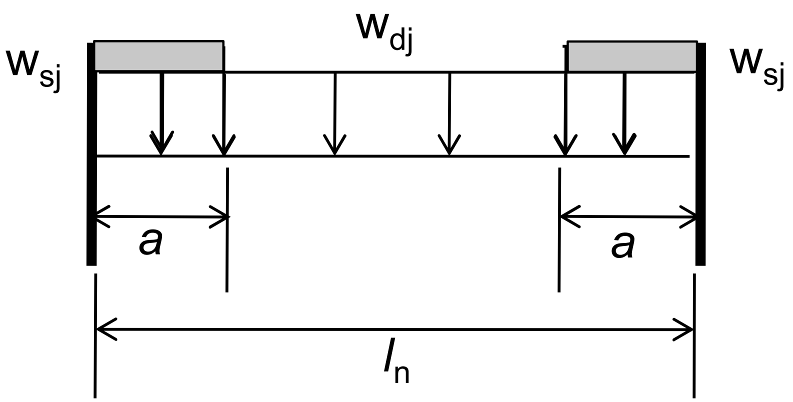
Basic Design Steps

❑ Step 2: Calculations of Loads

- Loads in an interior span of a two-way joist system can be calculated as follows:



a = b/2 – c/2 – half joist width



$W_{sh} = W_{sj} + W_{dj}$

$W_{sj} = W_{sh} - W_{dj}$

W_{sh} = Dead load of solid head

W_{dj} = Dead load of joist

W_{sj} = Dead load of solid head excluding joist

Basic Design Steps

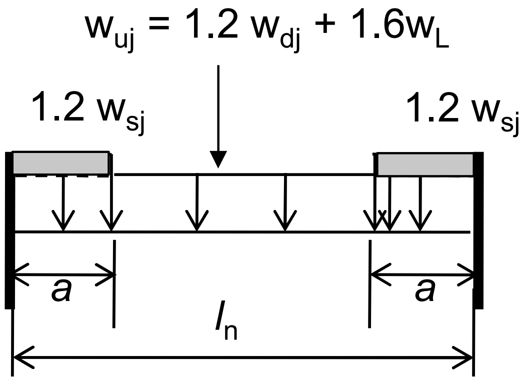
❑ Step 2: Calculations of Loads

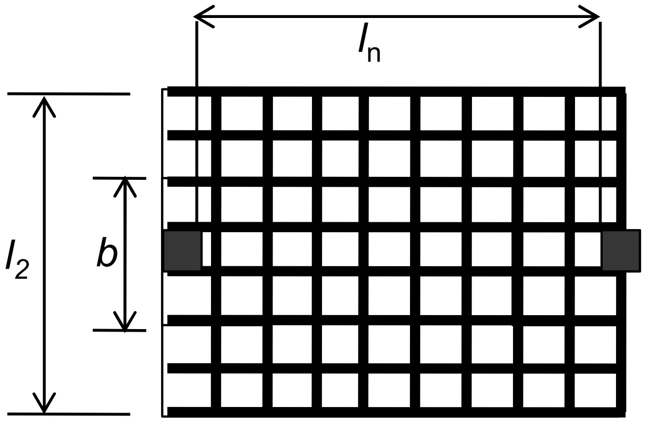
- Factored loads can be calculated as:
  - If w_L = live load (load/area) and w_{dj} = dead load of joists, then
    - Factored load due to joists w_j

$$w_{uj} = 1.2w_{dj} + 1.6w_L$$

    - Factored load due to w_{sj}

$$w_{usj} = 1.2w_{sj}$$





Basic Design Steps

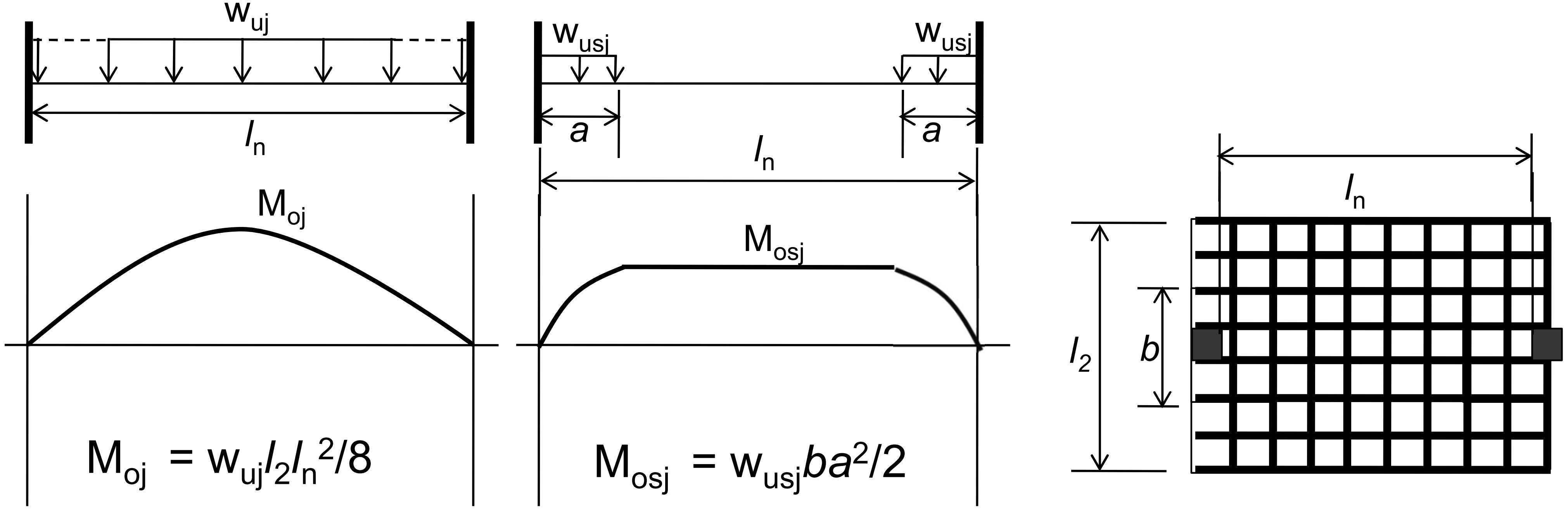
☐ **Step 3: Analysis**

- ACI code allows use of DDM for analysis of waffle slabs (ACI R8.10.1.2). In such a case, waffle slabs are considered as flat slabs, with the solid head acting as drop panels.

Basic Design Steps

❑ Step 3: Analysis

- Static moment calculation for DDM analysis:



$$M_{oj} = w_{uj} l_2 l_n^2 / 8$$

$$M_{osj} = w_{usj} b a^2 / 2$$

$$\mathbf{M_o = M_{oj} + M_{osj}}$$

Basic Design Steps

☐ Step 4: Determination of Reinforcement

- **Flexural reinforcement**

- The design of waffle slab for flexure is done like solid slab design.

Basic Design Steps

❑ Step 4: Determination of Reinforcement

❖ Punching Shear Reinforcement

- The solid head shall be checked against punching shear.
- The critical section for punching shear is taken at a section d/2 from face of the column, where d is the effective depth at solid head.

Basic Design Steps

❑ Step 4: Determination of Reinforcement

❖ Beam Shear Reinforcement

- Beam shear is not usually a problem in slabs including waffle slabs. However, for completion of design, beam shear may also be checked. Beam shear can cause problem in case where larger spans and heavier loads with relatively shallow waffle slabs are used.
- The critical section for beam shear is taken at a section d from face of the column, where d is the effective depth at solid head.

Basic Design Steps

- ☐ **Step 4: Determination of Reinforcement**

  - **Beam Shear Reinforcement**

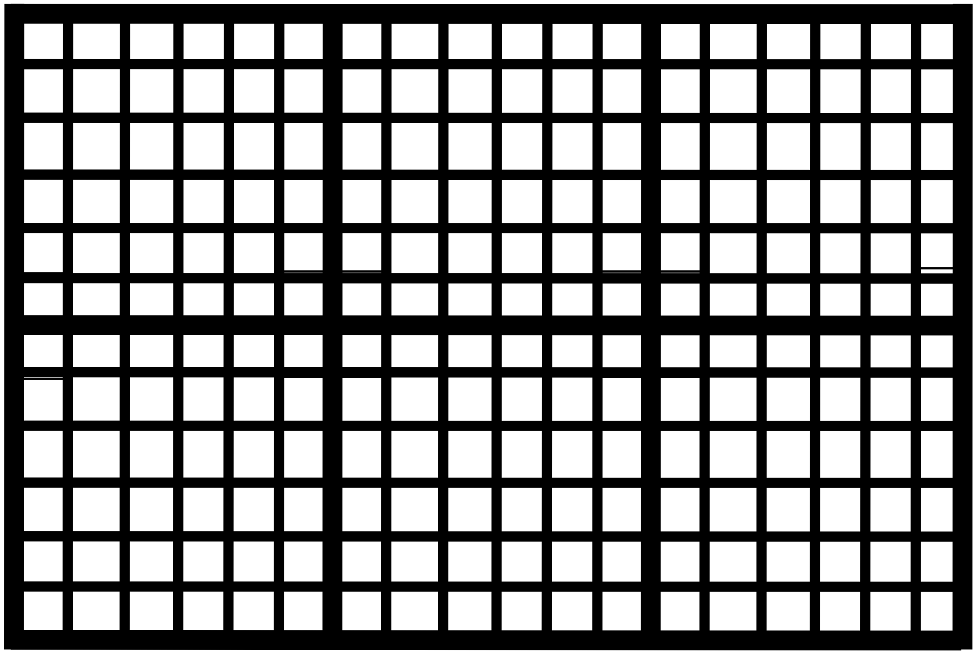
  - For joist construction, contribution of concrete to shear strength V_c shall be permitted to be 10 percent more than that specified in Chapter 22.

  - If required, one or two single legged stirrups are provided in the rib to increase the shear capacity of waffle slab.

Basic Design Steps

❑ Some Important Points

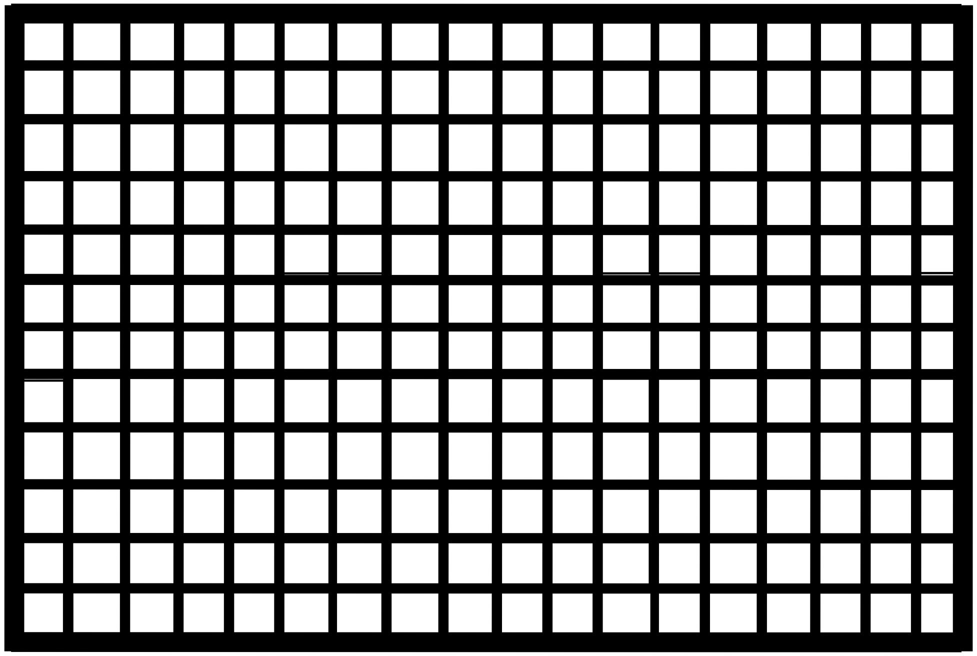
- For layouts that do not meet the standard 2-feet and 3-feet modules, it is preferable that the required additional width be obtained by increasing the width of the ribs framing into the solid column head.



Basic Design Steps

❑ Some Important Points

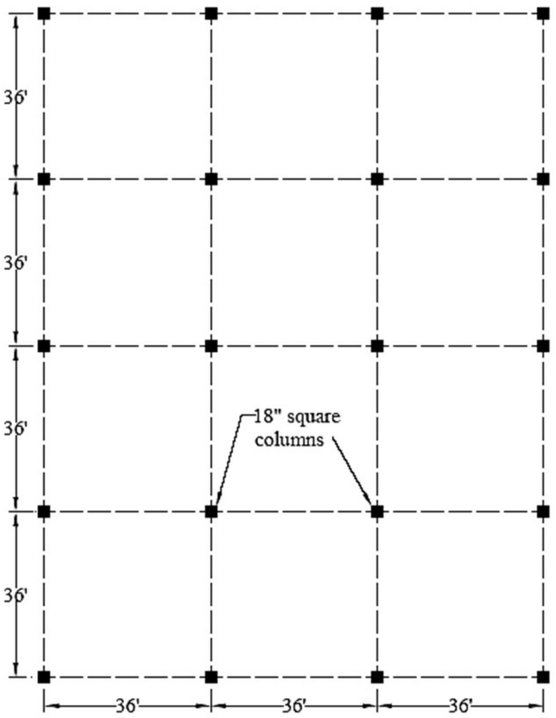
- The designer should sketch out the spacing for a typical panel and correlate with the column spacing as a part of the early planning.



Example 6.4

❑ Problem Statement

- ***Design*** the slab system of hall shown in figure as waffle slab, according to ACI 318. Use Direct Design Method for slab analysis. Take $f_c' = 4$ ksi and $f_y = 40$ ksi and live load = 100 psf



Example 6.4

❑ Solution

- A 108′ × 144′ building, divided into twelve (12) panels, supported at their ends on columns. Each panel is 36′ × 36′.
- The given slab system satisfies all the necessary limitations for Direct Design Method to be applicable.

Example 6.4

❑ Solution

➢ Step 1: Selection of Sizes

- **Columns**

Let all columns be 18″ × 18″.

- **Slab**

Adopt 30″ × 30″ standard dome.

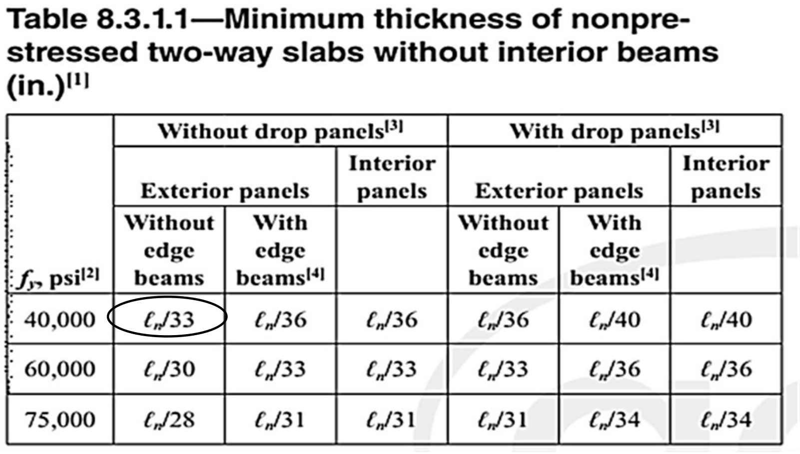


Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (in.)[1]

	Without drop panels[3]			With drop panels[3]		
	Exterior panels		Interior panels	Exterior panels		Interior panels
f_y, psi[2]	Without edge beams	With edge beams[4]		Without edge beams	With edge beams[4]	
40,000	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
60,000	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
75,000	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

Minimum equivalent flat slab thickness (h_f) can be found using ACI Table 8.3.1.1:

Exterior panel governs. Therefore,

$h_f = l_n/33$; $l_n = 36 - (2 \times 18/2)/12 = 34.5'$

$h_f = (34.5/30) \times 12 = 12.45''$

Example 6.4

❑ Solution

➢ Step 1: Selection of Sizes

❖ **Slab**

- The closest depth of doom that will fulfill the requirement of equivalent thickness of flat slab equal to 12.45″ is 12 in. with a slab thickness of 4 ½ in. for a dome size of 30-in.

Waffle flat slabs (30" × 30" voids at 3'-0")- Equivalent thickness	
Rib + Slab Depths (in.)	**Equivalent Thickness t_e (in.)**
8 + 3	8.61
8 + 4 ½	9.79
10 + 3	10.18
10 + 4 ½	11.37
12 + 3	11.74
12 + 4 ½	**12.95**
14 + 3	13.3
14 + 4 ½	14.54
16 + 3	14.85
16 + 4 ½	16.12
20 + 3	17.92
20 + 4 ½	19.26

Example 6.4

❑ Solution

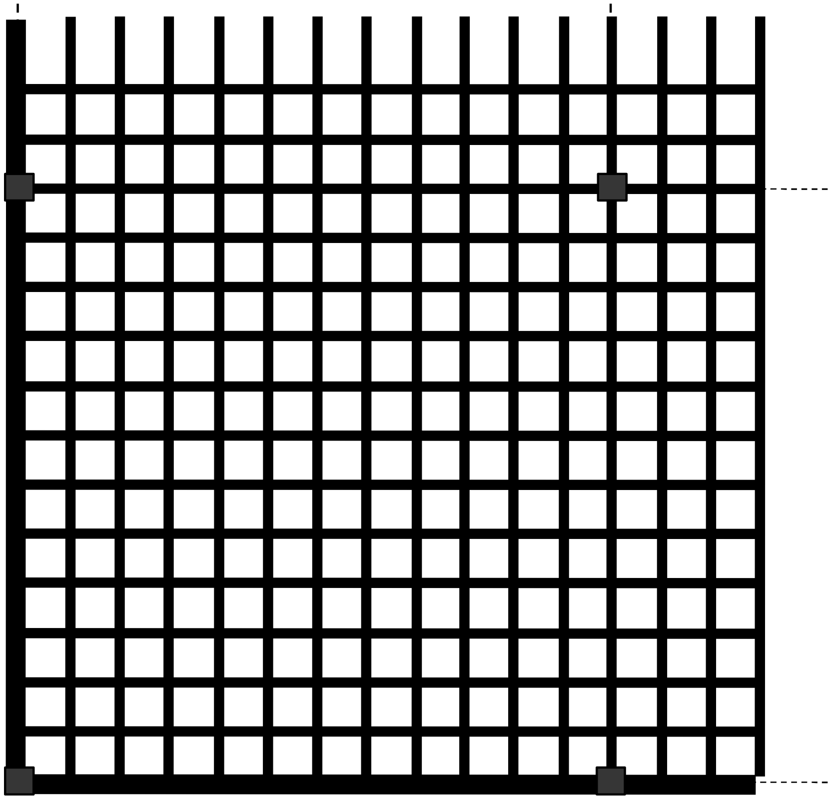
- **Step 1: Selection of Sizes**
  - **Slab (Planning of Joist layout)**

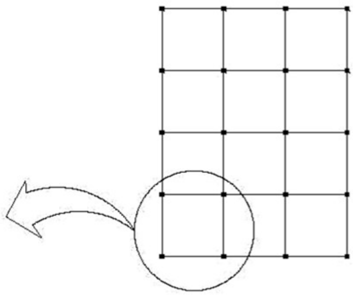
l = 36′-0″ = 432″
Standard module = 36″ × 36″

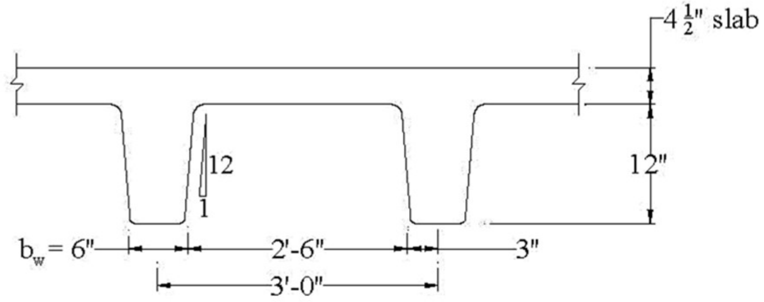
<u>No. of modules in 36′-0″:</u>
n = 432/36 = 12

<u>Joist width</u> = 6″

<u>Planning:</u>
First joist is placed on interior column centerline with progressive placing of other joists towards exterior ends of panel. To flush the last joist with external column, the width of exterior joist comes out to be 15″ (6″**+**Column size /2) as shown in plan view.







(b) Section of waffle slab

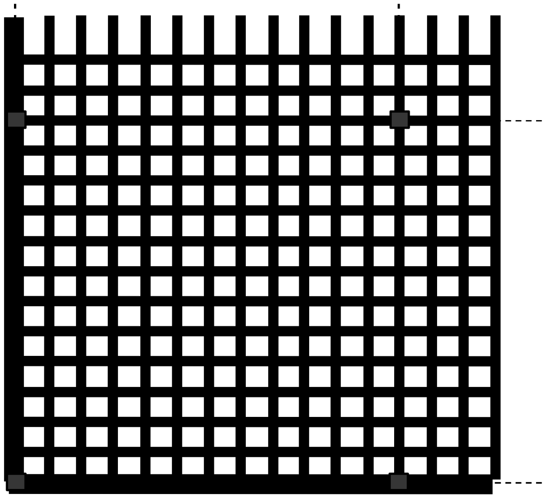
Example 6.4

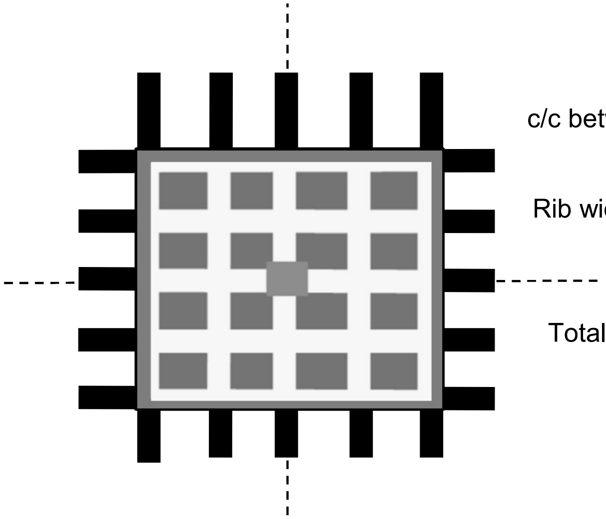
❑ Solution

➢ Step 1: Selection of Sizes

❖ Solid Head

- Solid head dimension from column centerline = l/6 = 36/6 = 6′
- Total required length of solid head = 2 x 6 = 12′
- As 3′ x 3′ module is selected, therefore 4 voids including joist width will make an interior solid head of 12.5′ x 12.5′. (Length of solid head = c/c distance between rib + rib width)
- Depth of the solid head = Depth of standard module = 12 + 4.5 = 16.5″





c/c between ribs= 4x3 = 12′

Rib width= 3+3 inches

Total = 12′ +0.5′
=12.5′

Example 6.4

❑ Solution

➢ Step 2: Calculation of Loads

- Floor (joist) dead load (w_{dj}) = 109 psf = 0.109 ksf

Standard Dome Dimensions and other Data (*Table 11-1, CRSI Design Handbook)*

Dome Size	Dome Depth (inches)	Volume of Void (ft^3)	Floor Dead Load (psf) per slab thickness	
			3 inches	4 ½ inches
30 inches	8	3.98	71	90
	10	4.92	80	99
	12	5.84	90	**109**
	14	6.74	100	119
	16	7.61	111	129
	20	9.3	132	151
19 inches	8	1.56	79	98
	10	1.91	91	110
	12	2.25	103	122
	14	2.58	116	134
	16	2.9	129	148

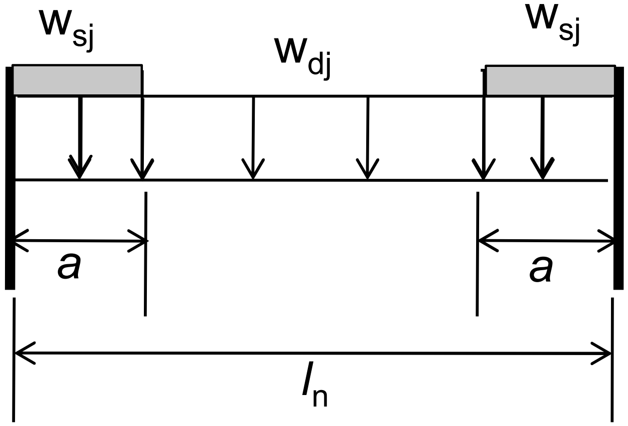
Example 6.4

❑ Solution

➢ Step 2: Calculation of Loads

- Solid head dead load $w_{sh} = \gamma_c h_{sh} = 0.15 \times \{(12 + 4.5)/12 = 0.20625$
- Solid Head dead load excluding joist (w_{sj}) $= w_{sh} - w_{dj}$

$= 0.20625 - 0.109 = 0.097$ ksf



Example 6.4

❑ Solution

➢ Step 2: Calculation of Loads

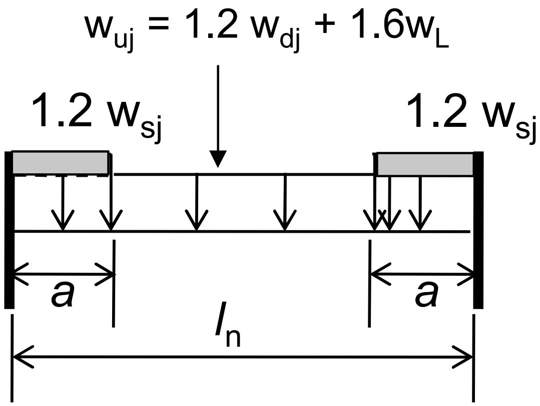
w_L = 100 psf = 0.100 ksf

Load due to joists plus LL (w_{uj})

$w_{uj} = 1.2\, w_{dj} + 1.6 w_L$

$= 1.2 \times 0.109 + 1.6 \times 0.100$

$= 0.291$ ksf



Load due to solid head (w_{usj})

$w_{usj} = 1.2 w_{sj}$

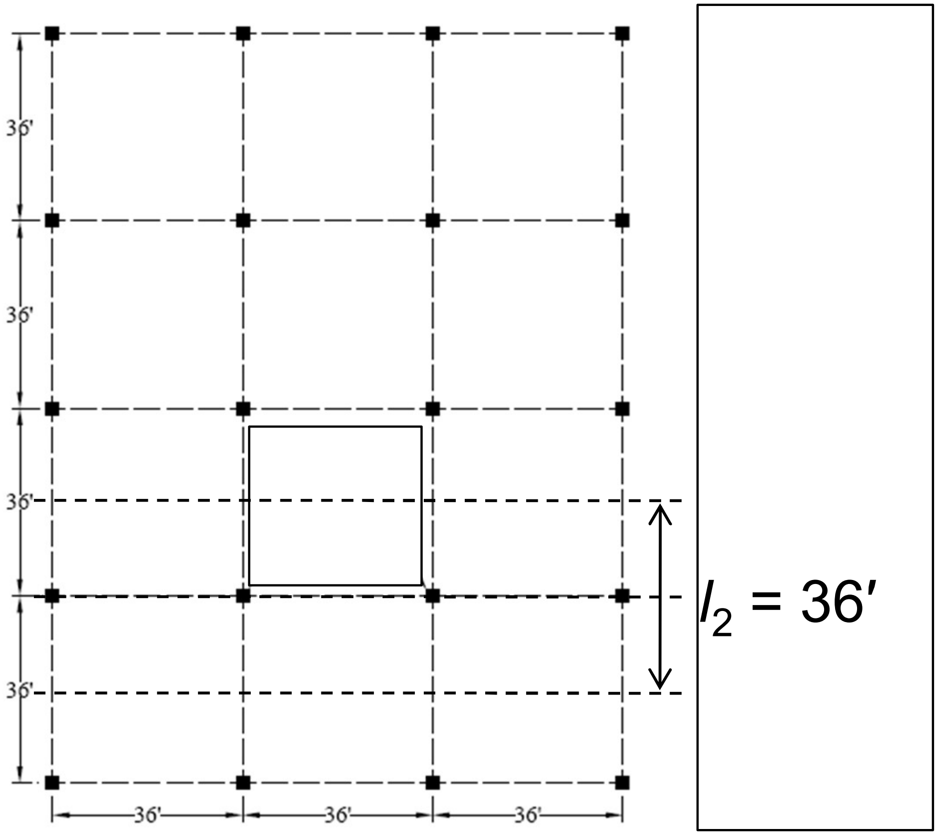
$= 1.2 \times 0.097 = 0.1164$ ksf

Example 6.4

❑ Solution

➢ Step 3: Frame Analysis (E-W Interior Frame)

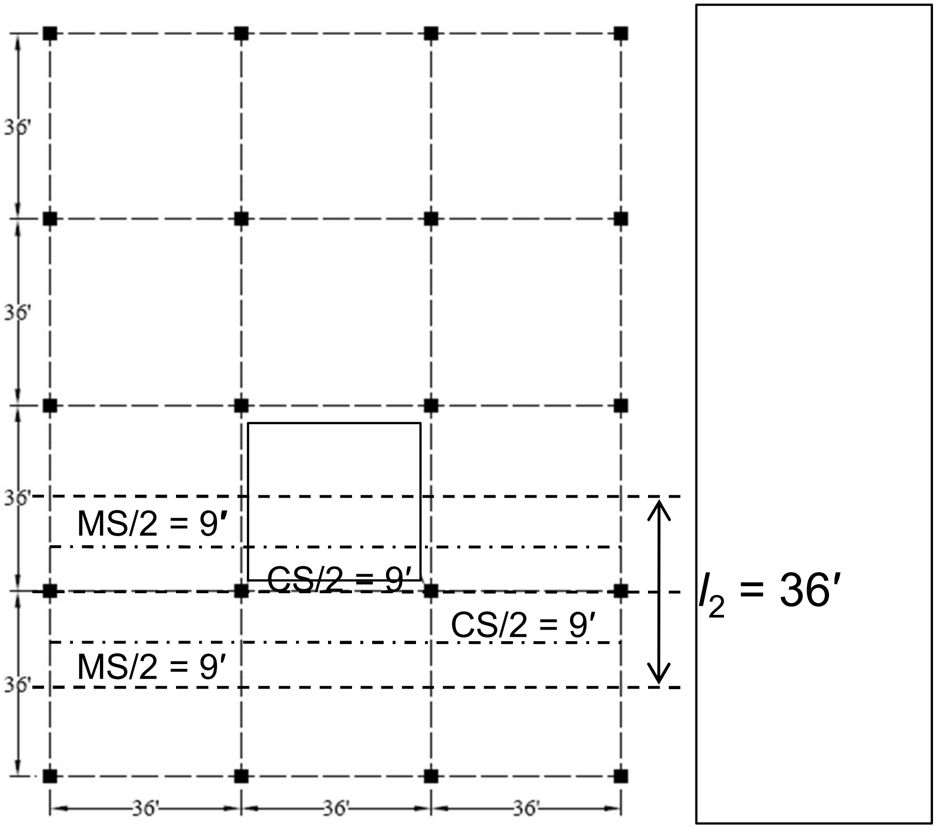
- ❖ **Marking E-W Interior Frame:**



Example 6.4

❑ Solution

- **Step 3: Frame Analysis (E-W Interior Frame)**
  - **Marking of Column and Middle Strips**



CS/2 = Least of $l_1/4$ or $l_2/4$

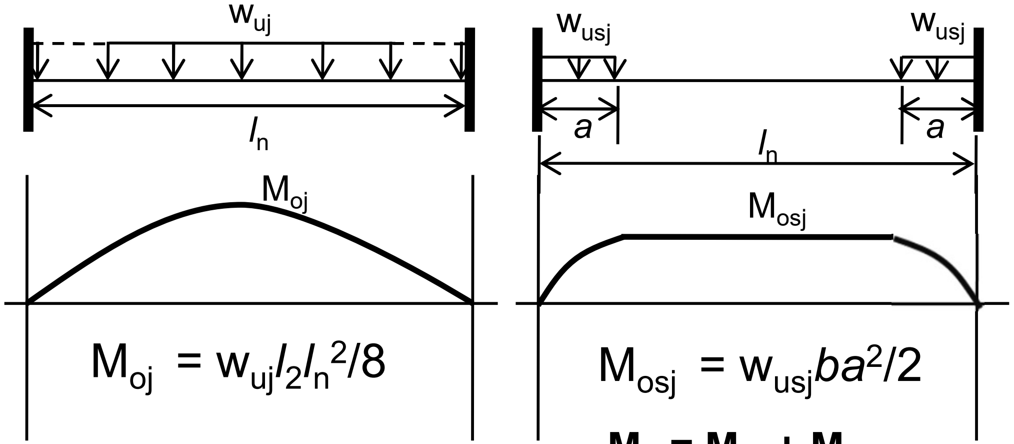
$l_2/4 = 36/4 = 9'$

Example 6.4

Solution

Step 3: Frame Analysis (E-W Interior Frame)

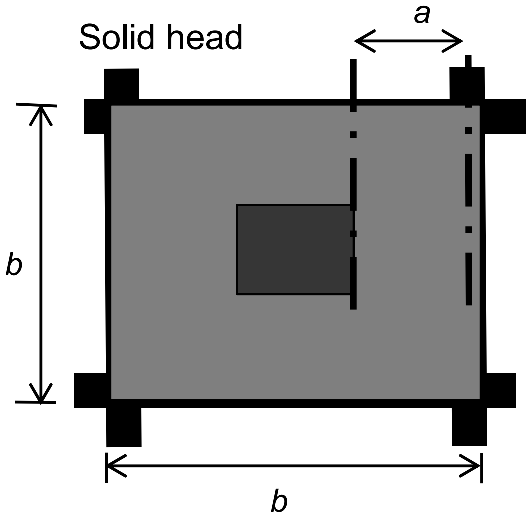
- **Static Moment Calculation**

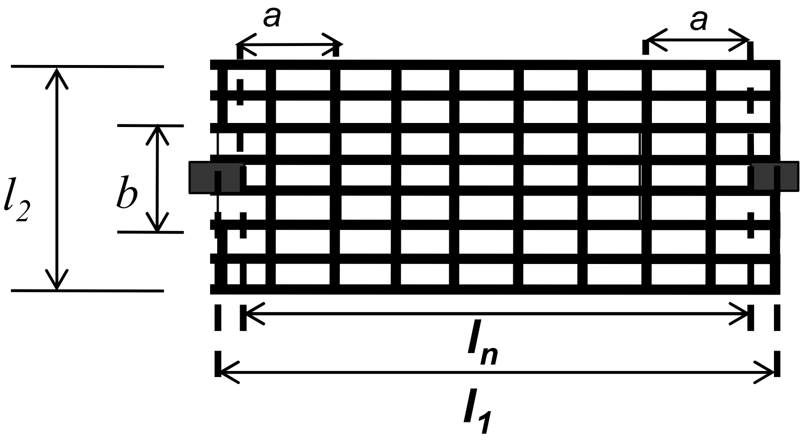


$M_{oj} = w_{uj} l_2 l_n^2/8$

$M_{osj} = w_{usj} b a^2/2$

$\mathbf{M_o = M_{oj} + M_{osj}}$





a = b/2 – c/2 – half joist width

a = 12.5/2 - 1.5/2 - 0.25
= 5.25 ft
b = 12.5 ft

Example 6.4

❑ Solution

➢ Step 3: Frame Analysis (E-W Interior Frame)

❖ Static Moment Calculation

M_{oj} (due to joists) = $w_{oj} l_2 l_n^2/8$

$= 0.291 \times 36 \times 34.5^2/8 = 1557.56$ ft-kip

M_{osj} (moment due to solid head excluding joists) = $w_{usj}\, ba^2/2$

$= 0.1164 \times 12.5 \times 5.25^2/2 = 20$ ft-kip

M_o (total static moment) = $M_{oj} + M_{osj} = 1557.56 + 20 = 1577.56$ ft-kip

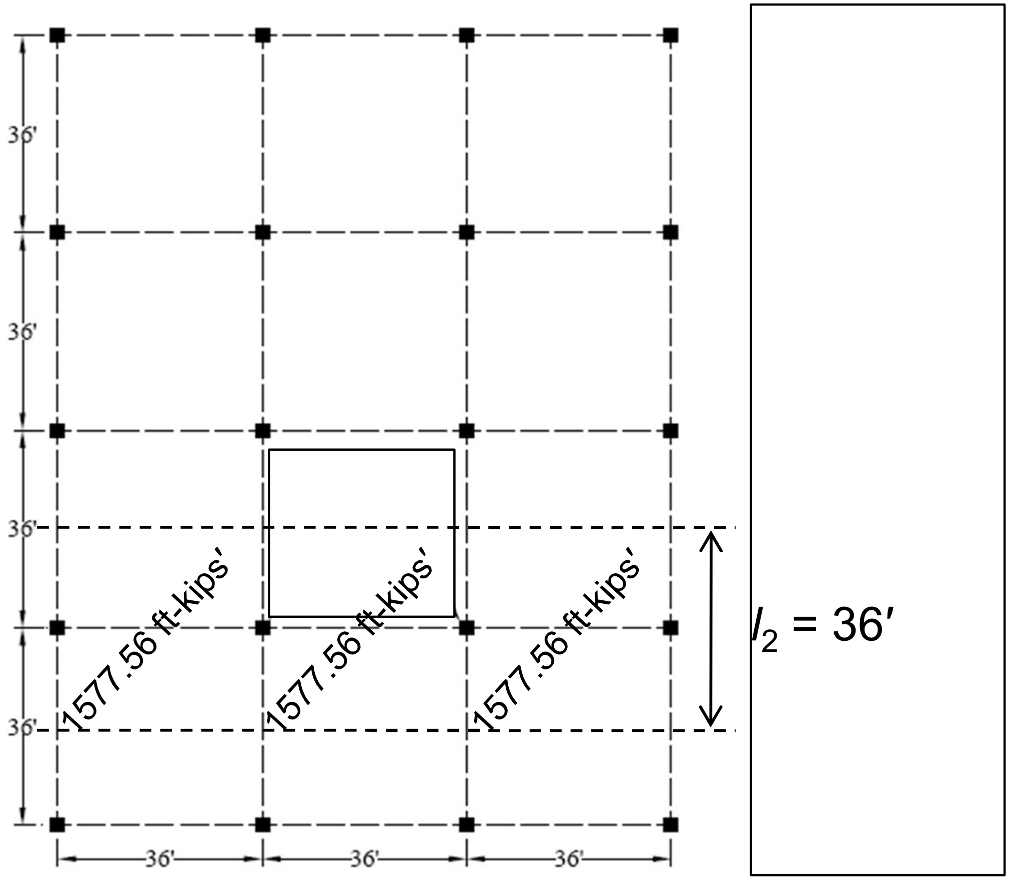
Note: Since normally M_{osj} is much smaller than M_{oj} , the former can be conveniently ignored in design calculations.

Example 6.4

❑ Solution

➢ Step 3: Frame Analysis (E-W Interior Frame)

❖ Static Moment Calculation

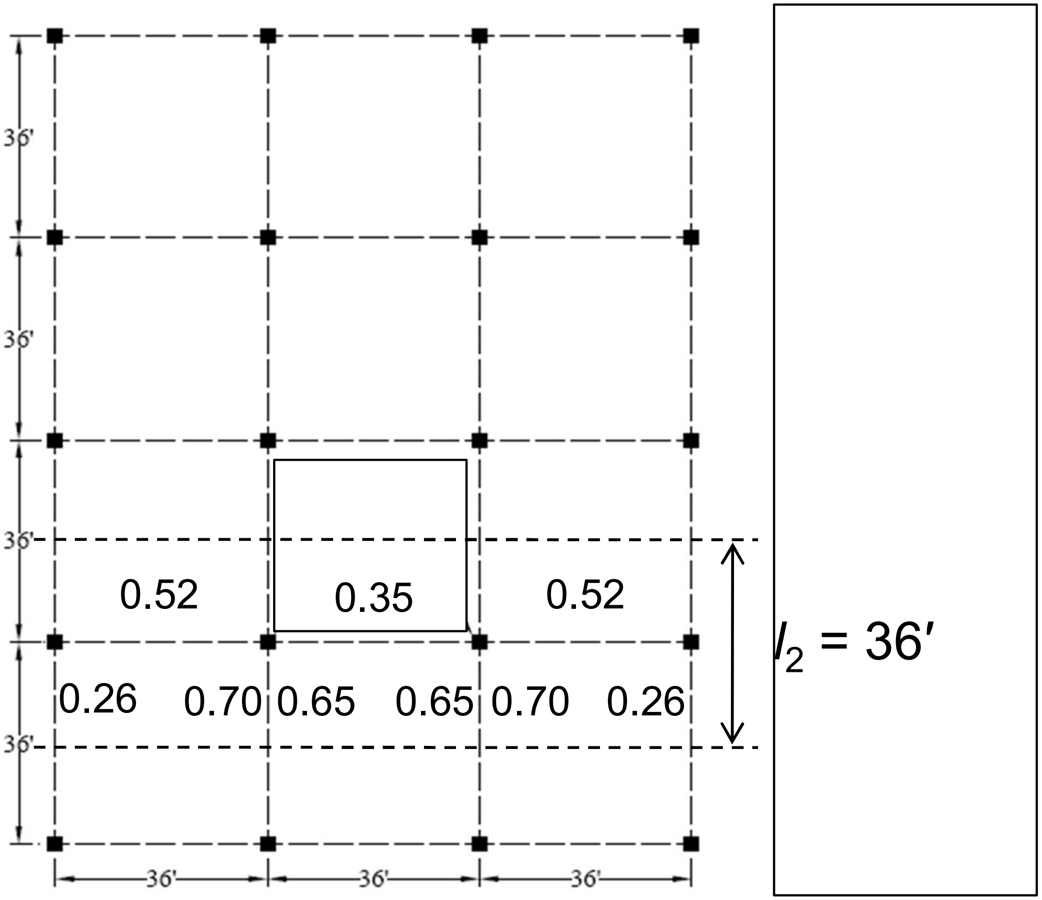


Example 6.4

❑ Solution

➢ Step 3: Frame Analysis (E-W Interior Frame)

- ❖ **Longitudinal Distribution of Total Static Moment (M_o)**



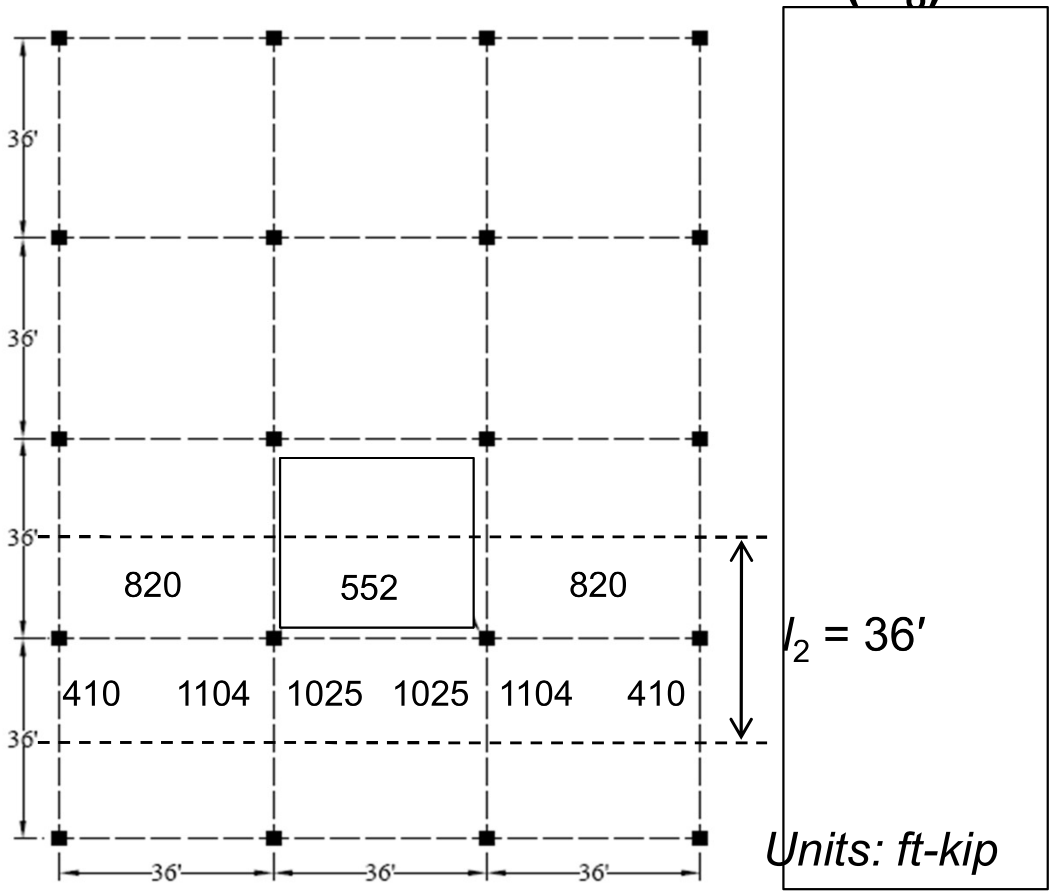
Example 6.4

❑ Solution

➢ Step 3: Frame Analysis (E-W Interior Frame)

- ❖ **Longitudinal Distribution of Total Static Moment (M_o)**

$M_L = M_o \times (D.F)_L$

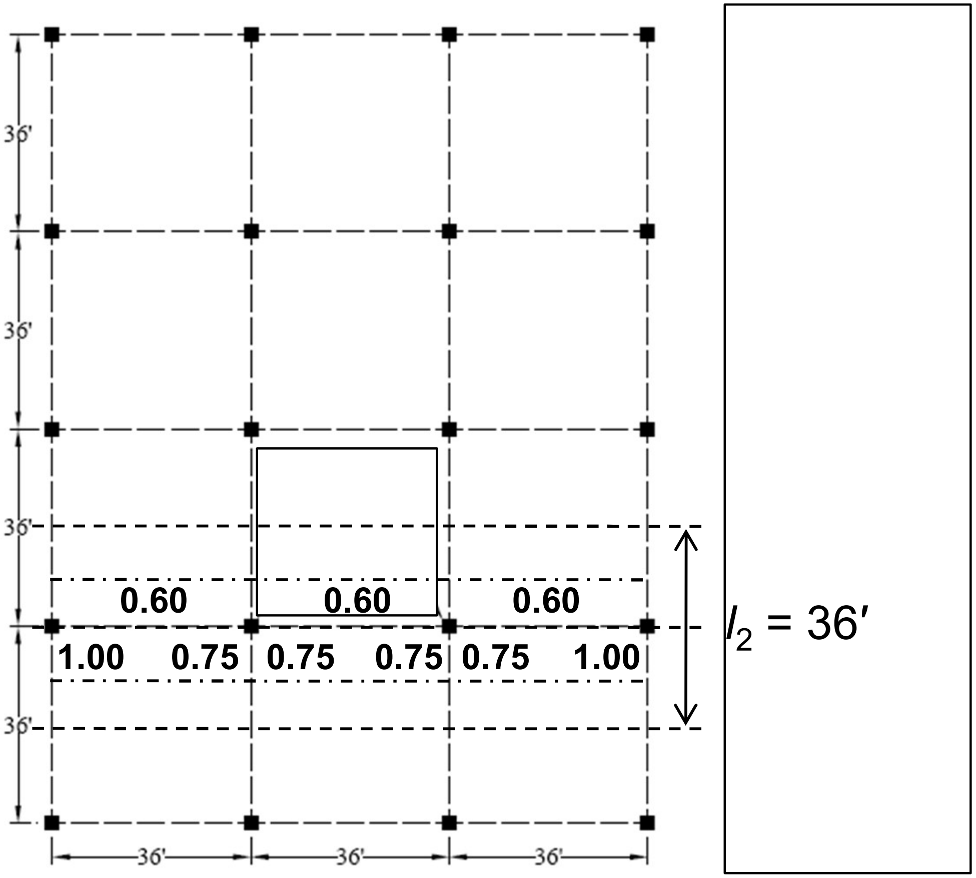


Example 6.4

❑ Solution

➢ Step 3: Frame Analysis (E-W Interior Frame)

❖ **Lateral Distribution of Longitudinal moment (L.M)**



Example 6.4

❑ Solution

- **Step 3: Frame Analysis (E-W Interior Frame)**

  - **Lateral Distribution of Longitudinal moment (L.M)**

$M_{L,ext-}$ = 410 kip-ft

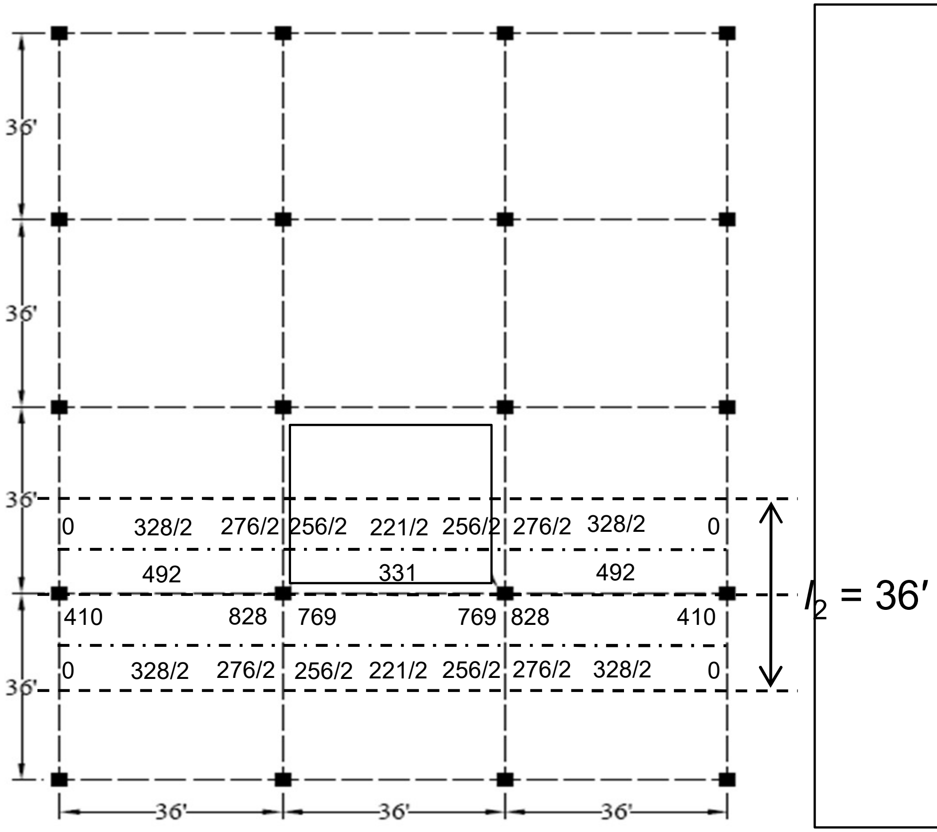
$M_{L,ext+}$ = 820 kip-ft

$M_{L,int-}$ = 1104 kip-ft

$M_{L,-}$ = 1025 kip-ft

$M_{L,+}$ = 552 kip-ft

$$M_{Lat} = M_L \times (D.F)_{Lat}$$



Example 6.4

❑ Solution

- **Step 3: Frame Analysis (E-W Interior Frame)**
  - **Lateral Distribution of Longitudinal moment (L.M)**

$M_{L,ext-}$ = 410 kip-ft

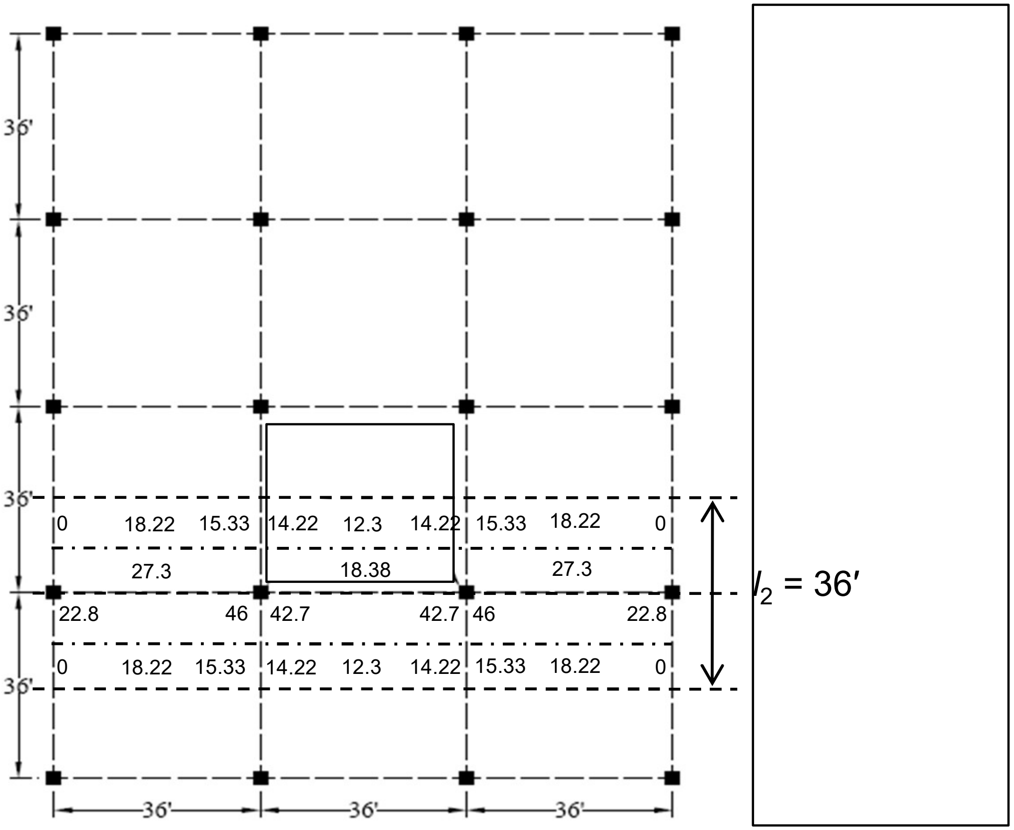
$M_{L,ext+}$ = 820 kip-ft

$M_{L,int-}$ = 1104 kip-ft

$M_{L,-}$ = 1025 kip-ft

$M_{L,+}$ = 552 kip-ft

M_{Lat} per foot = M_{lat}/strip width



Example 6.4

❑ Solution

➢ Step 3: Frame Analysis (E-W Exterior Frame)

❖ Static Moment Calculation

$l_2 = 18 + c/2$
$= 18 + (18/12)/2$
$= 18.75'$
c = column dimension

M_{oj} (due to joists) = $w_{oj} l_2 l_n^2/8$

$= 0.291 \times 18.75 \times 34.5^2/8 = 811.78$ ft-kip

M_{osj} (moment due to solid head excluding joists) = $w_{usj}\, ba^2/2$

$= 0.1164 \times 7 \times 5.25^2/2 = 11.23$ ft-kip

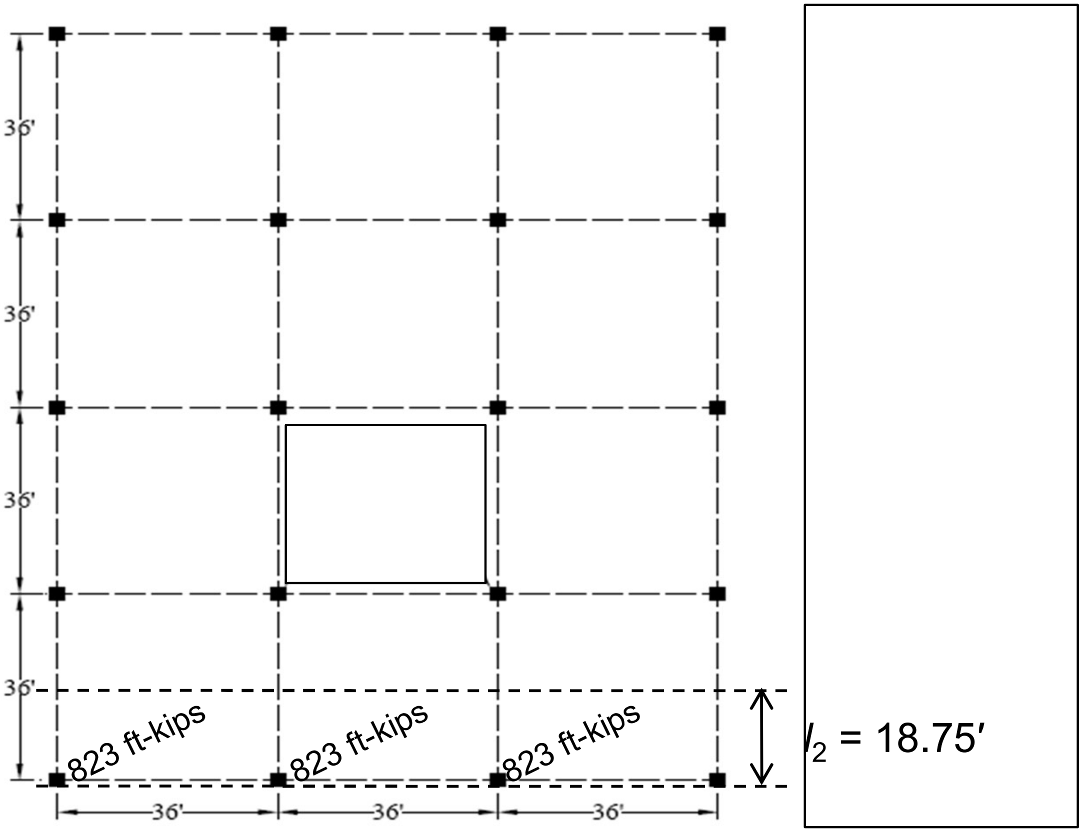
M_o (total static moment) = $M_{oj} + M_{osj} = 811.78 + 11.23 = 823$ ft-kip

Example 6.4

❑ Solution

➢ Step 3: Frame Analysis (E-W Exterior Frame)

❖ Static Moment Calculation

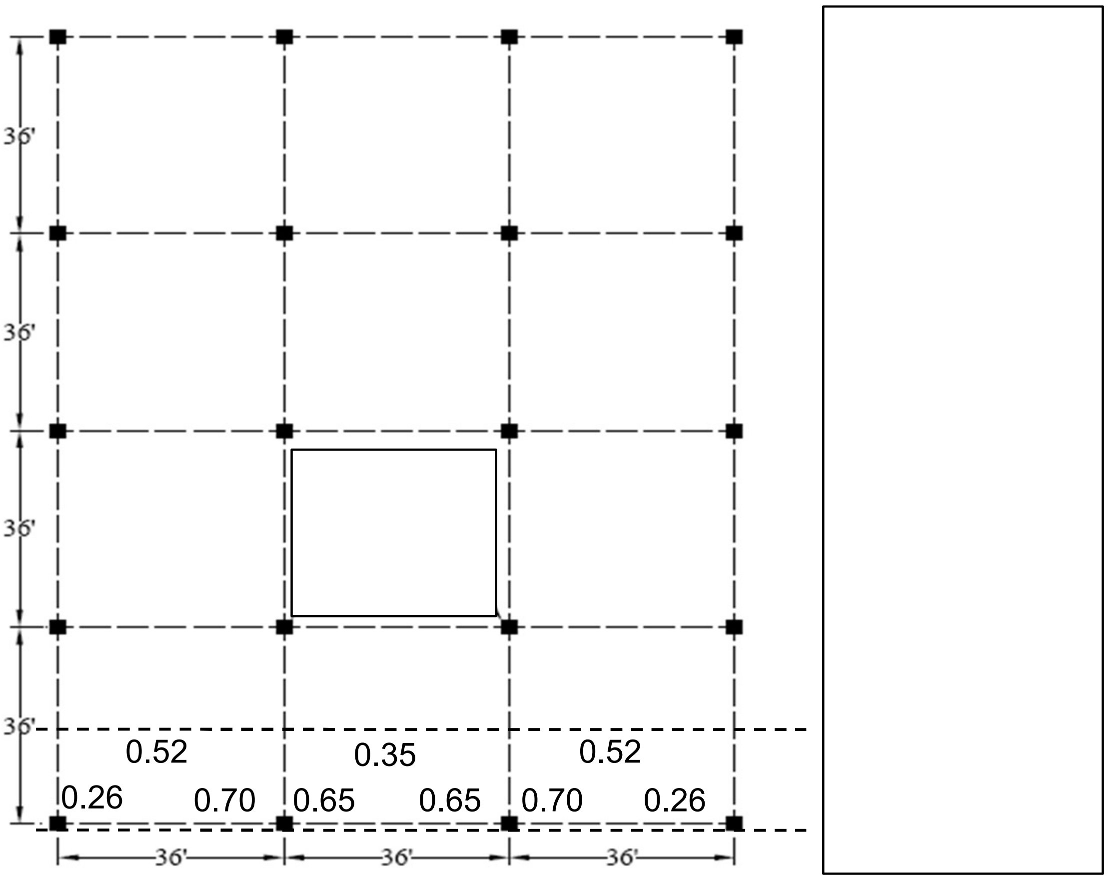


Example 6.4

❑ Solution

➢ Step 3: Frame Analysis (E-W Exterior Frame)

❖ Longitudinal Distribution of Total Static Moment

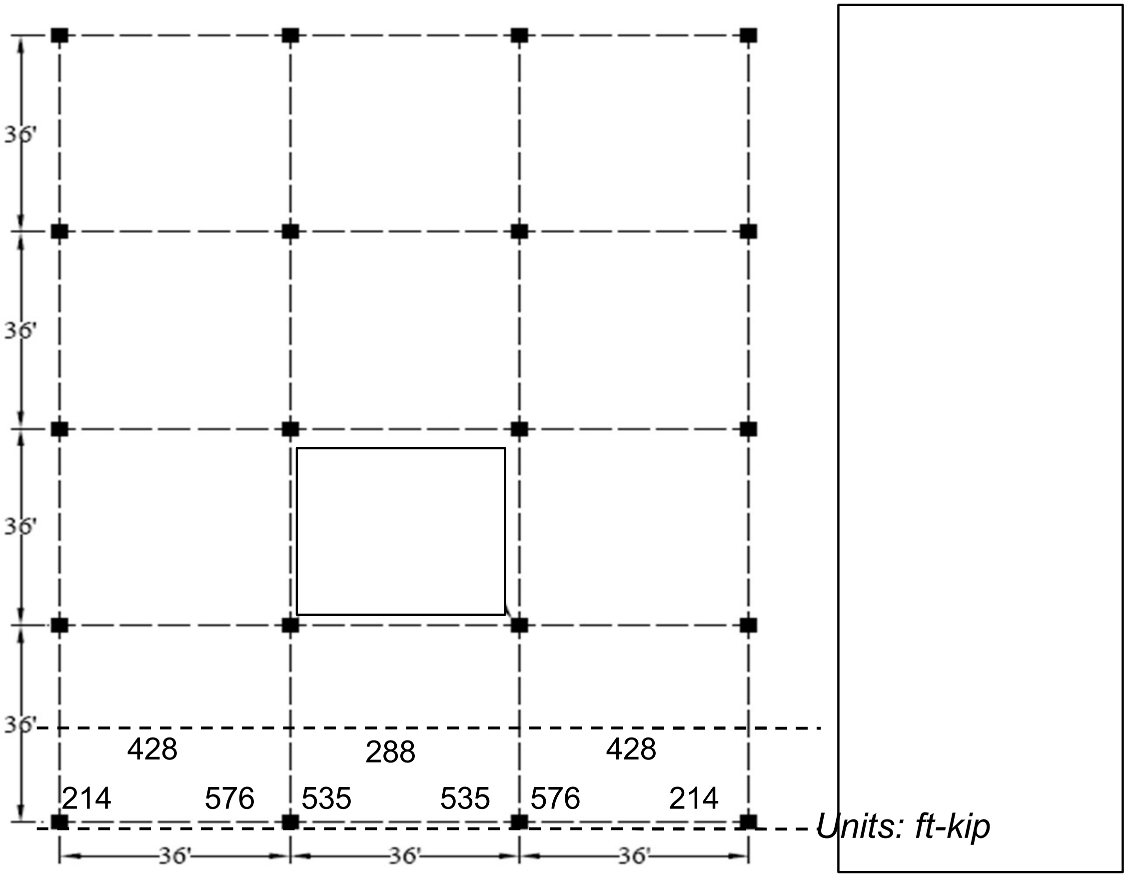


Example 6.4

Solution

- Step 3: Frame Analysis (E-W Exterior Frame)
  - Longitudinal Distribution of Total Static Moment

$$M_L = M_o \times (D.F)_L$$



Example 6.4

❑ Solution

➢ Step 3: Frame Analysis (E-W Exterior Frame)

- Lateral Distribution of Longitudinal Moment (L.M)

$M_{L,ext-}$ = 215 kip-ft

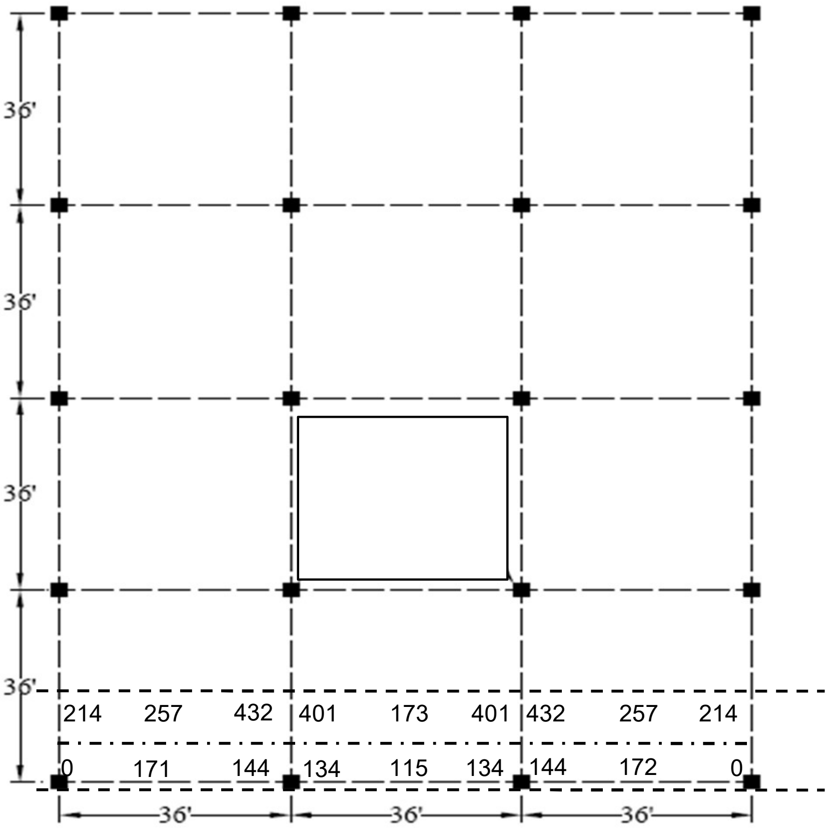
$M_{L,ext+}$ = 429 kip-ft

$M_{L,int-}$ = 578 kip-ft

$M_{L,-}$ = 536 kip-ft

$M_{L,+}$ = 289 kip-ft

$$M_{Lat} = M_L \times (D.F)_{Lat}$$



Example 6.4

❑ Solution

- **Step 3: Frame Analysis (E-W Exterior Frame)**
  - **Lateral Distribution of Longitudinal Moment (L.M)**

$M_{L,ext-}$ = 215 kip-ft

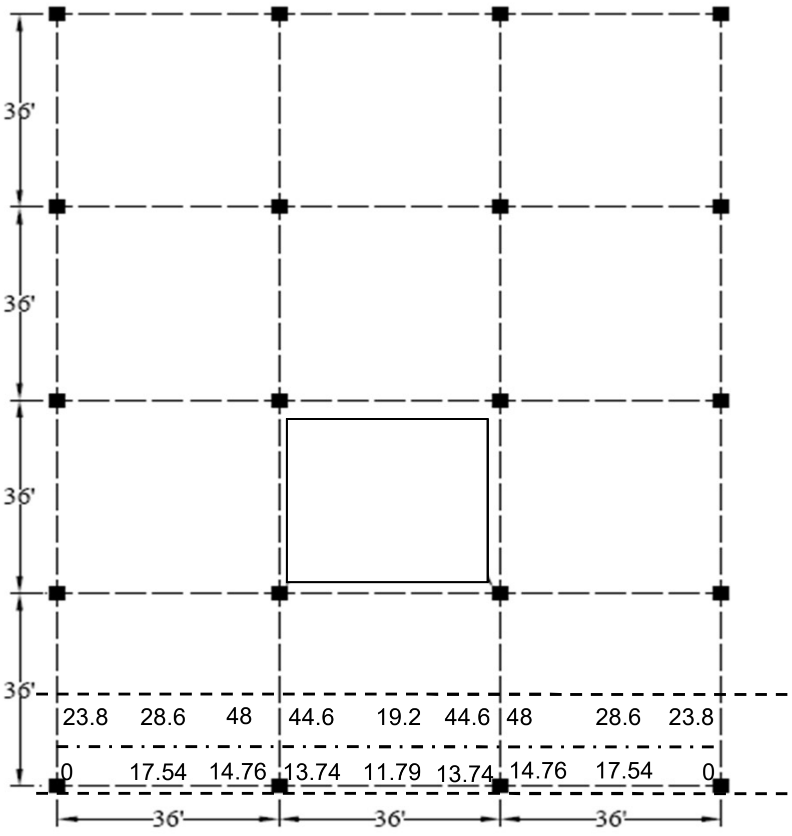
$M_{L,ext+}$ = 429 kip-ft

$M_{L,int-}$ = 578 kip-ft

$M_{L,-}$ = 536 kip-ft

$M_{L,+}$ = 289 kip-ft

M_{Lat} per foot = M_{lat}/strip width



Example 6.4

❑ Solution

➢ Step 3: Frame Analysis

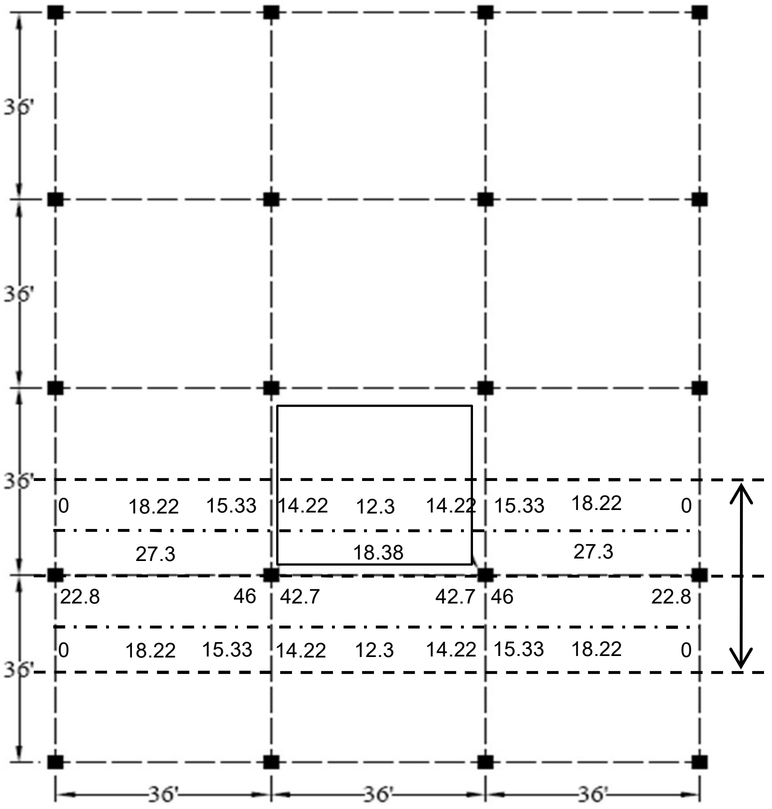
- Analysis of N-S Interior and Exterior Frame will be same as E-W respective frames due to square panels.

Example 6.4

❑ Solution

➢ Step 4: Determination of Reinforcement

- **E-W Interior Slab Strip**



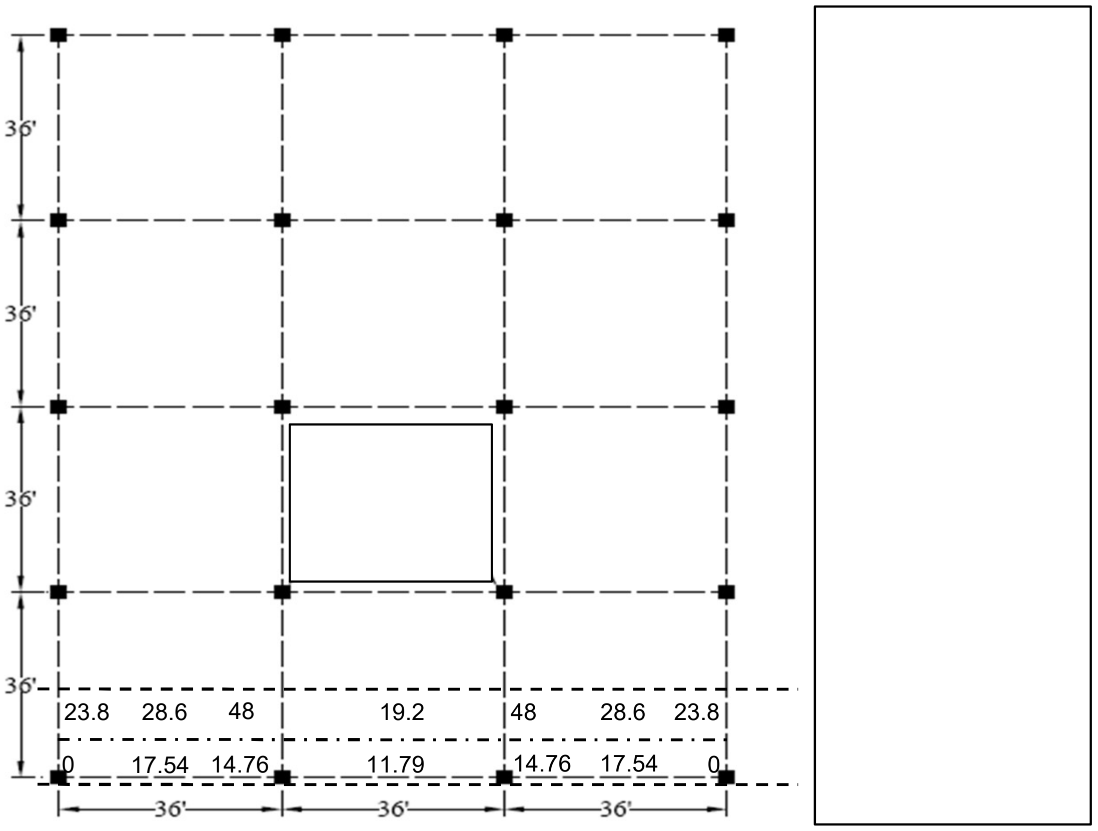
$l_2 = 36'$

Example 6.4

❑ Solution

➢ Step 4: Determination of Reinforcement

- ❖ E-W Exterior Frame



Example 6.4

❑ Solution

➢ Step 4: Determination of Reinforcement

- Design of N-S Interior and Exterior Frame will be same as E-W respective frames due to square panels and also for the reason that d_{avg} is used in design.
- d_{avg} = 16.5 – (0.75 inch (cover) – 0.75 inch (assumed bar diameter) = 15 inch
- This will be used for both directions positive as well as negative reinforcement.

Example 6.4

❑ Solution

- **Step 5: Detailing (E-W Frame)**

❖ Negative Reinforcement

M = 46 ft-k =46 x12 = 552 in-k

b = 12″ ; d = 15″

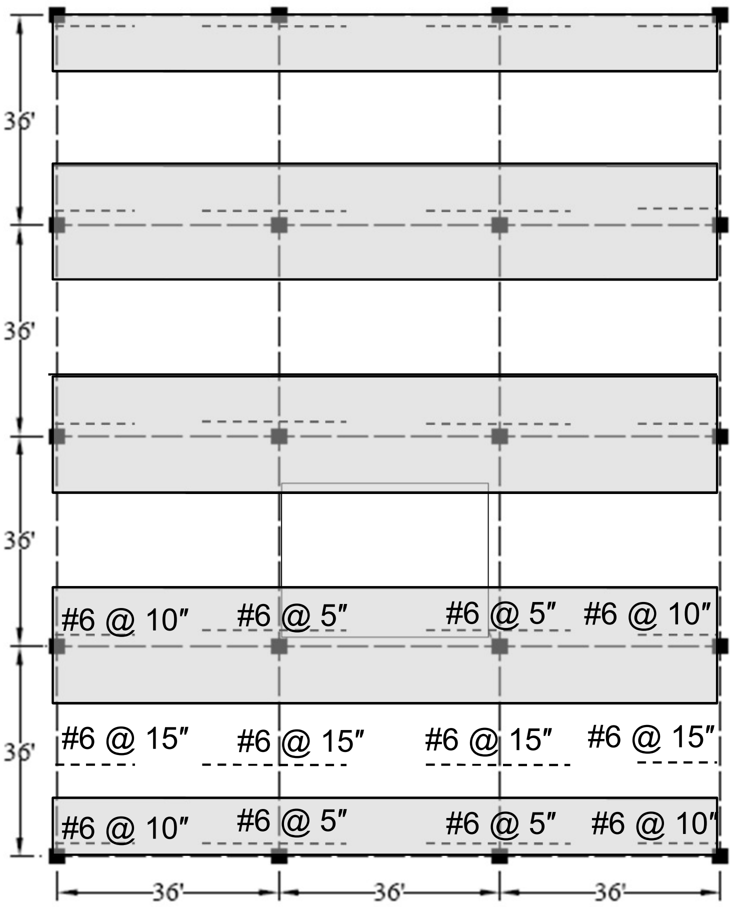
f'_c = 4 ksi ; f_y = 40 ksi

A_s = 1.06 in^2

S= 0.44/1.06 x 12 = 5 in.

Finally using #6 @ 5″ c/c & 10″ c/c in the column strips at all interior & exterior support, respectively.

In the middle strip, use #6 @ 15″ c/c.



Example 6.4

❑ Solution

➢ Step 5: Detailing (E-W Frame)

❖ Positive Reinforcement

For M = 27 ft-k =27x12 = 324 in-kip

b = 12″ ; d = 15″

f $'_c$ = 4 ksi ; f_y = 40 ksi

A_s = 0.612 in^2 . This is per foot reinforcement. For 18 feet col strip, this will be equal to 0.612 x 18 = 11.02 in^2

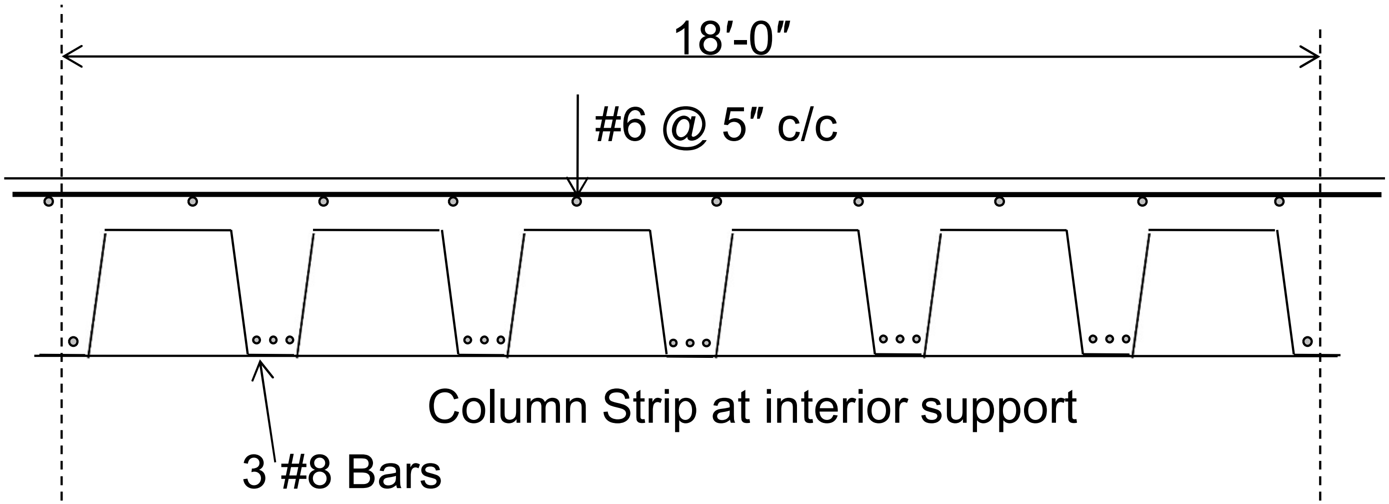
There are 6 joists in 18 feet width. Therefore, per rib reinforcement = 1.83

Using # 8 bars, 3 bars per joist rib will be provided in the column as well as middle strips.

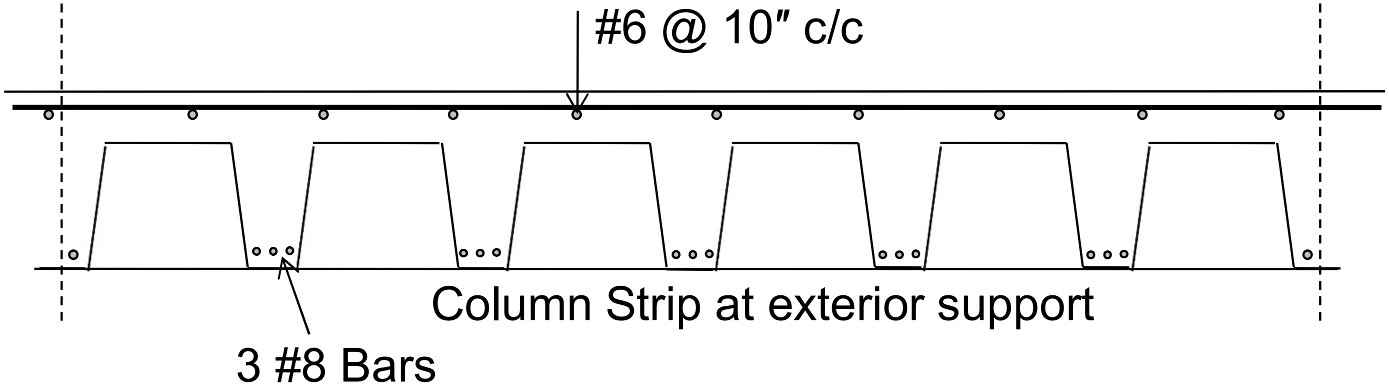
Example 6.4

❑ Solution

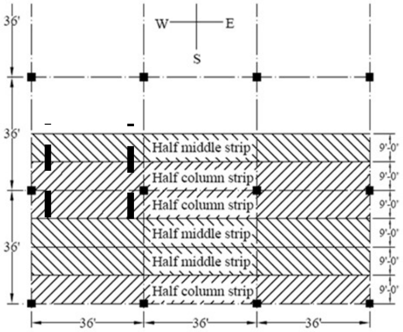
➢ Step 5: Detailing (E-W Interior Frame)



Column Strip at interior support



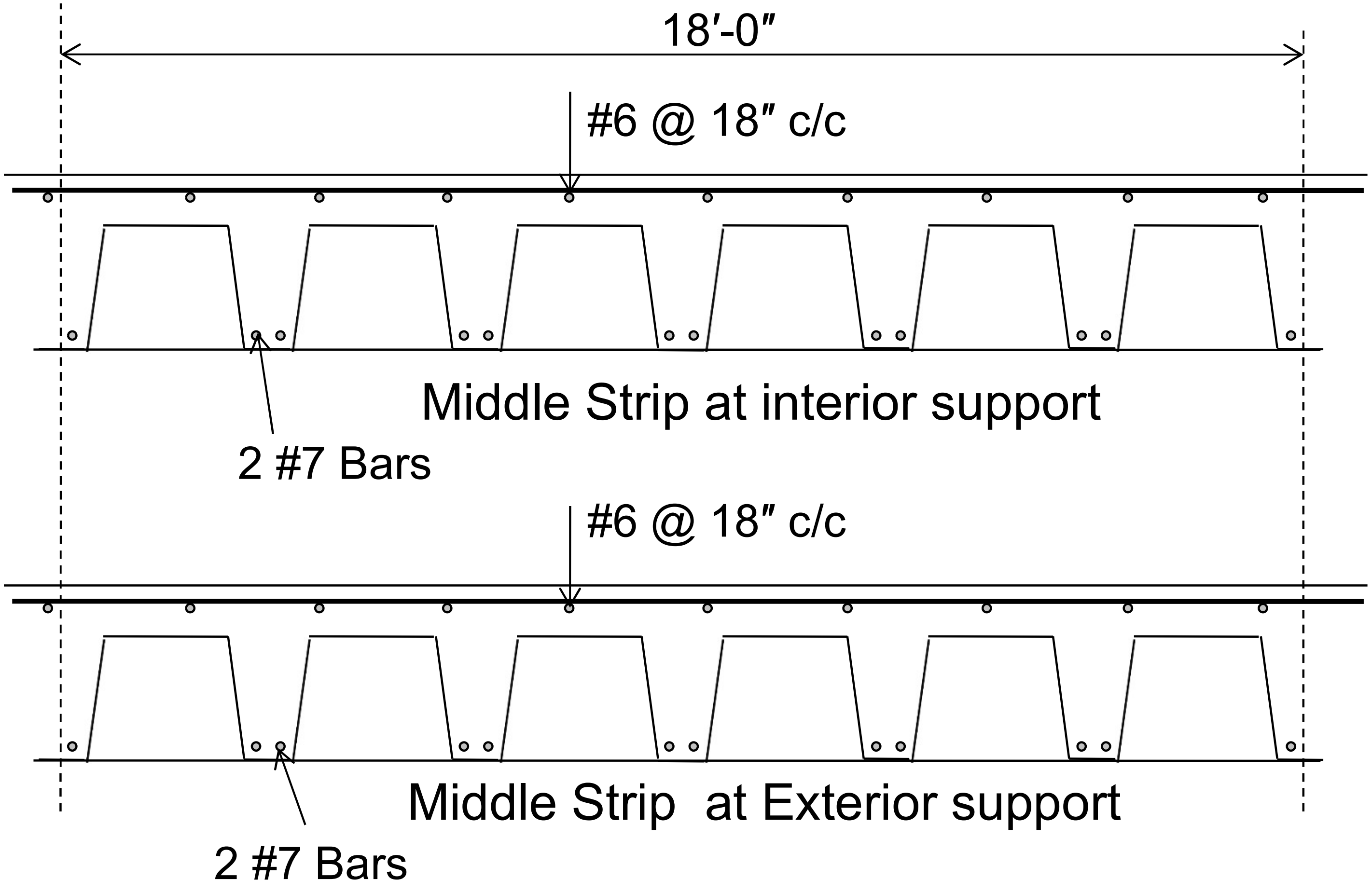
Column Strip at exterior support



Example 6.4

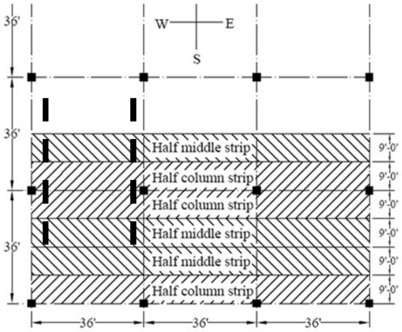
❑ Solution

➢ Step 5: Detailing (E-W Interior Frame)



Middle Strip at interior support

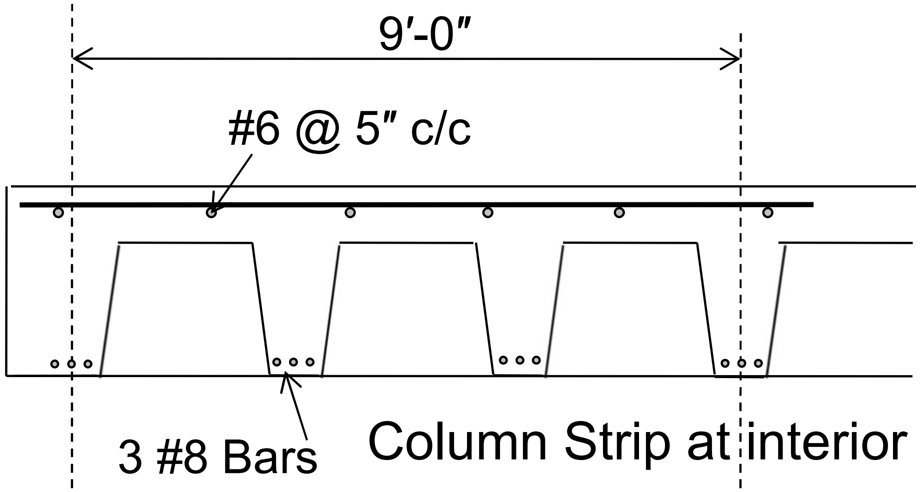
Middle Strip at Exterior support



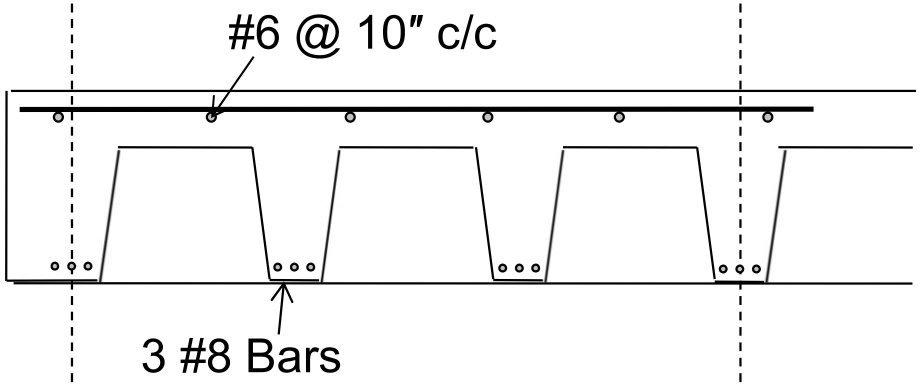
Example 6.4

❑ Solution

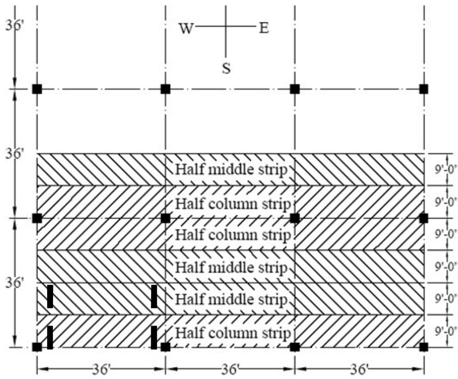
➢ Step 5: Detailing (E-W Exterior Frame)



Column Strip at interior support



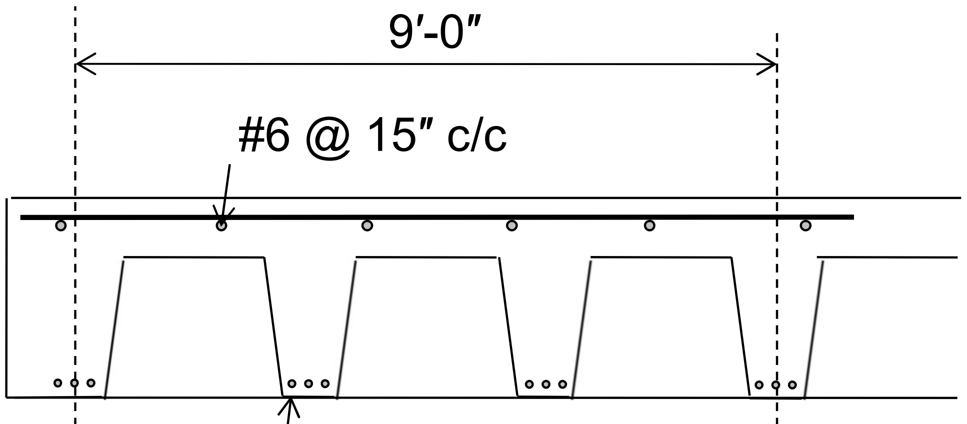
Column Strip at exterior support



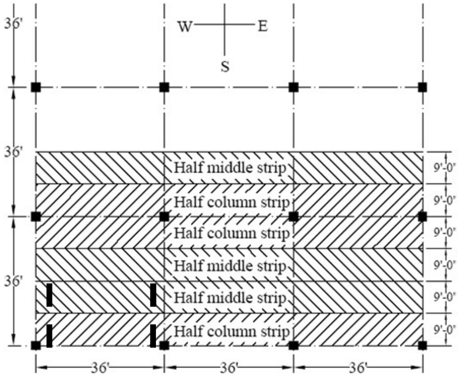
Example 6.4

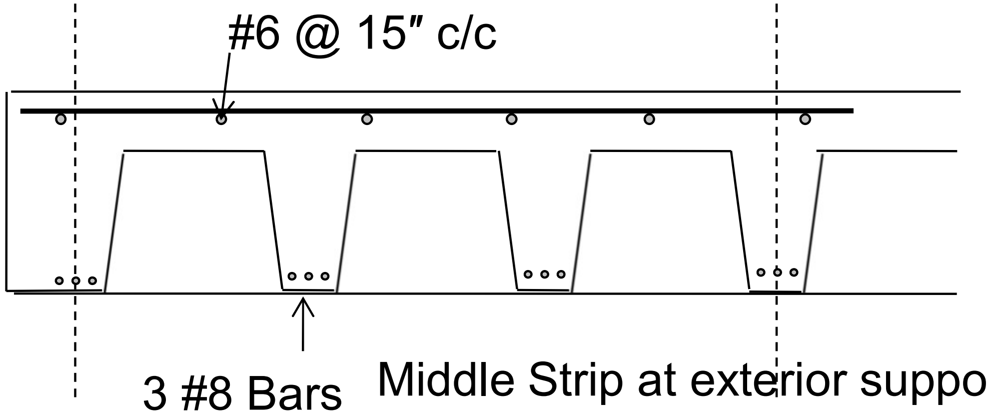
❑ Solution

- **Step 5: Detailing (E-W Exterior Frame)**



Middle Strip at interior support





Middle Strip at exterior support

Example 6.4

❑ Solution

➢ Step 4: Determination of Reinforcement

- Note: For the completion of design problem, the waffle slab should also be checked for beam shear and punching shear.

References

- Reinforced Concrete - Mechanics and Design (7th Ed.) by James MacGregor.
- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-19)
- Portland Cement Association (PCA 2002)