Lecture 11

Slenderness Effects in RC Structures

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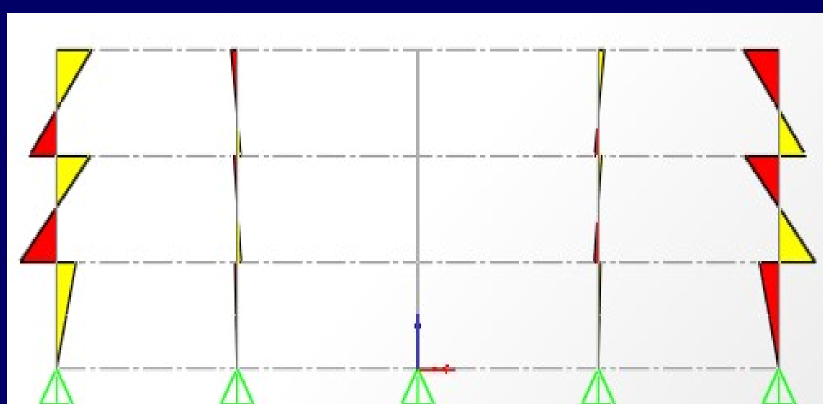
General

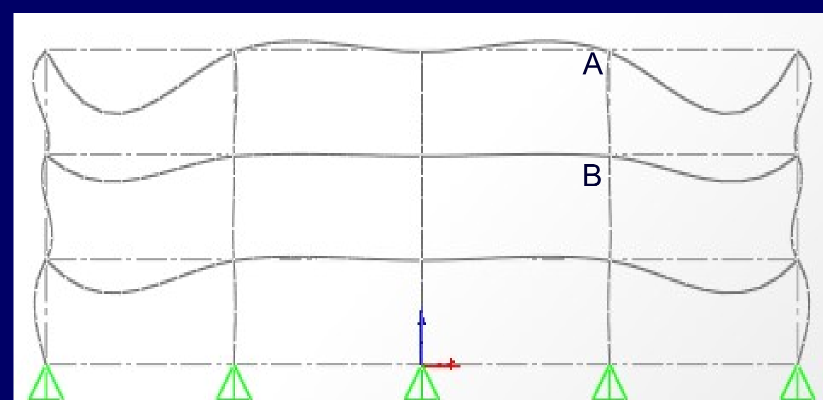
❑ Background

- For a frame under full dead and live load on all spans, the exterior columns experience notable bending from unbalanced loading, whereas interior columns exhibit no significant bending due to balanced loading.

- The columns may bend in single or double curvatures, depending on the load and end conditions.



Column bending moment diagram due to full dead and live load on all spans

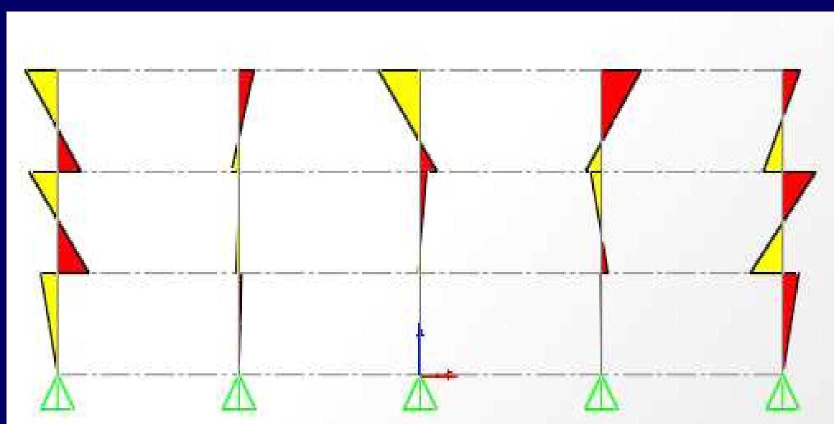


Deflected shape

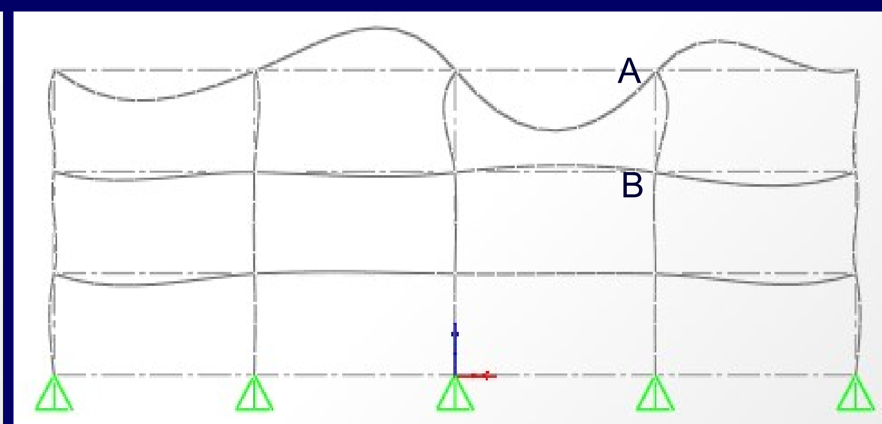
General

❑ Background

- The maximum column moment is obtained in column AB due to pattern live load shown.



Column bending moment diagram due to pattern live load

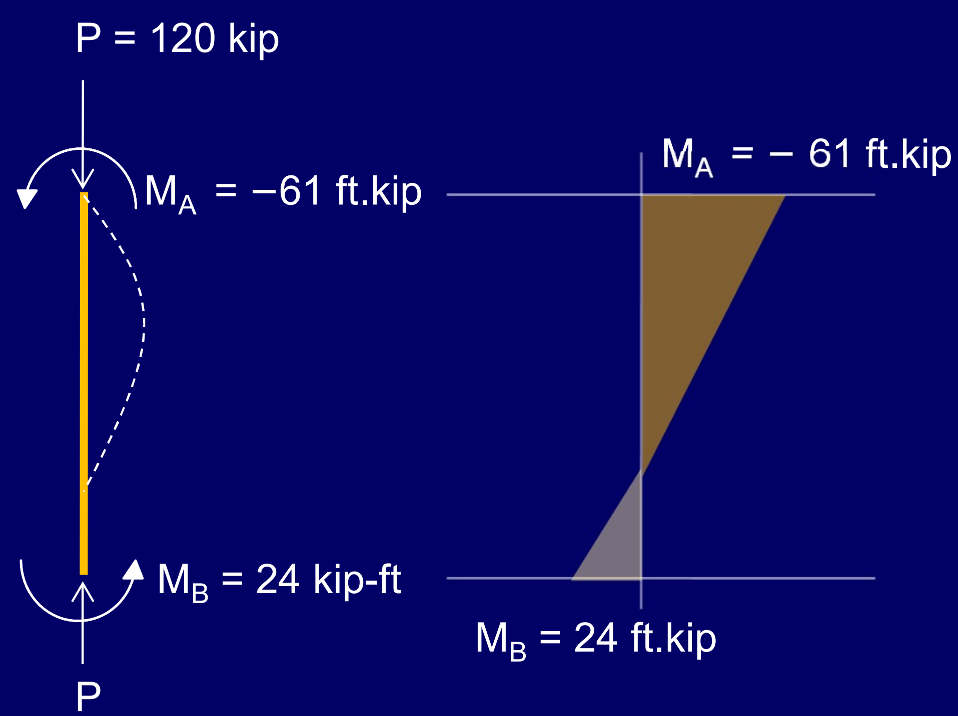


Deflected shape

General

❑ Background

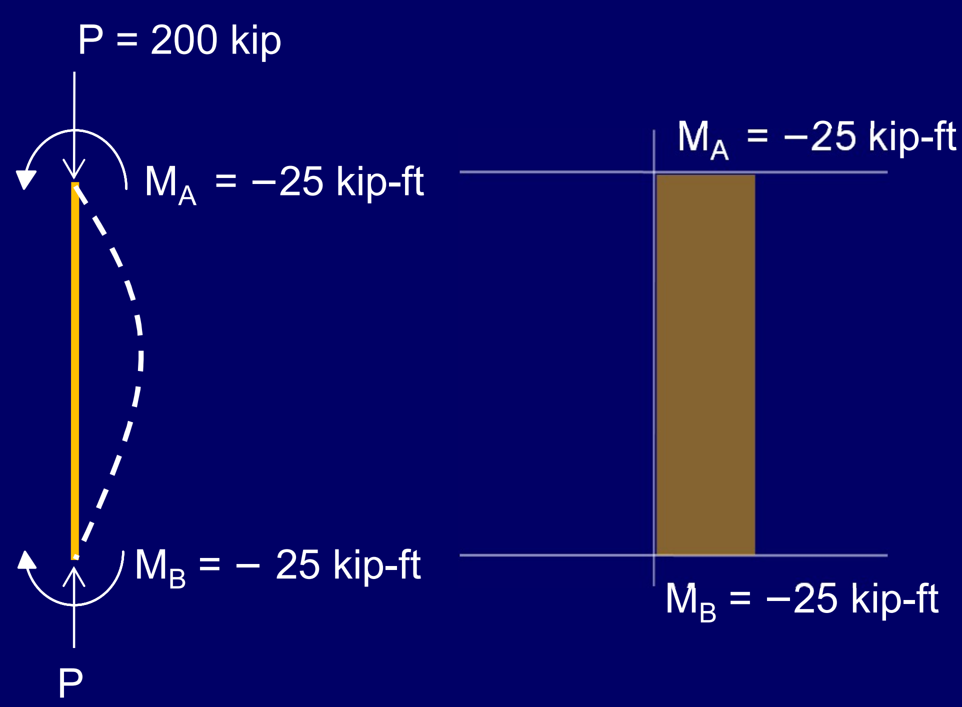
- For the given frame, the actual end moments and axial force in column AB due to dead plus pattern live loads are as shown in figure.



General

❑ Background

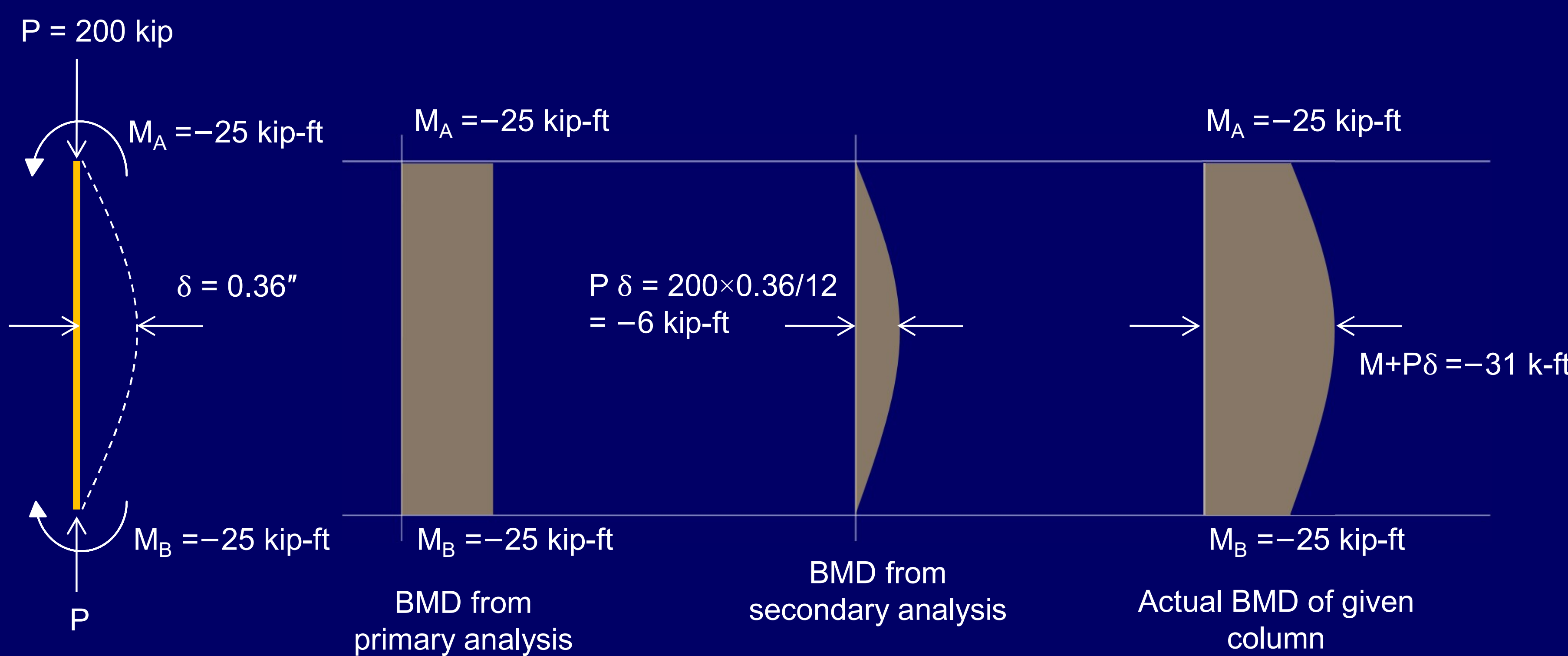
- However, for the purpose of clarification of concepts to follow, we assume that the column is subjected to equal end moments with single curvature.



General

❑ Background

- Due to lateral displacement of the column between its ends, axial load will cause additional secondary moment ($P\delta$) which will magnify column span moments.



General

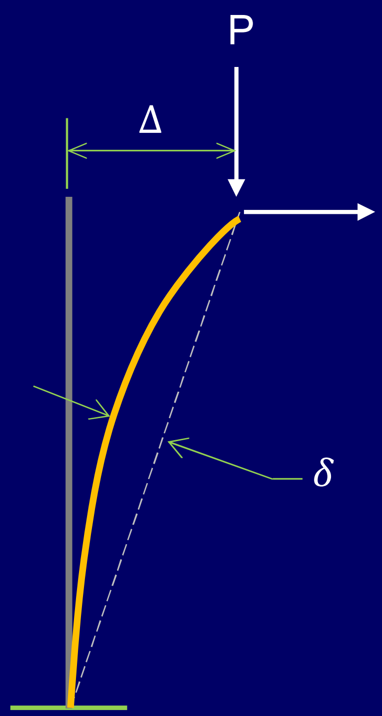
Background

- This means that column AB should have been designed for the magnified moment of 31 ft.kip ($M + P\delta$) instead of 25 ft.kip (M_A or M_B) if secondary effects are considered.
- Hence, conducting only a primary analysis, neglecting secondary effects, might yield misleading results in certain scenarios.

General

❑ Slenderness Effects in Columns

- The additional moments created by the column axial load acting on the deformed column are known as secondary moments or second-order moments or slenderness effects.
- There are two types of second-order moments:
  1. **PΔ Effect:** translation of the column ends
  2. **Pδ Effect:** deflection along the member.
- Slenderness effects gradually increase due to this geometric nonlinearity until the column stabilizes. However, if the column is too slender, it loses stability.



Secondary moments

General

❑ Degree of Slenderness

- To better understand the slenderness effects in columns, it is important to first know the concept of column slenderness.
- The degree of slenderness of a column is expressed in terms of slenderness ratio:

$$S.R = \frac{kl_u}{r}$$

Where;

k = effective length factor, depends on boundary condition of the column

l_u = unsupported column length

r = radius of gyration

General

Degree of Slenderness

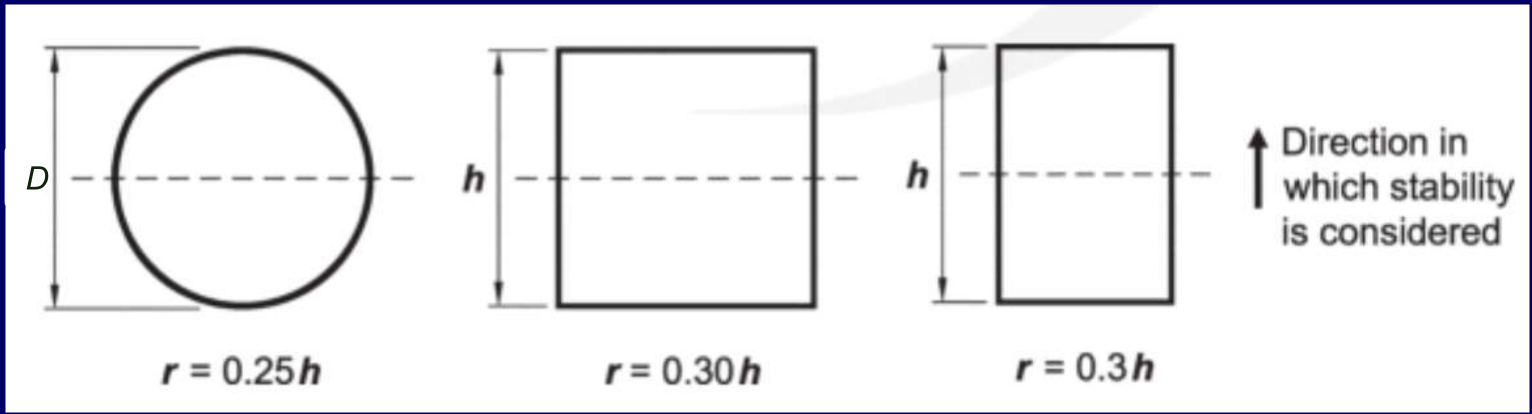
Radius of gyration (ACI 6.2.5.2)

- The radius of gyration r shall be permitted to be calculated by (a), (b) or (c).

a) $r = \sqrt{I_g/A_g}$

b) $r = 0.3h$ for rectangular columns

c) $r = 0.25D$ for circular columns

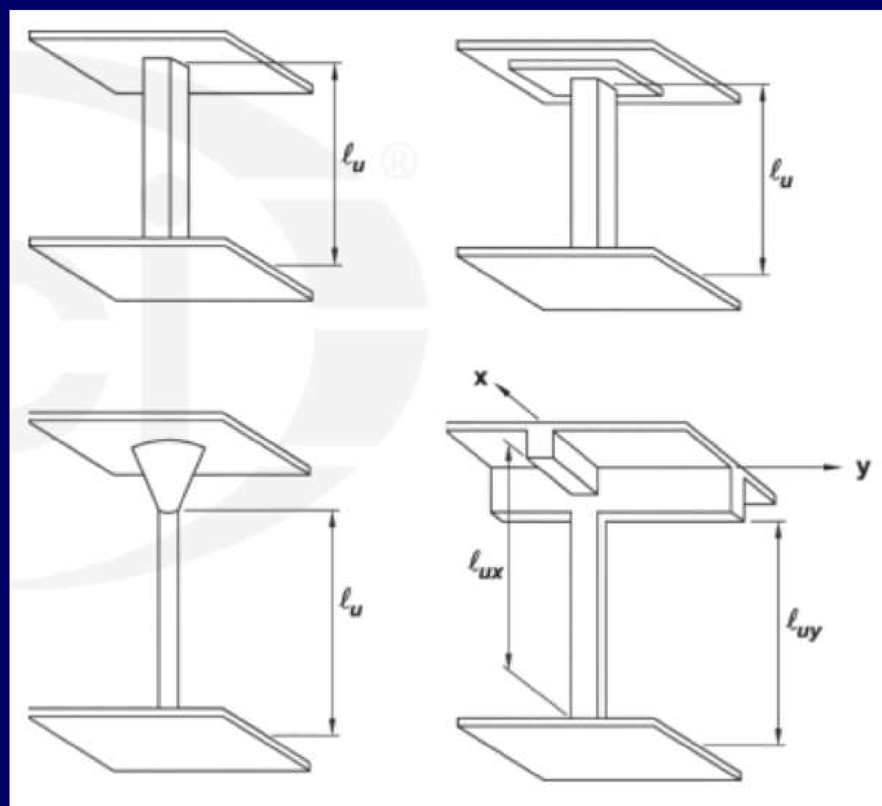


General

❑ Degree of Slenderness

❖ Unsupported Length

- The unsupported length l_u in the direction of analysis is shown below.

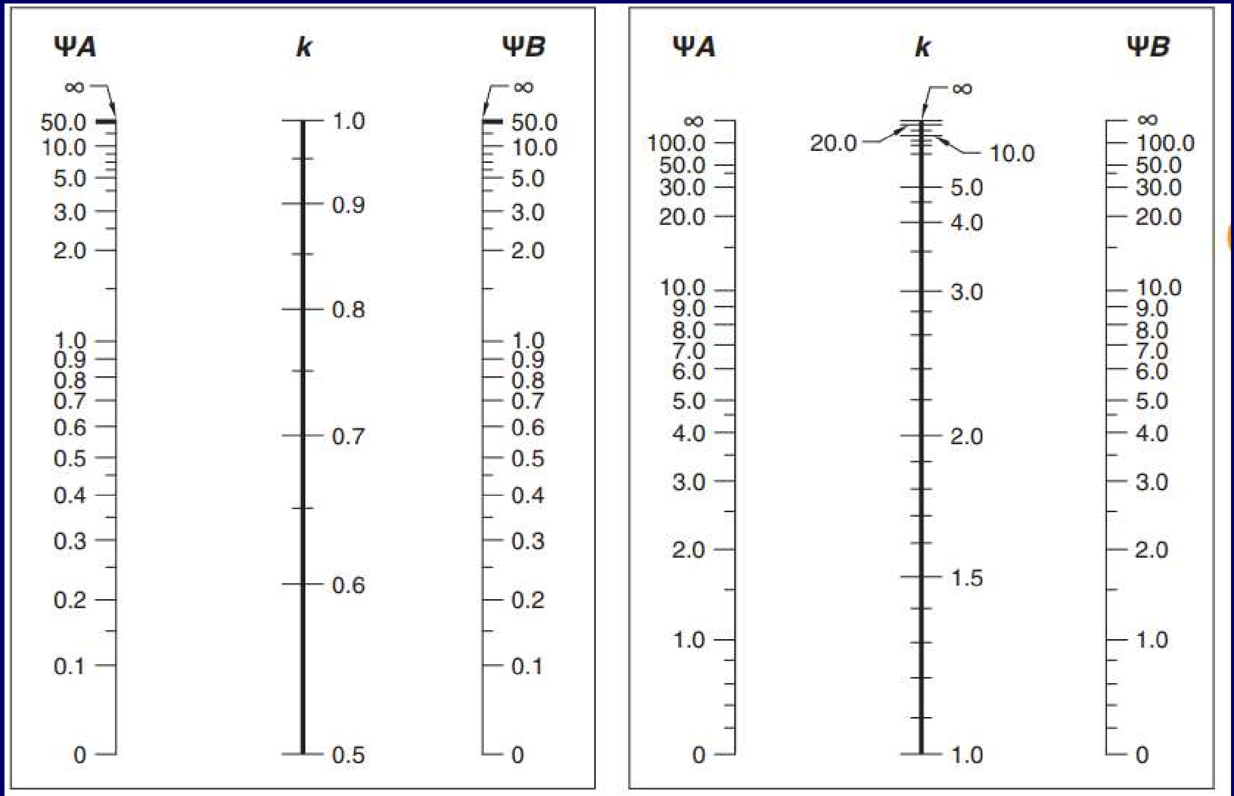


General

❑ Degree of Slenderness

❖ Effective Length Factor

- The Moreland alignment charts can be used to estimate values of k.



(a)
Nonsway frames

(b)
Sway frames

General

❑ Degree of Slenderness

Effective Length Factor – Determination of Ψ_A & Ψ_B

$$\Psi = \frac{\sum(EI/l)_{column}}{\sum(EI/l)_{beam}}$$

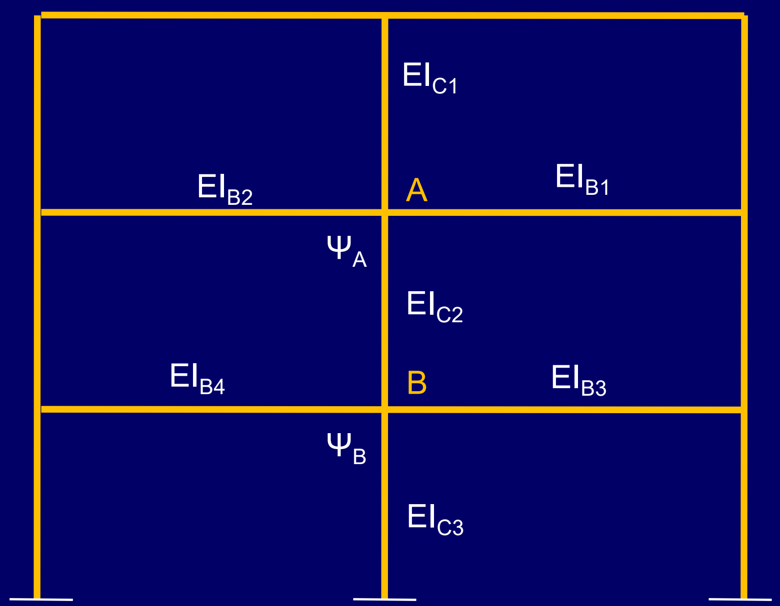
If E is same, then

$$\Psi = \frac{\sum(I/l)_C}{\sum(I/l)_B}$$

$$\Psi_A = \frac{I_{c1}/l_{c1} + I_{c2}/l_{c2}}{I_{B1}/l_{B1} + I_{B2}/l_{B2}}$$

And

$$\Psi_B = \frac{I_{c2}/l_{c2} + I_{c3}/l_{c3}}{I_{B2}/l_{B2} + I_{B3}/l_{B3}}$$



General

Degree of Slenderness

Effective Length Factor – Determination of Ψ_A & Ψ_B

- ACI recommends to use the following values for calculation of Ψ_A and Ψ_B.
  - Modulus of elasticity:, $E = 57000\sqrt{f_c'}$ (psi) [section 19.2.2]
  - Moment of Inertia, I: from Table 6.6.3.1.1

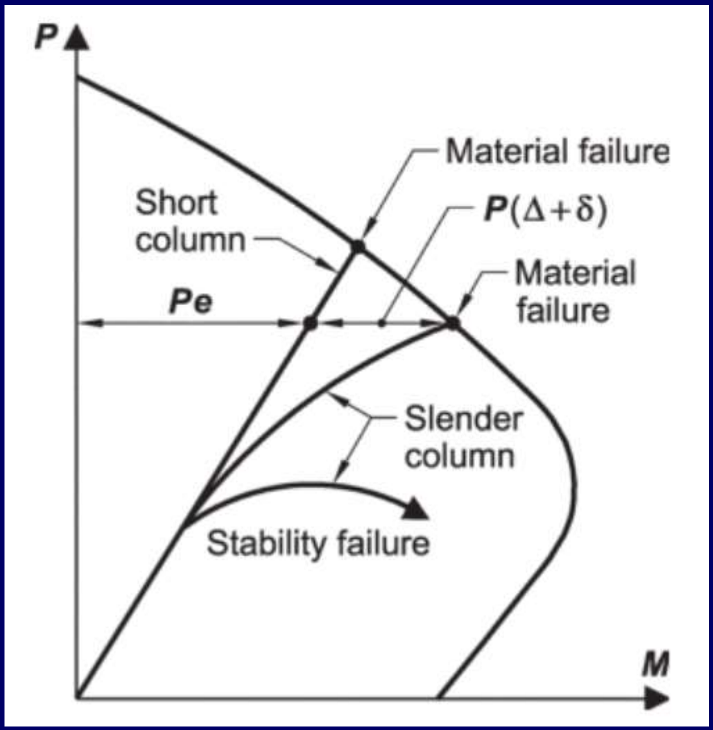
Table ACI 6.6.3.1.1	
Beams	$0.35I_g$
Columns	$0.70I_g$
Walls – uncracked	$0.70I_g$
Walls – cracked	$0.35I_g$
Flat plates and flat slabs	$0.25I_g$

General

Slenderness Effects in Columns

Effect of Slenderness on Column

- As the length l of a column is incrementally increased, a critical point will be reached where the column fails due to buckling under axial load alone, rather than crushing from the combined effects of axial load and bending moment.
- This type of failure mode is referred to as buckling failure or stability failure.



General

Effect of Slenderness on Column

- Critical buckling load P_c for a column can be calculated from the famous Euler equation:

$$P_c = \frac{\pi^2 EI}{(kl_u)^2}$$

- If $P > 0.75\,P_c$ (where 0.75 is the stiffness reduction factor as per ACI 6.7.1.1), the column will fail due to buckling.

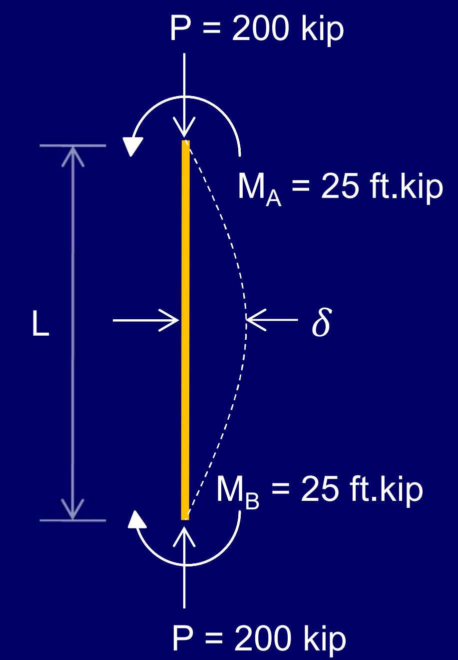
General

❑ Effect of Slenderness on Column

- As an example, the critical buckling load calculations for a 12″ square column with r = 3.6″, k = 1 and EI = 2.142 x 10^6 ksi are tabulated below.

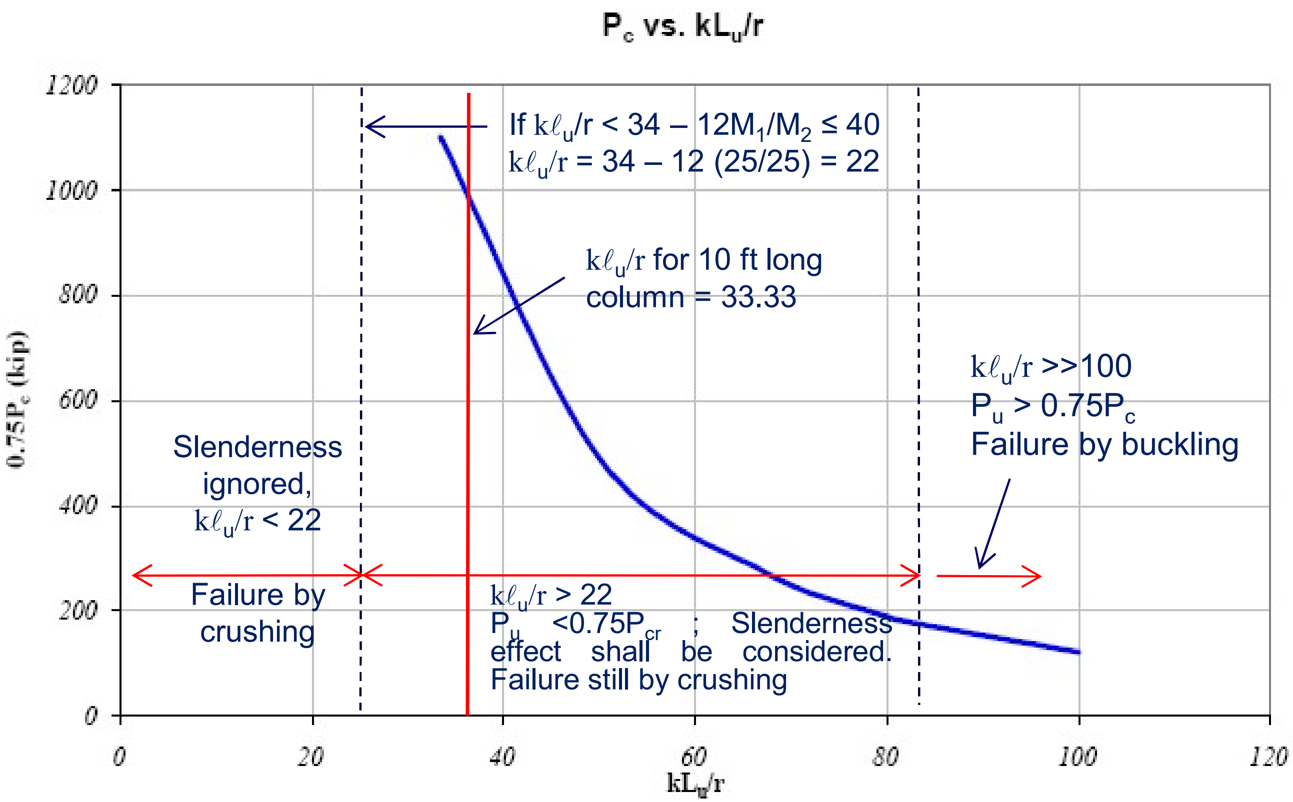
L (ft.)	δ (in.)	Pδ (ft.kip)	M+Pδ ft.kip)	$\frac{kl_u}{r}$	$0.75P_c$ (kip)
10	0.36	6	31	33.33	1101
15	0.8	13.33	38.33	50	489
20	1.41	25.5	50.5	67	275
23.4	**1.93**	**32.2**	**57.2**	**78**	**200**
25	2.2	36.6	61.6	83	176
30	3.2	53.33	78.33	100	122

Column will fail by buckling at L > 23.4 ft.



General

❑ Effect of Slenderness on Column



L (ft.)	$\frac{kl_u}{r}$	$0.75P_{cr}$ (kip)
10	33.33	1101
15	50	489
20	67	275
23.4	**78**	**200**
25	83	176
30	100	122

Failure stages of column corresponding to various slenderness ratios

General

❑ Analysis Procedures for Slenderness Effects

- ACI Code specifies the following three procedures for determination of secondary moments (slenderness effects).

1. Inelastic Analysis [section 6.8]
2. Elastic Second-order Analysis [section 6.7]
3. Elastic First-order Analysis with Moment Magnification Method [section 6.6.4.1 - 6.6.4.6]

General

❑ Analysis Procedures for Slenderness Effects

1. Inelastic Analysis

- ACI section 6.8 provides guidance for an inelastic second-order analysis that includes material nonlinearities, the duration of loads, shrinkage, creep and interactions with the foundation.

- An inelastic analysis procedure shall have been shown to result in calculation of strength and deformations that are in substantial agreement with results of physical tests of reinforced concrete components (ACI 6.8.1.2).

General

Analysis Procedures for Slenderness Effects

2. Elastic Second-order Analysis

- Due to the iterative nature of the analysis, the principle of superposition cannot be used to calculate second-order moments.
- Thus, it is necessary to applied loads must be factored and combined before use in conducting the analysis.
- Following geometric properties shall be used for this analysis.

Modulus of elasticity		$E = 57000\sqrt{f_c'}$
Moment of inertia, I	Beams	$0.35I_g$
	Columns	$0.70I_g$
	Walls – uncracked	$0.70I_g$
	Walls – cracked	$0.35I_g$
	Flat plates and flat slabs	$0.25I_g$
Area		$1.0A_g$

General

Analysis Procedures for Slenderness Effects

2. Elastic Second-order Analysis

- ACI section 6.7 provides more extensive guidance for conducting an elastic second-order analysis of frames including slender columns.
  - In the first step, elastic primary moments are determined using reduced stiffness properties of the structural members.
  - Then PΔ moments are evaluated using Δ from the primary analysis and added with primary moments to obtain secondary moments after first iteration.
  - The PΔ moments are again evaluated using Δ from the previous analysis and added with the last obtained moments to obtain secondary moments after second iteration.
  - The process is repeated till the results converge.

General

❑ Analysis Procedures for Slenderness Effects

3. **Moment Magnification Method (approximate)**

- In this method, secondary moments are calculated by magnifying elastic first-order moments using moment magnification factors prescribed by the Code (will be explained later).
- In this lecture, Moment Magnification Method will be used for determining slenderness effects in both non-sway and sway frames.

General

Consideration of Slenderness Effects

- According to ACI 6.2.5.1 , slenderness effects in columns shall be permitted to be neglected, if (a) or (b) is satisfied.

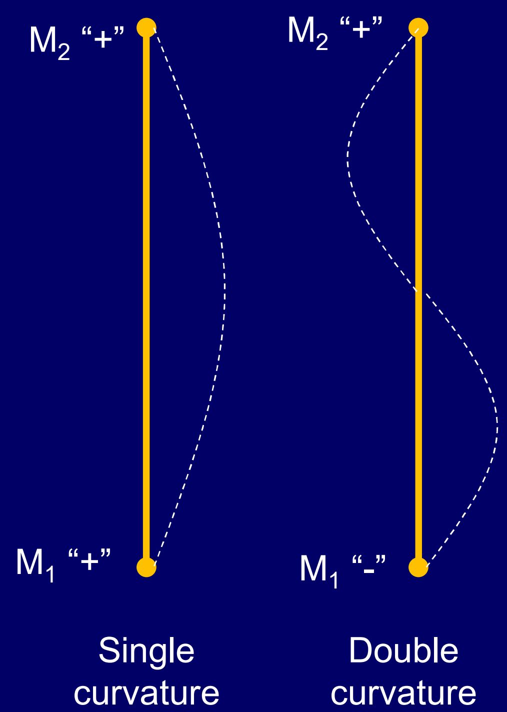
a) For columns of sway frames

$$\frac{kl_u}{r} \leq 22$$

b) For columns of non-sway frames

$$\frac{kl_u}{r} \leq min\,(34 - \frac{12M_1}{M_2}\,, 40)$$

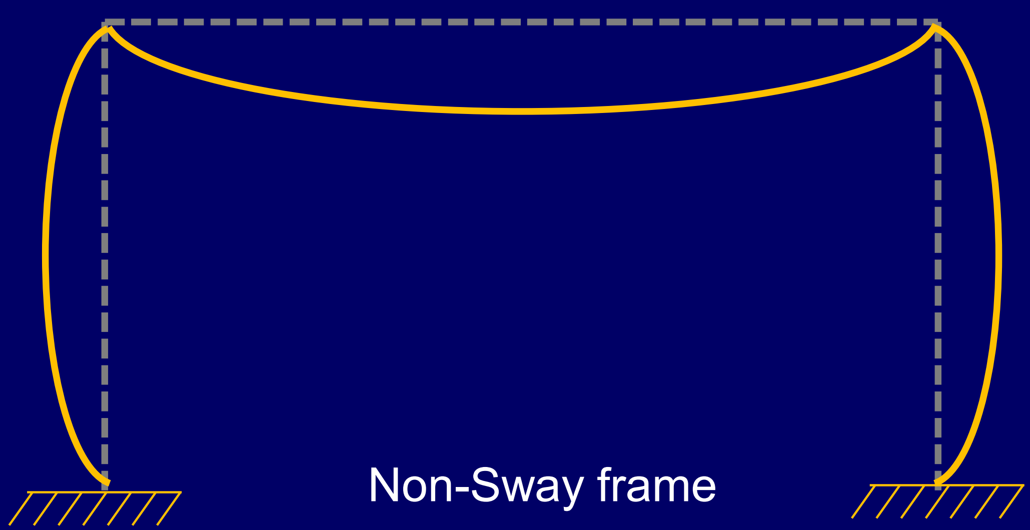
- M_1 and M_2 are smaller and larger end moments, respectively.
- M_2 is always positive while M_1 is positive when column bends with single curvature and negative when it bends with double curvature.



Sway and Non-Sway Frames

❑ Non-sway Frame

- A frame that is sufficiently braced by lateral bracing elements, exhibiting negligible lateral displacement is said to be Non-Sway frame. OR Frame essentially subjected only to gravity load only.

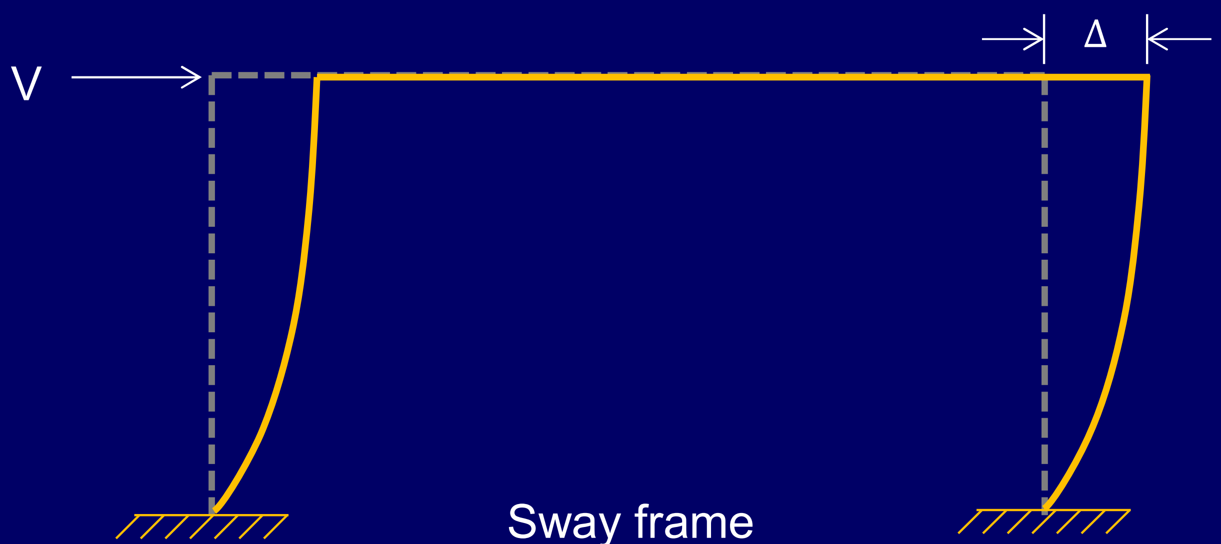


Non-Sway frame

Sway and Non-Sway Frames

Sway Frame

- A frame that lacks lateral bracing elements and undergo lateral displacement (side sway) is said to be Sway frame.
- Lateral loading, vertical eccentric loading, unsymmetrical EI, unequal vertical members, and unsymmetrical supports are the reason behind the swaying of frames.

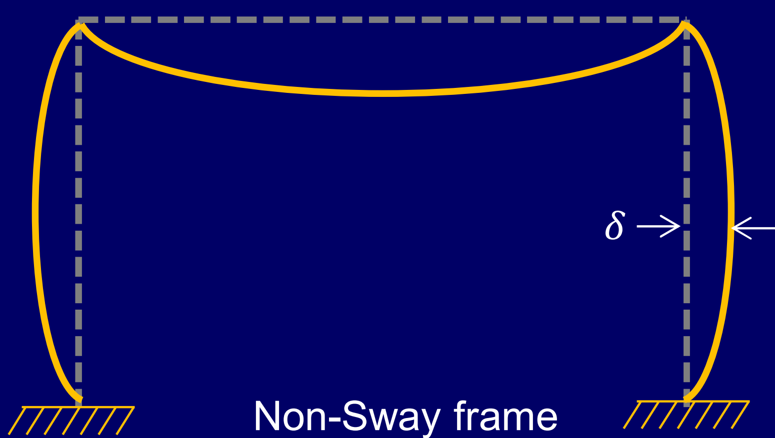


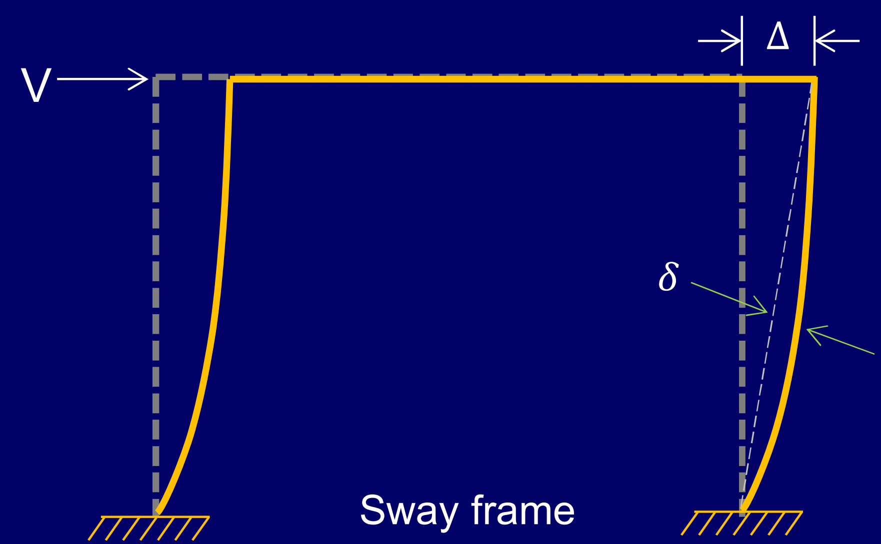
Sway frame

Sway and Non-Sway Frames

Slenderness effects in Non-Sway and Sway Frames

- In frames non-sway, the focus lies solely on the Pδ effect, with $\mathbf{P}\Delta$ being ignored.
- However, in frames that sway, both $\mathbf{P}\Delta$ and Pδ effects are need to be considered.





Sway and Non-Sway Frames

Classification Criteria

- ACI 318 provides three methods to declare story or frames as non-sway or sway.

Method 1 [ACI 6.2.5.1]

- Columns are non-sway if the gross lateral stiffness of the walls (bracing elements) in a story is at least 12 times the gross lateral stiffness of the columns in that story in the direction considered.

$$(EI)_{lat,brace} \geq 12(EI)_{lat,col}$$

Sway and Non-Sway Frames

Classification Criteria

- **Method 2** [ACI 6.6.4.3 (a)]
  - Columns are non-sway if the increase in column end moments due to second-order effects does not exceed 5 percent of the first-order end moments i-e:

$$\frac{M_{2n\ \ order}}{M_{1st\ order}} \leq 1.05$$

- **Method 3** [ACI 6.6.4.3 (b)]
  - Columns are non-sway if the stability index Q does not exceed 0.05.

$$Q = \frac{\sum P_u \Delta_o}{V_u l_c} \leq 0.05$$

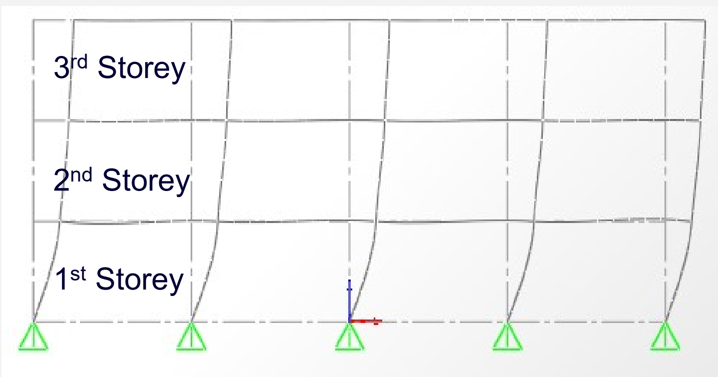
  - The terms in Q are explained next.

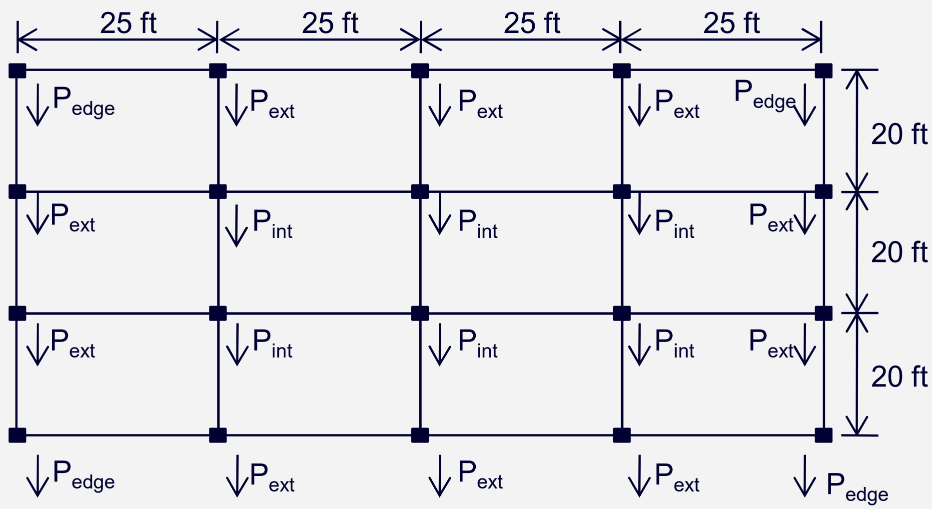
Sway and Non-Sway Frames

❑ Classification Criteria

❖ **Method 3** [ACI 6.6.4.3 (b)]

- **Calculation of $\sum P_u$**





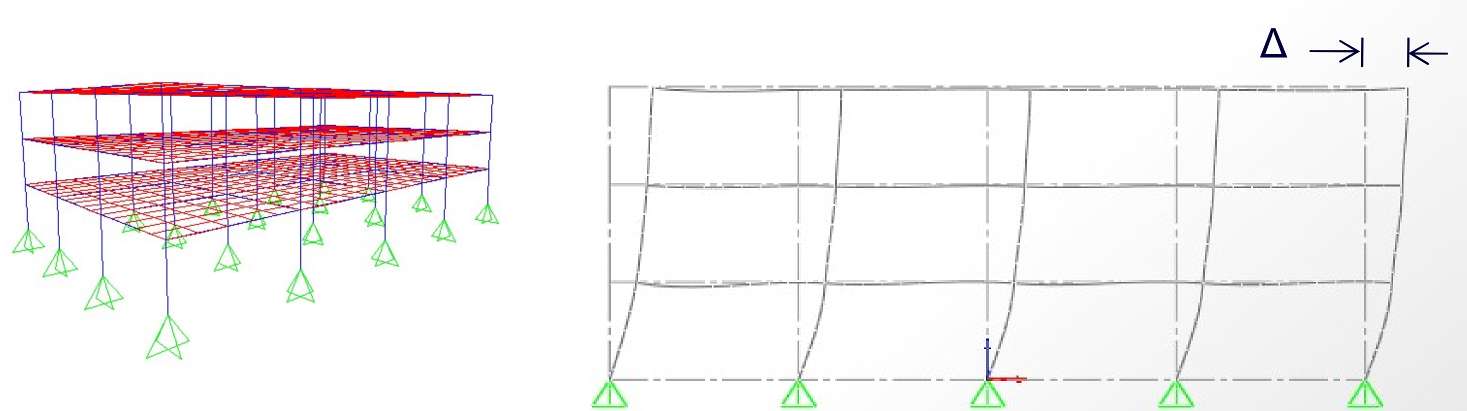
$$Q = \frac{\sum P_u \Delta_o}{V_u l_c}$$

$\sum P_u$ = sum of axial loads of all columns in a story due to gravity load only.
For “1.2D+1.0L+1.0 Lateral”, $P_u = 1.2P_D + 1.0P_L$, where P_D and P_L are axial loads in a column due to dead and live loads, respectively.
For “0.9D+1.0Lateral”, $P_u = 0.9P_D$, where P_D is axial load in a column due to dead load.

Sway and Non-Sway Frames

Classification Criteria

- **Method 3** [ACI 6.6.4.3 (b)]
  - **Calculation of $\sum P_u$**
  - $\sum P_u$ is used since all columns in a story deflect laterally by an amount Δ.



3D model

Elevation view

Sway and Non-Sway Frames

Classification Criteria

- **Method 3** [ACI 6.6.4.3 (b)]
  - **Calculation of $\sum P_u$**
  - It is important to mention at this point that:
    - Secondary effect in a column of non-sway story is independent of presence of other columns because the PΔ effects are produced in this column only due to load on it (independent of all other columns).
    - Secondary effect in a column of a sway story is dependent on the presence of other columns in that story because all columns provide resistance through their combined stiffness against lateral drift Δ.

Sway and Non-Sway Frames

Classification Criteria

Method 3 [ACI 6.6.4.3 (b)]

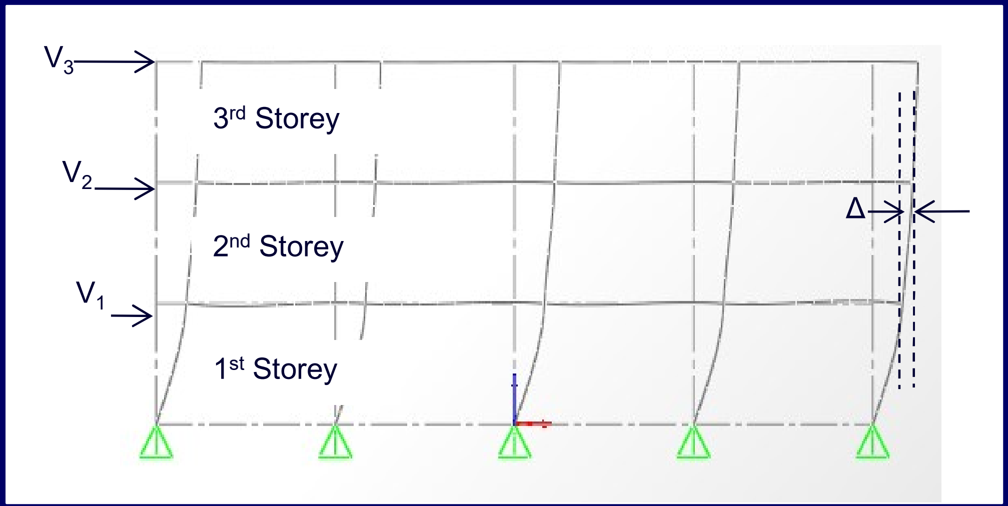
- **Calculation of Δ_o**

- Generally, unfactored loads are applied on a structural model. In the frame shown, if Δ is due to unfactored story shears V_1 and V_2, then load factor needs to be multiplied to obtain Δ_o.

$$\Delta_o = \gamma_L \times \Delta$$

γ_L = 1.0 (for Earthquake)

γ_L = 1.6 or 0.8 (for wind)

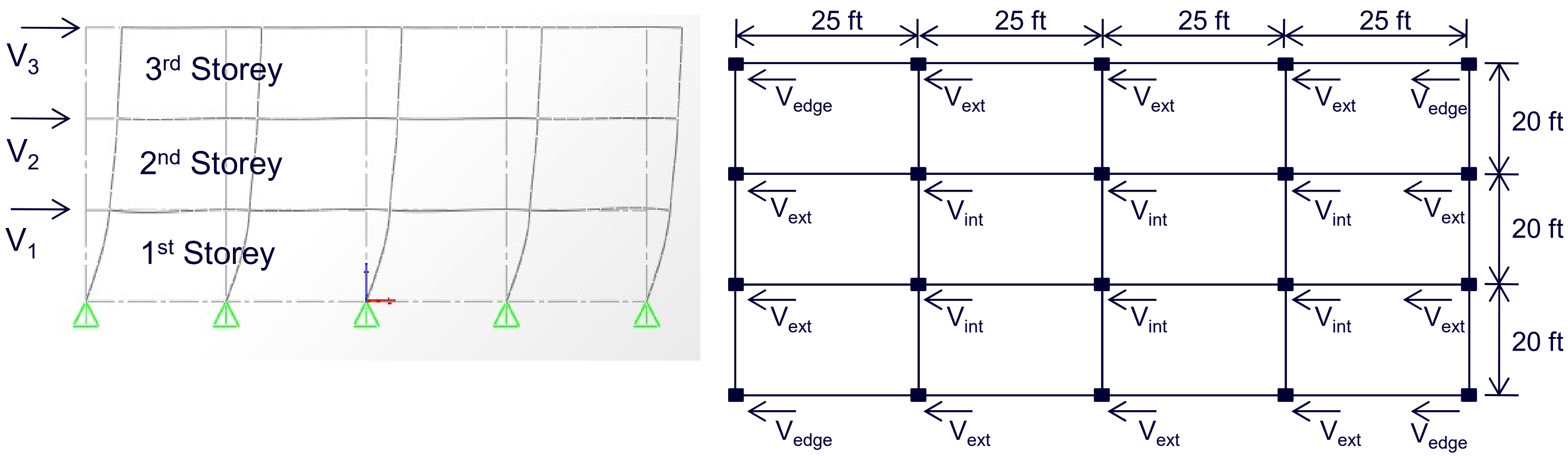


Sway and Non-Sway Frames

Classification Criteria

Method 3 [ACI 6.6.4.3 (b)]

- Calculation of V_u



V_u= sum of shears in all columns in a storey due to lateral load only.
For "1.2D+1.0L+1.0Lateral", V_u=1.0V_E, where V_E is shear in a column due to earthquake loads.
For "0.9D+1.0Lateral", V_u=1.0V_E

Sway and Non-Sway Frames

❑ Conclusion of Discussion

- Determine stability Index Q for classifying Frames.
- If Q $\leq$ 0.05, the story is declared as non-sway. In the non-sway story, the slenderness effects can be neglected if:

$$\frac{kl_u}{r} \leq min\,(34 - \frac{12M_1}{M_2}\,, 40)$$

- If kl_u/r exceed the above limit, the (PΔ effect) is ignored, and the secondary moment resulting from member curvature (Pδ effect) can be determined using any of the following analysis methods.

1. Inelastic analysis
2. Second-order analysis
3. Moment magnification method for non-sway frames

Sway and Non-Sway Frames

Conclusion of Discussion

- If Q > 0.05, the story is declared as sway. In the sway story, the slenderness effects can be neglected if:

$$\frac{kl_u}{r} \leq 22$$

- If kl_u/r exceed the above limit, the secondary moments resulting from translation (PΔ effect) and member curvature (Pδ effect) can be determined using any of the following analysis methods.

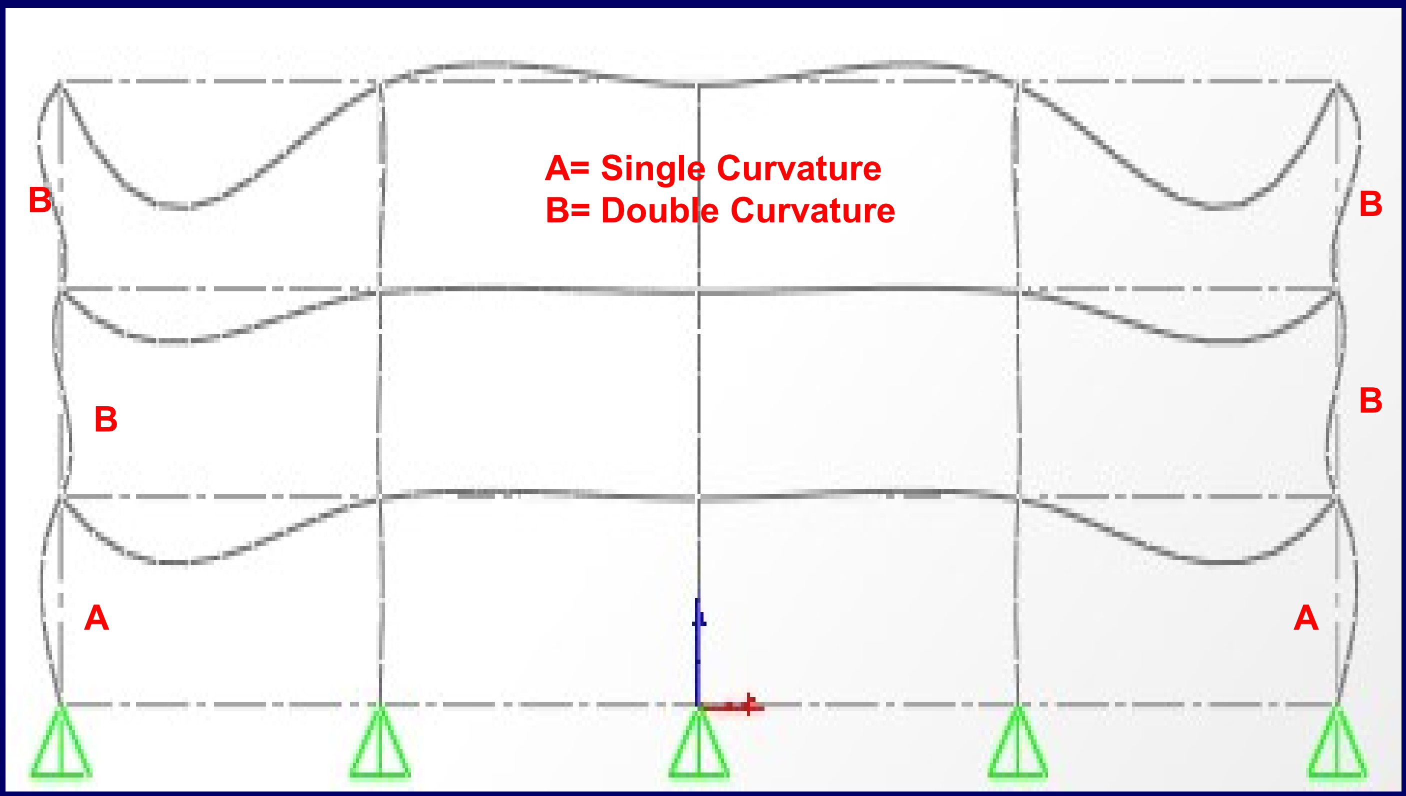
1. Inelastic analysis
2. Second-order analysis
3. Moment magnification method for sway frames

Slenderness Effects in Non-Sway Frames

Slenderness Effects in Non-Sway Frames

❑ Moment Magnification in Non-Sway Frames

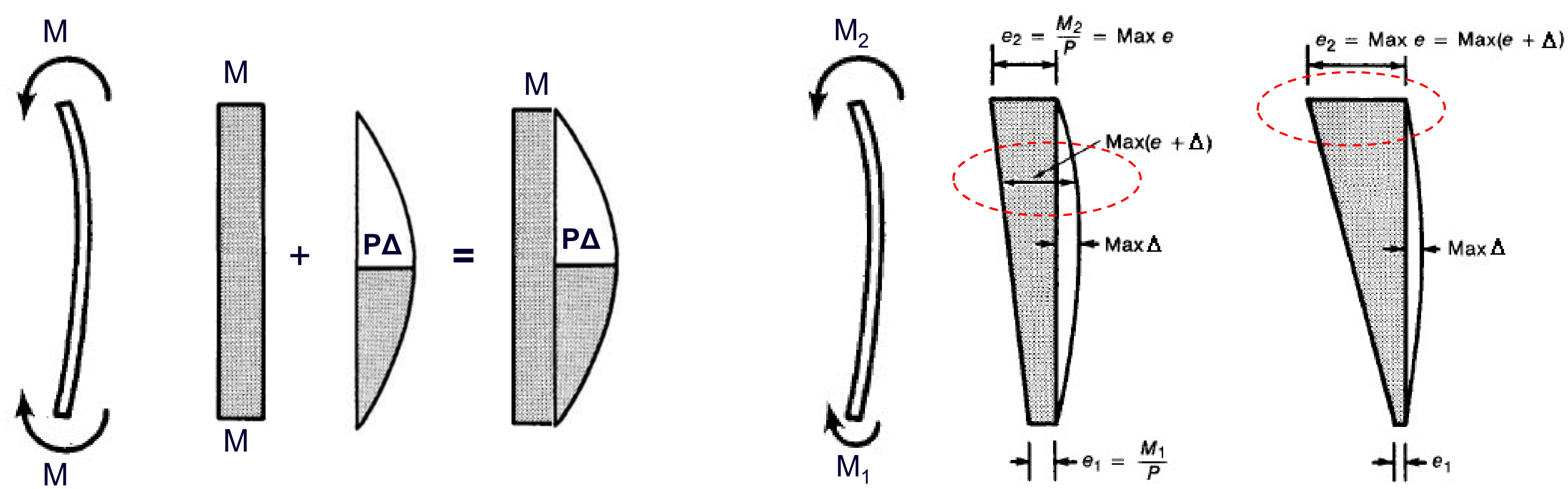
- Frames essentially subjected only to gravity load (no lateral load) are known as non-sway frames.



Slenderness Effects in Non-Sway Frames

❑ Moment Magnification in Non-Sway Frames

- The moment magnification in case of non sway frames depends on the locations of primary and secondary moments.
- In the case of end moments in opposite direction or single curvature the magnification will occur as shown below:



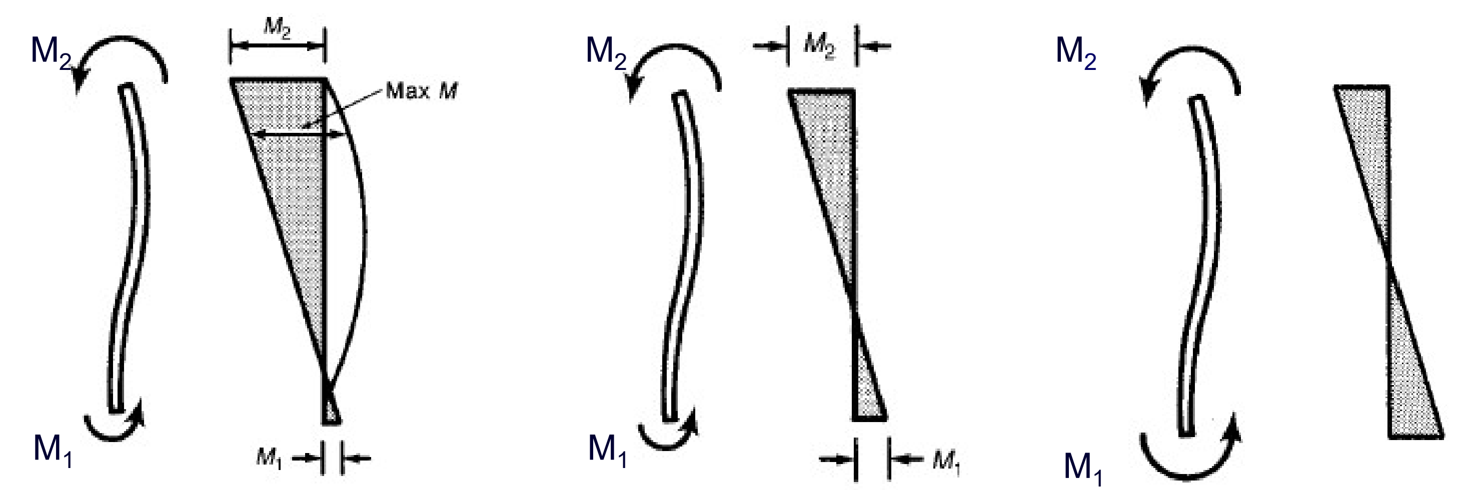
Equal end moments in opposite direction.
(PΔ Effects are most critical)

Unequal end moments in opposite direction.
(PΔ Effects may or may not be critical)

Slenderness Effects in Non-Sway Frames

❑ Moment Magnification in Non-Sway Frames

- In the case of end moments in same direction or double curvature the magnification will occur as shown below:



Unequal end moments in same direction
(No appreciable PΔ Effects in most cases)

Slenderness Effects in Non-Sway Frames

❑ Determination of Slenderness Effects

❖ Moment Magnification for Non-sway Frames (ACI 6.6.4.5)

- In non-sway frames, the secondary moments due to translation of column ends ($P\Delta$) are negligible. Hence, the secondary moments due member curvature ($P\delta$) are calculated using the following equation.

$$M_c = \delta_{ns} M_2$$

Where;

- M_c = magnified Moment due to primary as well as secondary moments ($P\delta$).
- M_2 = the larger end moment
- δ_{ns} = the magnification factor (described next)

Slenderness Effects in Non-Sway Frames

❑ Determination of Slenderness Effects

❖ Moment Magnification for Non-sway Frames (ACI 6.6.4.5)

- The magnification factor δ_{ns} can be calculated as:

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75\,P_c}} \geq 1$$

$$C_m = 0.6 + 0.4(M_1/M_2) \geq 0.4$$

- C_m is a correction factor relating the actual moment diagram to an equivalent uniform moment diagram.
- For columns with transverse loads applied between supports, $C_m = 1$

M_2 shall not be taken less than $M_{2,min}$

$$M_{2,min} = P_u(0.6 + 0.03h)$$

(ACI 6.6.4.5.4)

h = dimension of column in the direction of analysis (inches).

$$P_c = \frac{\pi^2}{(kl_u)^2} EI$$

Use Moreland chart for non-sway frames

$$EI = \frac{0.4E_c I_g}{1 + \beta_{dns}}$$

For simplifications, β_{dns} can be taken as 0.6 (ACI R6.6.4.4.4) and equation is reduced to **0.25 $E_c I_g$**

$$\beta_{dns} = \frac{\textit{Maximum factored axial sustained load}}{\textit{Maximum factored axial load associated with the same load combination}}$$

Example 11.1

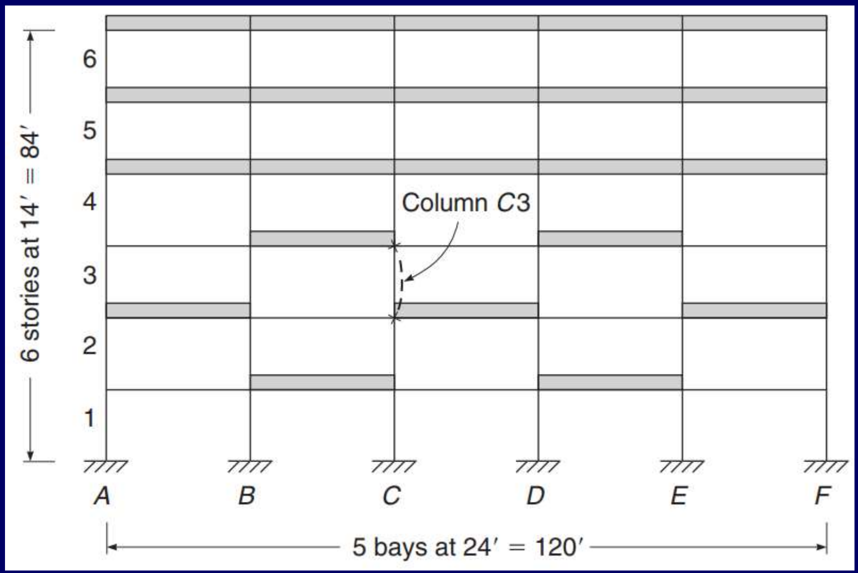
❑ Problem Statement

Design column C3 highlighted in figure using moment magnification method. The column is laterally braced (non-sway) against sidesway. The specified material strengths are $f_c' = 4000$ psi and $f_y = 60{,}000$ psi.

Service Loads on Column C3		
Actions	**Dead Load**	**Live Load**
P (ft)	230	173
M_2 (ft.kip)	2	108
M_1 (ft.kip)	-2	100

All beams: 48″ x 12″

All interior columns: (18″x18″)



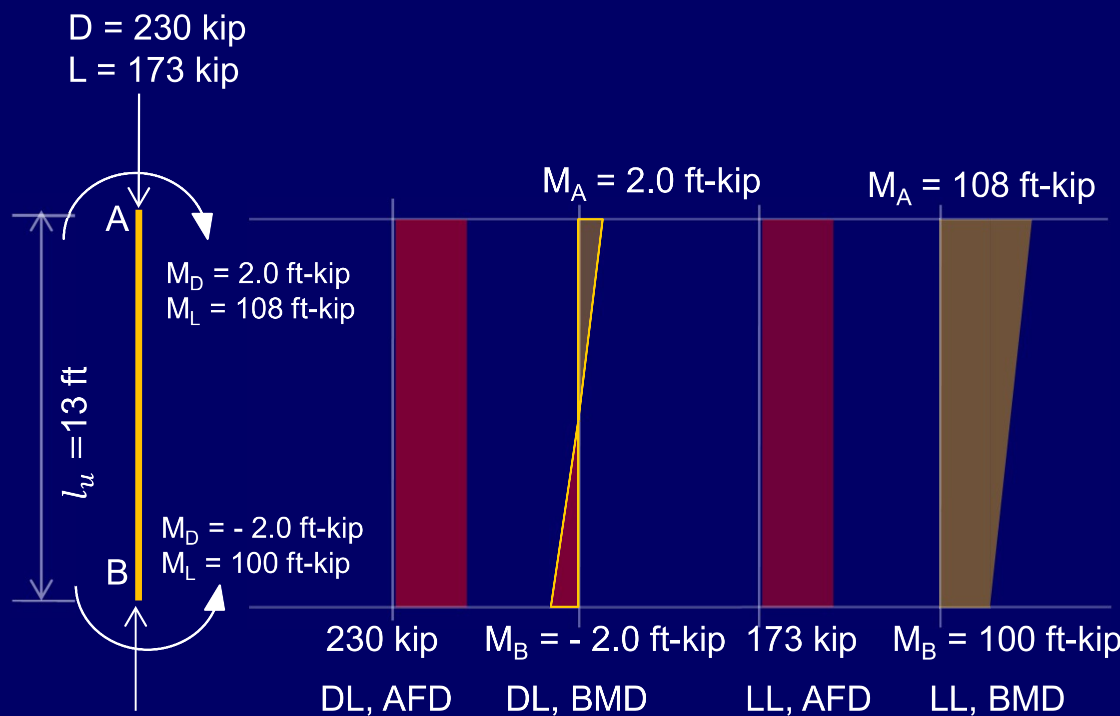
Example 11.1

❑ Solution

➢ Step 1: Primary Analysis

Factored Cases	P (kip)	M_A (ft.kip)	M_B (ft.kip)
1.4D	322	2.8	-2.8
1.2D	276	2.4	-2.4
1.6L	276.8	172.8	160

Combinations	P (kip)	M_{ugA} (ft.kip)	M_{ugB} (ft.kip)
1.2D+1.6L	552.8	175.2	157.6



Therefore,

P_u = 552.8 kips

M_1 = +157.6 kip-ft (single curvature)

M_2 = 175.2 kip-ft > $M_{2,min}$ = 552.8(0.6 + 0.03×18)/12 = 52.51 ft-kip

Example 11.1

Solution

Step 2: Check for Consideration of Slenderness Effects

The slenderness effects can be ignored for non-sway frames if:

$$\frac{kl_u}{r} \leq min\left(34 - \frac{12M_1}{M_2}, 40\right)$$

Substituting values, we get

$$\frac{kl_u}{r} = \frac{1 \times (13 \times 12)}{0.3(18)} = 28.89$$

$$34 - \frac{12M_1}{M_2} = 34 - \frac{12 \times 157.6}{175.2} = 23.21 < 40$$

$\frac{kl_u}{r} = 28.89 > 23.21 \rightarrow$ Slenderness effects need to be considered

Example 11.1

Solution

Step 2: Check for Consideration of Slenderness Effects

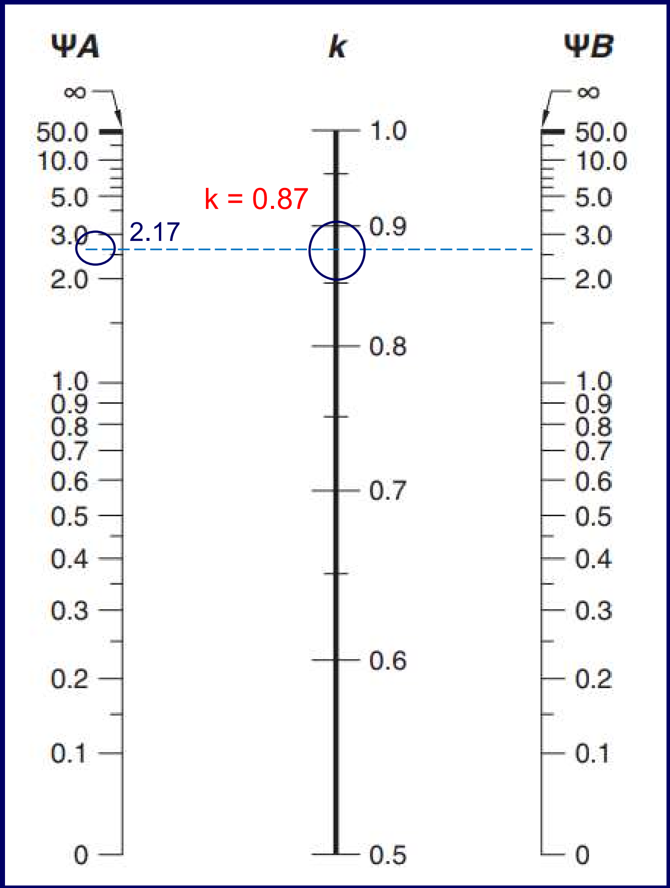
Calculation of Effective Length Factor , k

$$\left(\frac{I}{l}\right)_{beam} = \frac{2^{*}\left[0.35 \times \left(\frac{48 \times 12^3}{12}\right)\right]}{(24 \times 12)} = 16.8\ in^3$$

$$\left(\frac{I}{l}\right)_{col} = \frac{0.7 \times \left(\frac{18 \times 18^3}{12}\right)}{(14 \times 12)} = 36.45\ in^3$$

$$\Psi_A = \Psi_B = \frac{I_c/l_c}{I_B/l_B} = \frac{36.45}{16.8} = 2.17$$

From Moreland′s chart, k = 0.87



[*] The moment of inertia of T- Beam can be taken as 2 times the moment of inertia of rectangular beam (ACI R6.6.3.1.1).

Example 11.1

❑ Solution

➢ Step 3: Calculate Magnified Moment M_c due to $P\delta$

$$M_c = \delta_{ns} M_2$$

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75\ P_c}} \geq 1$$

$$\delta_{ns} = \frac{0.96}{1 - \frac{552.8}{0.75(4500)}} = 1.15$$

Subsisting this value, we get

$$M_c = \delta_{ns} M_2 = 1.15 \times 175.2$$

$$M_c = 201.48 \text{ ft. kip}$$

$$C_m = 0.6 + 0.4(M_1/M_2) \geq 0.4$$

$$C_m = 0.6 + 0.4(157.6/175.2) = 0.96$$

$$\beta_{dns} = \frac{1.2D}{1.2D + 1.6L} = \frac{276}{552.8} = 0.50$$

NOTE: We could also have taken β_{dns} = 0.6 for simplification.

$$EI = \frac{0.4 E_c I_g}{1 + \beta_{dns}} = 8.4\ \times\ 10^9\ in^2.lb$$

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} = 4500 \text{ kip}$$

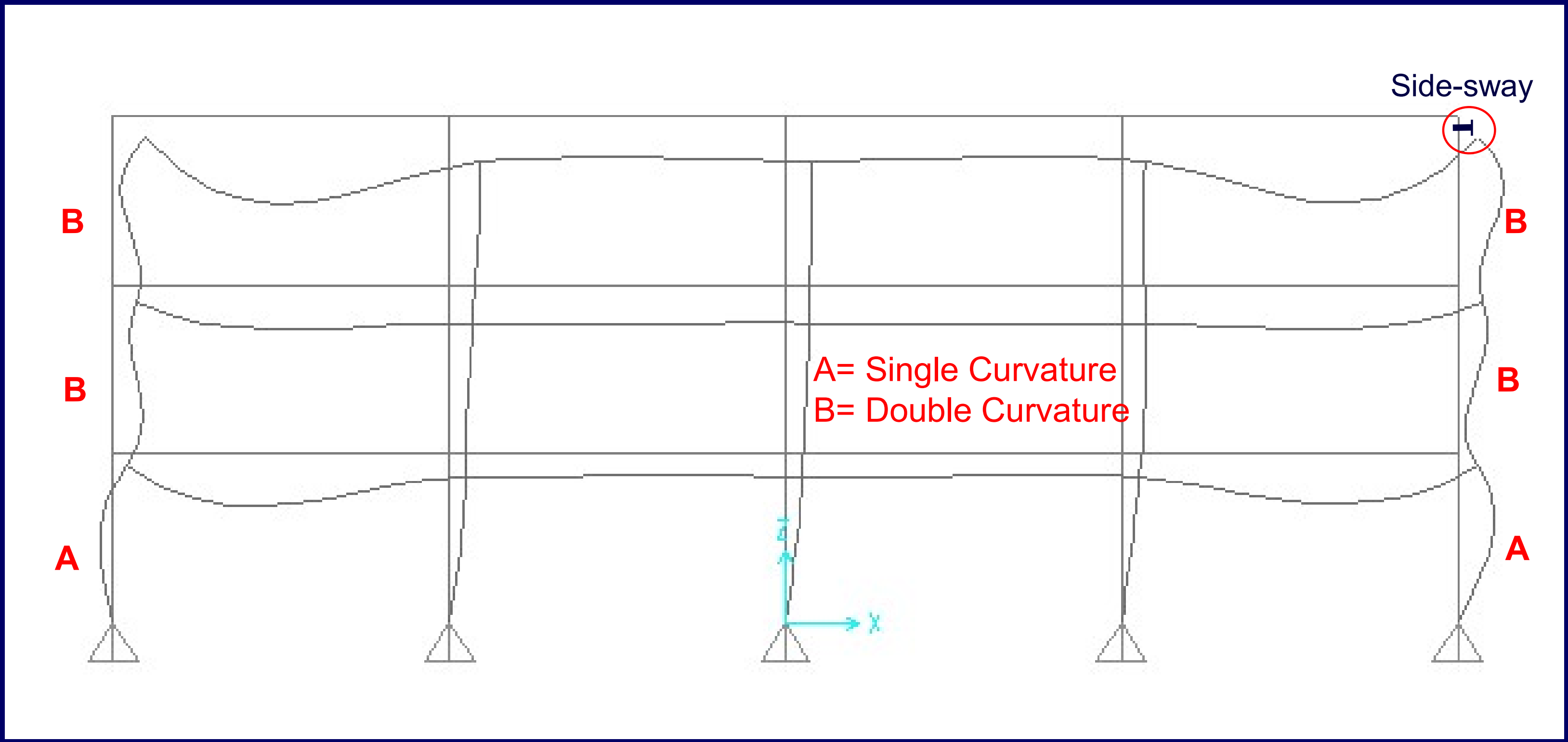
Hence, the column shall be designed for 201.48 ft. Kip instead of 175.2 ft-kip.

Slenderness Effects in Sway Frames

Slenderness Effects in Sway Frames

❑ Moment Magnification in Sway Frames

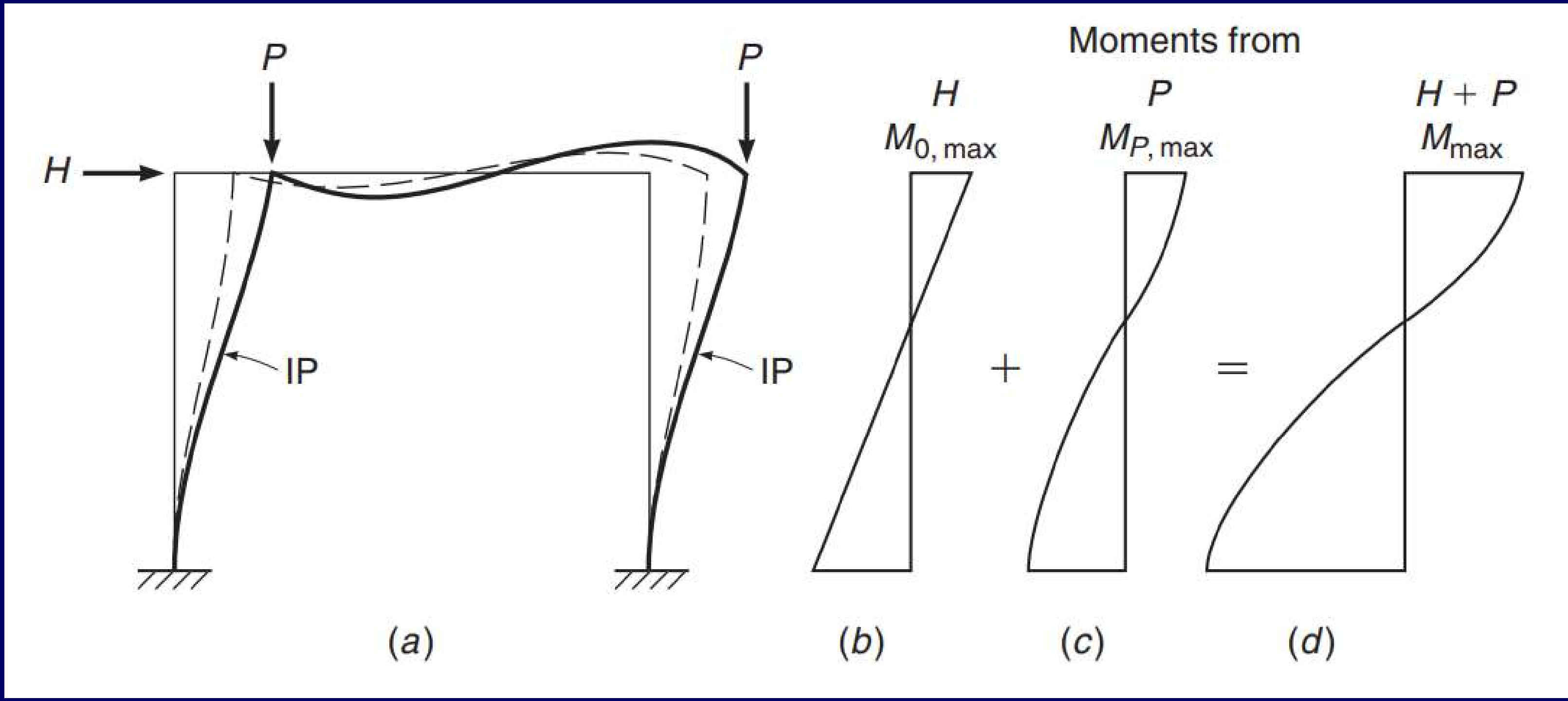
- Frames subjected to lateral loads are generally called as sway frames.



Slenderness Effects in Sway Frames

Moment Magnification in Sway Frames

- Effect of end moments and curvature on secondary effects in sway frames is shown below.

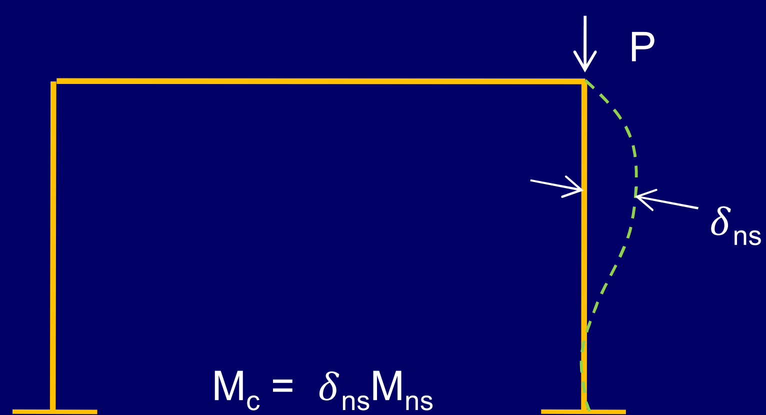


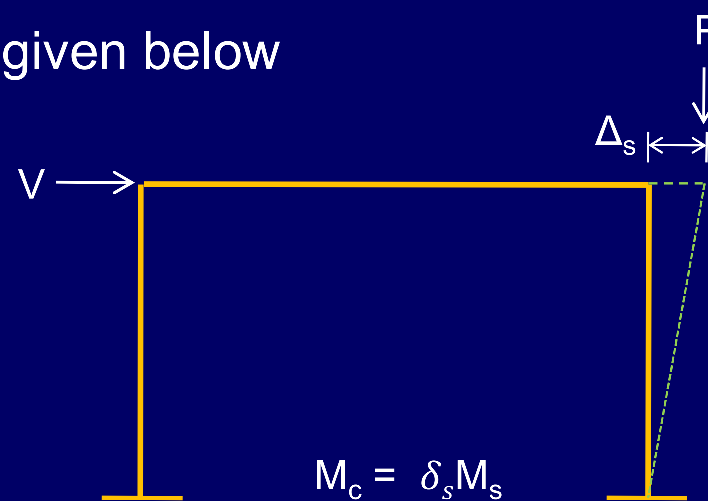
Maximum values of primary and secondary moments occur at the same locations, at the ends of the columns, they are therefore fully additive, leading to a large moment magnification in contrast to non sway frames.

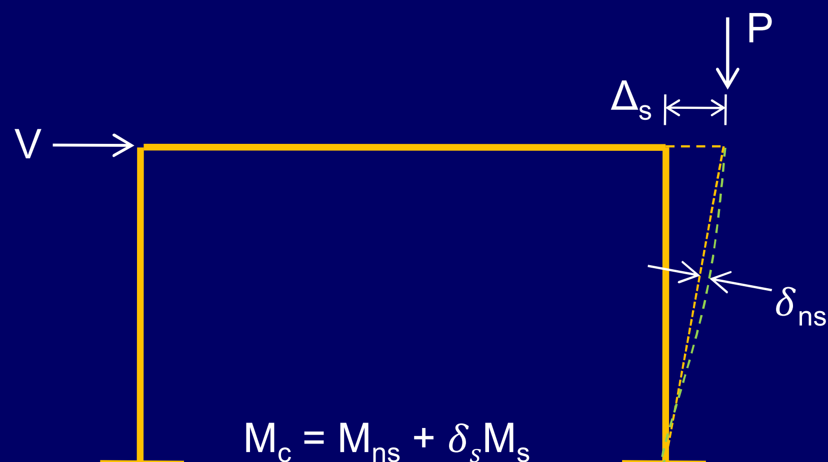
Slenderness Effects in Sway Frames

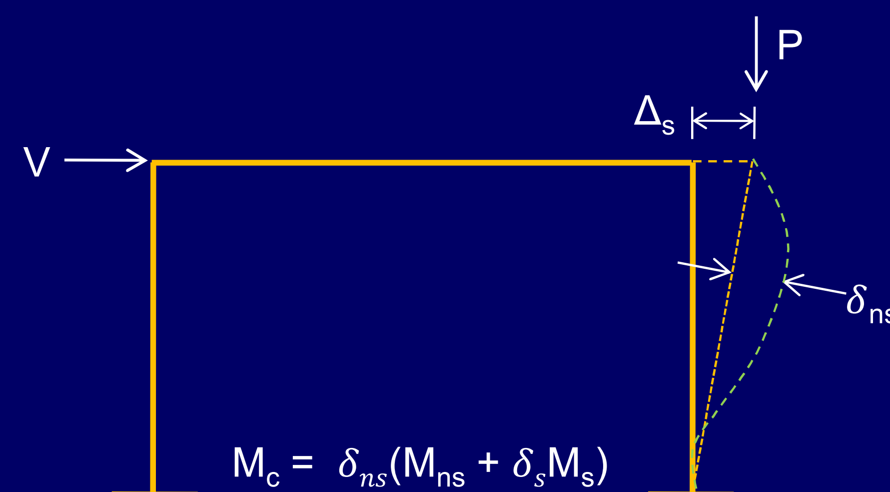
❑ Moment Magnification in Sway Frames

- Various possible scenarios are given below









Slenderness Effects in Sway Frames

Determination of Slenderness Effects

Moment Magnification for Sway Frames (ACI 6.6.4.6)

1. **Secondary Moments due to $P\Delta$ Effect**

- The $P\Delta$ effect are determined using following equation:

$$M_1 = M_{1ns} + \delta_s M_{1s}$$

$$M_2 = M_{2ns} + \delta_s M_{2s}$$

Where;

- M_{1ns}, M_{2ns} = first order moments due to gravity loads.
- M_{1s}, M_{2s} = first order moments due to lateral loads.
- δ_s = moment magnifier for sway frames (described next)

Slenderness Effects in Sway Frames

Determination of Slenderness Effects

Moment Magnification for Sway Frames (ACI 6.6.4.6)

1. **Secondary Moments due to $P\Delta$ Effect**

- The moment magnification factor for sway frames δ_s can be determined using any of the following methods:

- Method 1

$$\delta_s = \frac{1}{1 - Q} \geq 1$$

- This method closely predicts secondary effects as long as $\delta_s \leq 1.5$.
- If $\delta_s > 1.5$, method 2 shall be used.

$$Q = \frac{\sum P_u \Delta_o}{V_u l_c}$$

Where;

- $\sum P_u$ = total factored vertical load
- Δ_o = first order relative deflection between top and bottom of the story
- V_u = total story shear
- l_c = length of compressive member measured c/c of joint in frame.

Slenderness Effects in Sway Frames

Determination of Slenderness Effects

Moment Magnification for Sway Frames (ACI 6.6.4.6)

1. **Secondary Moments due to PΔ Effect**

- Method 2

$$\delta_s = \frac{1}{1 - \frac{\sum P_u}{0.75 \sum P_c}} \geq 1$$

Where;

- $\sum P_u$ is the summation for all the vertical loads in a story of a 3D structure.
- $\sum P_c$ is the summation for all sway resisting columns in a story of a 3D structure.

Slenderness Effects in Sway Frames

❑ Determination of Slenderness Effects

❖ Moment Magnification for Sway Frames (ACI 6.6.4.6)

2. **Secondary Moments due to Pδ Effect**

- Once the moments due to PΔ effect are determined, the second step is to calculate secondary moments along the member length (Pδ).
- The magnified moment due to Pδ effect can be determined using the non-sway frame procedure substituting the magnified moments of PΔ.

$$M_c = \delta_{ns}(M_2)$$

where; $M_2 = M_{ns} + \delta_s M_s$. Substituting this, we get

$$\underline{M_c} = \delta_{ns}(M_{ns} + \delta_s M_s)$$

The magnified moment M_C now includes both PΔ and Pδ effects

Slenderness Effects in Sway Frames

❑ Stepwise Procedure for Determining Secondary Moments

1. Perform primary analysis
2. Calculate stability index Q to classify frames as sway or non-sway
   - The following load combinations are considered
     - **Case 1: Gravity Loads Only**

       1.2D + 1.6L ……………..……….. (i)

     - **Case 2: Gravity Plus Lateral Load**

       1.2D + 1.0L + 1.0E ………………... (ii)

       0.9D + 1.0E ……………………… (iii)

Q will be calculated for these combinations only

Slenderness Effects in Sway Frames

❑ Stepwise Procedure for Determining Secondary Moments

3. Determine the slenderness ratio using k = 1.0 and compare it to code-specified limits to decide whether to neglect or consider secondary effects.

   - If slenderness effects needs to be considered, the actual value of effective length factor *k* is determined by finding values of Ψ_A and Ψ_B and then using relevant Moreland chart.

$$\left.\begin{aligned}\Psi_A &= \frac{I_{c1}/l_{c1} + I_{c2}/l_{c2}}{I_{B1}/l_{B1} + I_{B2}/l_{B2}} \\ \Psi_B &= \frac{I_{c2}/l_{c2} + I_{c3}/l_{c3}}{I_{B2}/l_{B2} + I_{B3}/l_{B3}}\end{aligned}\right\}$$

Where; $I_c = 0.7I_g$

$I_b = 0.35I_g$

Slenderness Effects in Sway Frames

Stepwise Procedure for Determining Secondary Moments

4. Analyze the frame for non-sway case under gravity loads only.
   - For this, $M_c = \delta_{ns} M_{ns}$ and load combination: 1.2D + 1.6L
5. Analyze the frame for sway case under gravity plus lateral loads
   - For this, two load combinations $1.2D + 1.0L \pm E$ and $0.9D \pm E$ are used
   - Moments are divided in two groups:
     i. Moments in column due to gravity loads: $M_{ns} = 1.2M_D + 1.0M_L$
     ii. Moments in column due to lateral loads: $M_s = 1.0E$
   - Magnified moments due to $P\Delta$ effect are calculated using

$$M_1 = M_{ns1} + \delta_s M_{s1}$$

$$M_2 = M_{ns2} + \delta_s M_{s2}$$

Slenderness Effects in Sway Frames

❑ Stepwise Procedure for Determining Secondary Moments

5. Analyze the frame for sway case under gravity plus lateral loads

- Magnified moment including both $P\delta$ and $P\Delta$ effects are calculated as follows:

$$M_c = \delta_{ns} M_2 \quad ; \; where \;\; M_2 = M_{2ns} + \delta_s M_{2s}$$

- The value of δ_{ns} in this case is determined by substituting magnified moments in C_m and taking $EI = 0.4E_c I_g$ in critical buckling formula.

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75\,P_c}} \geq 1$$

$$C_m = 0.6 + 0.4(M_1/M_2) \;\geq 0.4$$

$$P_c = \frac{\pi^2}{(kl_u)^2} EI$$

Here;

$M_1 = M_{1ns} + \delta_s M_{1s}$

$M_2 = M_{2ns} + \delta_s M_{2s}$

$EI = 0.4E_c I_g$

Slenderness Effects in Sway Frames

❑ Stepwise Procedure for Determining Secondary Moments

6. Check for the stability of structural system using code-prescribed method.

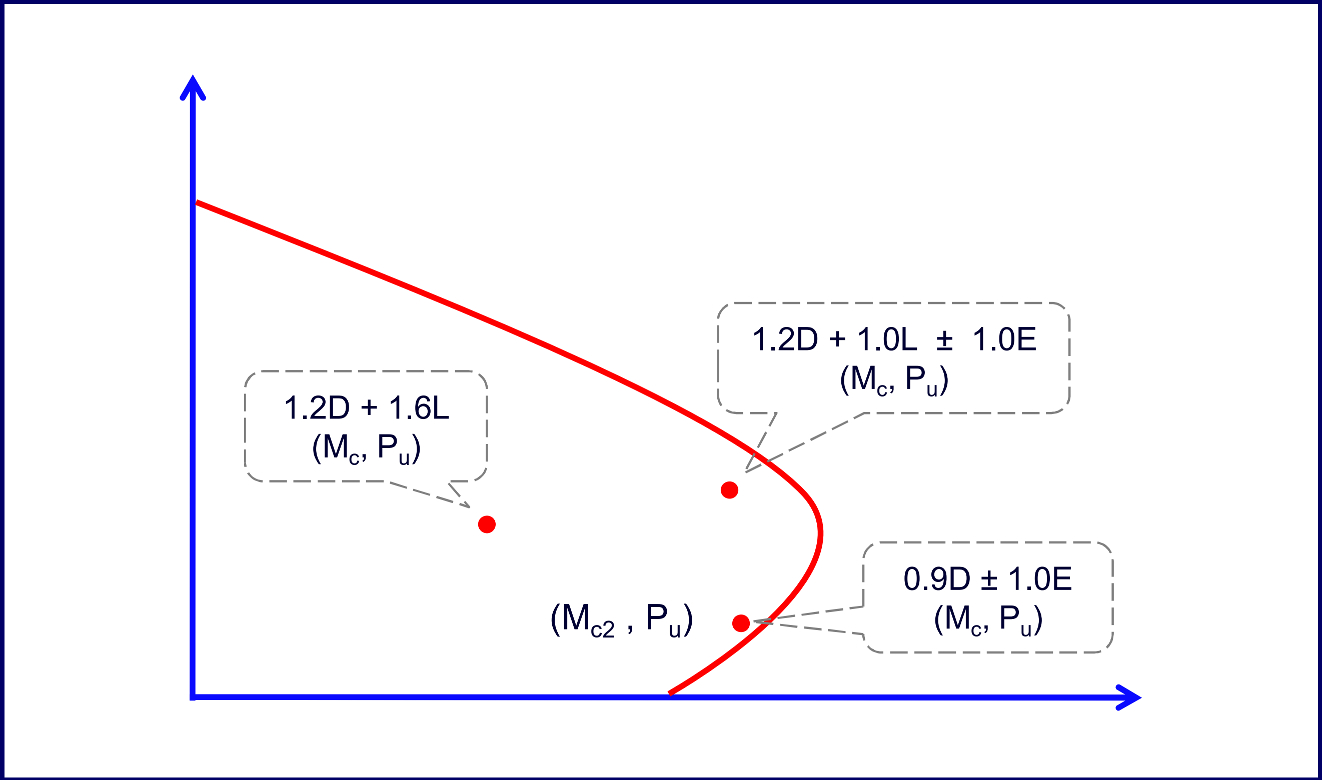
   - Second-order moments shall not exceed 40 percent of first-order (primary) moments (ACI 6.2.5.3).

$$M_{2nd\ order}/M_{1st\ order} \leq 1.4$$

Slenderness Effects in Sway Frames

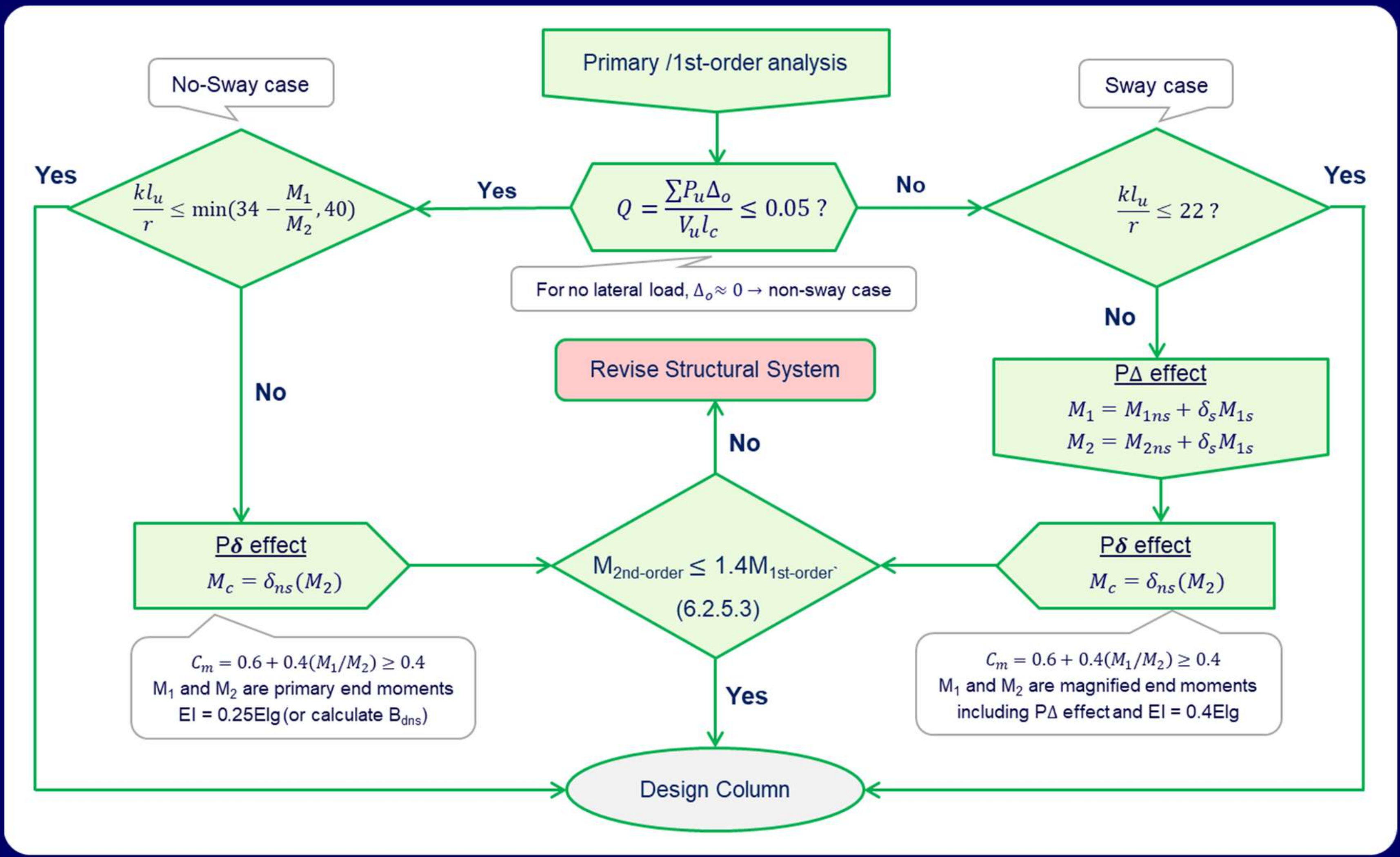
❑ Stepwise Procedure for Determining Secondary Moments

7. Design the column for axial loads and moments resulting from all load combinations as previously discussed. The design corresponding to critical situation is adopted.



Slenderness Effects in Sway Frames

❑ Workflow for Determining Column Slenderness Effects

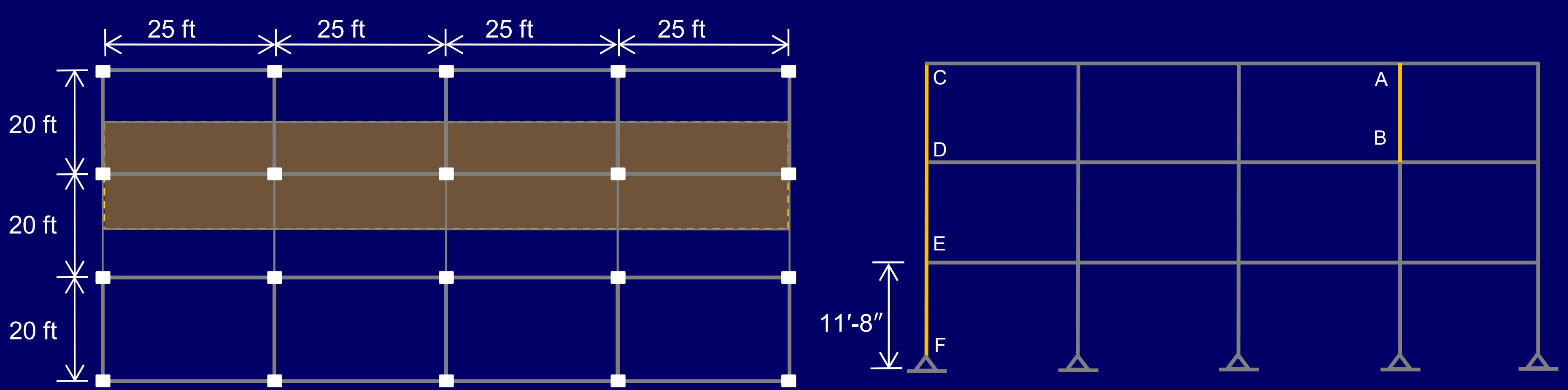


Example 11.2

❑ Problem Statement

Evaluate the slenderness effects in columns AB, CD and EF for the multi-story reinforced concrete frame shown below.

- All columns: 14″x14″ ; All beams: 14″x20″
- Story height: 11′-8″ (c/c) ; E = 3600 ksi

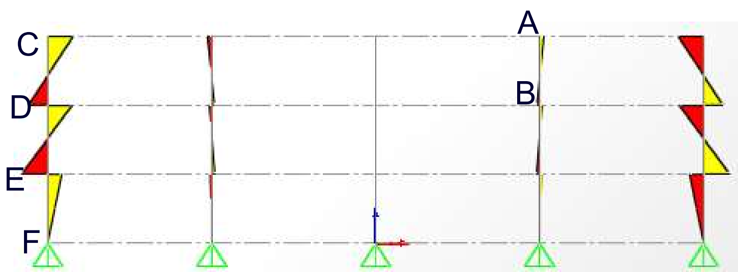


Example 11.2

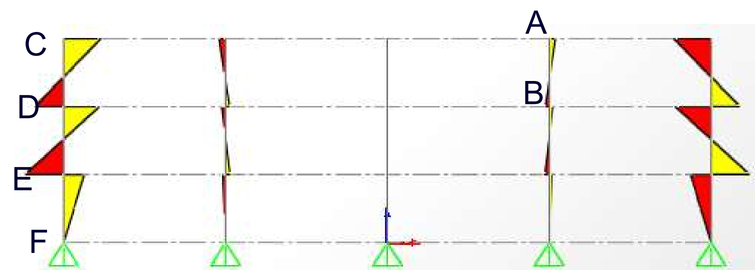
❑ Solution

➢ Step 1: Primary Analysis

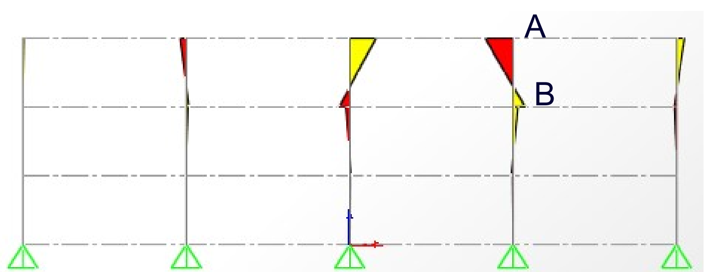
- The first-order linear elastic order analysis was carried out in the analysis software. The bending moments diagrams for various loadings are shown below.



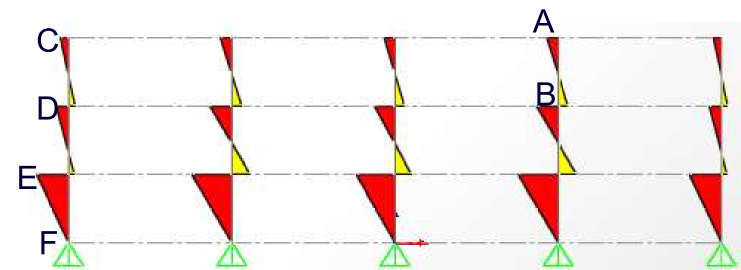
Service DL BMD shape for columns



Service LL BMD shape for columns



Service Pattern LL BMD shape for column AB



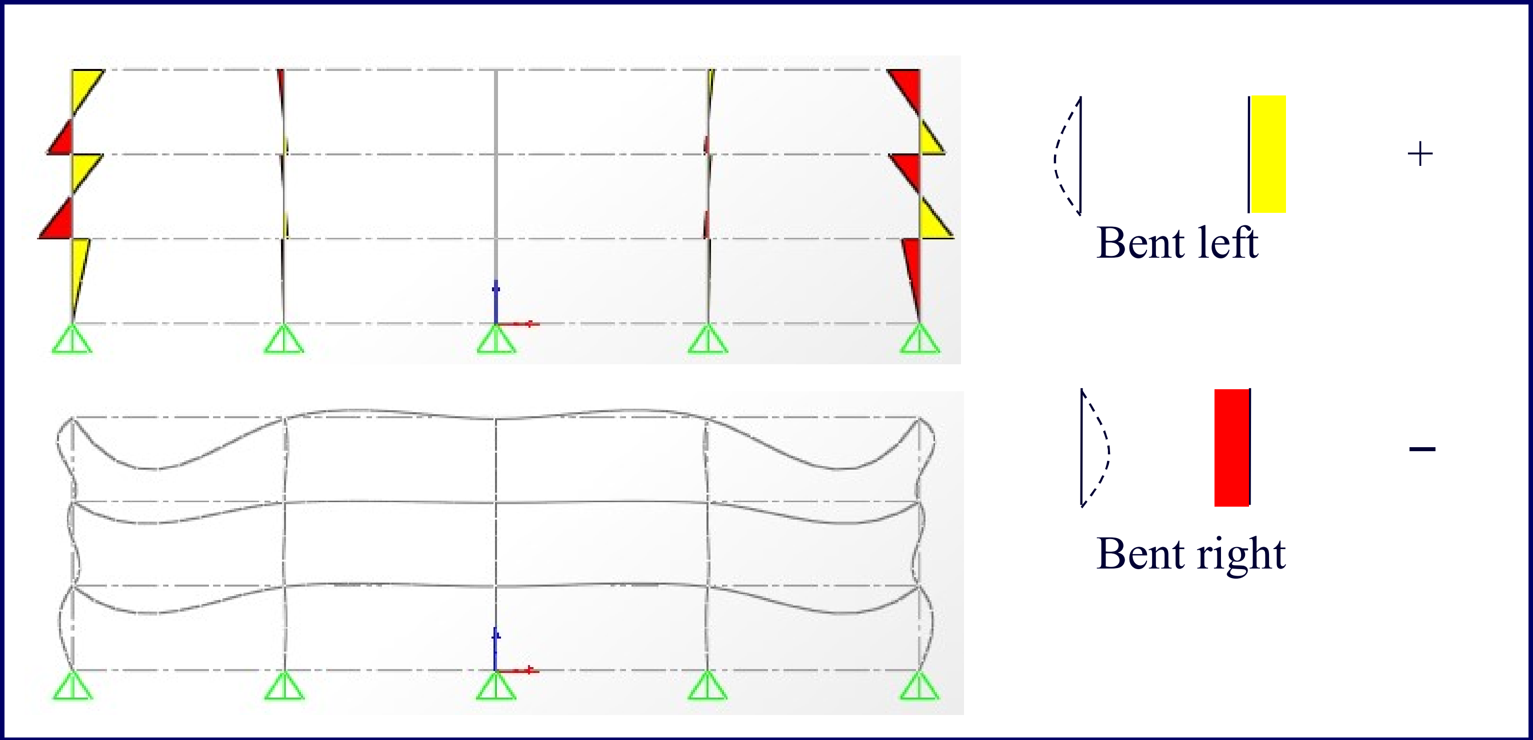
Lateral EQ Load BMD shape

Example 11.2

❑ Solution

➢ Step 1: Primary Analysis

- Sign Convention for bending Moment Diagram



Example 11.2

❑ Solution

➢ Step 1: Primary Analysis

❖ Results Obtained From Primary Analysis

Column	Service Load Effects					
	P_D (kip)	P_L (kip)	P_E (kip)	M_D (ft-kip)	M_L (ft-kip)	M_E (ft-kip)
AB	65	80	0	5.78	-37	-65
CD	31	33	0	44	60	-45
EF	94	102	28	20	27	-175

Example 11.2

Solution

Step 1: Primary Analysis

- The following Load Combinations would be considered for combining the previously calculated load effects.

  - **Case 1: Gravity Loads Only**

    1.2D + 1.6L ………………….. (i)

  - **Case 2: Gravity Plus Lateral Load**

    1.2D + 1.0L + 1.0E ……………….. (ii)

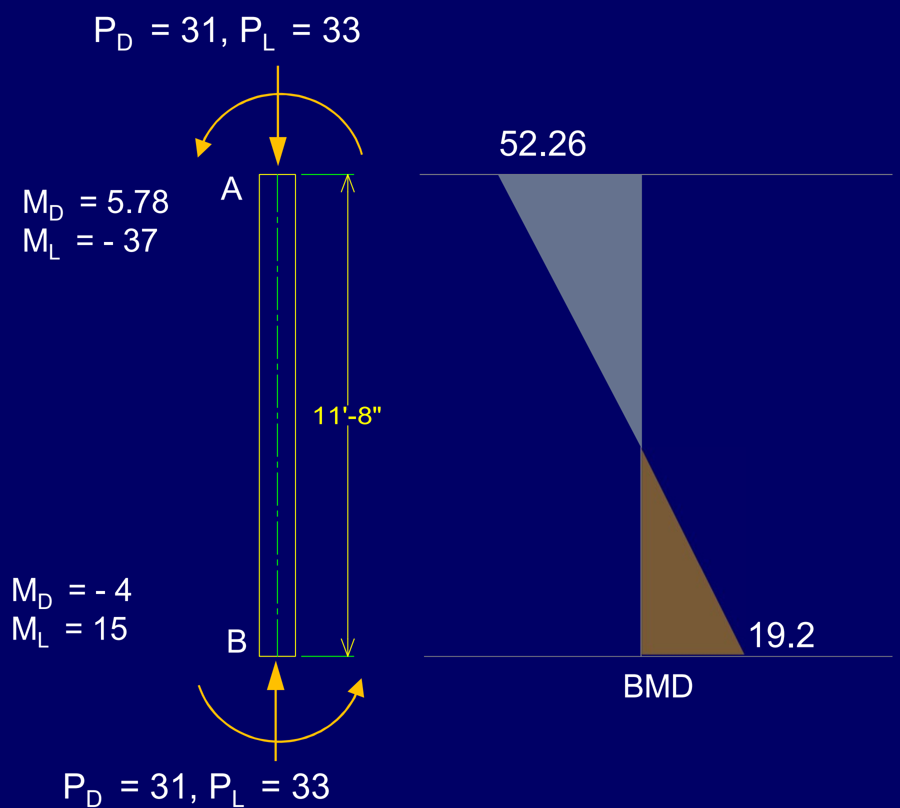
    0.9D + 1.0E ……………………… (iii)

Example 11.2

❑ Solution

➢ Step 1: Primary Analysis – (1.2D + 1.6L)

❖ **Calculation of M_{ns} and M_s for Column AB**



P_u = 206 kip

M_1 = - 19.2 ft-kip

(double curvature)

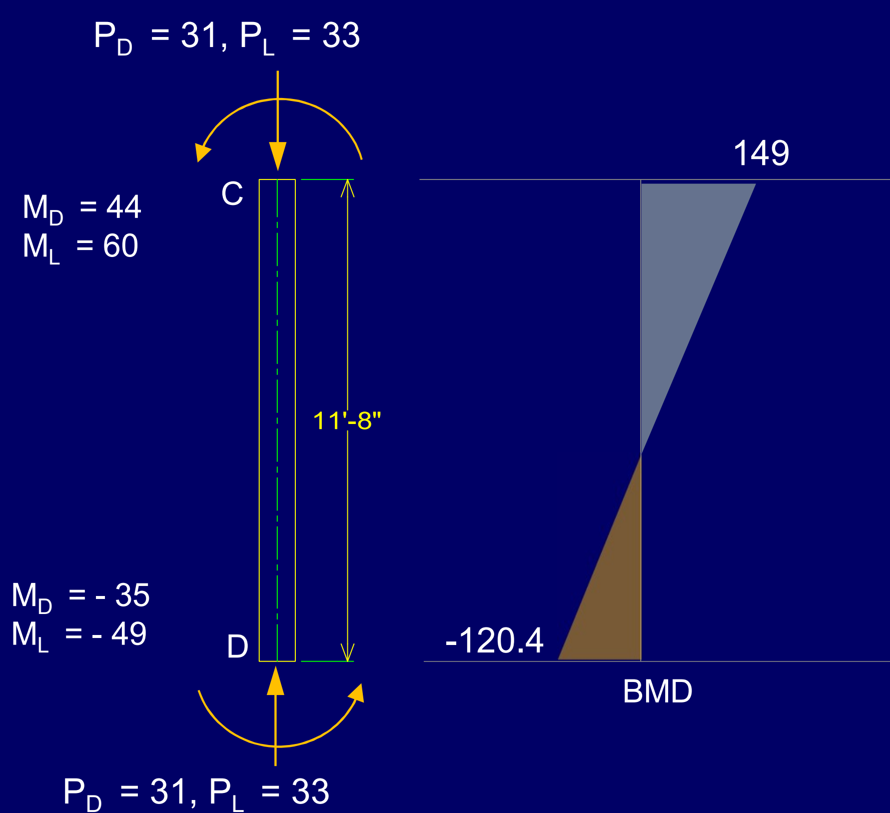
M_2 = 52.26 ft-kip

Example 11.2

❑ Solution

➢ Step 1: Primary Analysis – (1.2D + 1.6L)

❖ Calculation of M_{ns} and M_s for Column CD



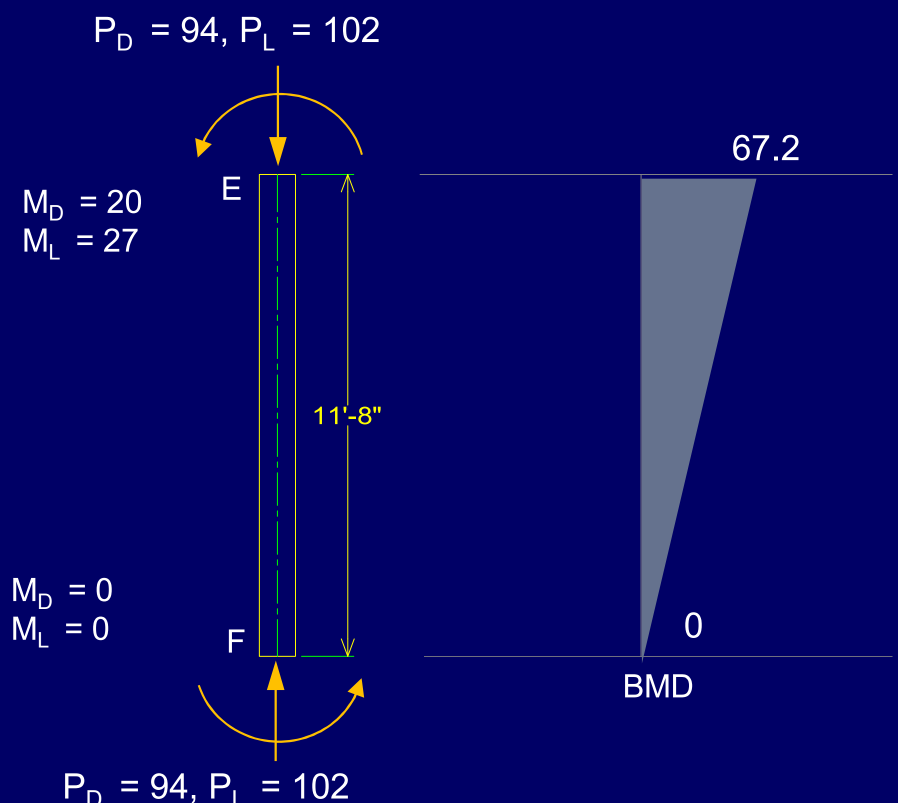
P_u = 90 kip
M_1 = - 120.4 ft-kip
(double curvature)
M_2 = 149 ft-kip

Example 11.2

❑ Solution

➢ Step 1: Primary Analysis – (1.2D + 1.6L)

❖ Calculation of M_{ns} and M_s for Column EF



P_u = 276 kip

M_1 = 0 ft-kip

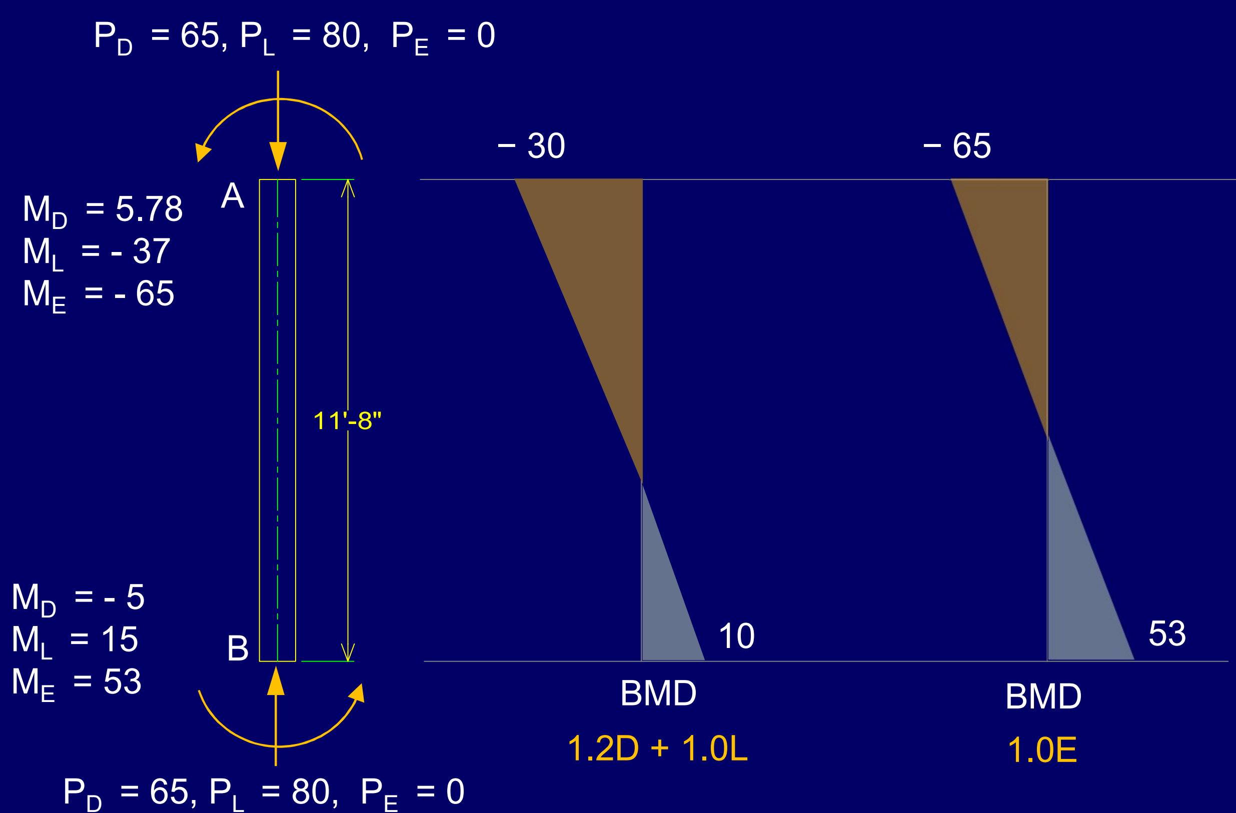
M_2 = 67.2 ft-kip

Example 11.2

❑ Solution

➢ Step 1: Primary Analysis – (1.2D + 1.0L + 1.0E)

❖ **Calculation of M_{ns} and M_s for Column AB**



P_u = 158 kip

M_{1ns} = 10 ft-kip

M_{1s} = 53 ft-kip

M_{2ns} = − 30 ft-kip

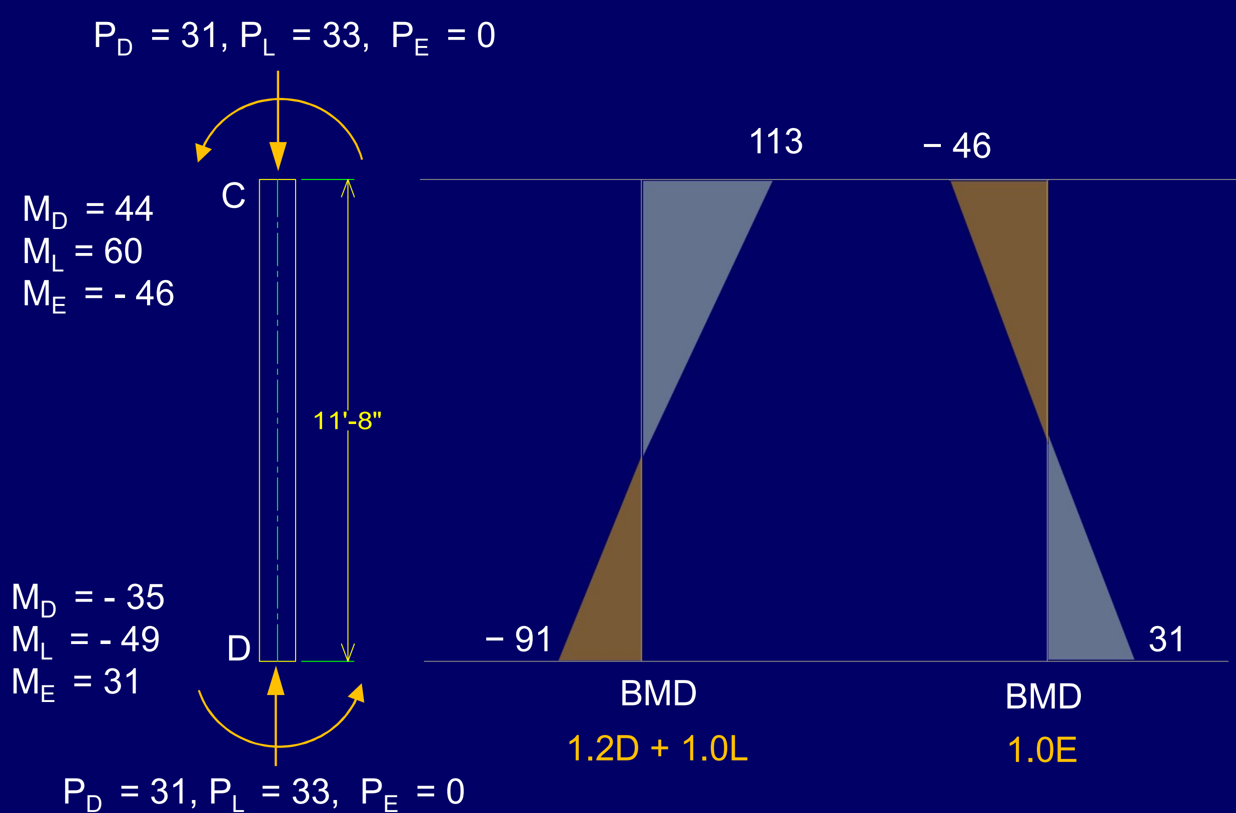
M_{2s} = − 65 ft-kip

Example 11.2

❑ Solution

➢ Step 1: Primary Analysis – (1.2D + 1.0L + 1.0E)

❖ **Calculation of M_{ns} and M_s for Column CD**



P_u = 70.2 kip

M_{1ns} = − 91 ft-kip

M_{1s} = 31 ft-kip

M_{2ns} = 113 ft-kip

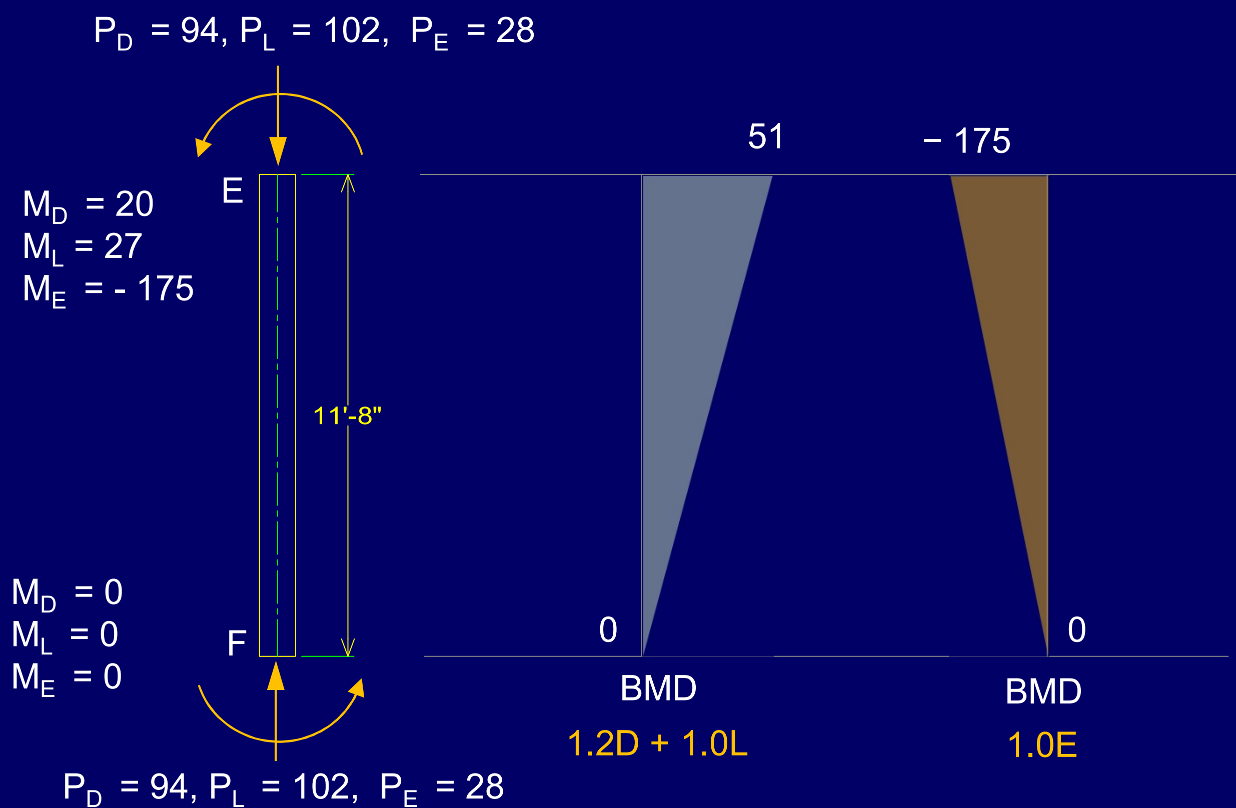
M_{2s} = − 46 ft-kip

Example 11.2

❑ Solution

➢ Step 1: Primary Analysis – (1.2D + 1.0L + 1.0E)

❖ **Calculation of M_{ns} and M_s for Column EF**



P_u = 214.8 kip

M_{1ns} = 0 ft-kip

M_{1s} = 0 ft-kip

M_{2ns} = 51 ft-kip

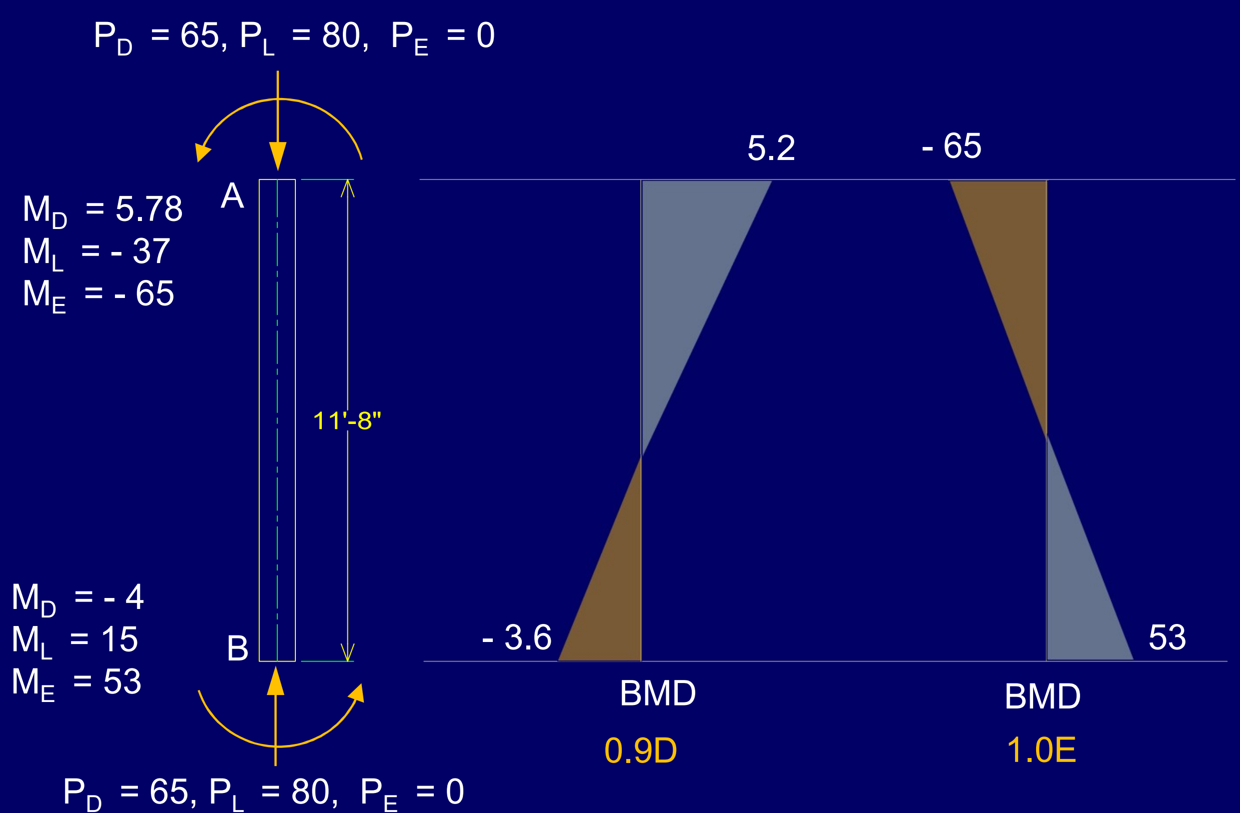
M_{2s} = − 175 ft-kip

Example 11.2

❑ Solution

➢ Step 1: Primary Analysis – (0.9D + 1.0E)

❖ **Calculation of M_{ns} and M_s for Column AB**



P_u = 58.5 kip

M_{1ns} = − 3.6 ft-kip

M_{1s} = 53 ft-kip

M_{2ns} = 5.2 ft-kip

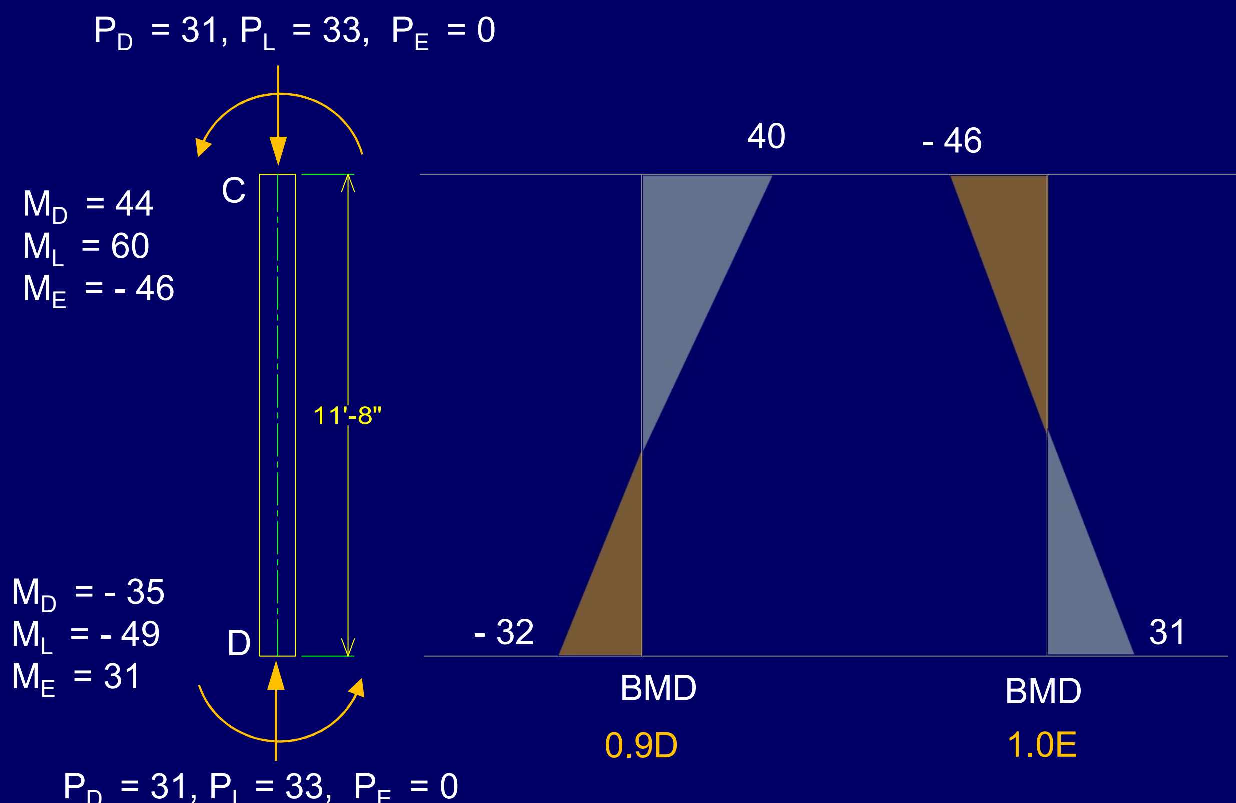
M_{2s} = − 65 ft-kip

Example 11.2

❑ Solution

➢ Step 1: Primary Analysis – (0.9D + 1.0E)

❖ **Calculation of M_{ns} and M_s for Column CD**



P_u = 27.9 kip

M_{1ns} = − 32 ft-kip

M_{1s} = 31 ft-kip

M_{2ns} = 40 ft-kip

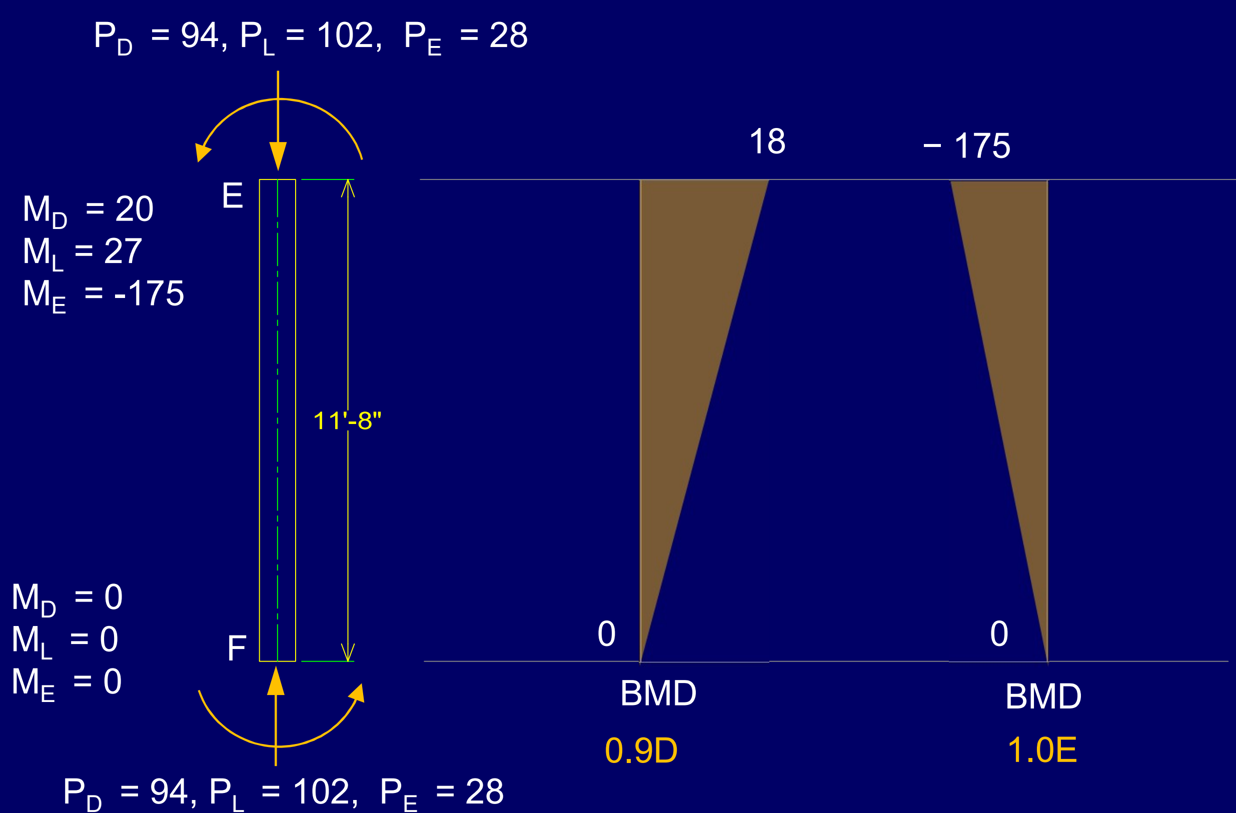
M_{2s} = − 46 ft-kip

Example 11.2

❑ Solution

➢ Step 1: Primary Analysis – (0.9D + 1.0E)

❖ Calculation of M_{ns} and M_s for Column EF



P_u = 84.6 kip

M_{1ns} = 0 ft-kip

M_{1s} = 0 ft-kip

M_{2ns} = 18 ft-kip

M_{2s} = −175 ft-kip

Example 11.2

❑ Solution

➢ **Step 1: Primary Analysis – Summary of Calculations**

Load Combination: 1.2D + 1.6L			
Column	**P_u (kip)**	**M_1 (ft-kip)**	**M_2 (ft-kip)**
AB	206	−19.2	52.26
CD	90	−120.4	149
EF	276	0	67.2

Example 11.2

❑ Solution

➢ **Step 1: Primary Analysis – Summary of Calculations**

Column	Load Combination: 1.2D + 1.0L + 1.0E				
	P_u (kip)	M_{1ns} (ft-kip)	M_{1s} (ft-kip)	M_{2ns} (ft-kip)	M_{2s} (ft-kip)
AB	158	10	53	− 30	− 65
CD	70.2	− 91	31	113	− 46
EF	214.8	0	0	51	− 175

Column	Load Combination: 0.9D + 1.0E				
	P_u (kip)	M_{1ns} (ft-kip)	M_{1s} (ft-kip)	M_{2ns} (ft-kip)	M_{2s} (ft-kip)
AB	58.5	− 3.6	53	5.2	− 65
CD	27.9	− 32	31	40	− 46
EF	84.6	0	0	18	− 175

Example 11.2

Solution

Step 2: Calculation of Stability Index Q

- A story is considered non-sway if stability index Q ≤ 0.05; otherwise, it is classified as sway.

$$Q = \frac{\sum P_u \Delta_o}{V_u l_c}$$

- **Case 1: Gravity Loads Only**

  1.2D + 1.6L ………………… (i)

  Already non-sway case. No need for calculating Q.

- **Case 2: Gravity Plus Lateral Load**

  1.2D + 1.0L + 1.0E ……………... (ii)

  0.9D + 1.0E ……………………. (iii)

  Q needs to be calculated for declaring sway or non-sway stories.

Example 11.2

❑ Solution

➢ **Step 2: Calculation of Stability Index Q (1.2D + 1.0L + 1.0E)**

❖ **Calculation of $\sum P_u$ for columns AB &CD**

No. of interior columns = 6

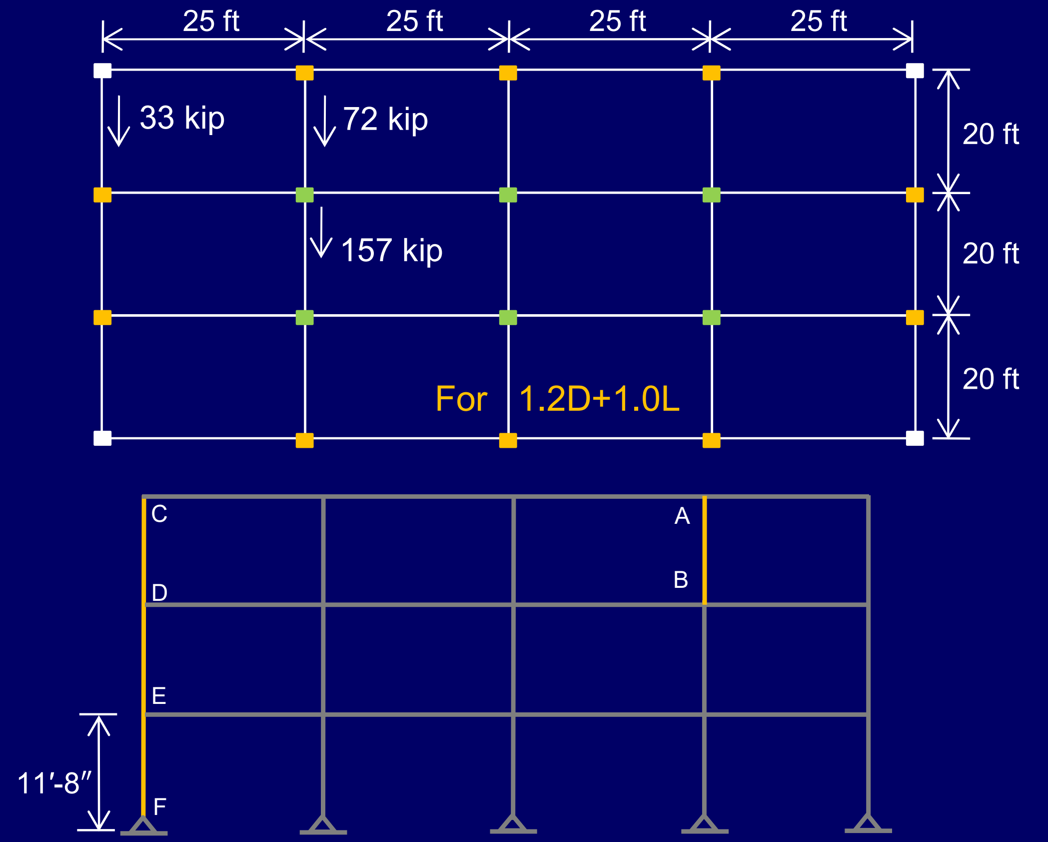
No. of edge columns = 10

No. of corner columns = 4

Now,

$\sum P_u = 4 \times 33 + 10 \times 72 + 6 \times 157$

$= 1794$ kip



Example 11.2

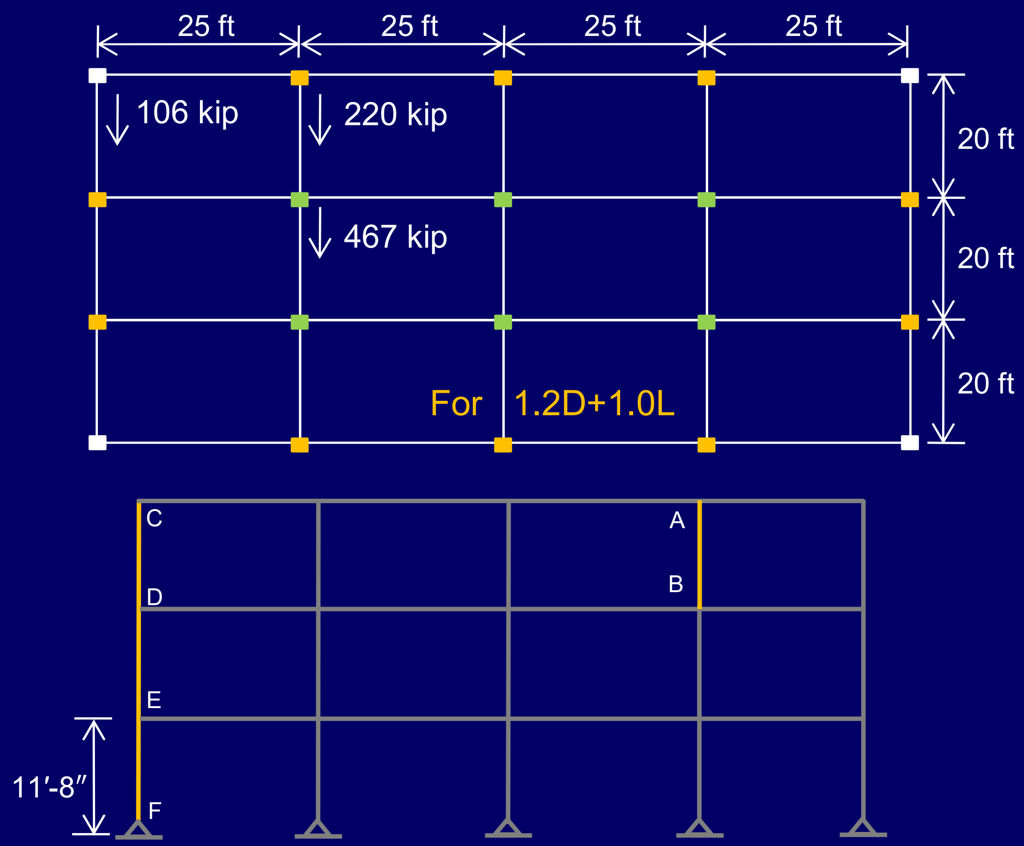
❑ Solution

➢ **Step 2: Calculation of Stability Index Q (1.2D + 1.0L + 1.0E)**

❖ **Calculation of $\sum P_u$ for column EF**

$\sum P_u$ = 4×106 + 10×220 + 6×467

= 5426 kip



Example 11.2

❑ Solution

➢ **Step 2: Calculation of Stability Index Q (1.2D + 1.0L + 1.0E)**

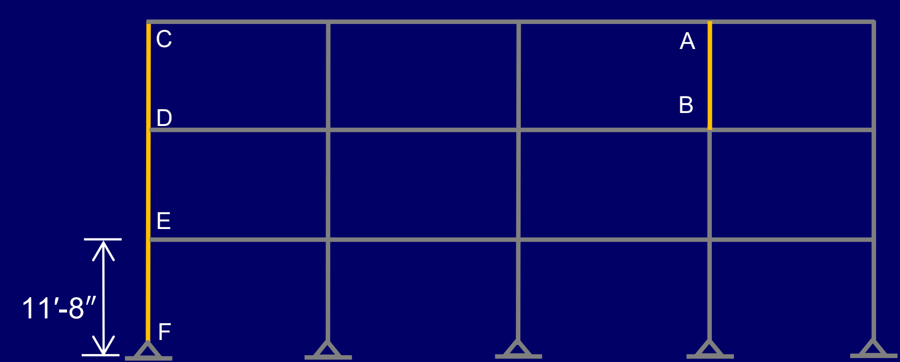
❖ **Calculation of V_u & Δ_o for columns AB & CD**

$V_u = 4\times6 + 6\times10 + 4\times7.8 + 6\times11.9$

$= 187$ kip

And

$\Delta_o = 0.36''$ (from analysis software)



Example 11.2

❑ Solution

➢ **Step 2: Calculation of Stability Index Q (1.2D + 1.0L + 1.0E)**

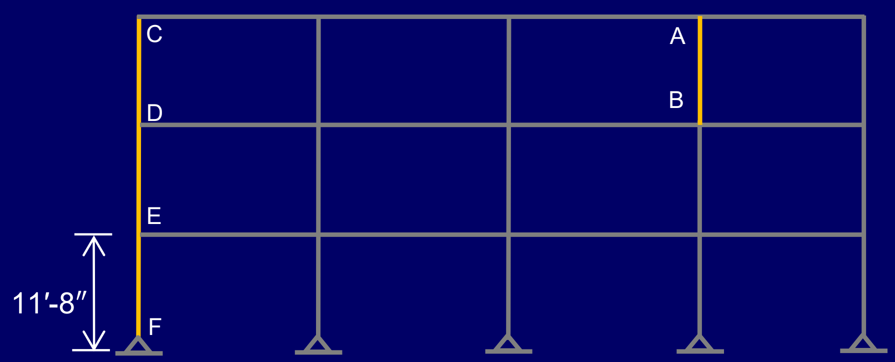
❖ **Calculation of V_u & Δ_o for column EF**

$V_u = 4\times16 + 6\times20 + 4\times17 + 6\times21$

$= 378$ kip

And

$\Delta_o = 1.9''$ (from analysis software)



Example 11.2

Solution

Step 2: Calculation of Stability Index Q (1.2D + 1.0L + 1.0E)

Column	$\sum P_u$ (kip)	l_c (in.)	V_u (kip)	Δ_o (in.)	$Q = \frac{\sum P_u \Delta_o}{V_u l_c}$	Remarks
AB	1794	140	187	0.36	0.03	Q < 0.05 → non-sway
CD	1794	140	187	0.36	0.03	Q < 0.05, → non-sway
EF	5426	140	378	1.9	0.19	Q > 0.05 → Sway

- Hence for columns AB and CD that are part of a non-sway story, $P\Delta$ will be ignored and magnified moment due to Pδ effect only is determined as;

$$M_c = \delta_{ns}(M_{ns} + \delta_s M_s)\ ; \quad \delta_s = 1$$

- For column EF that is part of sway story, the magnified moment considering both $P\Delta$ and Pδ effects will be calculated using:

$$M_c = \delta_{ns}(M_{ns} + \delta_s M_s)\ ; \quad \delta_s \neq 1$$ (needs to be calculated)

Example 11.2

❑ Solution

➢ **Step 2: Calculation of Stability Index Q (0.9D + 1.0E)**

❖ **Calculation of $\sum P_u$ for columns AB &CD**

No. of interior columns = 6

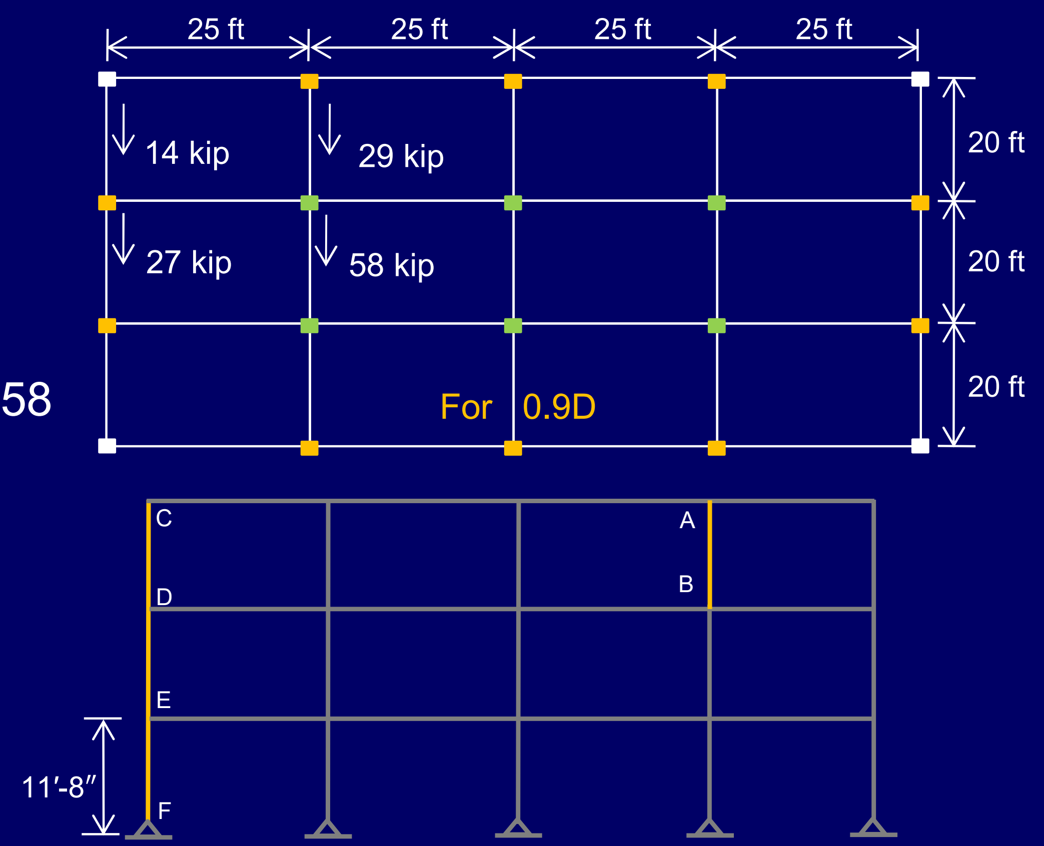
No. of edge columns = 10

No. of corner columns = 4

Now,

$\sum P_u = 4\times14 + 6\times29 + 4\times27 + 6\times58$

$= 686$ kip



Example 11.2

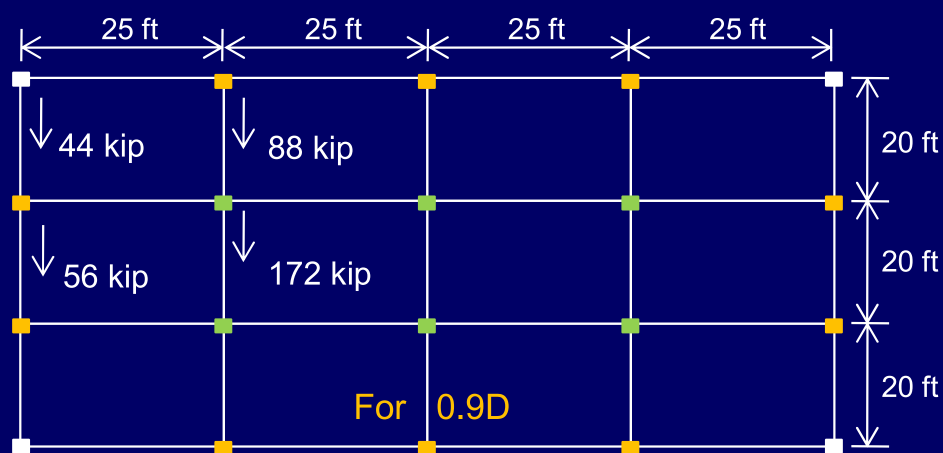
❑ Solution

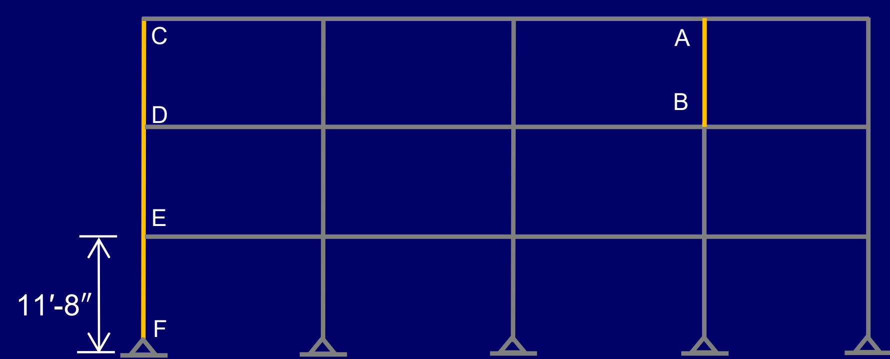
➢ Step 2: Calculation of Stability Index Q (0.9D + 1.0E)

❖ **Calculation of $\sum P_u$ for column EF**

$$\sum P_u = 4\times44 + 6\times88 + 4\times56 + 6\times172$$

$$= 1960 \text{ kip}$$





Example 11.2

❑ Solution

- **Step 2: Calculation of Stability Index Q (0.9D +1.0E)**

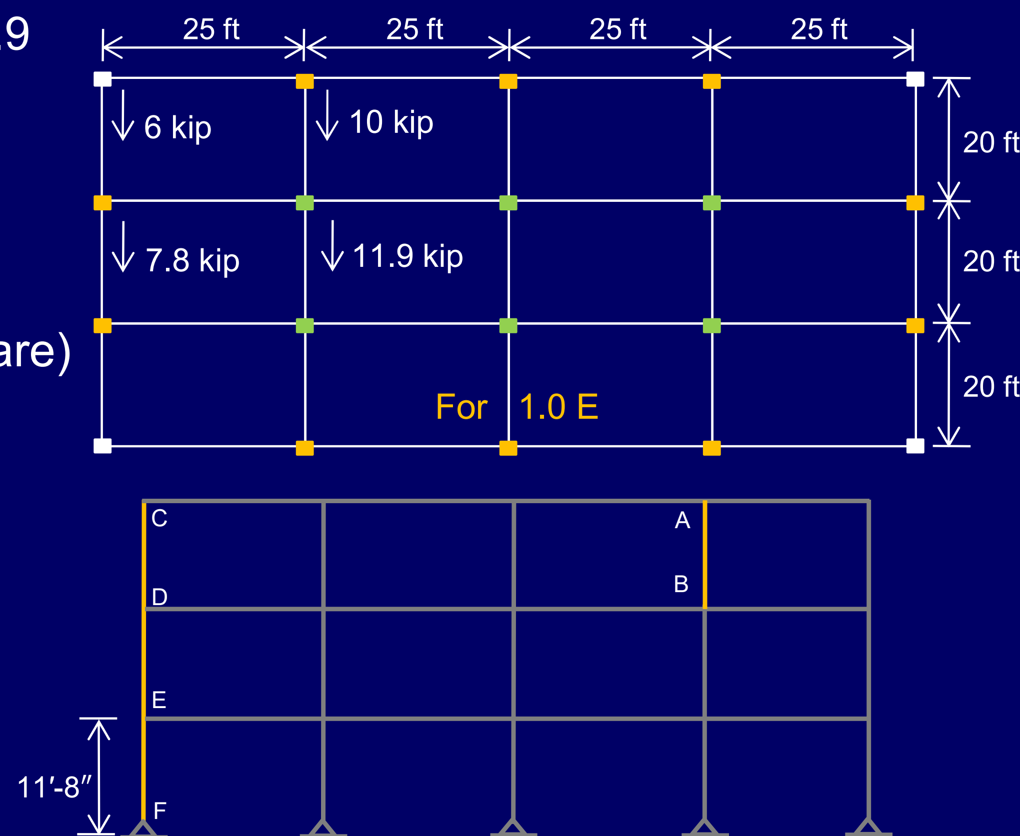
  - **Calculation of Vu & Δ_o for columns AB & CD**

$V_u = 4\times6 + 6\times10 + 4\times7.8 + 6\times11.9$

$= 187$ kip

And

$\Delta_o = 0.36''$ (from analysis software)



Example 11.2

❑ Solution

➢ **Step 2: Calculation of Stability Index Q (0.9D + 1.0E)**

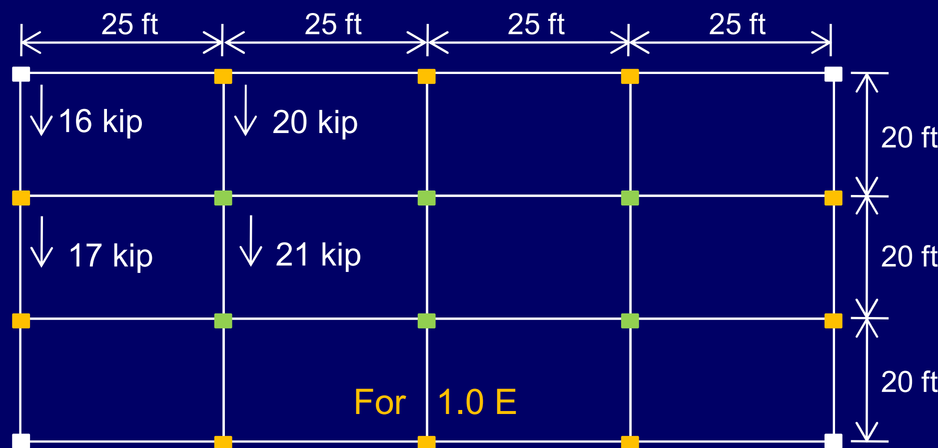
❖ **Calculation of Vu & Δ_o for column EF**

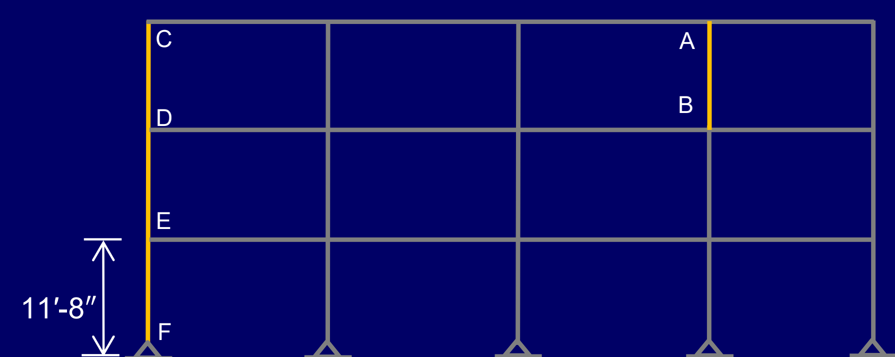
V_u = 4×16 + 6×20 + 4×17 + 6×21

= 378 kip

And

Δ_o = 1.9″ (from analysis software)





Example 11.2

❑ Solution

➢ Step 2: Calculation of Stability Index Q (0.9D + 1.0E)

Column	$\sum P_u$ (kip)	l_u (ft)	V_u (kip)	Δ_o (in.)	$Q = \frac{\sum P_u \Delta_o}{V_u l_c}$	Remarks
AB	686	140	187	0.36	0.01	Q < 0.05 → non-sway
CD	686	140	187	0.36	0.01	Q < 0.05, → non-sway
EF	1960	140	378	1.9	0.07	Q > 0.05 → sway

- Hence for columns AB and CD that are part of a non-sway story, $P\Delta$ will be ignored and magnified moment due to Pδ effect is determined as;

$$M_c = \delta_{ns}(M_{ns} + \delta_s M_s)\ ; \quad \delta_s = 1$$

- For column EF that is part of sway story, the magnified moment considering both $P\Delta$ and Pδ effects will be calculated using:

$$M_c = \delta_{ns}(M_{ns} + \delta_s M_s)\ ; \quad \delta_s \neq 1$$ (needs to be calculated)

Example 11.2

❑ Solution

➢ Step 3: Magnified Moments for Case 1 (1.2D + 1.6L)

❖ **Check if the slenderness effects can be ignored**

Assuming k = 1, determine slenderness ratio and check with code-specified limit

Column	l_u (in.)	$r = 0.3h$ (in.)	kl_u/r	$\frac{M_1}{M_2}$	$min\left(34 - \frac{12M_1}{M_2}, 40\right)$	Remarks
AB	120	0.3(14) = 4.2	28.57	−19.2/52.26 = − 0.37	min(38.44, 40) = 38.4	$\frac{kl_u}{r} < 38.4$ Neglect slenderness effects
CD	120	0.3(14) = 4.2	28.57	−120.4/149 = − 0.81	min(43.7, 40) = 40	$\frac{kl_u}{r} < 40$ Neglect slenderness effects
EF	120	0.3(14) = 4.2	28.57	0/67.2 = 0	min(34, 40) = 34	$\frac{kl_u}{r} < 34$ Neglect slenderness effects

Example 11.2

Solution

- **Step 3: Magnified Moments for Case 2 (1.2D + 1.0L + 1.0E)**

  - **Check if the slenderness effects can be ignored**

Assuming k = 1, determine slenderness ratio and check with code-specified limit.

Slenderness effects in sway frames can be neglected if

$$\frac{kl_u}{r} \leq 22$$

$$\frac{kl_u}{r} = \frac{1 \times 120}{0.3(14)} = 28.57$$

Since 28.57 > 22 , the slenderness effects cannot be neglected.

Now calculate the actual value of effective length factor k using Moreland Chart

Example 11.2

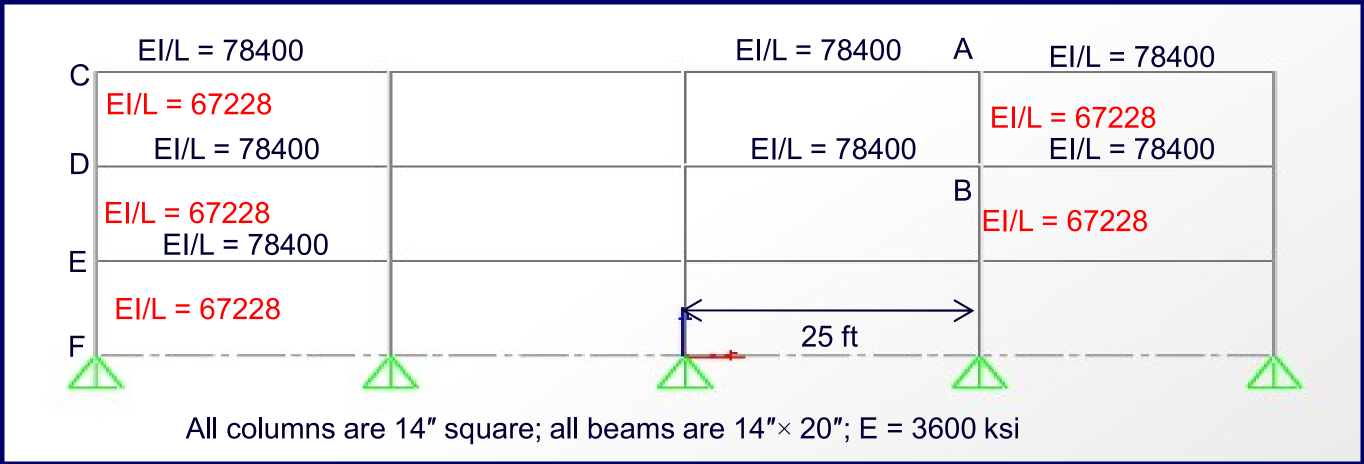
❑ Solution

➢ Step 3: Magnified Moments for Case 2 (1.2D + 1.0L + 1.0E)

❖ **Calculate effective length factor k**

$$\Psi_A = \frac{67228}{2 \times 78400} = 0.43 \quad ; \Psi_B = \frac{2 \times 67228}{2 \times 78400} = 0.85 \quad ; \Psi_C = \frac{67228}{78400} = 0.85$$

$$\Psi_D = \frac{2 \times 67228}{78400} = 1.71 \quad ; \quad \Psi_E = \frac{2 \times 67228}{78400} = 1.71 \; ; \; \Psi_F = \infty$$



Example 11.2

❑ Solution

➢ Step 3: Magnified Moments for Case 2 (1.2D + 1.0L + 1.0E)

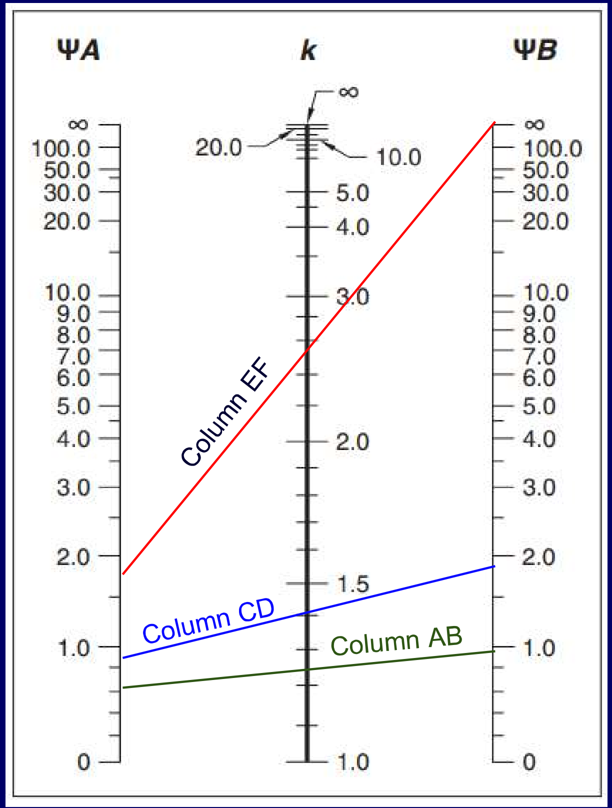
❖ Calculate effective length factor k

The values of k for using Moreland Alignment Chart are given below.

$k_{AB} = 1.25$

$k_{CD} = 1.4$

$k_{EF} = 2.7$



Example 11.2

❑ Solution

➢ Step 3: Magnified Moments for Case 2 (1.2D + 1.0L + 1.0E)

❖ **Calculate magnified moments due to $P\Delta$**

$$M_1 = M_{1ns} + \delta_s M_{1s}$$

$$M_2 = M_{2ns} + \delta_s M_{2s}$$

$$\delta_s = \frac{1}{1 - Q}$$

$$M_{2,min} = P_u(0.6 + 0.03h)$$

Column	P_u (kip)	Q	δ_s	M_{1ns} (ft.kip)	M_{1s} (ft.kip)	M_{2ns} (ft.kip)	M_{2s} (ft.kip)	M_1 (ft.kip)	M_2 (ft.kip)	M_{min} (ft.kip)
AB	158	0.03	1	10	53	− 30	− 65	63	− 95	13.4
CD	70.2	0.03	1	− 91	31	113	− 46	− 60	67	6.0
EF	214.8	0.19	1.23	0	0	51	−175	0	− 215.3	18.3

Example 11.2

❑ Solution

➢ Step 3: Magnified Moments for Case 2 (1.2D + 1.0L + 1.0E)

- ❖ **Calculate magnified moments due to $P\delta$ and $P\Delta$ effects**

$$M_c = \delta_{ns} M_2 \ ; \ \delta_{ns} = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}} \geq 1$$

$$P_c = \frac{\pi^2 EI}{(kl_u)^2}$$

$$EI = 0.4E_c I_g = 0.4 \times 3600 \left(\frac{14 \times 14^3}{12}\right) = 4.61 \times 10^6 \text{ kip.in}^2$$

And

$$C_m = 0.6 + 0.4(M_1/M_2) \geq 0.4$$

Example 11.2

❑ Solution

➢ **Step 3: Magnified Moments for Case 2 (1.2D + 1.0L + 1.0E)**

❖ **Calculate magnified moments due to $P\delta$ and $P\Delta$ effects**

Column	P_u (kip)	k	$0.75P_c$ (kip)	M_1/M_2	C_m	$\delta_{ns} \geq 1$	$M_c = \delta_{ns} max\,(M_1, M_2)$ (ft.kip)
AB	158	1.25	1516.6	63/ − 95 = − 0.66	0.34 < 0.4	0.45 < 1	1 × (− 95) = − 95
CD	70.2	1.4	1209.0	− 60/67 = − 0.90	0.24 < 0.4	0.42 < 1	1 × 67 = 67
EF	214.8	2.7	325.1	0/− 215.3 = 0	0.6 > 0.40	1.77	1.77 × (− 215.3) = − 381.1

Example 11.2

Solution

➢ Step 3: Magnified Moments for Case 2 (1.2D + 1.0L - 1.0E)

❖ Calculate magnified moments due to $P\delta$ and $P\Delta$ effects

Column	P_u (kip)	Q	δ_s	M_{1ns} (ft.kip)	M_{1s} (ft.kip)	M_{2ns} (ft.kip)	M_{2s} (ft.kip)	M_1 (ft.kip)	M_2 (ft.kip)	M_{min} (ft.kip)
AB	158	0.03	1	10	− 53	− 30	+ 65	− 43	35	5
CD	70.2	0.03	1	− 91	− 31	113	+ 46	− 122	159	2.4
EF	214.8	0.19	1.23	0	0	51	+175	0	266.3	7.2

Column	P_u (kip)	k	$0.75P_c$ (kip)	M_1/M_2	$C_m \geq 0.4$	$\delta_{ns} \geq 1$	$M_c = \delta_{ns} max\,(M_1, M_2)$ (ft.kip)
AB	158	1.25	1516.6	− 43/35 = − 1.22	0.11 < 0.4	0.45 < 1	1 × (− 43) = − 43
CD	70.2	1.4	1209.0	46/− 122 = − 0.38	0.45	0.48 < 1	1 × 159 = 159
EF	214.8	2.7	325.1	0/266.3 = 0	0.6	1.77	1.77 × 266.3 = 471.4

Example 11.2

❑ Solution

➢ **Step 3: Magnified Moments for Case 2 (0.9D + 1.0E)**

- In the subsequent slides, the magnified moments for the second load combination 0.9D + 1.0E. Will be determined by repeating the same procedure as adopted for 1.2D + 1.0E.

Example 11.2

❑ Solution

➢ Step 3: Magnified Moments for Case 2 (0.9D + 1.0E)

- Calculate magnified moments due to $P\delta$ and $P\Delta$ effects

Column	P_u (kip)	Q	δ_s	M_{1ns} (ft.kip)	M_{1s} (ft.kip)	M_{2ns} (ft.kip)	M_{2s} (ft.kip)	M_1 (ft.kip)	M_2 (ft.kip)	M_{min} (ft.kip)
AB	58.5	0.01	1	− 3.6	53	5.2	− 65	49.4	− 59.8	5.0
CD	27.9	0.01	1	− 32	31	40	− 46	− 1.0	− 6.0	2.4
EF	84.6	0.07	1.08	0	0	51	− 175	0	− 138.0	7.2

Column	P_u (kip)	k	$0.75P_c$ (kip)	M_1/M_2	$C_m \geq 0.4$	$\delta_{ns} \geq 1$	$M_c = \delta_{ns} max\,(M_1, M_2)$ (ft.kip)
AB	58.5	1.25	1516.6	− 59.8/49.4 = − 0.83	0.93	0.97 < 1	1 × (− 59.8) = − 59.8
CD	27.9	1.4	1209.0	− 1/− 6 = 0.17	0.53	0.54 < 1	1 × (− 6) = − 6
EF	84.6	2.7	325.1	0/− 138 = 0	0.6	0.81 < 1	1 × (− 138) = − 138

Example 11.2

Solution

Step 3: Magnified Moments for Case 2 (0.9D - 1.0E)

- Calculate magnified moments due to $P\delta$ and $P\Delta$ effects

Column	P_u (kip)	Q	δ_s	M_{1ns} (ft.kip)	M_{1s} (ft.kip)	M_{2ns} (ft.kip)	M_{2s} (ft.kip)	M_1 (ft.kip)	M_2 (ft.kip)	M_{min} (ft.kip)
AB	58.5	0.01	1	− 3.6	− 53	5.2	+ 65	56.6	70.2	5.0
CD	27.9	0.01	1	− 32	− 31	40	+ 46	− 63.0	− 86.0	2.4
EF	84.6	0.07	1.08	0	0	51	+ 175	0	240	7.2

Column	P_u (kip)	k	$0.75P_c$ (kip)	M_1/M_2	$C_m \geq 0.4$	$\delta_{ns} \geq 1$	$M_c = \delta_{ns} max\,(M_1, M_2)$ (ft.kip)
AB	58.5	1.25	1516.6	− 56.6/70.2 = − 0.81	0.28 < 0.4	0.42 < 1	1 × 70.2 = 70.2
CD	27.9	1.4	1209.0	− 63/− 86 = 0.73	0.9	0.92 < 1	1 × − 86 = − 86
EF	84.6	2.7	325.1	0/240 = 0	0.6	0.81 < 1	1 × 240 = 240

Example 11.2

❑ Solution

➢ Step 3: Magnified Moments – Summary

- The summary of magnified moments for all the load combinations is shown below.

Load Combination	Column AB		Column CD		Column EF	
	P_u (kip)	M_c (kip-ft)	P_u (kip)	M_c (kip-ft)	P_u (kip)	M_c (kip-ft)
1.2D + 1.6L	**206**	**52.3**	**90**	**149**	**276**	**67.2**
1.2D + 1.0L + 1.0E	**158**	**− 95**	70.2	67	214.8	− 381.1
1.2D + 1.0L − 1.0E	158	− 43	**70.2**	**159**	**214.8**	**471.4**
0.9D + 1.0E	58.5	− 59.8	27.9	− 6	84.6	− 138
0.9D − 1.0E	**58.5**	**70.2**	**27.9**	**− 86**	**84.6**	**240**

Example 11.2

❑ Solution

➢ Step 4: Check Stability of Structure

- Second-order moments shall not exceed 40 percent of first-order (primary) moments (ACI 6.2.5.3).

$$M_{2nd\ order}/M_{1st\ order} \leq 1.4$$

Column	1.2D + 1.0L ± 1.0E			0.9D ± 1.0E		
	$M_{1st\text{-}order}$ (kip-ft)	$M_{2nd\text{-}order}$ (kip-ft)	$\frac{M_{2nd}}{M_{1st}}$	$M_{1st\text{-}order}$ (kip-ft)	$M_{2nd\text{-}order}$ (kip-ft)	$\frac{M_{2nd}}{M_{1st}}$
AB	− 95	− 95	1	70.2	70.2	1
CD	159	159	1	86	− 86	1
EF	226	471.4	2.08 > 1.4	193	240	1.2

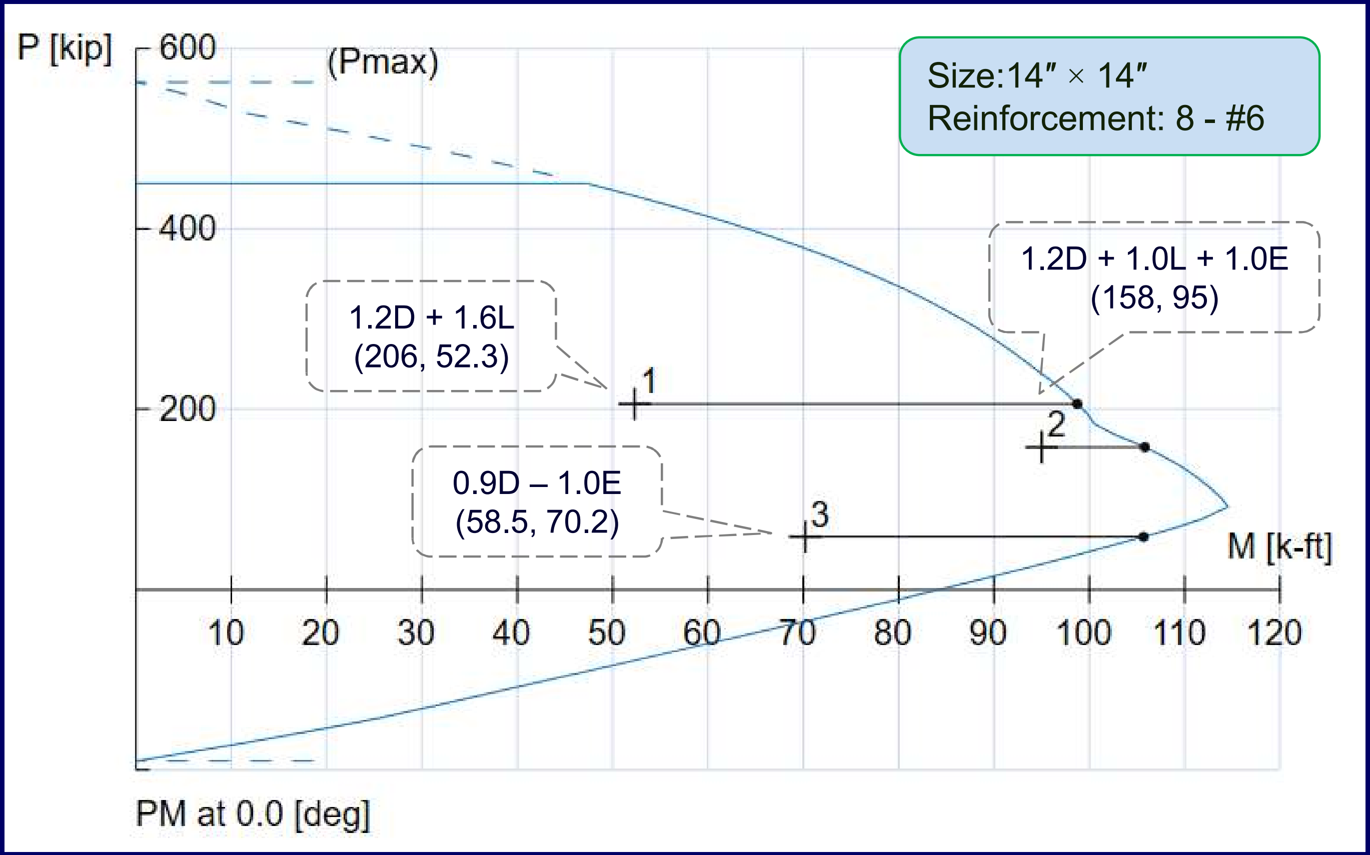
Column EF is unstable and requires revision of its size.

Example 11.2

❑ Solution

➢ Step 5: Determination of Reinforcement

❖ Determination of Longitudinal Reinforcement for Column AB

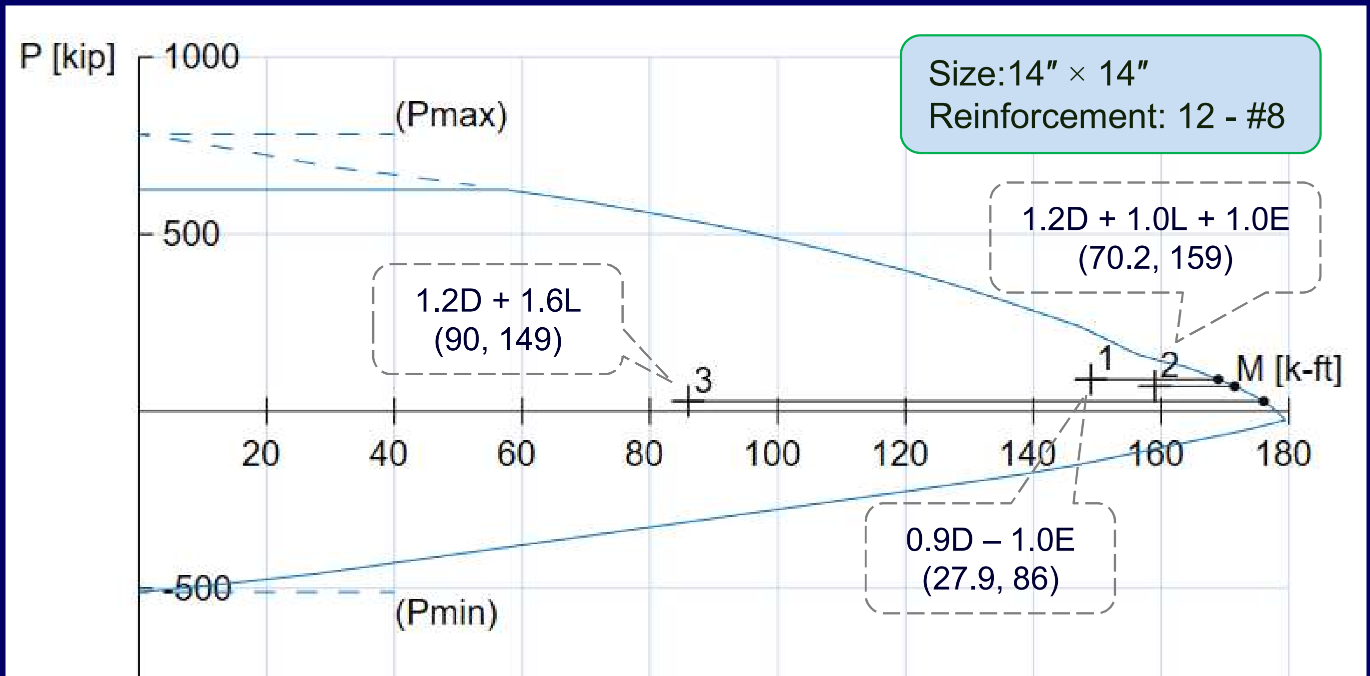


Example 11.2

❑ Solution

➢ Step 5: Determination of Reinforcement

❖ Determination of Longitudinal Reinforcement for Column CD



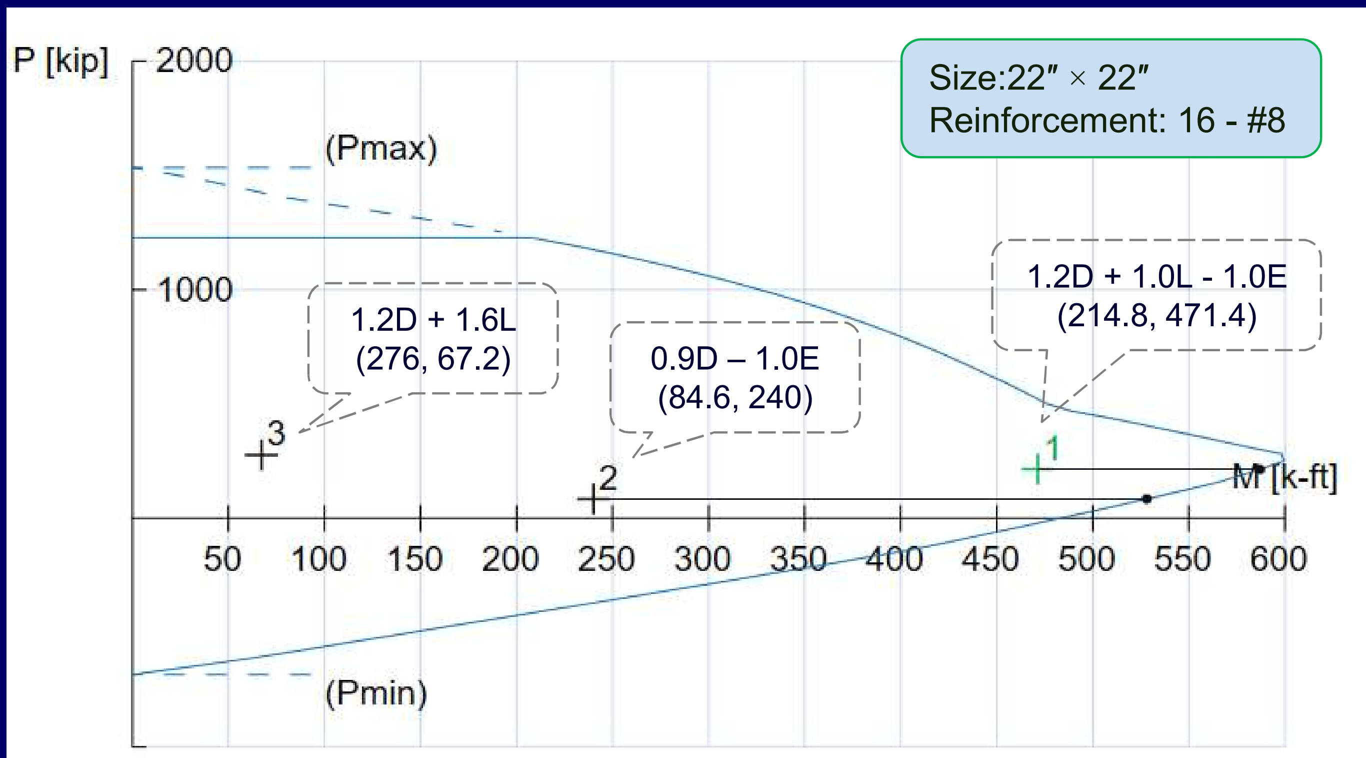
NOTE: The provided reinforcement of 12 - #8 bars (4.84%) is within the limits set by the code, it is higher than the typical practice range of 1% to 2%. The sized of column should be increased to reduce the reinforcement ratio. However, in this example, the column size is kept same.

Example 11.2

❑ Solution

➢ Step 5: Determination of Reinforcement

❖ Determination of Longitudinal Reinforcement for Column EF

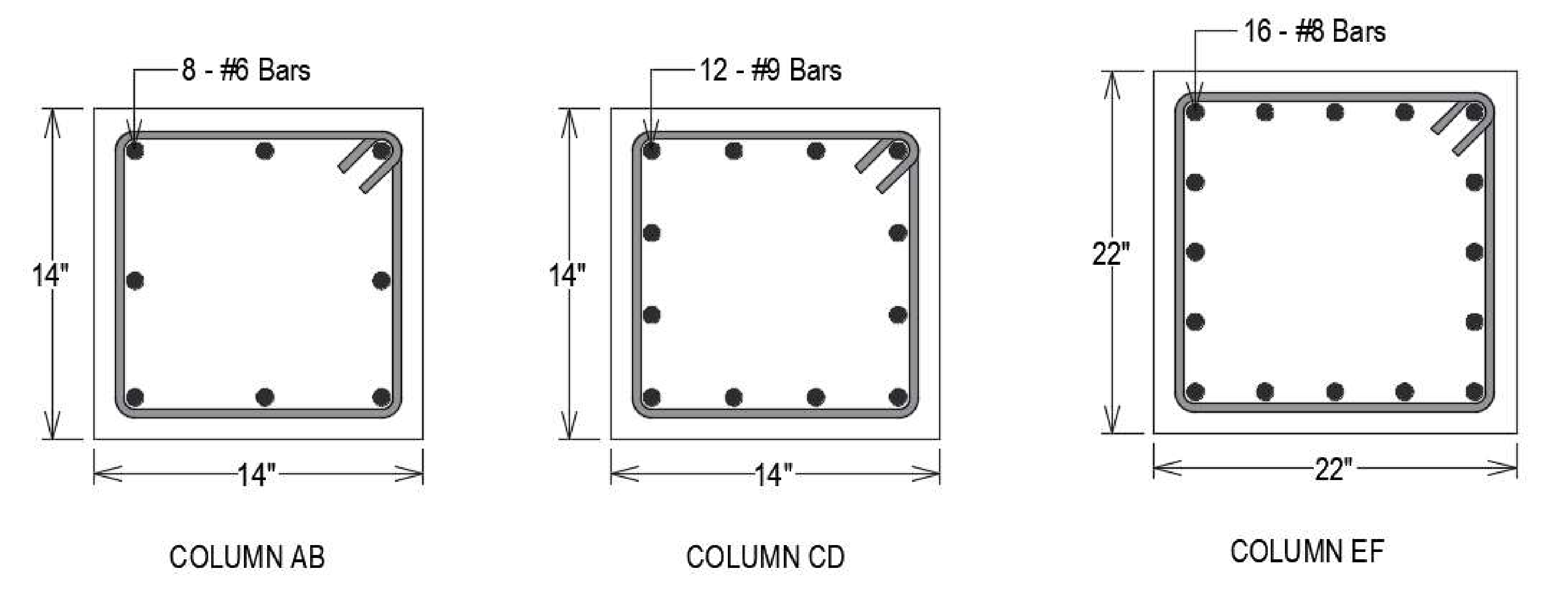


NOTE: The size of column has been increased from 14″ × 14″ to 22″ × 22″

Example 11.2

❑ Solution

➢ Step 6: Drafting



NOTE: The drawing shows detailing of longitudinal reinforcement only. Transverse reinforcement shall be calculated as per requirements and provided (which have been omitted in this example).

References

- Reinforced Concrete - Mechanics and Design (7th Ed.) by James MacGregor.
- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-19)
- Portland Cement Association (PCA 2002)