Lecture 12

Design of RC Shallow Foundations

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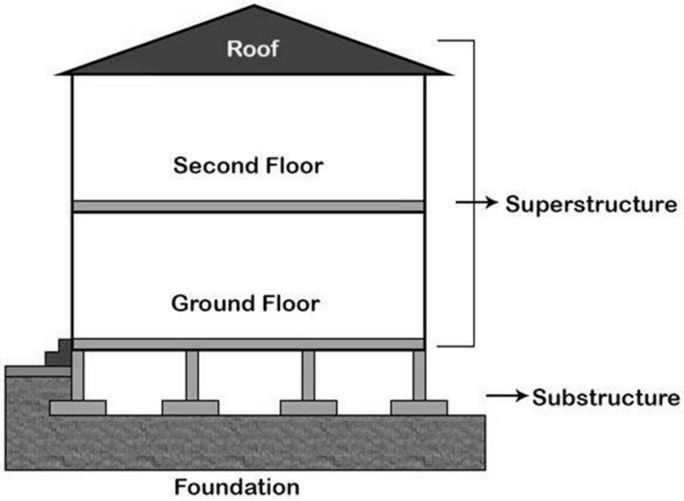
- General
- General Requirements for Footing Design
- ACI Provisions for Shallow Foundations
- Design of Isolated Column Footings
- Design of Combined Footing
- Design of Mat Footing

General

❑ Foundation

- The substructure, or foundation, is the part of a structure that is usually placed below the surface of the ground and that transmits the load to the underlying soil or rock.
- Foundation is regarded as the most important component of engineered systems.



General

❑ Classification of Foundation

- Foundations can be divided into two broad categories depending on the depth of foundation:

1. **Shallow Foundations**
   - Load transfer occurs at shallower depths.
   - Examples: Isolated, Wall, Combined, Mat footings.
2. **Deep Foundations**
   - Load transfer occurs at deeper depths.
   - Examples: Piles, drilled piers, drilled caissons.

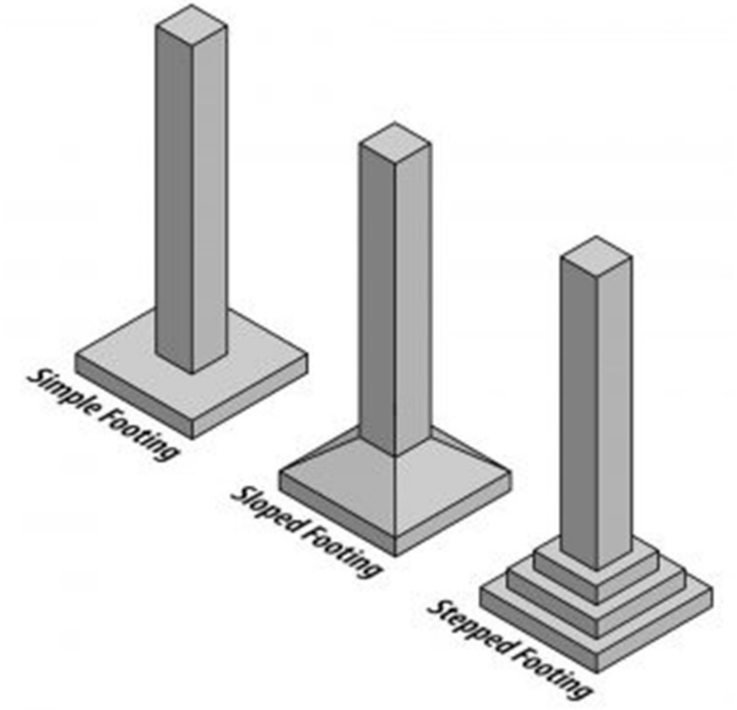
General

❑ Classification of Foundation

1. Types of Shallow Foundations

I. Isolated Column Footing

- Isolated column footing carrying a single column is usually called spread footing.
- Sometimes spread footings are tapered or stepped to save materials.



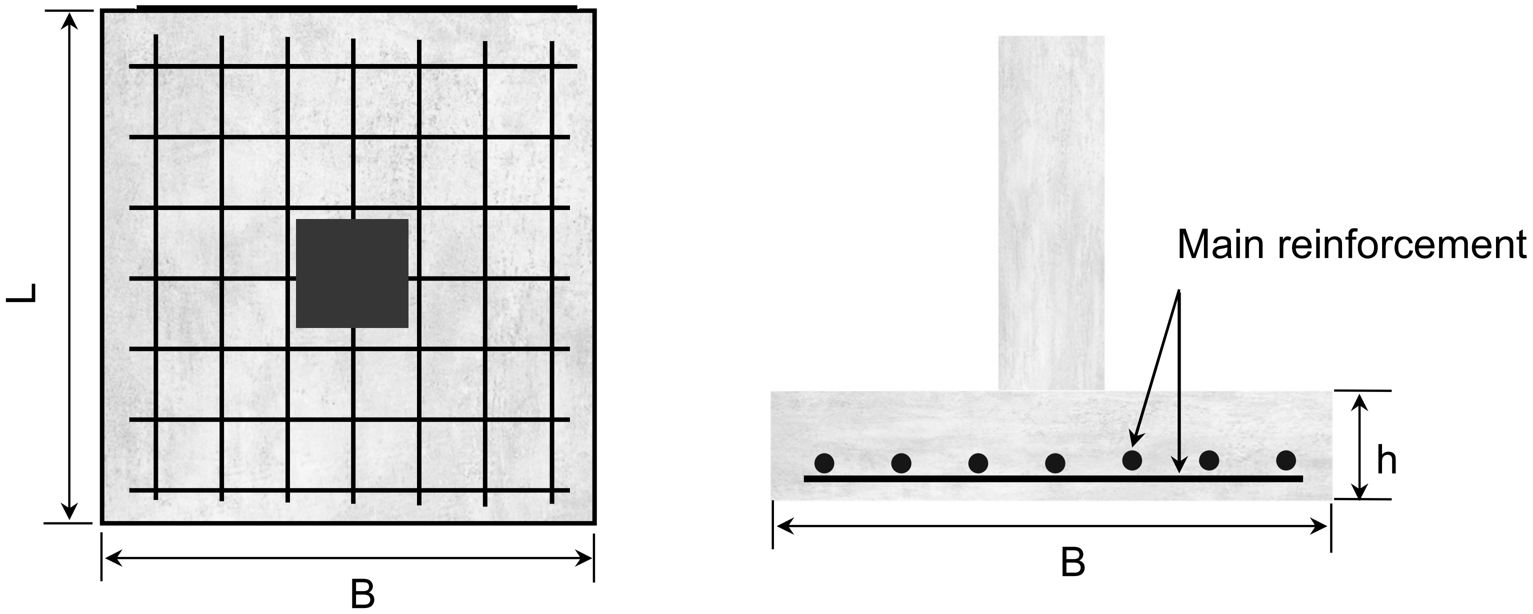
General

❑ Classification of Foundation

1. Types of Shallow Foundations

I. Isolated Column Footing

- Isolated column footing/spread footing will typically have reinforcement in two orthogonal directions at the bottom of the footing for flexure.



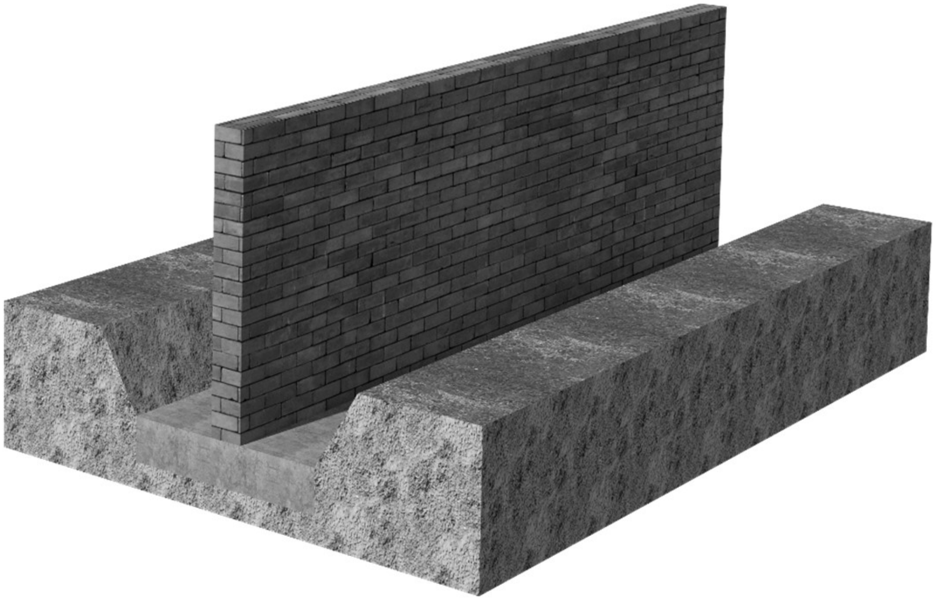
General

❑ Classification of Foundation

1. Types of Shallow Foundations

II. Wall Footing (Strip Footing)

- Wall footings or strip footings display essentially one-dimensional action, cantilevering out on each side of the wall.



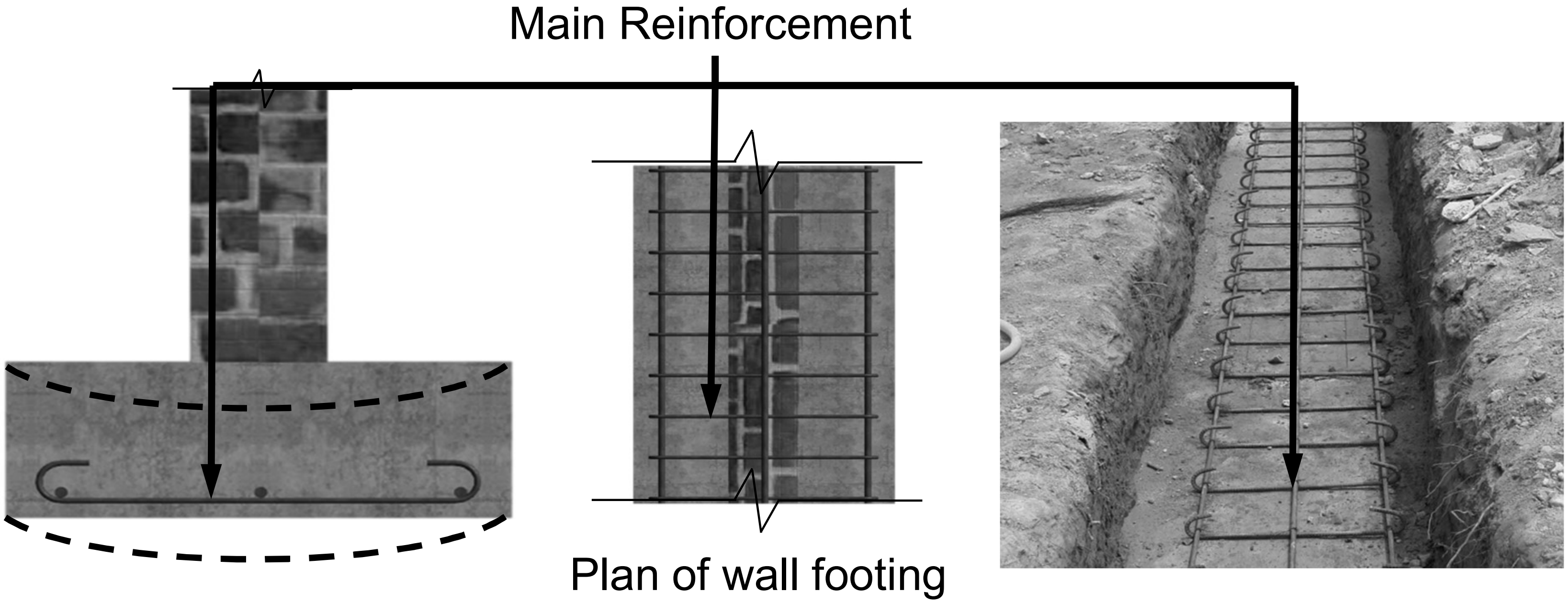
General

❑ Classification of Foundation

1. Types of Shallow Foundations

II. Wall Footing (Strip Footing)

- Reinforcement for flexure is placed at the bottom of the footing perpendicular to the wall, as shown.



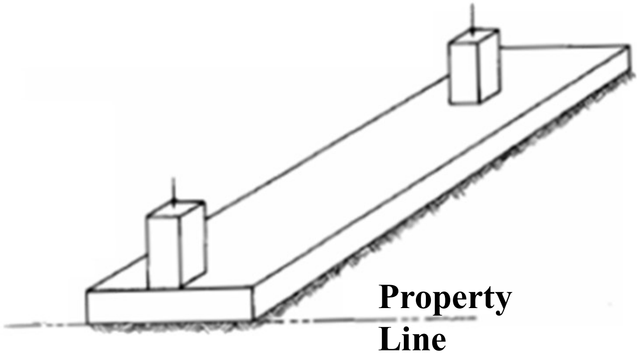
General

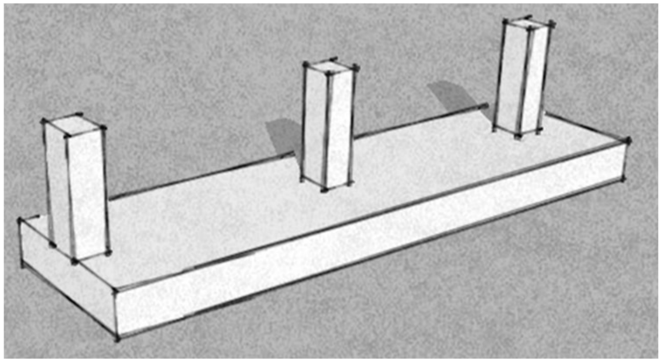
❑ Classification of Foundation

1. Types of Shallow Foundations

III. Combined Footing

- A combined footing is a type of footing supporting two or more than two columns.
- A combined footing can be
  - i. Two columns footing
  - ii. Column Strip or Multiple Columns Footing





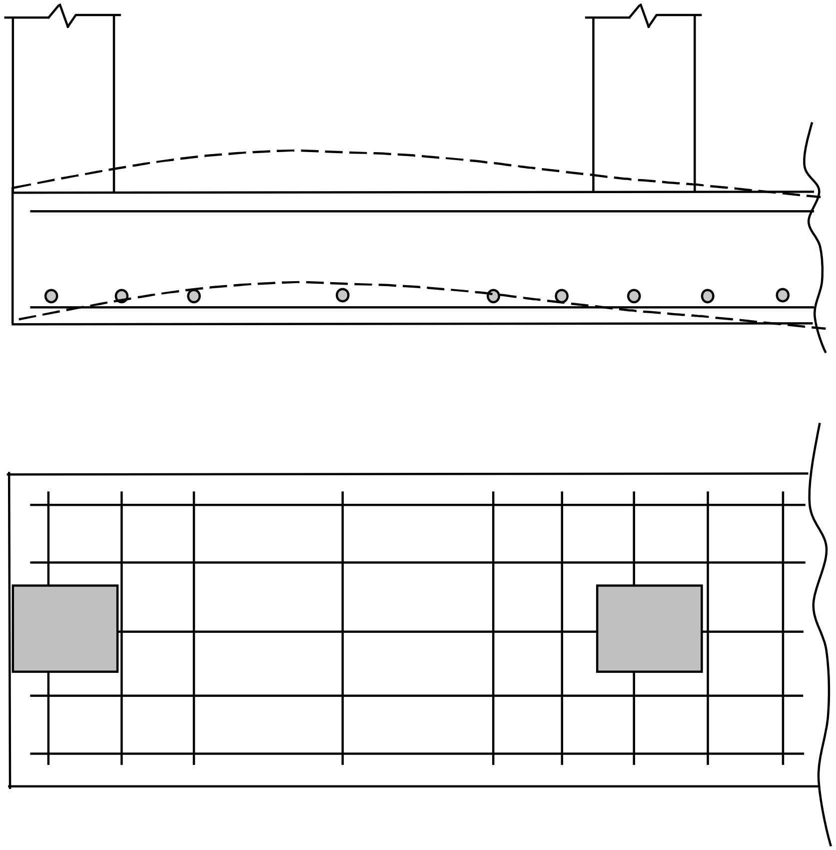
General

❑ Classification of Foundation

1. Types of Shallow Foundations

III. Combined Footing

- Main reinforcement in long (longitudinal) direction is provided at the top of footing.
- Main reinforcement in short (transverse) direction is provided at the bottom of the footing.



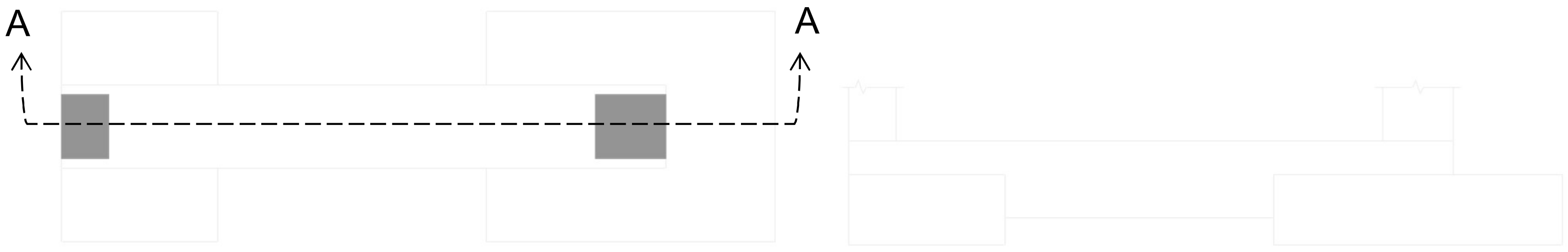
General

❑ Classification of Foundation

1. Types of Shallow Foundations

IV. Cantilever or Strap Footing

- These are similar to combined footings, except that the footings of exterior and interior columns are built independently.
- They are joined by a strap beam to transmit the effect of the bending moment produced by the eccentric wall column load to the interior column footing area.



Plan

Section A-A

General

❑ Classification of Foundation

1. Types of Shallow Foundations

v. Mat footing

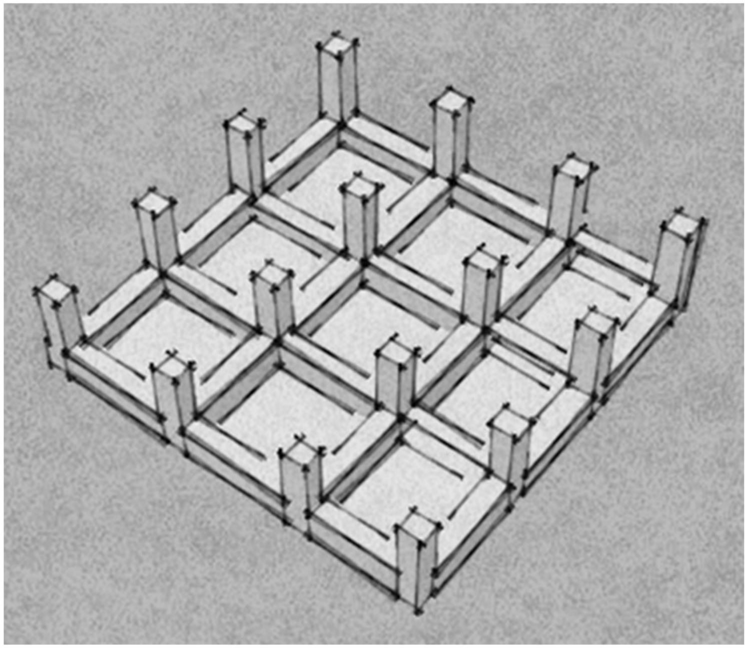
- A mat or raft foundation transfers the loads from all the columns in a building to the underlying soil.
- Mat foundations are used when excessive loads are supported on a limited area or when very weak soils are encountered.
- Mat footings are essentially inverted slabs and hence they have as much configurations as typical slab systems have.

General

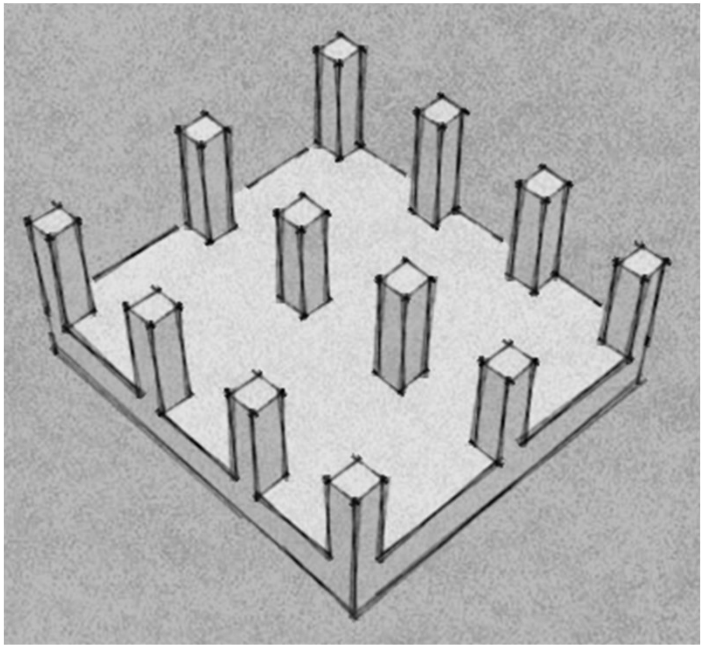
❑ Classification of Foundation

1. Types of Shallow Foundations

v. Mat footing



Mat Footing with Beams



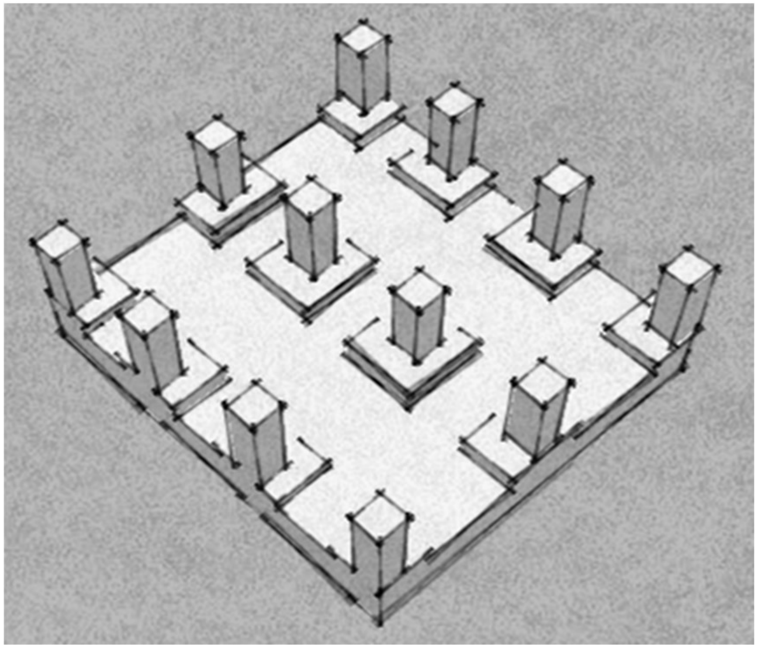
Mat Footing without Beams

General

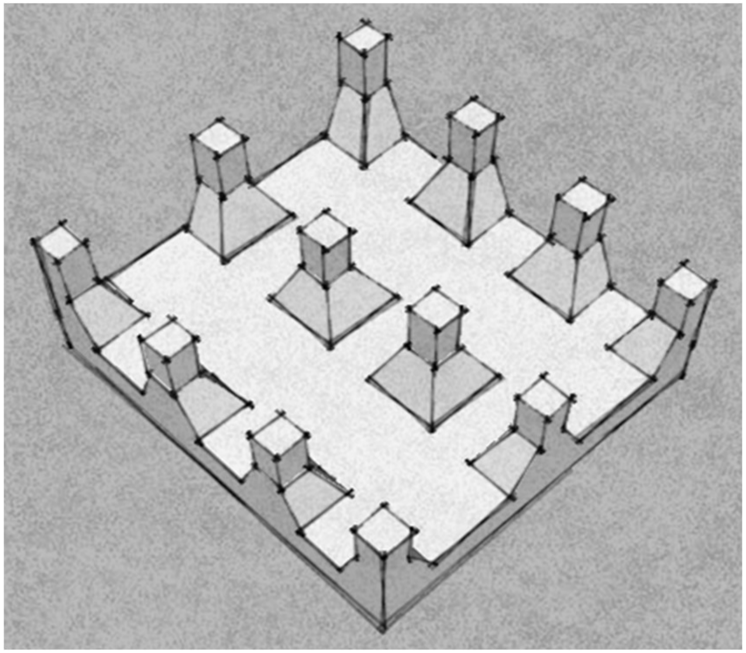
❑ Classification of Foundation

1. Types of Shallow Foundations

v. Mat footing



Mat Footing with Drop panels



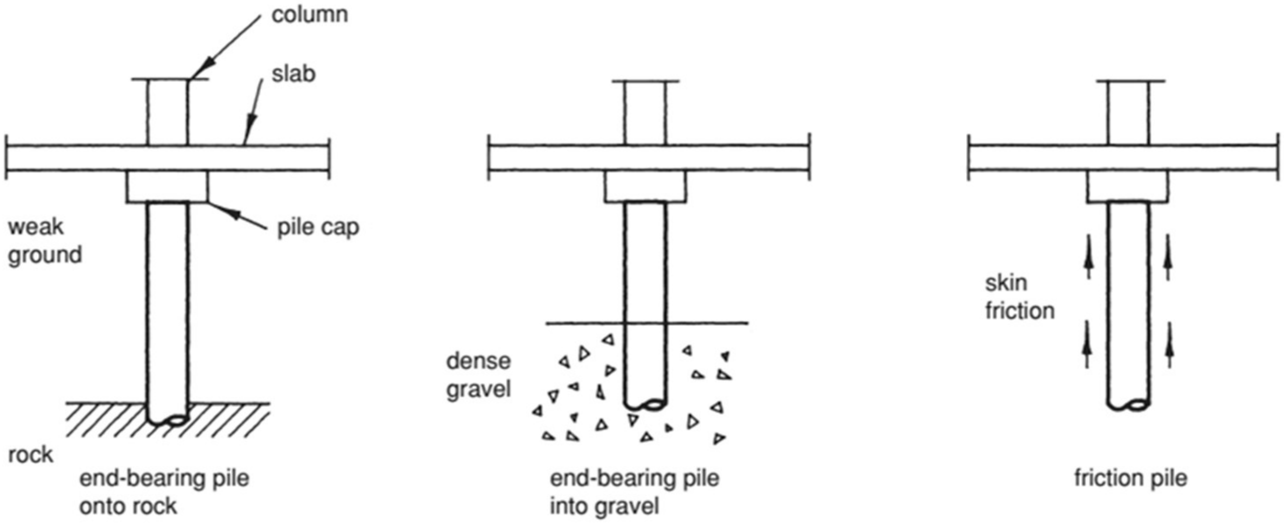
Mat Footing with Column capitals

General

❑ Classification of Foundation

2. Deep Foundation

- This type of foundation is essential when the supporting ground consists of structurally unsound layers of materials to large depths.
- The piles maybe either end bearing, skin friction, or both.



General

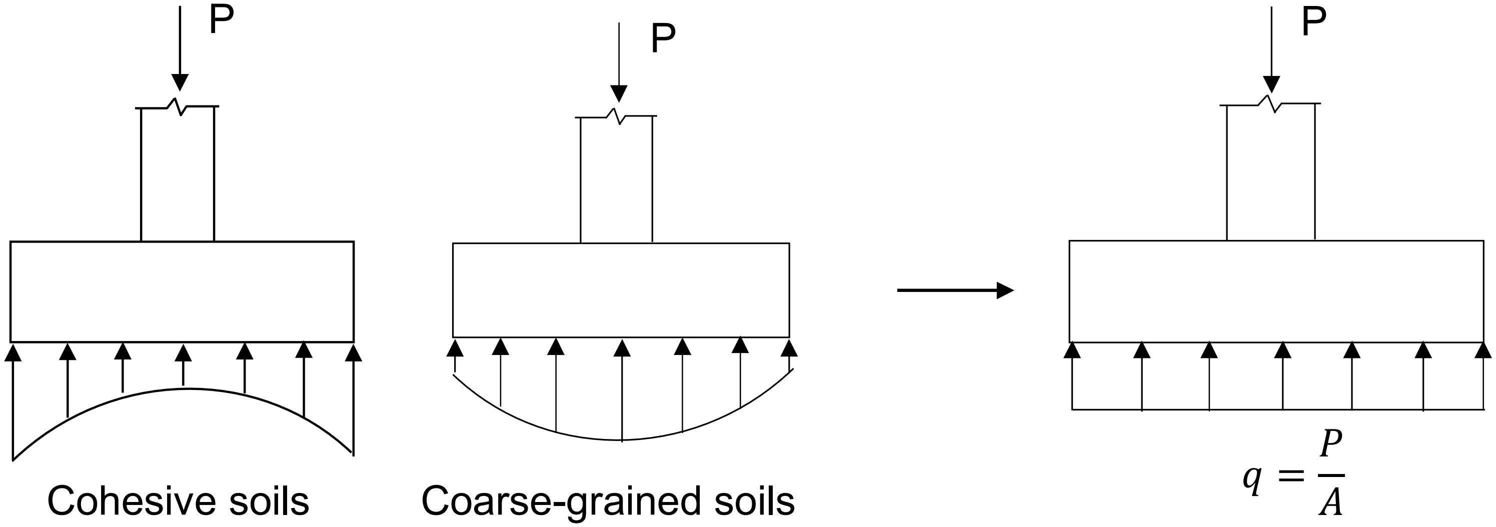
❑ Selection of Foundation

- The type of foundation is selected in consultation with geotechnical engineer.
- Factors to be considered are:
  - Soil strength
  - Soil type
  - Variability of soil type over the area and with increasing depth
  - Susceptibility of the soil and the building to deflections.
  - Construction methods

General Requirements for Footing Design

❑ Soil Pressure Distribution

- The soil pressure distribution depends upon Footing rigidity and the Soil type.
- It is customary to assume that soil pressures are linearly distributed.



$q = \frac{P}{A}$

ACI Provisions for Shallow Foundations

❑ Design Criteria

- The design and detailing of:
  - One-way foundations (strip footings, one-way combined footings, and grade beams) shall be in accordance with chapter 13 section 13.3 and the applicable provisions of Chapter 7 (one-way slabs) and Chapter 9 (beams).
  - Two-way isolated footings shall be in accordance with chapter 13 section 13.3 and the applicable provisions of Chapter 7(one-way slabs) and Chapter 8 (two-way slabs).
  - Two-way combined footings and Mat foundations shall be in accordance with applicable provisions of chapter 8.

ACI Provisions for Shallow Foundations

☐ Dimensions

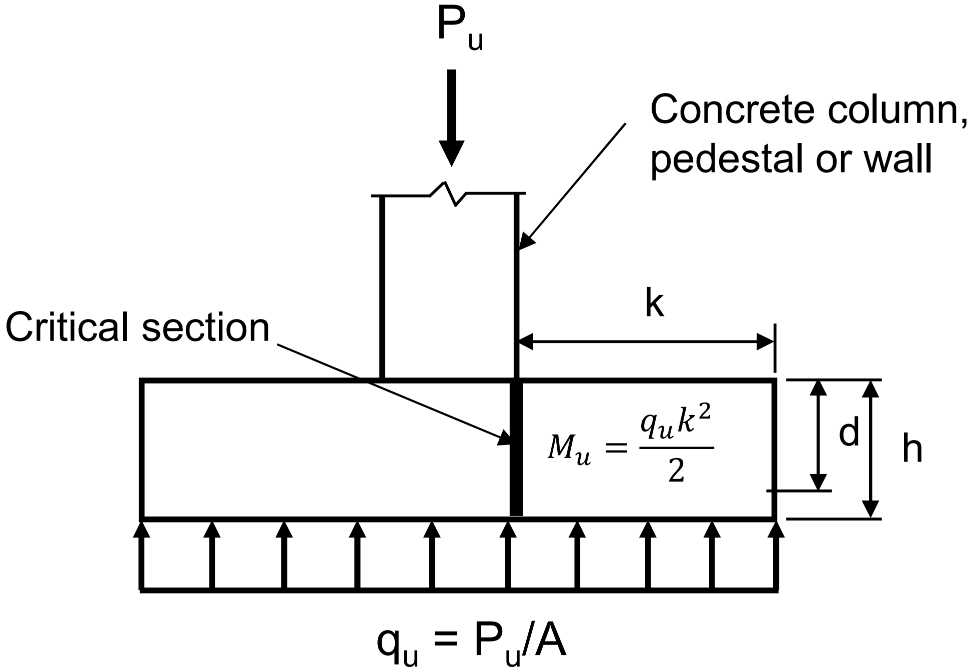
- The base area shall be determined based on unfactored forces (ACI R13.2.6.1).
- The thickness shall be calculated based on based on factored forces (because these are determined using strength design method which utilizes factored loads).
- Minimum thickness shall be selected such that the effective depth of bottom reinforcement is at least 6 in (ACI, 13.3.1.2).

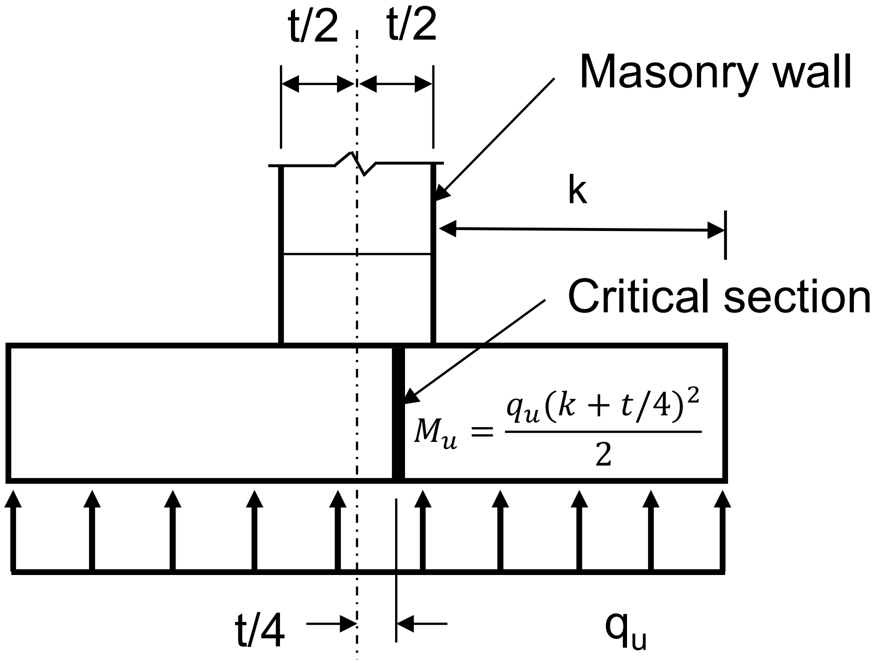
ACI Provisions for Shallow Foundations

❑ Design Considerations in Flexure

❖ Critical Sections (AC Table 13.2.7.1)

The maximum factored moment is calculated at critical section defined in accordance with Table 13.2.7.1.

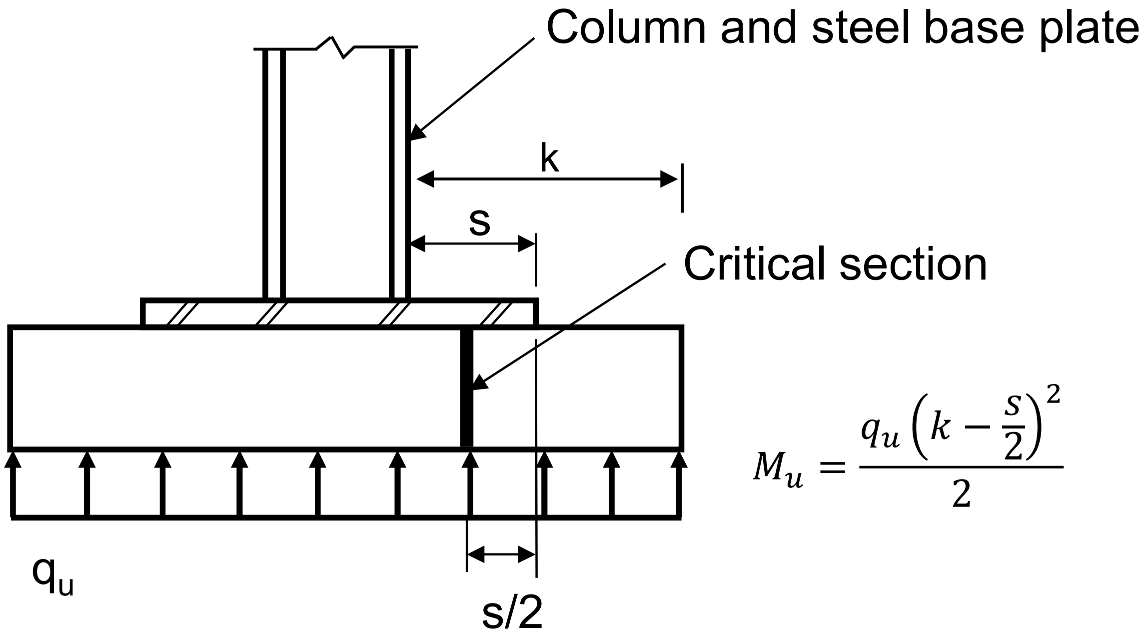




ACI Provisions for Shallow Foundations

❑ Design Considerations in Flexure

- Critical Sections (AC Table 13.2.7.1)



$$M_u = \frac{q_u\left(k - \frac{s}{2}\right)^2}{2}$$

ACI Provisions for Shallow Foundations

❑ Design Considerations in Flexure

- Minimum reinforcement shall be as per as per section 7.6.1.1.

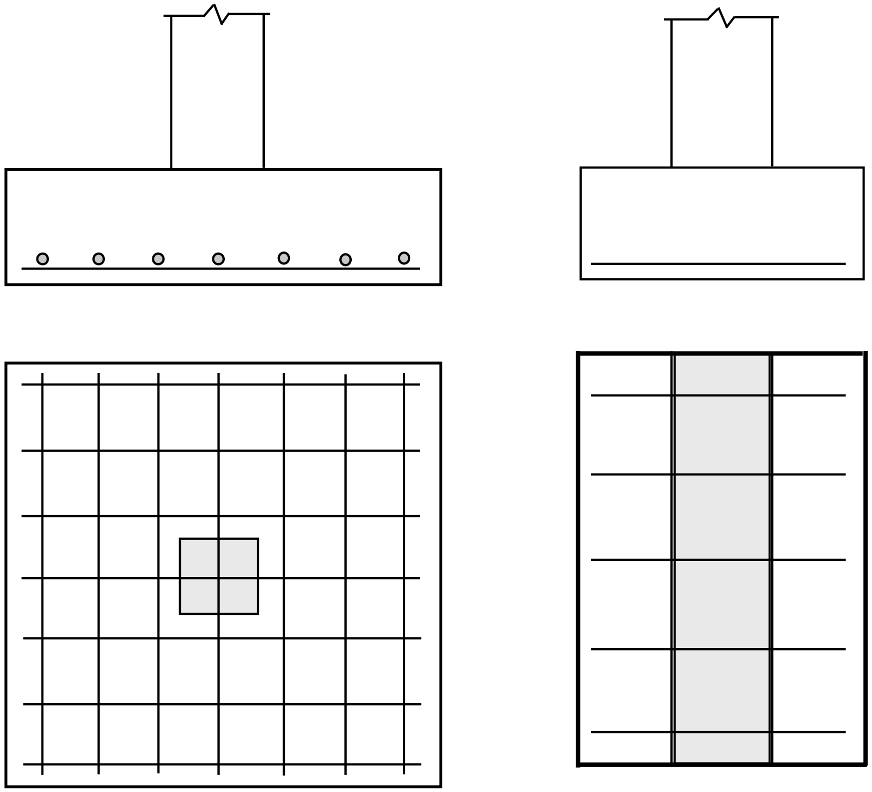
$$A_{min} = 0.0018A_g$$

- Maximum spacing shall not exceed the smaller of 3h or 18″.

ACI Provisions for Shallow Foundations

❑ Distribution of Reinforcement

- In one-way footings and two-way square footings, reinforcement shall be distributed uniformly across entire width of footing (ACI 13.2.2.2 and 13.3.3.2).



ACI Provisions for Shallow Foundations

❑ Distribution of Reinforcement

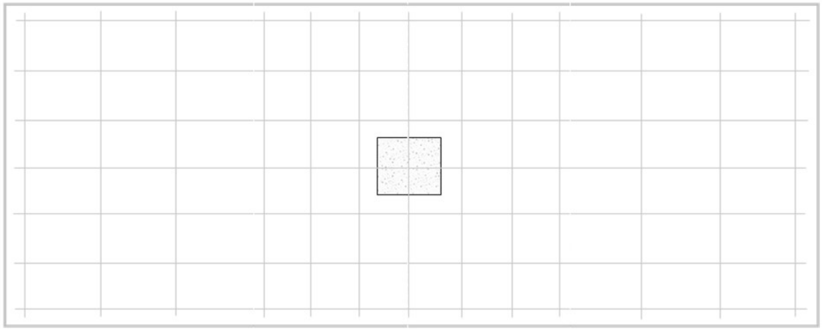
- ACI 13.3.3.3 states that in two-way rectangular footings, reinforcement shall be distributed in accordance with ACI 13.3.3.3 (a) and ACI 13.3.3.3(b).
  - Uniform distribution of flexural reinforcement in long direction.
  - Concentrated flexural reinforcement in short direction, $\gamma_s \, A_{SL}$

$$\gamma_s = \frac{2}{\beta + 1}$$

$$A_{s1} = \frac{2}{\beta + 1} A_{SL}$$

$$A_{s2} = \frac{A_{SL} - A_{S1}}{\beta + 1}$$

Where $\beta = L/B$



ACI Provisions for Shallow Foundations

❑ Design Considerations in Shear

- The behavior of footings in shear is similar to beams and supported slabs.
- The thickness of the footing for shear is based on the more severe of the following two conditions.
    - One-way shear (also called beam shear)
    - Two-way shear (also called punching shear)

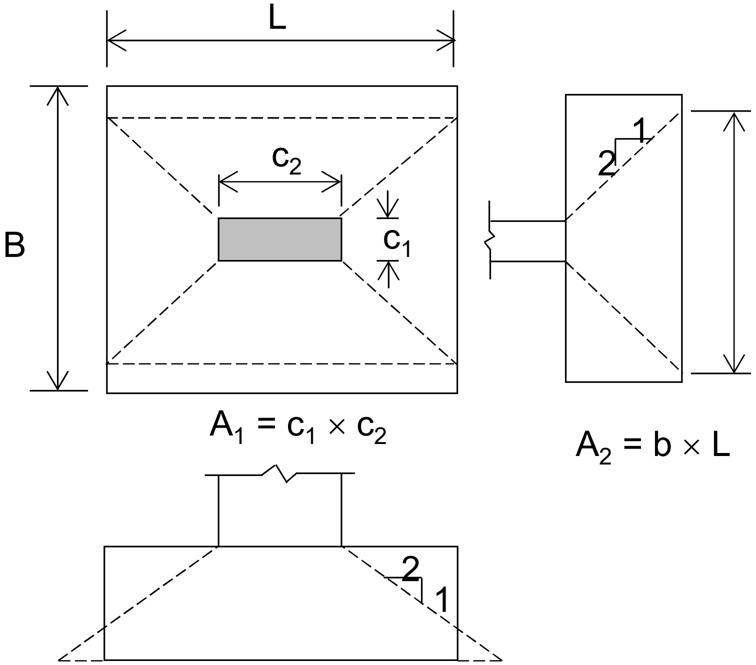
ACI Provisions for Shallow Foundations

❑ Transfer of Column Forces to the Base

- Factored column forces are transferred to the footing by bearing on concrete and through reinforcement. Nominal bearing strength B_n of footing shall be calculated in accordance with Table 22.8.3.2.

Table 22.8.3.2 – Nominal Bearing Strength	
Geometry bearing area	B_n
Supporting surface is wider on all sides than the loaded area	$\min\left[\sqrt{\frac{A_2}{A_1}}, 2\right] 0.85 f_c' A_1$
Other cases	$0.85 f_c' A_1$

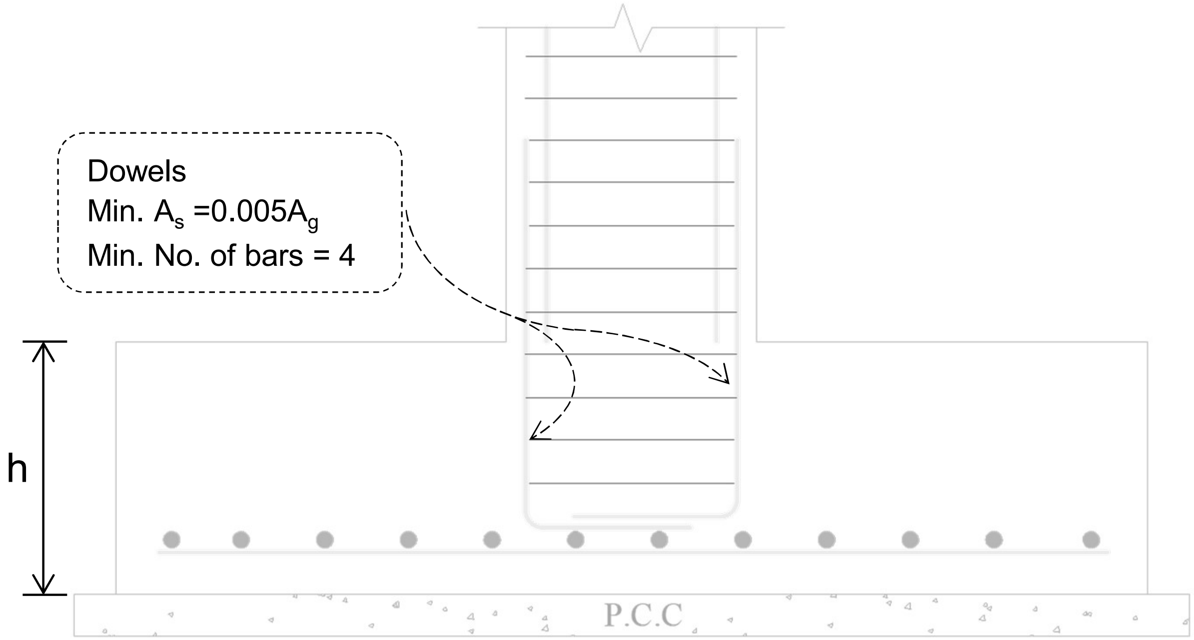
$\emptyset B_n \geq P_u$ (where $\emptyset = 0.65$)



ACI Provisions for Shallow Foundations

❑ Transfer of Column Forces to the Base

- If $P_u > \emptyset B_n$ then bearing reinforcement is provided:
  1. by Extending the longitudinal bars of column and anchoring them into the footing or
  2. In the form properly anchored dowels.



Design of Isolated Column Footings

➢ Step 1: Selection of Sizes

❖ Thickness

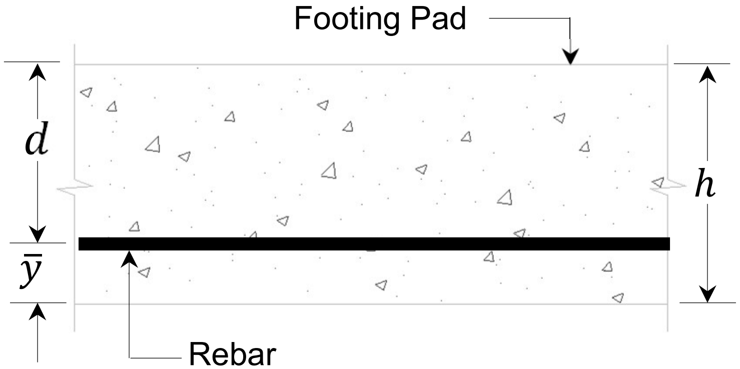
- Assume thickness *h* of the footing that satisfies the shear requirements.
- Also, calculate effective depth of footing "d"

$$d = h - \bar{y} = h - \left(Cc + \frac{d_b}{2}\right)$$

Where;

C_c = Clear cover which is 3in.

d_b = Diameter of rebar to be used



$\bar{y} = C_c + d_b/2$

Design of Isolated Column Footings

Step 1: Selection of Sizes

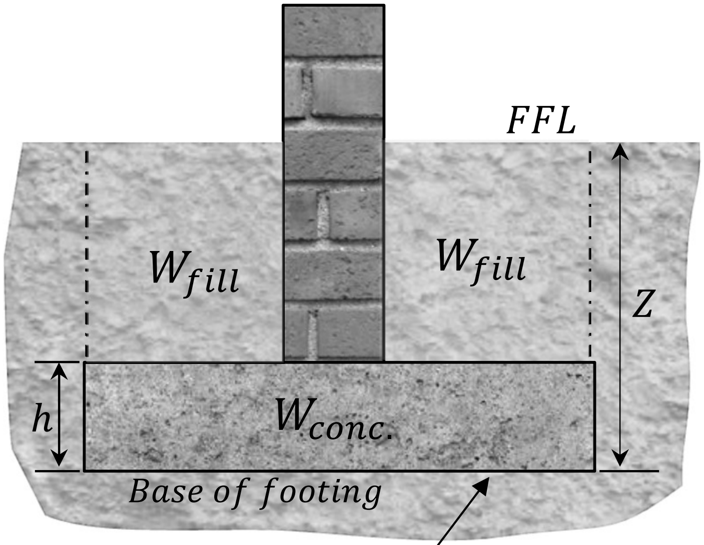
Required Bearing Area

- Determine bearing area using service loads

$$A = \frac{D + L}{q_e}$$

Where;

$$q_e = q_a - \gamma_c h - \gamma_{fill}(Z - h)$$



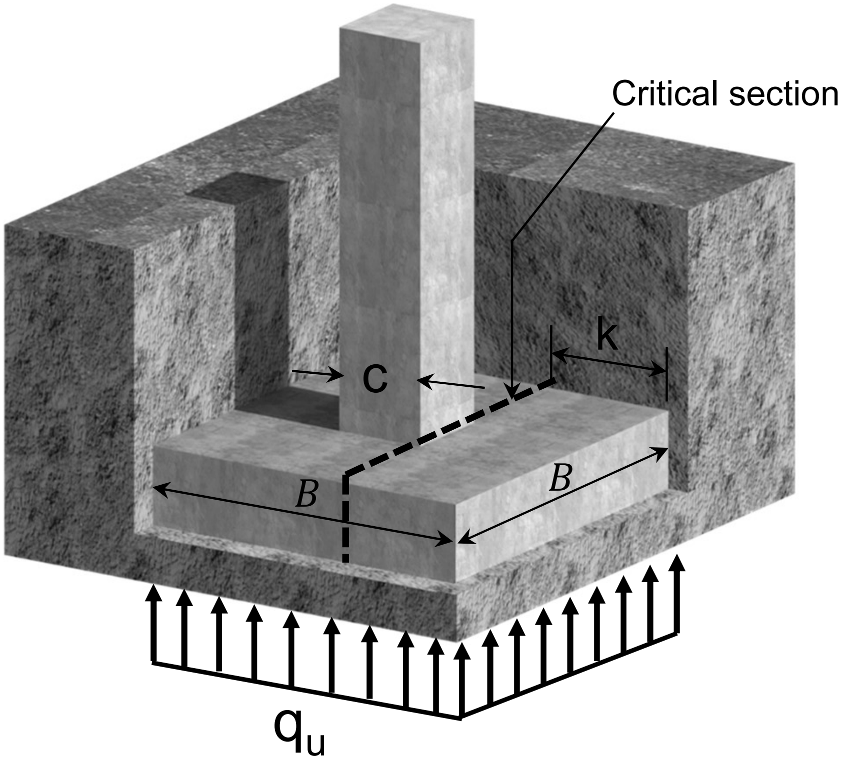
q_a is taken at this level

Design of Isolated Column Footings

Step 2: Calculation of Loads (bearing pressure)

- Calculate factored load using the following formula

$$q_u = \frac{1.2D + 1.6L}{A_{pvd}}$$



Design of Isolated Column Footings

Step 3: Analysis

Shear Force

$V_u = q_u B(k - d)$ (One-way shear)

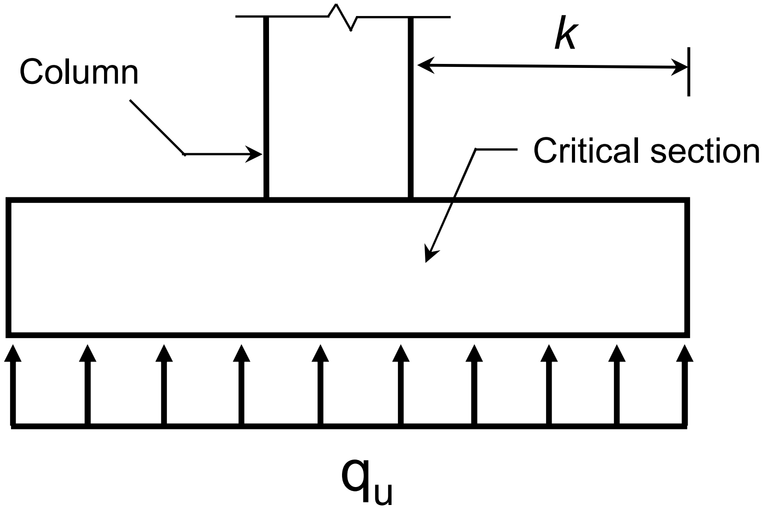
$V_u = q_u[BL - (c_1 + d)(c_2 + d)]$ (Two-way shear)

Bending Moment

$M_u = \frac{q_u b k^2}{2}$ (Concrete wall)

$M_u = \frac{q_u b \left(k + \frac{t}{4}\right)^2}{2}$ (Masonry wall)

$M_u = \frac{q_u B k^2}{2}$ (Column footing)



Design of Isolated Column Footings

➢ Step 4: Applying Shear Check

- Calculate concrete capacity for one-way shear or two-way shear and compare with demand;

$$\emptyset V_c = 2\emptyset\sqrt{f_c{}'}bd$$

(Concrete capacity for One-way shear)

$$\emptyset V_c = min\begin{cases} \emptyset\left(2+\dfrac{4}{\beta}\right)\sqrt{f_c'}b_od \\ \emptyset\left(2+\dfrac{\alpha_s d}{b_o}\right)\sqrt{f_c'}b_od \\ \emptyset 4\sqrt{f_c'}b_od \end{cases}$$

(Concrete capacity for Two-way shear)

- If $\emptyset V_c \geq V_u$ → depth of footing is sufficient
- If $\emptyset V_c < V_u$ → increase the depth of footing

Design of Isolated Column Footings

Step 5: Determination of Flexural Reinforcement

- Determine steel area either by Trial and Success method, or using Direct method as follows:

$$a = d - \sqrt{d^2 - \frac{2.614M_u}{f_c'b}} \quad and \quad A_s = \frac{M_u}{\emptyset f_y\left(d - \frac{a}{2}\right)}$$

Step 6: Check for Minimum Flexural Reinforcement

- Check the calculated area in previous step with minimum reinforcement limit

$$A_{s,min} = 0.0018bh$$

Design of Isolated Column Footings

Step 7: Detailing of Reinforcement

Flexural Bars

$$S = \frac{12A_b}{A_s}$$

$$S_{max} = min\,of\;\; 3h\; or\; 18"$$

Shrinkage and Temperature Bars

$$A_{S+T} = 0.0018bh$$

$$No.\; of\; bars = \frac{A_{S+T}}{A_b}$$

$$S_{max} = min\,of\; 5h\; or\; 18"$$

Where;

b = 12 in. for wall footing and

b = B (Width of footing in inches)

h = thickness of footing (in)

Design of Isolated Column Footings

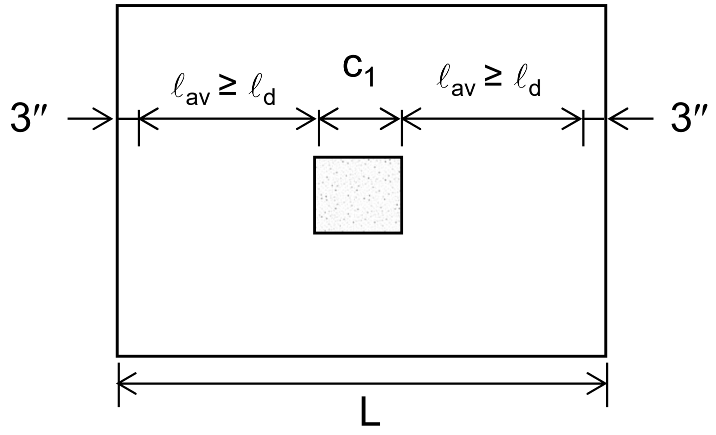
➢ Step 7: Detailing of Reinforcement

❖ Development of Reinforcement

- The flexural reinforcement must extend beyond the column face by a minimum of l_d.

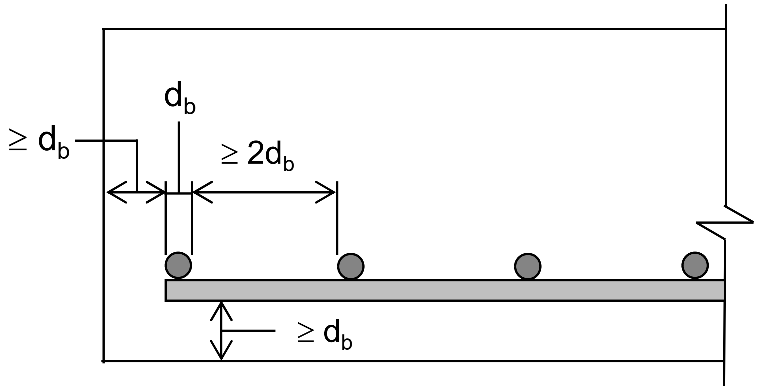
$$l_d = \frac{f_y}{25\sqrt{f_c'}} d_b \text{ (for \#6 and smaller bars)}$$

$$l_d = \frac{f_y}{20\sqrt{f_c'}} d_b \text{ (for \#7 and larger bars)}$$



- Available length l_{av} in a given direction is calculated as;

$$l_{av} = \frac{L - c_1}{2} - 3$$



Design of Isolated Column Footings

- **Step 8: Drafting**
  - Provide a neat and clear drawing in both plan and sectional views showing all necessary details such as:
    - Dimensional Details
      - Width, thickness, depth of base from NSL etc.)
    - Reinforcement Details
      - Direction and spacing of main bars and distribution bars
      - Development length and hook lengths

Design of Isolated Column Footings

❑ Example 12.1 – Design of Wall Footing

A 12-in thick concrete wall carries a service dead load of 10 kips/ft and a service live load of 12.5 kips/ft. The loads are acting at the base of the wall. The allowable bearing capacity, q_a, is 5000 psf at the level of the base of the footing, which is 5 ft below the finish floor level. The density of soil is 120 pcf.

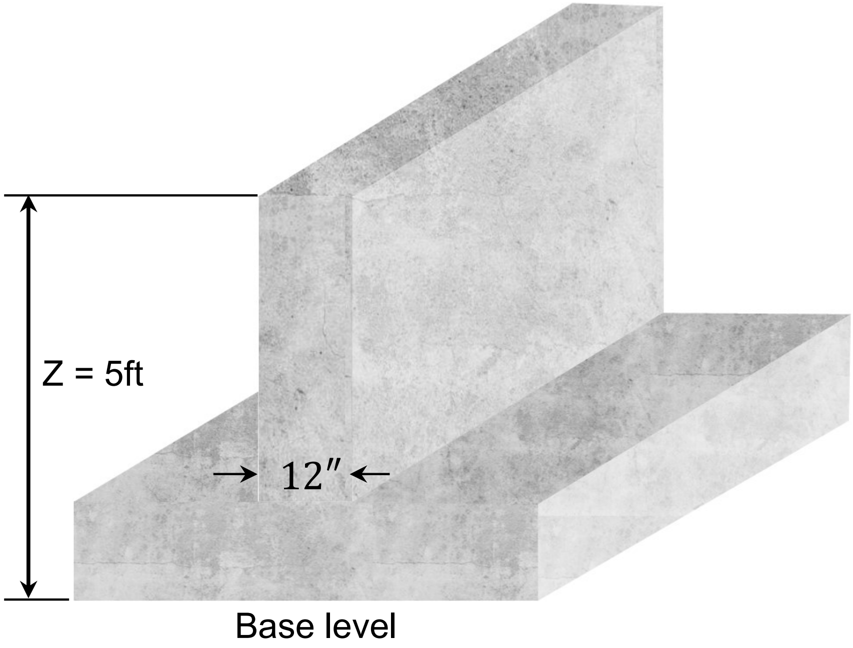
Design the wall footing using $f_c' = 3500\ psi$ and $f_y = 60{,}000\ psi$.

Design of Isolated Column Footings

❑ Example 12.1 – Solution

We have given the following data

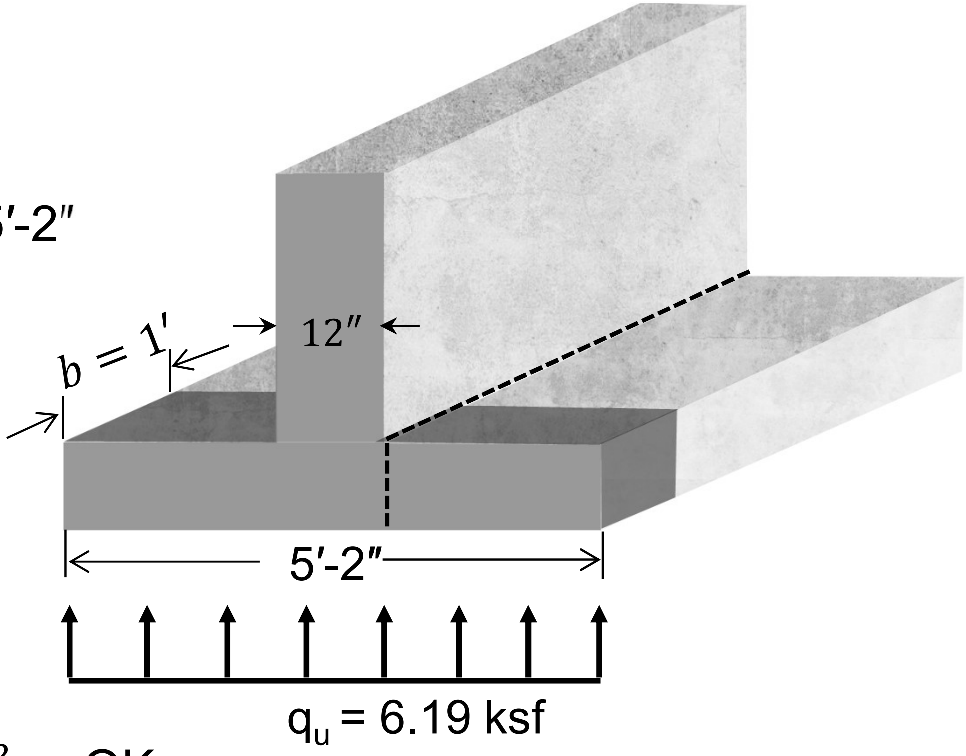
- Wall thickness = 12 in.
- Service dead load = 10 kips
- Service live load = 12.5 kips
- Depth of base from FFL = 5ft
- Allowable bearing capacity = 5ksf
- Unit weight of concrete = 0.150kcf
- Unit weight of backfill = 0.120kcf
- $f_c' = 3.5ksi$ and $f_y = 60ksi$



Design of Isolated Column Footings

❑ Example 12.1 – Solution

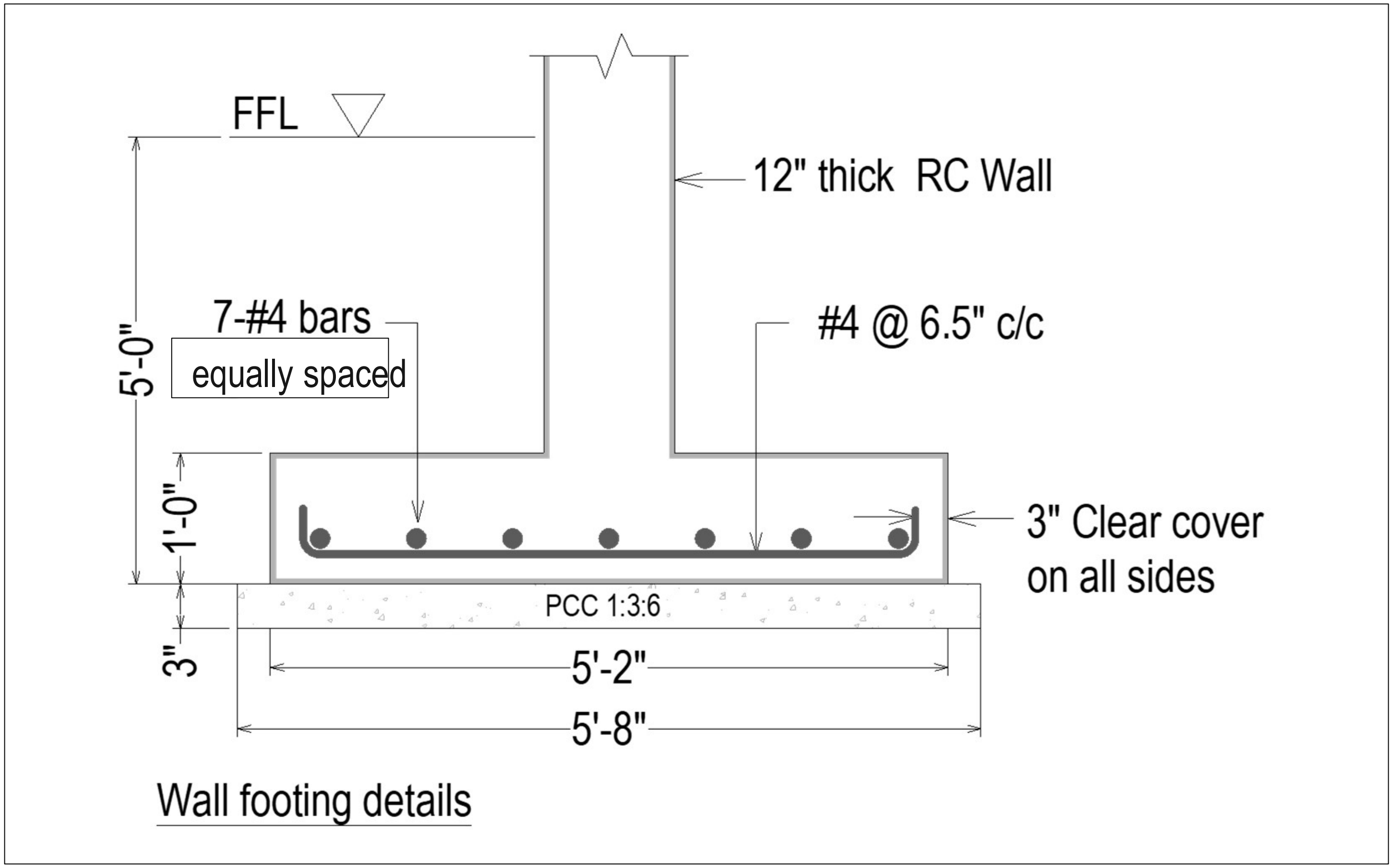
- Thickness, h = 12″
- Bearing Area = 5.15 ft^2 → $B = $ 5′-2″
- $q_u = $ 6.19 ksf
- $V_u = $ 8.38 kip
- $M_u = $ 161.2 in.kip/ft
- $\emptyset V_c = $ 9.32 kip > V_u → OK
- $A_s = $ 0.35 in^2 > $A_{s,min} = 0.26\ in^2$ → OK
- Spacing: #4 @ 6.86 c/c < $S_{max} = $ 18″ → OK



NOTE: For step-by-step solution, please refer to **Lecture 09 of RCD – I** (available on my website).

Design of Isolated Column Footings

❑ Example 12.1 – Solution



Wall footing details

Design of Isolated Column Footings

❑ Example 12.2 – Design of Square Column Footing

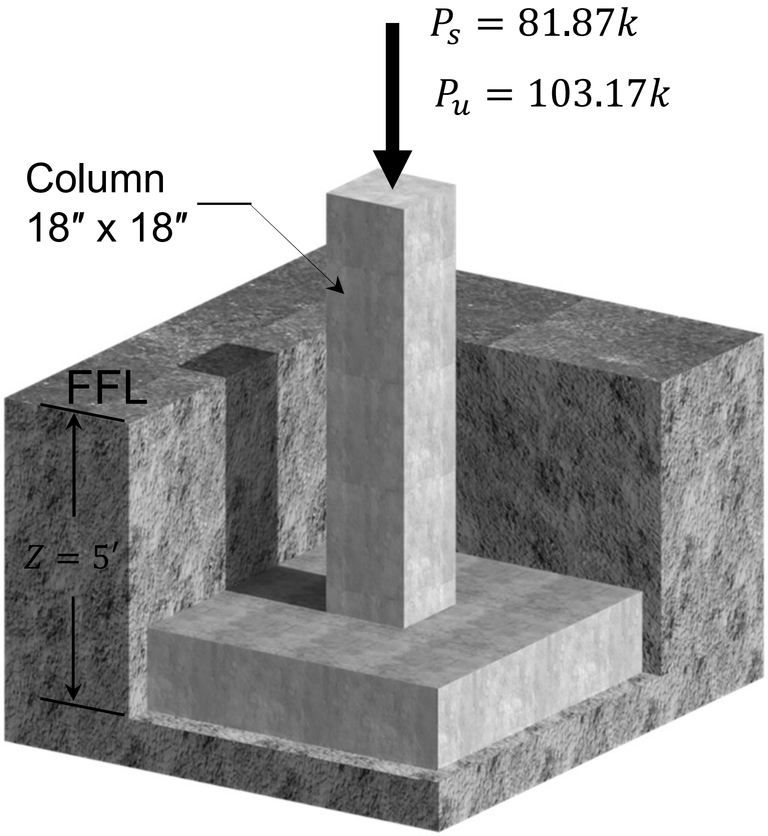
An 18″ square column with concrete of $f_c' = 3000\, psi$, reinforced with 8,#8 bars of $f_y = 60{,}000\, psi$, supports a service load of 81.87 kips (factored load = 103.17 kips). The load is acting at the base of column. The same concrete and steel is also used in the footing. The allowable soil pressure at the level of the base of the column footing is 2.204 ksf.

Design a square footing with base 5′ below ground level. Take unit weight of backfill as 100 psf.

Design of Isolated Column Footings

❑ Example 12.2 – Solution

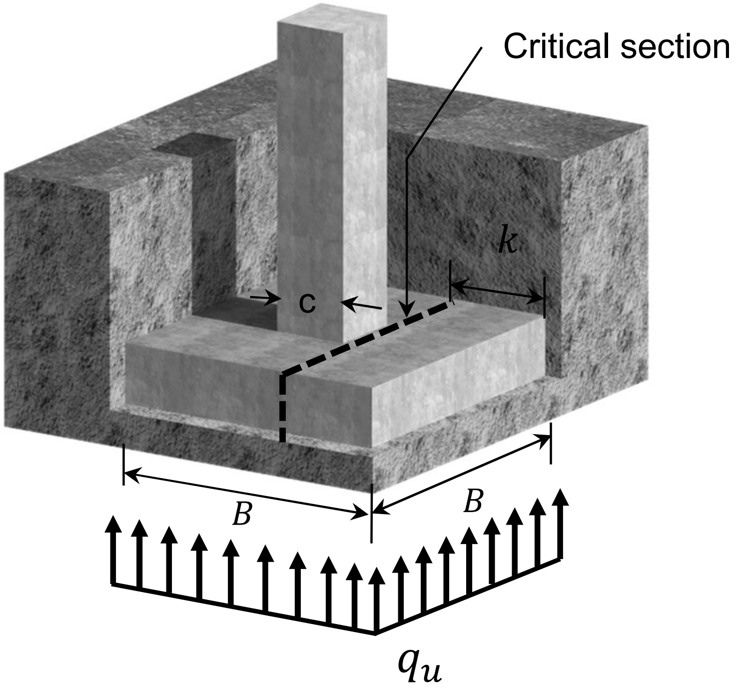
- Service load = 81.87 kips
- Factored load = 103.17 kips
- Size of column: 18″ x 18″
- Depth of base from FFL = 5ft
- Allowable bearing capacity = 2.204ksf
- Unit weight of concrete = 0.150kcf
- Unit weight of backfill = 0.100kcf
- $f_c' = 3\ ksi$ and $f_y = 60\ ksi$



Design of Isolated Column Footings

❑ Example 12.2 – Solution

- Thickness, h = 15″
- Bearing Area = 49.92 ft^2 → $B = 7'$
- $q_u = 2.11$ ksf
- $V_u = 90.64$ kip
- $M_u = 670.19$ in.kip
- $\emptyset V_c = 222.98$ kip > V_u → OK
- $A_s = 1.09$ in^2 < $A_{s,min} = 2.27$ in^2 → take $A_{s,min}$
- Detailing: 12 - #4



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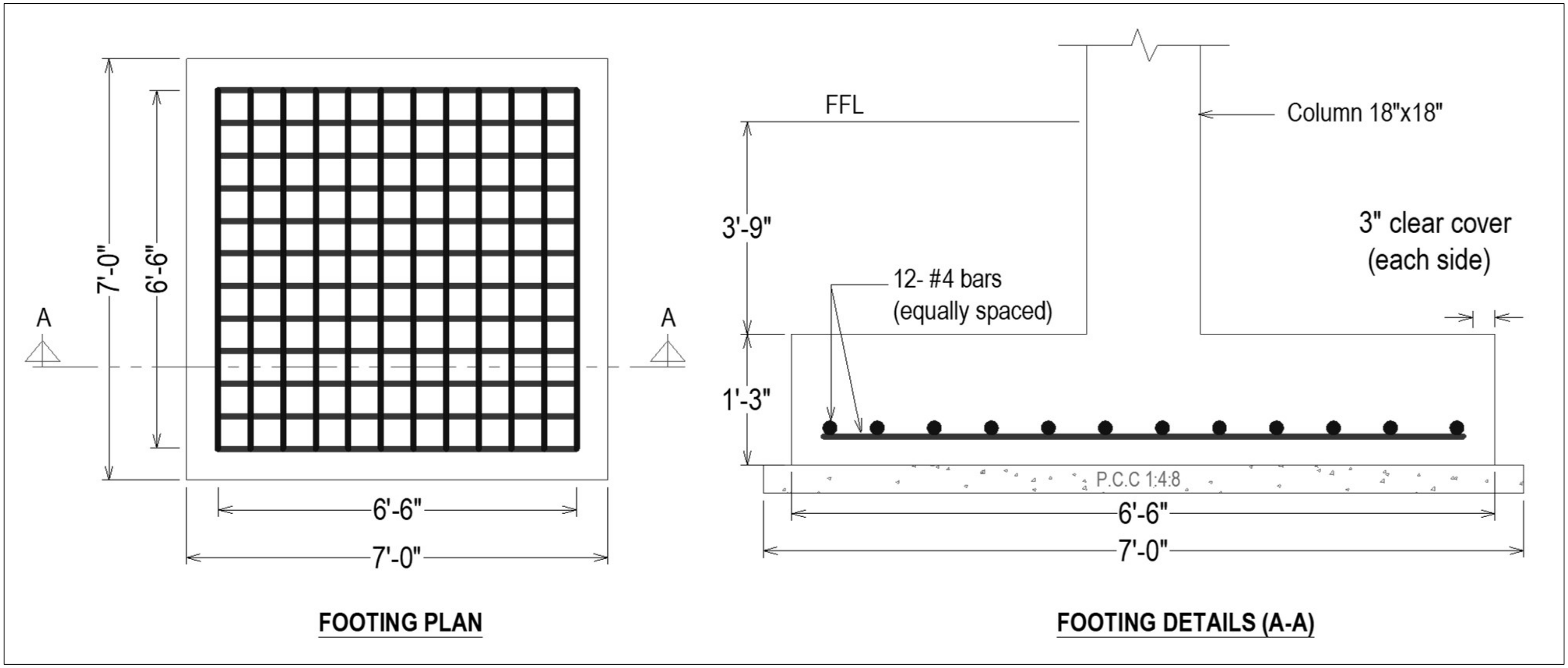
Design of Isolated Column Footings

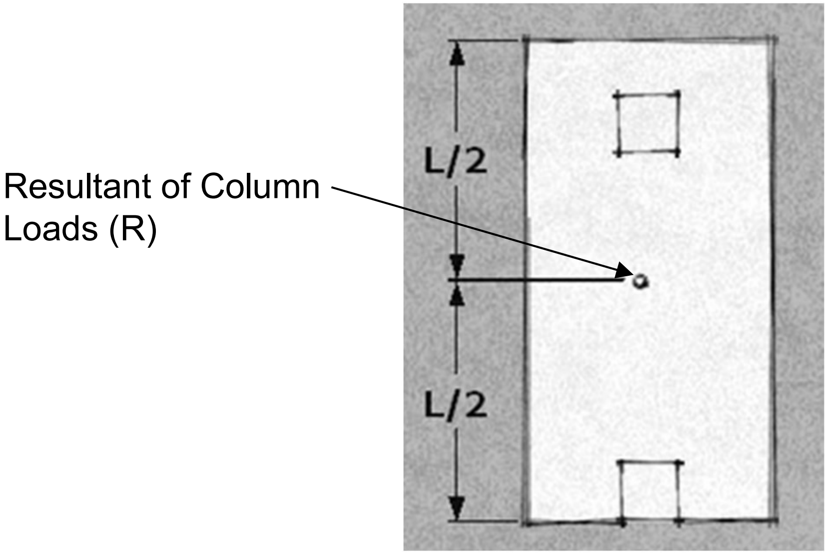
❑ Example 12.2 – Solution

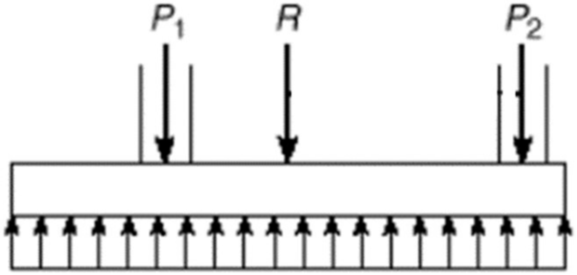


Design of Combined Footing

❑ Geometric Configuration

- The shape of combine footing is chosen such that the centroid of the area in contact with soil coincides with the resultant of the column loads supported by the footing.
- This ensure uniform bearing pressure over the entire area and forestalls the tendency of a footing to tilt

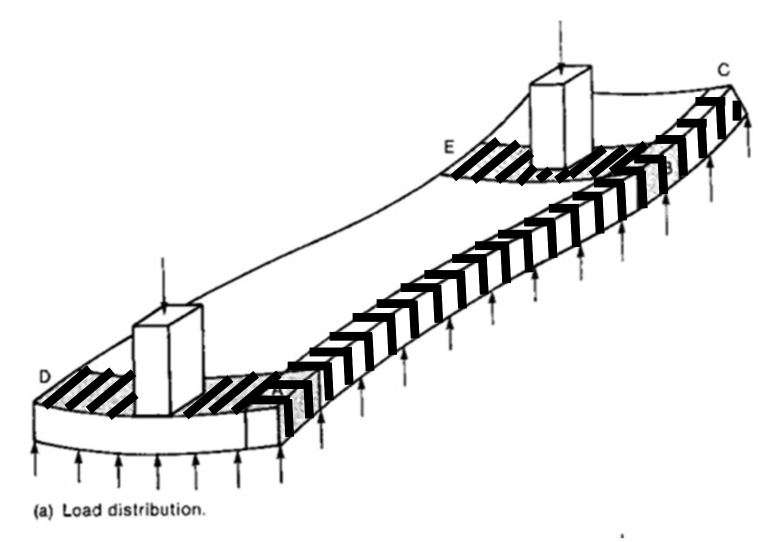




Design of Combined Footing

❑ Structural Action

- The soil pressure is assumed to act on longitudinal beam strip.
- These transmit the load to the hypothetical cross beams, which transmit the upward soil pressure to the columns.

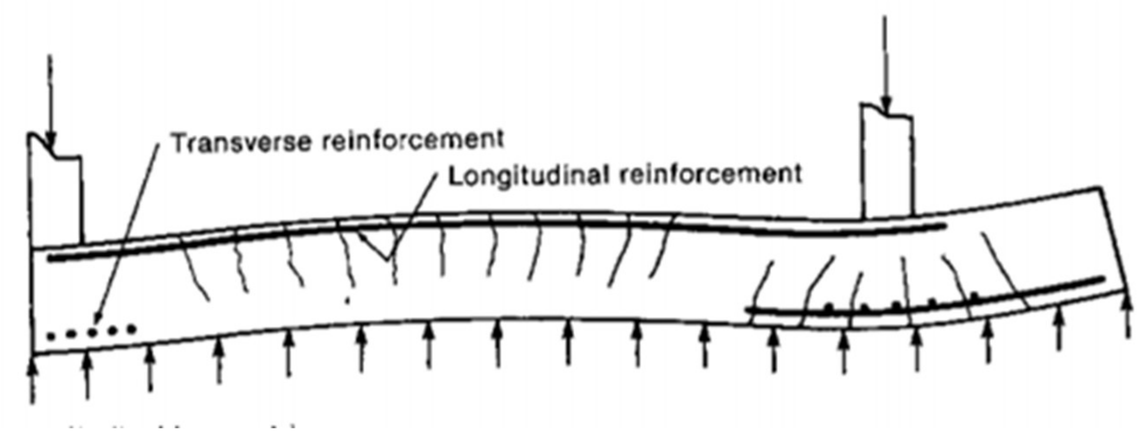


(a) Load distribution.

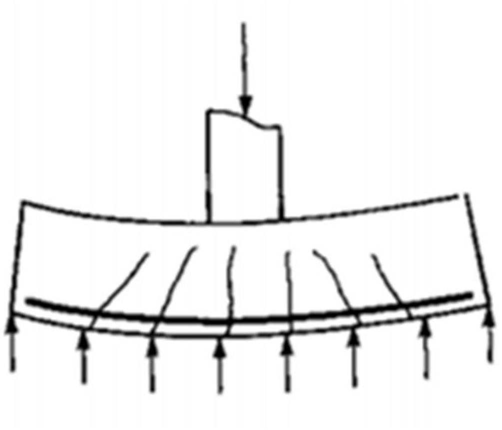
Design of Combined Footing

❑ Reinforcement Distribution

- The deflected shape and reinforcement of longitudinal and transverse beam strips are shown below:



Longitudinal Beam Strip



Transverse Beam Strip

Design of Combined Footing

❑ Design Procedure

➢ **Step 1: Estimate the Size of Footing**

- Assume thickness h of the footing which must satisfy the shear requirements.
- Determine the net permissible soil pressure,

$$q_e = q_a - W_{fill}$$

- Determine required area of footing,

A = Service load/ q_e

Design of Combined Footing

❑ Design Procedure

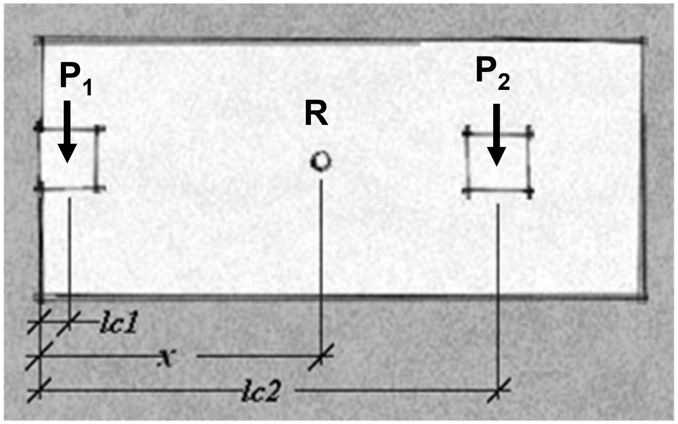
➢ Step 1: Estimate the Size of Footing

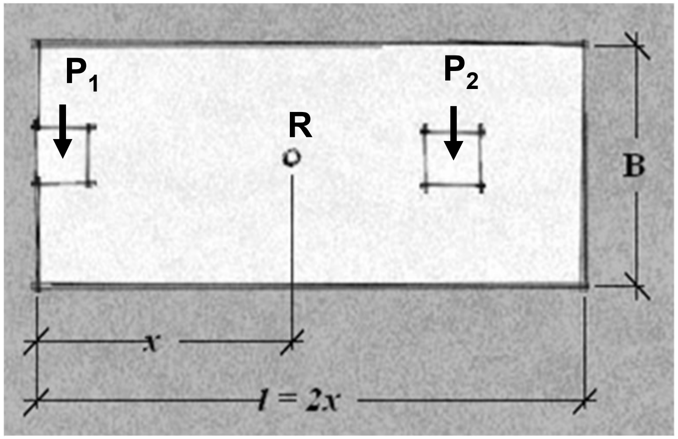
- Determine the location of resultant of column loads from the exterior face of the exterior column:

$$x = \frac{(l_{c1} \times P_1) + (l_{c2} \times P_2)}{P_1 + P_2}$$

- Length of footing will be equal to '2x'. Determine width of footing as follows:

$$B = \frac{P}{L \times q_e}$$





Design of Combined Footing

❑ Design Procedure

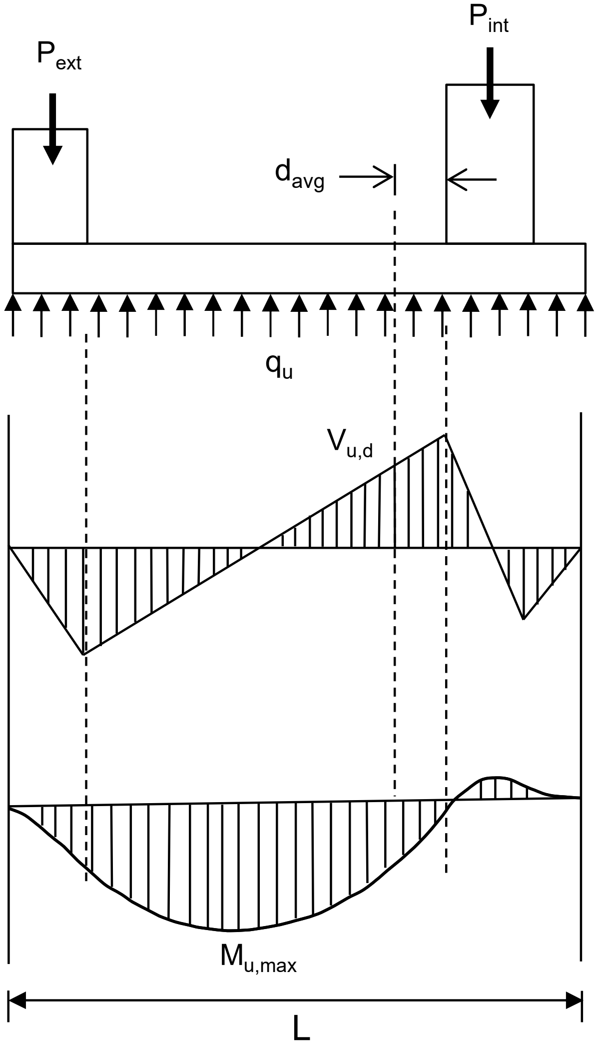
➢ Step 2: Analysis in Longitudinal Direction

- Determine ultimate and net soil pressures

$$q_u = \frac{1.2\mathrm{D} + 1.6\mathrm{L}}{A_{provided}}$$

$$q_n = q_u \times B$$

- Determine maximum shear force and bending moment:
  - V_u at distance *d* from the face of support
  - M_u at critical section

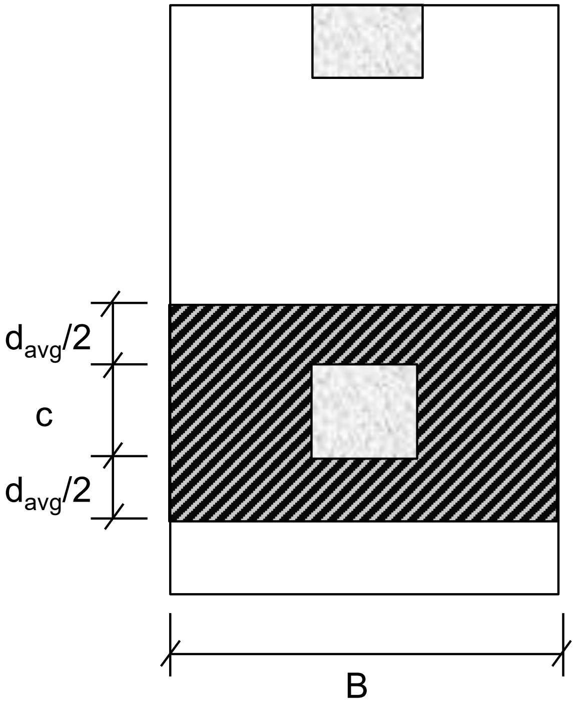


Design of Combined Footing

❑ Design Procedure

- **Step 3: Analysis in Transverse Direction**

- Determine effective width of transverse beam = c + d
- Determine net upward force per linear foot = P_{int} /B



Design of Combined Footing

❑ Design Procedure

➢ Step 4: Check for One-way Shear

- **Shear Demand**

  V_u at distance *d* from the face of support.

- **Shear Capacity**

$$\emptyset V_c = 2\emptyset\sqrt{f_c'}\,Bd_{avg}$$

Design of Combined Footing

❑ Design Procedure

➢ Step 5: Check for Two-way Shear

- **Punching Shear Demand**
- **Punching Shear Capacity**

$$\emptyset V_c = min\left[2+\frac{4}{\beta}, 2+\frac{\alpha_s d_{avg}}{b_o}, 4\right]\emptyset\sqrt{f_c'}b_o d_{avg}$$

Design of Combined Footing

❑ Design Procedure

➢ Step 6: Determination of Flexural Reinforcement

$$a = d - \sqrt{d^2 - \frac{2.614 M_u}{f_c' b_w}}$$

$$A_s = \frac{M_u}{\emptyset f_y \left(d - \frac{a}{2}\right)}$$

➢ Step 7: Reinforcement Detailing and Drafting

- Detail the reinforcement satisfying spacing and development length requirements.

Design of Combined Footing

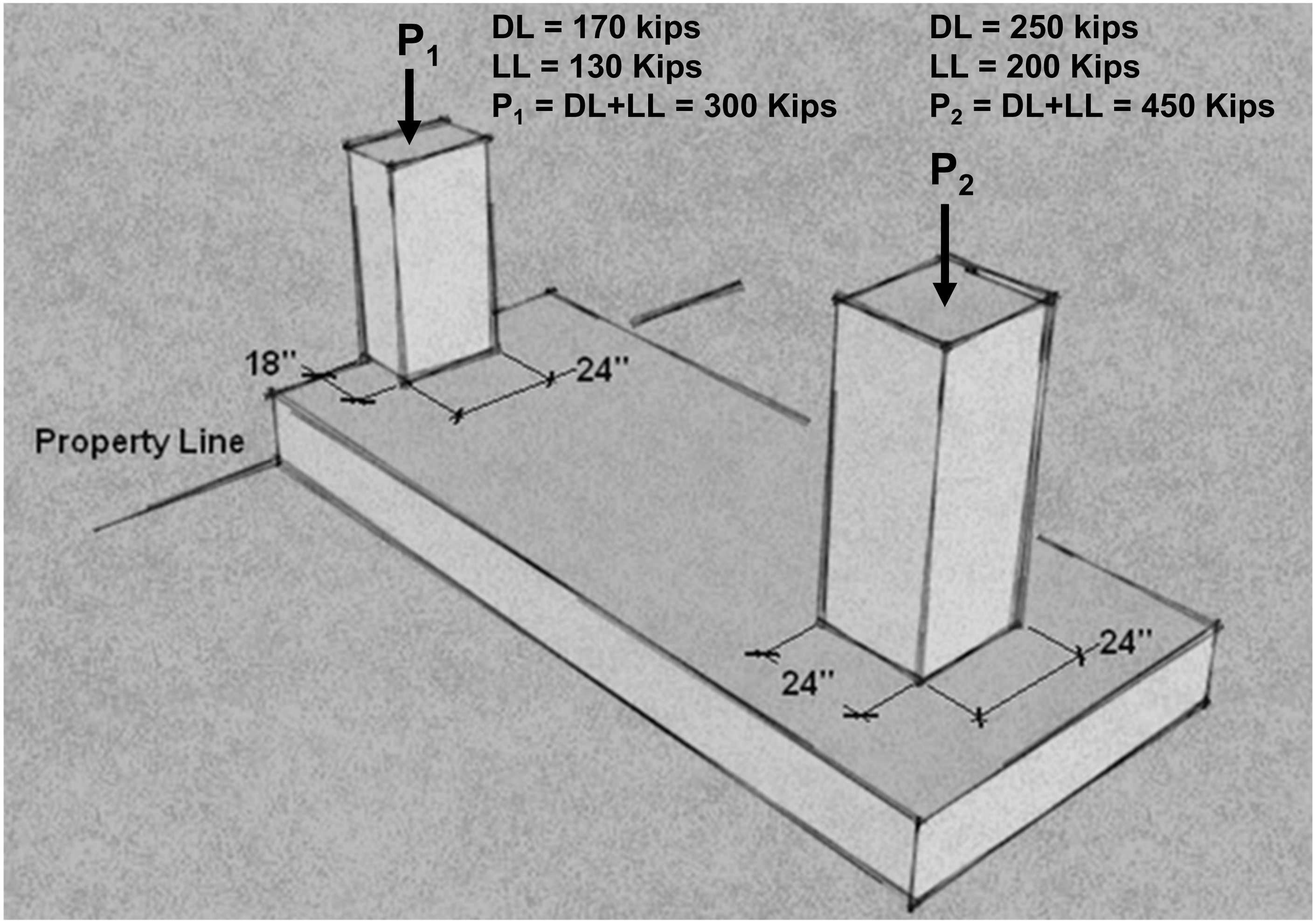
- **Example 12.2**

An exterior 24 × 18 in. column with D = 170 kips, L = 130 kips, and an interior 24 × 24 in. column with D = 250 kips, L = 200 kips are to be supported on a combined rectangular footing whose outer end cannot protrude beyond the outer face of the exterior column. The distance center to center of the column is 18 ft. and the allowable bearing pressure of soil is 6000 psf. The bottom of footing is 6 ft below grade and a surcharge of 100 psf is specified on the surface.

Design the footing using f_c' = 3000 psi and f_y = 60,000 psi

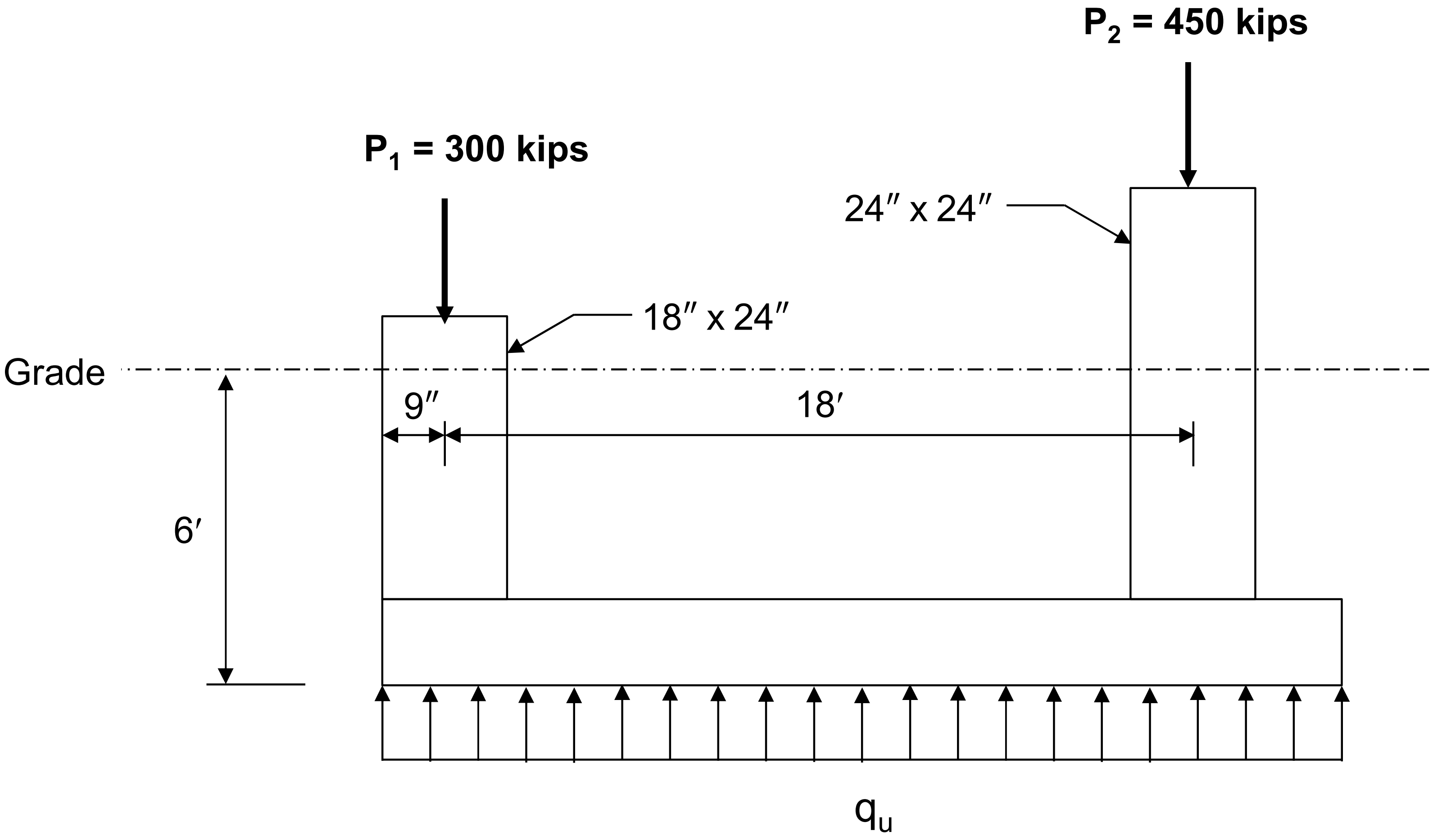
Design of Combined Footing

❑ Example 12.2



Design of Combined Footing

- **Example 12.2**



Design of Combined Footing

❑ Example 12.2 – Solution

➢ Step 1: Estimate Size of Footing and Net Pressure

- Assume thickness h of the footing which must satisfy the shear requirements.

Let h = 3′-5″ ft.

Assuming #8 bar: $d_{avg} = h - C_c - d_b = 41 - 3 - 1 = 37''$

- The space between the bottom of the footing and the surface will be occupied partly by concrete and partly by backfill. An average unit weight of (150+100)/2 = 125 pcf can be assumed.

$$q_e = 6 - (6 \times 0.125 + 0.100) = 5.15\ \mathrm{ksf}$$

- Determine the required area of footing, A

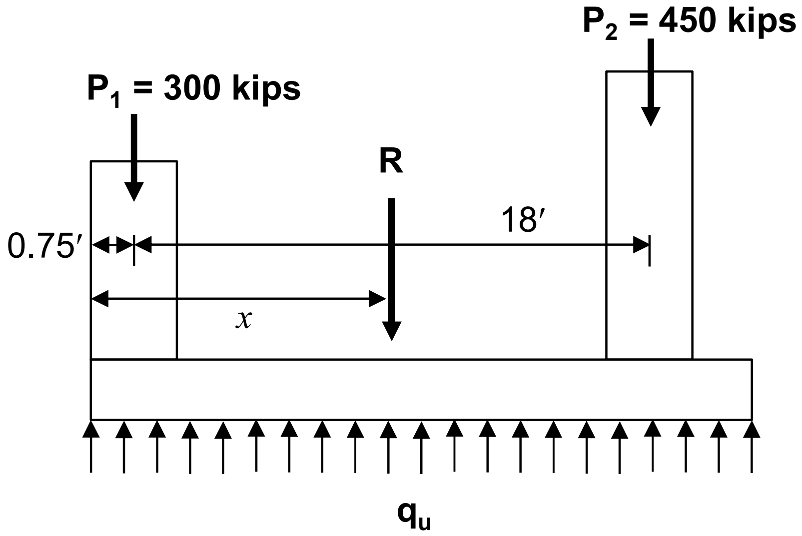
$$A = \frac{300 + 450}{5.15} = 145.6\ \mathrm{ft}^2$$

Design of Combined Footing

❑ Example 12.2 – Solution

➢ Step 1: Estimate Size of Footing and Net Pressure

- Determine the location of resultant of column loads from the exterior face of the exterior column:



$$x = \frac{(l_{c1} \times P_1) + (l_{c2} \times P_2)}{P_1 + P_2}$$

$$= \frac{(0.75 \times 300) + (18.75 \times 450)}{300 + 450}$$

$$x = 11.55'$$

Therefore,

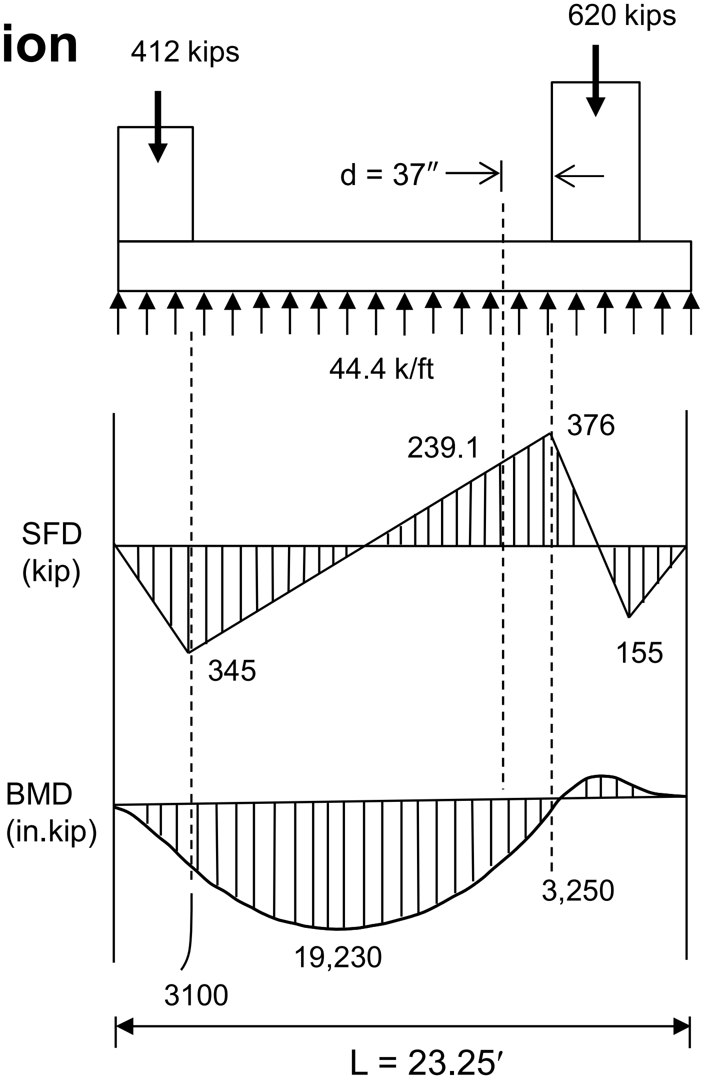
Length of footing, L = 2x = 2×11.55 = 23.1 ft. (select L = 23 ft. 3 in.)

Width, B = $(P_1+P_2)/Lq_e$ = (750)/ (23.25 × 5.15) = 6.3 ft. (select B = 6 ft. 6 in.)

Design of Combined Footing

❑ Example 12.2 – Solution

➢ Step 2: Analysis in Longitudinal Direction



Total upward soil pressure is given by

$$q_u = \frac{1.2D + 1.6L}{A_{provided}}$$

$$= \frac{1.2(170 + 250) + 1.6(130 + 200)}{23.25 \times 6.5} = 6.83 \text{ ksf}$$

And the net upward soil pressure is

$q_n = q_u \times B = 6.83 \times 6.5 = 44.4$ k/ft.

From figure,

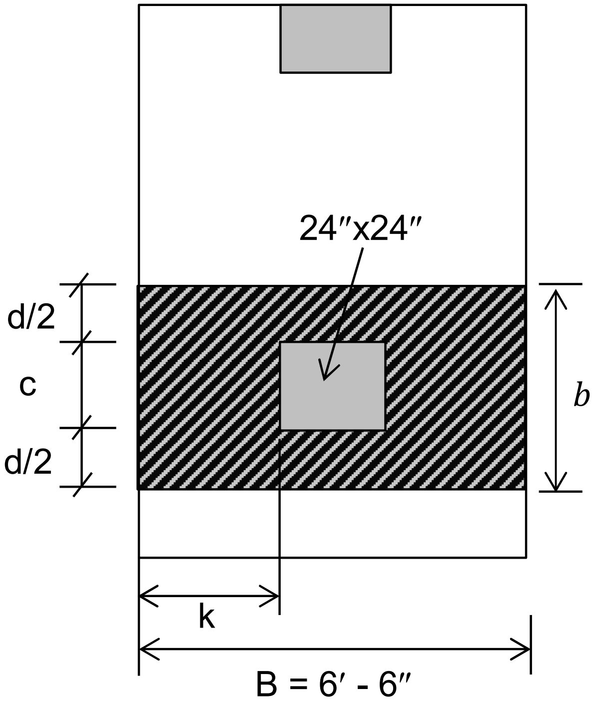
$M_u = 19230 \text{ in. kip}$

$V_{u,int} = 239.1 \text{ kip}$

Design of Combined Footing

❑ Example 12.2 – Solution

➢ Step 3: Analysis in Transverse Direction



Width of transverse beam under interior column is:

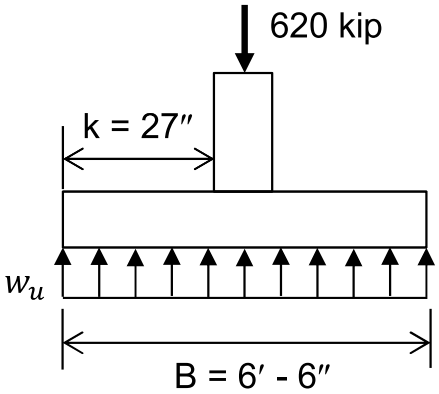
$$b = c + d_{avg} = 24 + 37 = 61 \text{ in}$$

The net upward load per linear foot of transverse beam is calculated as;

$$w_u = \frac{P_{int}}{B} = \frac{620}{6.5} = 95.4 \text{ kip/ft}$$

The moment at the edge of interior column is:

$$M_{int} = \frac{w_u k^2}{2} = \frac{95.4 \times (2.25)^2}{2} \times 12 = 2897.8 \text{ in. kip}$$



Design of Combined Footing

Example 12.2 – Solution

Step 4: Check for One-way Shear

- **Shear Demand**

$V_{u,int}$ = 239.1 (from Step 2)

- **Shear Capacity**

$$\emptyset V_c = 2\emptyset\sqrt{f_c'}\, Bd_{avg}$$

Substituting values, we get

$$\emptyset V_c = 0.75 \times 2 \times \sqrt{3000} \times 78 \times 37$$

$$\emptyset V_c = 237.1\, kip \approx V_u \rightarrow$$ OK!

Design of Combined Footing

❑ Example 12.2 – Solution

➢ Step 5: Check for Two-way Shear

- **Shear Demand**

For interior column:

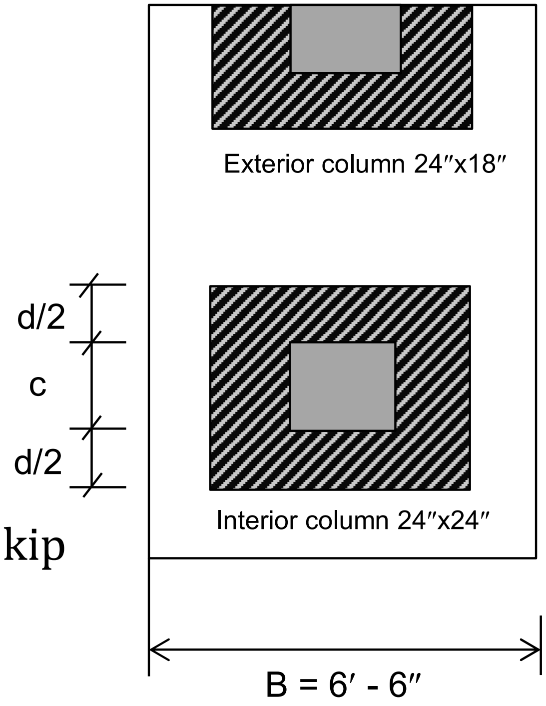
$$V_{u,int} = P_{int} - q_u[c_s + d_{avg})(c_L + d_{avg})]$$

$$V_{u,int} = 620 - 6.83\left[\left(\frac{24+37}{12}\right)\left(\frac{24+37}{12}\right)\right] = 443.5 \text{ kip}$$

For exterior column:

$$V_{u,ext} = P_{ext} - q_u[c_L + d_{avg})(c_s + d_{avg}/2)]$$

$$V_{u,ext} = 412 - 6.83\left[\left(\frac{24+37}{12}\right)\left(\frac{18+37}{2\times 12}\right)\right] = 332.43 \text{ kip}$$



Design of Combined Footing

❑ Example 12.2 – Solution

➢ Step 5: Check for Two-way Shear

- **Shear Capacity – Interior Column**

$b_o = 2(c_S + d) + 2(c_L + d) = 2(24+37) + 2(24+37) = 244''$

$$\emptyset\sqrt{f_c'}b_od = 0.75\sqrt{3000} \times 244 \times 37/1000 = 370.86$$

β_c = longer side of column/shorter side of column = 24/24 = 1

α_s = 40 for interior column

$$\emptyset V_c = min\left[2 + \frac{4}{\beta}, 2 + \frac{\alpha_s d}{b_o}, 4\right]\emptyset\sqrt{f_c'}b_od = min\left[2 + \frac{4}{1}, 2 + \frac{40 \times 37}{244}, 4\right]370.86$$

$\emptyset V_c = min[6\,, 8.1\,, 4] = 4 \times 370.86 = 1483.44$ kip > V_u = 443.5 kip → OK

Design of Combined Footing

❑ Example 12.2 – Solution

➢ Step 5: Check for Two-way Shear

- **Shear Capacity – Exterior Column**

$b_o = 2(c_L + d/2) + (c_s + d) = 2(18+37/2) + (24+37) = 134''$

$$\emptyset\sqrt{f_c'}b_o d = 0.75\sqrt{3000} \times 134 \times 37/1000 = 203.67$$

β_c = longer side of column/shorter side of column = 24/18 = 1.33

α_s = 30 for exterior column

$$\emptyset V_c = min\left[2 + \frac{4}{\beta}, 2 + \frac{\alpha_s d}{b_o}, 4\right]\emptyset\sqrt{f_c'}b_o d = min\left[2 + \frac{4}{1.33}, 2 + \frac{30 \times 37}{134}, 4\right]203.67$$

$\emptyset V_c = min[5, 10.3, 4] = 4 \times 203.67 = 814.68$ kip > V_u = 332.43 kip → OK

Design of Combined Footing

❑ Example 12.2 – Solution

➢ Step 6: Determination of Flexural Reinforcement

❖ Longitudinal Direction

$$a = d - \sqrt{d^2 - \frac{2.614M_u}{f_c'B}} = 37 - \sqrt{37^2 - \frac{2.614 \times 19230}{3 \times 78}} = 3.03''$$

$$A_s = \frac{M_u}{\emptyset f_y\left(d - \frac{a}{2}\right)} = \frac{19230}{0.9 \times 60\left(37 - \frac{3.03}{2}\right)} = 10.0\ in^2 \text{ (Using 10 \#9 bars)}$$

For the portion of longitudinal beam that cantilevers beyond interior column, $A_{s,min}$ controls:

$$A_{s,min} = \frac{200}{f_y}Bd = \frac{200}{60000} \times 78 \times 37 = 9.62\ \text{in}^2 \text{ (Using 10 \#9 bars)}$$

Design of Combined Footing

❑ Example 12.2 – Solution

➢ Step 6: Determination of Flexural Reinforcement

- **Transverse Direction**

$$a = d - \sqrt{d^2 - \frac{2.614M_u}{f_c'B}} = 37 - \sqrt{37^2 - \frac{2.614 \times 2900}{3 \times 61}} = 0.56''$$

$$A_s = \frac{M_u}{\emptyset f_y\left(d - \frac{a}{2}\right)} = \frac{2900}{0.9 \times 60\left(37 - \frac{0.56}{2}\right)} = 1.46\ in^2$$

$$A_{s,min} = \frac{200}{f_y}Bd = \frac{200}{60000} \times 61 \times 37 = 7.52\ \text{in}^2$$ (Using 10 #8 bars)

Design of Combined Footing

❑ Example 12.2 – Solution

➢ Step 7: Detailing of Reinforcement

- **Check Development Length**

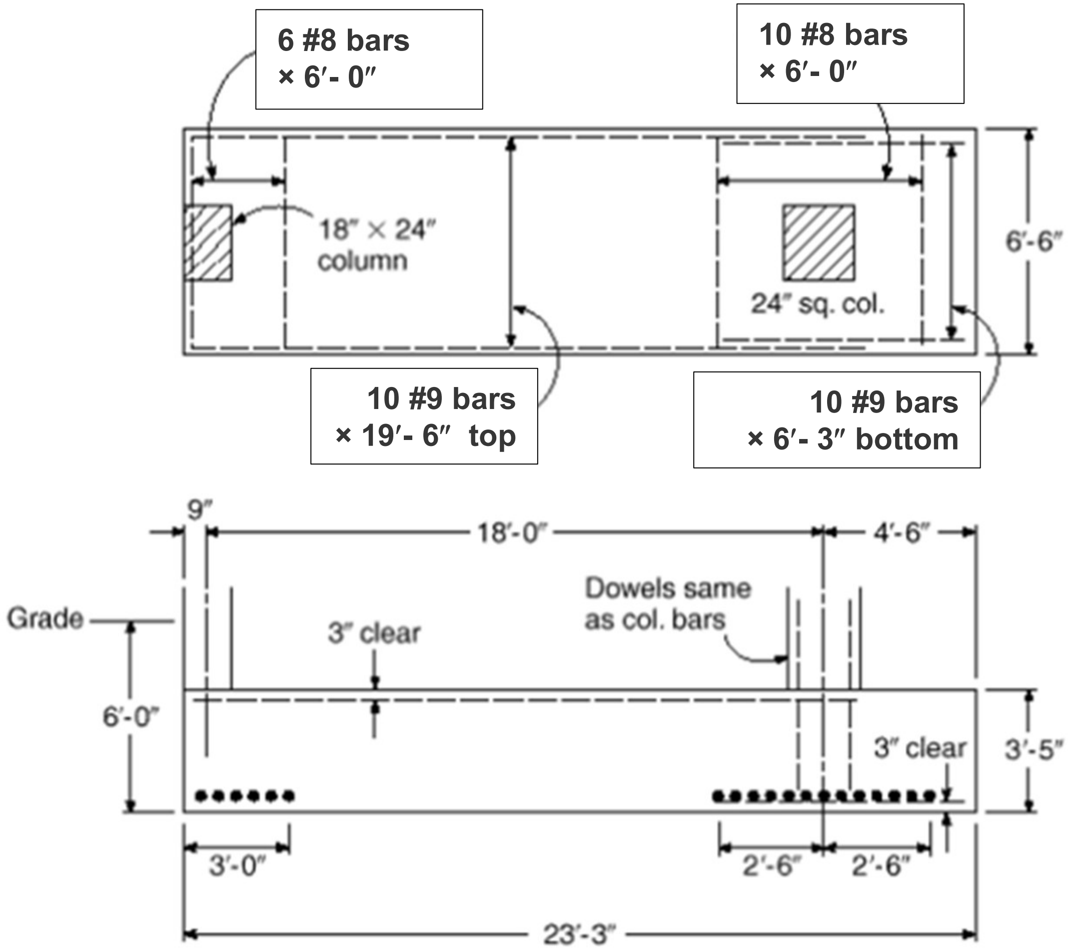
$$l_d = \frac{f_y d_b}{20\sqrt{f_c'}} = \frac{60000 \times 1}{20\sqrt{3000}} = 55''(\ 4.56')$$

From bending moment diagram in longitudinal direction, the distance from the point of max. moment to the nearer left end of the bars is equal to 9.05 ft. which is considerably larger than the required min. development length. Hence the selected reinforcement is adequate for both bending and bond.

Design of Combined Footing

❑ Example 12.2 – Solution

- **Step 8: Drafting**



Design of Mat Foundations

❑ Design Procedure

➢ **Step 1: Selection of Sizes**

- Proportion the base dimensions (B and L) of the mat so that the maximum pressure q at the base of the mat due to service loads is less than or equal to the permissible soil pressure q_p.

$$q = \frac{\sum P}{BL}\left(1 \pm \frac{6e_x}{B} \pm \frac{6e_y}{L}\right) \leq q_p$$

- Determine minimum thickness of slab based on shear strength requirements.
  - One-way shear
  - Two-way shear

Design of Mat Foundations

❑ Design Procedure

➢ Step 2: Analysis

- Determine maximum bending moments using factored loads.
- Analysis methods for factored bending moment determination
  - Rigid or conventional
  - Flexible

ACI 336.2R-88 contains provisions for analysis and design of Mat foundations.

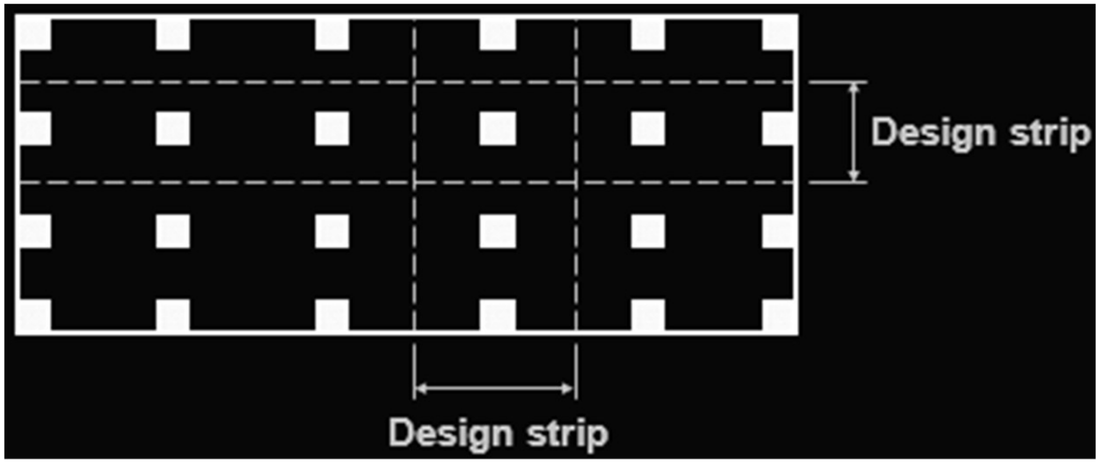
Design of Mat Foundations

❑ Design Procedure

- **Step 2: Analysis**

- **Rigid or Conventional**

- In the rigid or conventional method of analysis, the mat is treated as a rigid body and design strips are established in both directions.
- These strips are analyzed as combined footings with multiple columns.



Design of Mat Foundations

❑ Design Procedure

- **Step 2: Analysis**

- **Approximate Flexible Method**

- If the mat does not meet the rigidity requirements of the conventional method, it must be designed as a flexible plate.

- The required mat depth is computed based on shear requirements and radial tangential moments, shear forces, and deflections are computed using charts.

Design of Mat Foundations

Design Procedure

- **Step 2: Analysis**
- **Computer Analysis**
- Computer analysis for mat foundations is typically based on an approximation where the mat is divided into several discrete or finite elements. The common solution methods are:
  - Finite Element Method
  - Finite Difference Method
  - Finite Grid Method

Design of Mat Foundations

❑ Design Procedure

- **Step 3: Determination of Flexural Reinforcement**
  - The required flexural reinforced is determined using the Strength Design Method.
  - Like footings, mat foundations should be designed as tension-controlled sections.
- **Step 4: Development of Reinforcement**
- **Step 5: Check Transfer of Force at base of supported Member**

References

- Reinforced Concrete - Mechanics and Design (7th Ed.) by James MacGregor.
- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-19)
- Portland Cement Association (PCA 2002)