

Lecture 03

Design of Two-way Slab Systems with Beams

By:

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Lecture Contents

- General
- Analysis of Two-way Slabs
- ACI Code Provisions for Two-way Slabs
- Design Examples
- Homework
- References



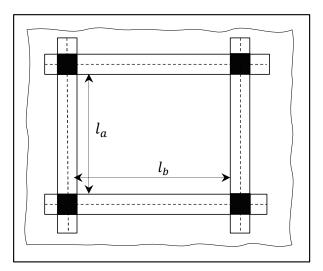
Learning Objectives

- □ At the end of this lecture, students will be able to
 - Classify one-way and two-way slab systems
 - Employ ACI coefficient method for two-way slab analyses
 - Analyze and Design two-way slabs for flexure
 - Compare manual and Finite Element Analysis (FEA) results
 - **Design** a typical house & regular building for gravity loads



☐ Introduction

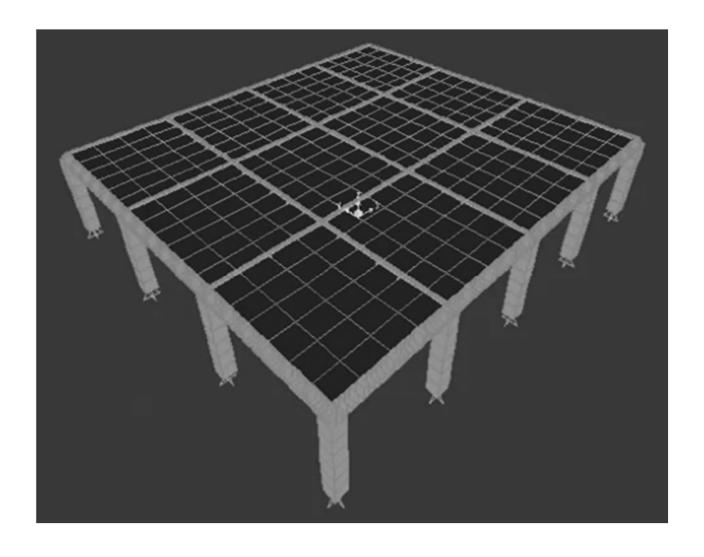
- When the ratio of long to short span in a slab supported on all sides is less than 2, then the bending is in two directions.
 Such a slab is termed as a two-way slab.
- In two-way slabs, the shorter side receives more demand than the longer side.



Slab supported on all sides, but $\beta = l_b/l_a < 2$

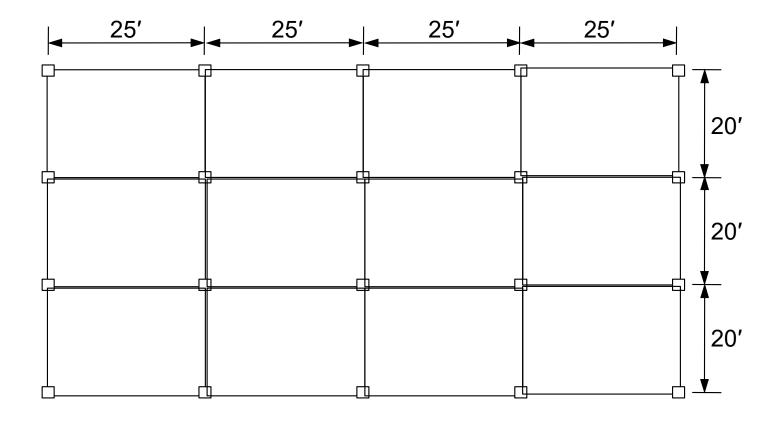


□ Bending Behavior of Two-way Slabs



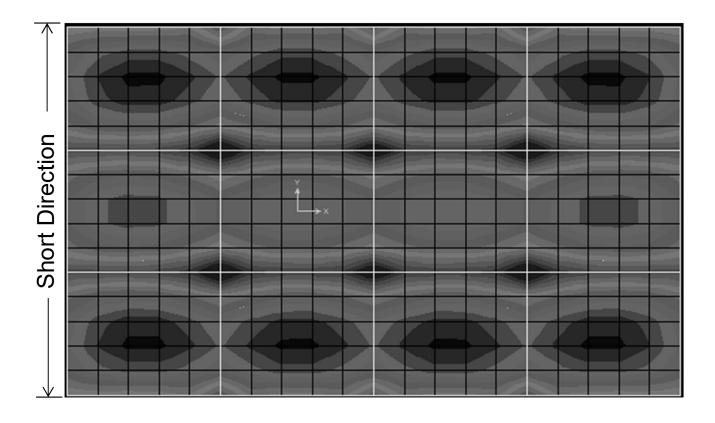


- □ Bending Behavior of Two-way Slabs
 - Consider the typical floor plan as shown below





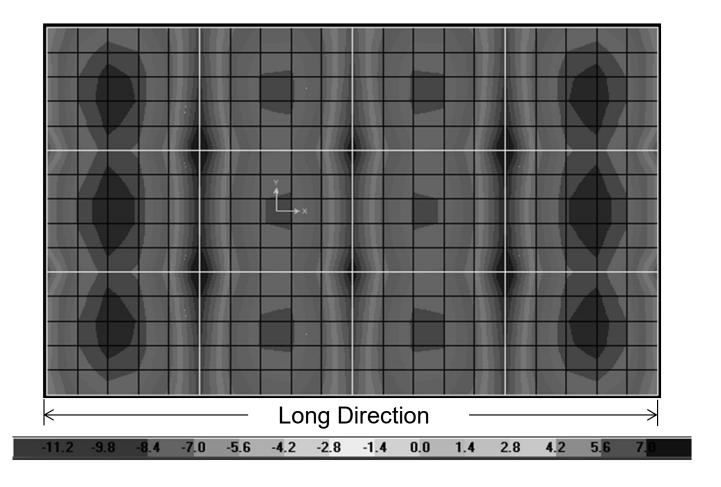
- **Bending Behavior of Two-way Slabs**
 - Short Direction Moments



8.80 -7.70 -6.60 -5.50 -4.40 -3.30 -2.20 -1.10 0.00 1.10 2.20 3.30 4.40 5.



- **Bending Behavior of Two-way Slabs**
 - Long Direction Moments





Moment Coefficient Method

- The Moment Coefficient Method, first introduced in the ACI Code in 1963, is applicable to two-way slabs with walls, steel beams, and relatively deep, stiff edge beams supporting each slab panel on its four sides $(h = 3h_f)$.
- Although, not included in 1977 and later versions of ACI code, its continued use is permissible under Section 8.2.1. ACI 318-19 Code.
- The procedure for using this method is explained in the following slides.

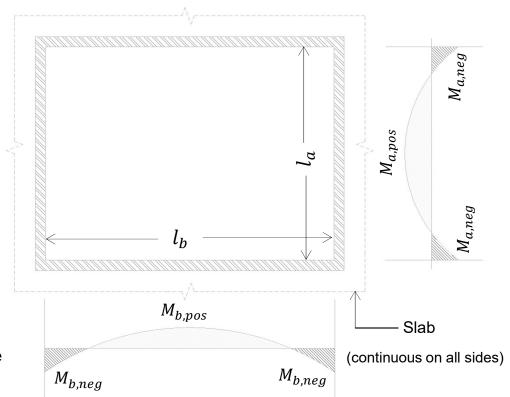


Moment Coefficient Method

- The figure below shows the four critical locations where bending moments for a two-way slab panel are calculated.
 - 1) $M_{a,neg}$
 - $M_{b,neg}$
 - 3) $M_{a,pos}$
 - 4) $M_{b,pos}$

Note:

 l_a and l_b are the clear lengths of the short and long sides, respectively.





Moment Coefficient Method

- These four bending moments are calculated in the following manner.
 - $1) M_{a,neg} = C_a w_u l_a^2$
 - $2) M_{b,neg} = C_b w_u l_b^2$
 - 3) $M_{a,pos} = C_{a,dl} w_{u,dl} l_a^2 + C_{a,ll} w_{u,ll} l_a^2$
 - 4) $M_{b,pos} = C_{b,dl} w_{u,dl} l_b^2 + C_{b,ll} w_{u,ll} l_b^2$

where;

 w_u = total factored load

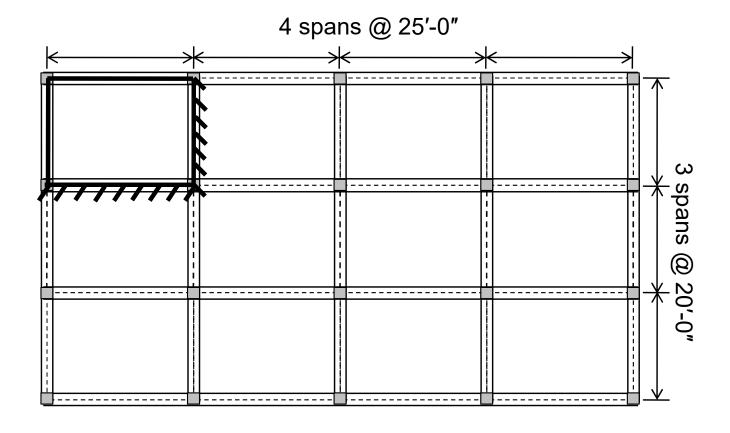
 $w_{u,dl}$ = total factored dead load

 $w_{u,ll}$ = total factored live load

 C_a , C_b , $C_{a,dl}$, $C_{b,dl}$, $C_{b,dl}$, $C_{b,ll}$ = coefficients obtained from ACI Tables.

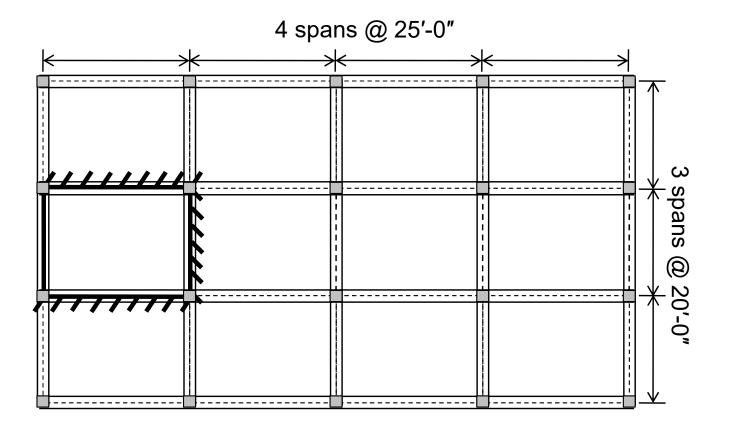


- **Moment Coefficient Method**
 - Various Cases of Slab Panel
 - Depending on the support conditions, several cases are possible



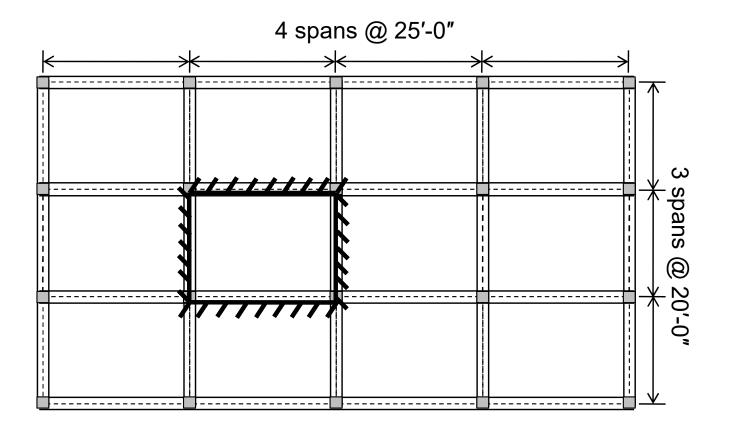


- **Moment Coefficient Method**
 - Various Cases of Slab Panel
 - Depending on the support conditions, several cases are possible



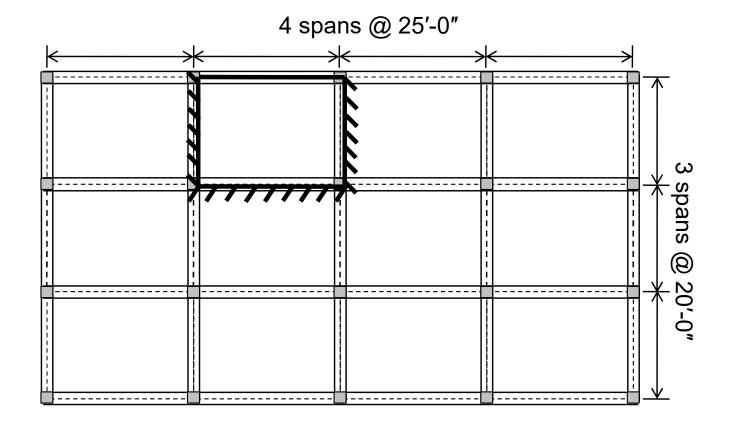


- **Moment Coefficient Method**
 - Various Cases of Slab Panel
 - Depending on the support conditions, several cases are possible





- **Moment Coefficient Method**
 - Various Cases of Slab Panel
 - Depending on the support conditions, several cases are possible





□ Moment Coefficient Method

ACI Moment Coefficients Tables

Table	Table A1: Coefficients ($C_{a, Negative}$) For Negative Moment in Slab along Short Direction									
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	
0.50	0.000	0.086	0.000	0.094	0.090	0.097	0.000	0.089	0.088	
0.55	0.000	0.084	0.000	0.092	0.089	0.096	0.000	0.085	0.086	
0.60	0.000	0.081	0.000	0.089	0.088	0.095	0.000	0.080	0.085	
0.65	0.000	0.077	0.000	0.085	0.087	0.093	0.000	0.074	0.083	
0.70	0.000	0.074	0.000	0.081	0.086	0.091	0.000	0.068	0.081	
0.75	0.000	0.069	0.000	0.076	0.085	0.088	0.000	0.061	0.078	
0.80	0.000	0.065	0.000	0.071	0.083	0.086	0.000	0.055	0.075	
0.85	0.000	0.060	0.000	0.066	0.082	0.083	0.000	0.049	0.072	
0.90	0.000	0.055	0.000	0.060	0.080	0.079	0.000	0.043	0.068	
0.95	0.000	0.050	0.000	0.055	0.079	0.075	0.000	0.038	0.065	
1.00	0.000	0.045	0.000	0.050	0.075	0.071	0.000	0.033	0.061	



■ Moment Coefficient Method

ACI Moment Coefficients Tables

Table	Table A2: Coefficients ($C_{b,\ Negative}$) For Negative Moment in Slab along Long Direction									
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	
0.50	0.000	0.006	0.022	0.006	0.000	0.000	0.014	0.010	0.003	
0.55	0.000	0.007	0.028	0.008	0.000	0.000	0.019	0.014	0.005	
0.60	0.000	0.010	0.035	0.011	0.000	0.000	0.024	0.018	0.006	
0.65	0.000	0.014	0.043	0.015	0.000	0.000	0.031	0.024	0.008	
0.70	0.000	0.017	0.050	0.019	0.000	0.000	0.038	0.029	0.011	
0.75	0.000	0.022	0.056	0.024	0.000	0.000	0.044	0.036	0.014	
0.80	0.000	0.027	0.061	0.029	0.000	0.000	0.051	0.041	0.017	
0.85	0.000	0.031	0.065	0.034	0.000	0.000	0.057	0.046	0.021	
0.90	0.000	0.037	0.070	0.040	0.000	0.000	0.062	0.052	0.025	
0.95	0.000	0.041	0.072	0.045	0.000	0.000	0.067	0.056	0.029	
1.00	0.000	0.045	0.076	0.050	0.000	0.000	0.071	0.061	0.033	



■ Moment Coefficient Method

ACI Moment Coefficients Tables

Table A3	Table A3: Coefficients ($C_{a, dl}$) For Dead Load Positive Moment in Slab along Short Direction									
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	
0.50	0.095	0.037	0.080	0.059	0.039	0.061	0.089	0.056	0.023	
0.55	0.088	0.035	0.071	0.056	0.038	0.058	0.081	0.052	0.024	
0.60	0.081	0.034	0.062	0.053	0.037	0.056	0.073	0.048	0.026	
0.65	0.074	0.032	0.054	0.050	0.036	0.054	0.065	0.044	0.028	
0.70	0.068	0.030	0.046	0.046	0.035	0.051	0.058	0.040	0.029	
0.75	0.061	0.028	0.040	0.043	0.033	0.048	0.051	0.036	0.031	
0.80	0.056	0.026	0.034	0.039	0.032	0.045	0.045	0.032	0.029	
0.85	0.050	0.024	0.029	0.036	0.031	0.042	0.040	0.029	0.028	
0.90	0.045	0.022	0.025	0.033	0.029	0.039	0.035	0.025	0.026	
0.95	0.040	0.020	0.021	0.030	0.028	0.036	0.031	0.022	0.024	
1.00	0.036	0.018	0.018	0.027	0.027	0.033	0.027	0.020	0.023	



■ Moment Coefficient Method

ACI Moment Coefficients Tables

Table A	Table A4: Coefficients ($C_{b,dl}$) For Dead Load Positive Moment in Slab along Long Direction									
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	
0.50	0.006	0.002	0.007	0.004	0.001	0.003	0.007	0.004	0.002	
0.55	0.008	0.003	0.009	0.005	0.002	0.004	0.009	0.005	0.003	
0.60	0.010	0.004	0.011	0.007	0.003	0.006	0.012	0.007	0.004	
0.65	0.013	0.006	0.014	0.009	0.004	0.007	0.014	0.009	0.005	
0.70	0.016	0.007	0.016	0.011	0.005	0.009	0.017	0.011	0.006	
0.75	0.019	0.009	0.018	0.013	0.007	0.013	0.020	0.013	0.007	
0.80	0.023	0.011	0.020	0.016	0.009	0.015	0.022	0.015	0.010	
0.85	0.026	0.012	0.022	0.019	0.011	0.017	0.025	0.017	0.013	
0.90	0.029	0.014	0.024	0.022	0.013	0.021	0.028	0.019	0.015	
0.95	0.033	0.016	0.025	0.024	0.015	0.024	0.031	0.021	0.017	
1.00	0.036	0.018	0.027	0.027	0.018	0.027	0.033	0.023	0.020	



■ Moment Coefficient Method

ACI Moment Coefficients Tables

Table <i>i</i>	Table A5: Coefficients ($C_{a,ll}$) For Live Load Positive Moment in Slab along Short Direction									
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	
0.50	0.095	0.066	0.088	0.077	0.067	0.078	0.092	0.076	0.067	
0.55	0.088	0.062	0.080	0.072	0.063	0.073	0.085	0.070	0.063	
0.60	0.081	0.058	0.071	0.067	0.059	0.068	0.077	0.065	0.059	
0.65	0.074	0.053	0.064	0.062	0.055	0.064	0.070	0.059	0.054	
0.70	0.068	0.049	0.057	0.057	0.051	0.060	0.063	0.054	0.050	
0.75	0.061	0.045	0.051	0.052	0.047	0.055	0.056	0.049	0.046	
0.80	0.056	0.041	0.045	0.048	0.044	0.051	0.051	0.044	0.042	
0.85	0.050	0.037	0.040	0.043	0.041	0.046	0.045	0.040	0.039	
0.90	0.045	0.034	0.035	0.039	0.037	0.042	0.040	0.035	0.036	
0.95	0.040	0.030	0.031	0.035	0.034	0.038	0.036	0.031	0.032	
1.00	0.036	0.027	0.027	0.032	0.032	0.035	0.032	0.028	0.030	



■ Moment Coefficient Method

ACI Moment Coefficients Tables

Table	Table A6: Coefficients ($C_{b, ll}$) For Live Load Positive Moment in Slab along Long Direction									
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	
0.50	0.006	0.004	0.007	0.005	0.004	0.005	0.007	0.005	0.007	
0.55	0.008	0.006	0.009	0.007	0.005	0.006	0.009	0.007	0.006	
0.60	0.010	0.007	0.011	0.009	0.007	0.008	0.011	0.009	0.007	
0.65	0.013	0.010	0.014	0.011	0.009	0.010	0.014	0.011	0.009	
0.70	0.016	0.012	0.016	0.014	0.011	0.013	0.017	0.014	0.011	
0.75	0.019	0.014	0.019	0.016	0.013	0.016	0.020	0.016	0.013	
0.80	0.023	0.017	0.022	0.020	0.016	0.019	0.023	0.019	0.017	
0.85	0.026	0.019	0.024	0.023	0.019	0.022	0.026	0.022	0.020	
0.90	0.029	0.022	0.027	0.026	0.021	0.025	0.029	0.024	0.022	
0.95	0.033	0.025	0.029	0.029	0.024	0.029	0.032	0.027	0.025	
1.00	0.036	0.027	0.032	0.032	0.027	0.032	0.035	0.030	0.08	

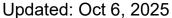


■ Moment Coefficient Method

ACI Moment Coefficients Tables

Table	Table A7: Ratio of Load "w" in Short Direction for Shear in Slab and Load on Supports									
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	
0.50	0.94	0.94	0.76	0.94	0.99	0.97	0.86	0.89	0.97	
0.55	0.92	0.92	0.69	0.92	0.98	0.96	0.81	0.85	0.95	
0.60	0.89	0.89	0.61	0.89	0.97	0.95	0.76	0.80	0.94	
0.65	0.85	0.85	0.53	0.85	0.96	0.93	0.69	0.74	0.92	
0.70	0.81	0.81	0.45	0.81	0.95	0.91	0.62	0.68	0.89	
0.75	0.76	0.76	0.39	0.76	0.94	0.88	0.56	0.61	0.86	
0.80	0.71	0.71	0.33	0.71	0.92	0.86	0.49	0.55	0.83	
0.85	0.66	0.66	0.28	0.66	0.90	0.83	0.43	0.49	0.79	
0.90	0.60	0.60	0.23	0.60	0.88	0.79	0.38	0.43	0.75	
0.95	0.55	0.55	0.20	0.55	0.86	0.75	0.33	0.38	0.71	
1.00	0.50	0.50	0.17	0.50	0.83	0.71	0.29	0.33	0.67	







ACI Code Provisions for Two-way Slabs

Minimum Slab Thickness (8.3.1.2)

The Minimum thickness of two-way slabs with beams spanning between supports on all sides shall be as per ACI Table 8.3.1.2.

α_{fm}	Minimum h, in.						
$\alpha_{fm} \leq 0.2$	Section 8.3.1 applies						
$0.2 \le \alpha_{fm} \le 2.0$	Greater of $\left[\frac{l_n\left(0.8 + \frac{f_y}{200,000}\right)}{36 + 5\beta(\alpha_{fm} - 0.2)}\right], 5''$						
$\alpha_{fm} > 2.0$	Greater of $ \left[\frac{l_n \left(0.8 + \frac{f_y}{200,000} \right)}{36 + 9\beta} \right], 3.5'' $						

- α_{fm} is the average value of α_f for all beams on edges of a panel. $\alpha_f = E_{cb} I_b / E_{cs} I_s$
- l_n is the clear span in the long direction, measured face-to-face of beams (in.).
- β is the ratio of clear spans in long to short directions of slab.



ACI Code Provisions for Two-way Slabs

■ Minimum Slab Thickness (8.3.1.2)

• If the beams are stiff enough, then α_{fm} can be taken greater than 2 and hence the third condition of the Table 8.3.1.2 would be governed.

$$h_{min} = \text{Greater of } \left[\frac{l_n \left(0.8 + \frac{f_y}{200,000} \right)}{36 + 9\beta} \right], 3.5''$$

Setting $l_n = l_b$ and $\beta = l_b/l_a$ the equation becomes

$$h_{min} = max \left[\frac{l_b \left(0.8 + \frac{f_y}{200,000} \right)}{36 + 9(l_b/l_a)} , 3.5'' \right]$$

This equation will be used onward to calculate minimum slab thickness.

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ACI Code Provisions for Two-way Slabs

☐ Minimum Flexural Reinforcement (8.6.1)

 The minimum reinforcement requirement for two-way slabs is identical to that of one-way slabs.

$$A_{s,min} = 0.0018A_g$$

☐ Spacing of Flexural Reinforcement (8.7.1)

- Maximum spacing s shall be the lesser of:
 - 2h and 18 in. at critical sections
 - 3h and 18 in. at other sections



Moment Coefficient Method

- Stepwise Procedure
- Calculate minimum slab depth
- Calculate loads
- Decide about case of slab
- Use tables to pick moment coefficients
- Calculate Moments
- Determine required reinforcement
- Apply reinforcement checks



Design Example 4.1 Design of Typical Single Story House



□ Problem Statement

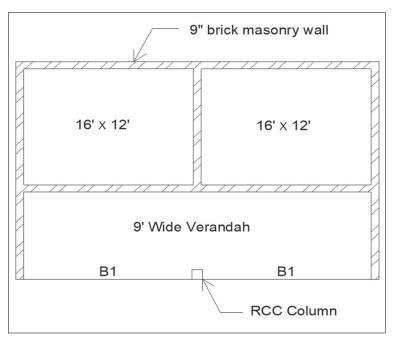
A single – story house with two bedrooms and a verandah is shown in figure on the next slide. The story height is 12 feet, and the thickness of the masonry wall is 9 inches. There is four inches of mud layer and two inches of tile brick over the slab. According to ASCE 7-10, the expected uniform service live load for the residential buildings is 40 psf. Material strengths are $f_C = 3$ ksi and $f_y = 60$ ksi. The allowable bearing capacity of the foundation soil is 1.0 TSF.

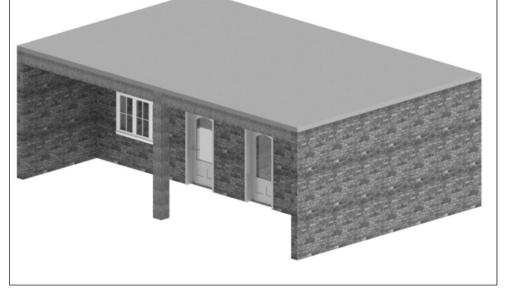
Design the slab, beam B1 and column





□ Problem Statement





Floor Plan

3D Model



☐ Given Data

Dimensions of Rooms: 16' x 12' (interior)

Story height, h = 12'

SDL: 4" Mud layer and 2" Tile layer

Live load: 40psf

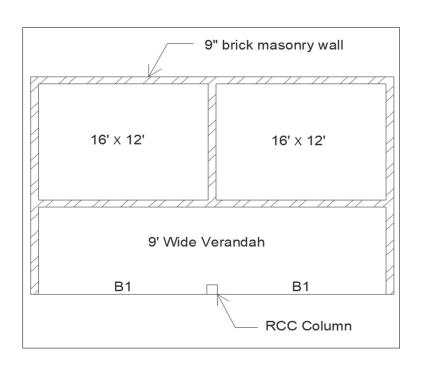
 $f_c' = 3$ ksi

 $f_{\rm v}=60~{\rm ksi}$

 $q_a = 2.204 \text{ ksf}$

□ Required Data

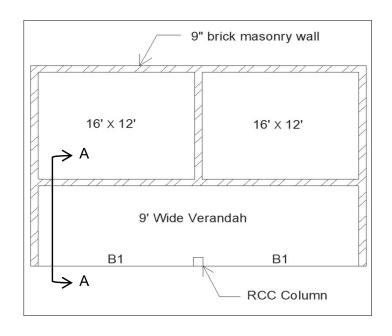
Design the Slab, Beam B1 and Column

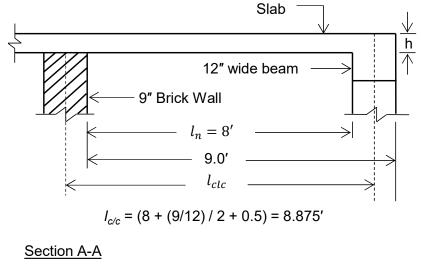




□ Solution

 As can be seen from the figure that the slab over the rooms is a twoway slab case whereas the verandah slab is a one-way slab.







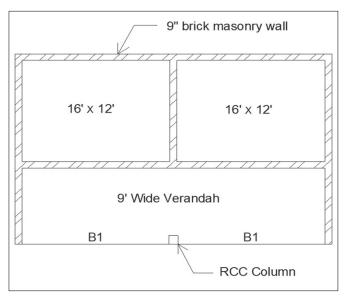
☐ Solution

- Slab Design
- Step 1: Selection of Sizes
 - For Two-way Slabs: Assuming $\alpha_{fm} > 2.0$, the minimum thickness is given by:

$$h_{min} = max \left[\frac{l_n \left(0.8 + \frac{f_y}{200,000} \right)}{36 + 9\beta} , 3.5'' \right]$$

Substituting values, we get

$$h_{min} = max \left[\frac{16\left(0.8 + \frac{60,000}{200,000}\right)}{36 + 9\left(\frac{16}{12}\right)} , 3.5'' \right] = 4.4''$$





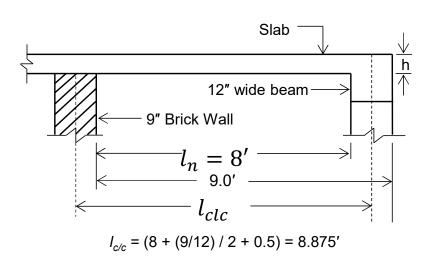
- □ Solution
 - Slab Design
 - > Step 1: Selection of Sizes
 - For One-way Slab: For one-end continuous slabs, we have

$$h_{min} = \frac{l}{24} \left(0.4 + \frac{f_{\mathcal{Y}}}{100000} \right)$$

Substituting values, we get

$$h_{min} = \frac{8.875}{24} \left(0.4 + \frac{60,000}{100,000} \right) = 4.4''$$

Finally take h = 5"



Section A-A



- **□** Solution
 - Slab Design
 - > Step 2: Calculation of loads

Material	Thickness h (in.)	Unit weight γ (kcf)	$W = h \times \gamma \text{ (ksf)}$
Concrete Slab	5	0.15	(5/12) × 0.15 = 0.0625
Mud	4	0.12	(4/12) × 0.12 = 0.04
Tile	2	0.12	(2/12) × 0.12= 0.02
То	tal dead load =	0.1225 ksf	

$$w_{u,dl} = 1.2D = 1.2 \times 0.1225 = 0.147 \text{ ksf}$$

 $w_{u,ll} = 1.6 \times 0.04 = 0.064 \text{ ksf}$
 $w_u = 0.147 + 0.064 = \mathbf{0.211 ksf}$



□ Solution

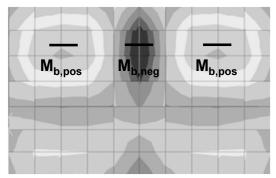
- Slab Design
- > Step 3: Analysis
 - The given system consists of both one way and two-way slabs. A system where a two-way slab is continuous with a one-way slab or vice versa can be called as a mixed slab system.
 - Strictly speaking, the ACI approximate analysis methods are not suitable for mixed systems.
 - In case of one-way slabs, the ACI approximate analysis is applicable where a one-way slab is continuous with a one-way slab.
 - In case of two-way slabs, the moment coefficient tables are applicable where a two-way slab is continuous with a two-way slab.

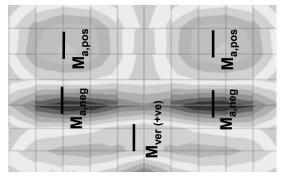


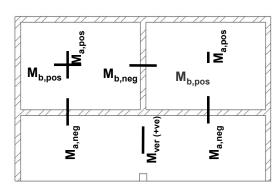
- **□** Solution
 - Slab Design
 - > Step 3: Analysis
 - The best approach to analyze a mixed system is to use Finite Element software.
 - However, such a system can also be analyzed manually by making certain approximations.
 - We will analyze this system using both methods.



- □ Solution
 - Slab Design
 - Step 3: Analysis (two-way slab)
 - Below are the Finite Element Analysis (FEA) results obtained using SAFE.







Moments in Long Direction

Moments in Short Direction

Two-way Slab Moments (in-kip/ft) for Rooms				One-way Slab Moment (in-kip/ft) for Verandah		
$M_{a(+)}$	$M_{b(+)}$	$M_{a(-)}$	$M_{b(-)}$	$M_{ver(+)}$	$M_{ver,ext(-)}$	
19.0	14.0	25.2	20.0	13.2	4.6	



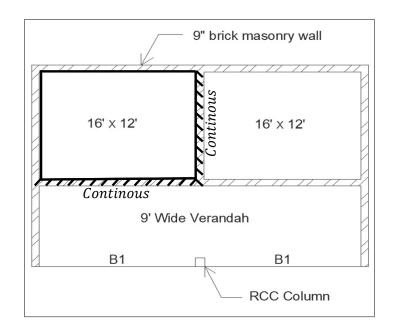
□ Solution

- Slab Design
- Step 3: Analysis (two-way slab)
- Select slab case

From the figure, the slab case is 4

$$m = \frac{l_a}{l_b} = \frac{12}{16} = 0.750$$

Now, with m = 0.75 and slab case 4, pickup the moment coefficients from the relevant Tables





- □ Solution
 - Slab Design
 - Step 3: Analysis (two-way slab)

Moment Coefficients

$$C_{a,neg} = 0.0760$$
.076

.076

	Table A1:	Coefficien	ts (C _{a, Negative}) For Nega	tive Mome	nt in Slab a	long Short	Direction	
		.11111							
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.000	0.086	0.000	0.094	0.090	0.097	0.000	0.089	0.088
0.55	0.000	0.084	0.000	0.092	0.089	0.096	0.000	0.085	0.086
0.60	0.000	0.081	0.000	0.089	0.088	0.095	0.000	0.080	0.085
0.65	0.000	0.077	0.000	0.085	0.087	0.093	0.000	0.074	0.083
0.70	0.000	0.074	0.000	0.081	0.086	0.091	0.000	0.068	0.081
0.75	0.000	0.069	0.000 ▶	0.076	0.085	0.088	0.000	0.061	0.078
0.80	0.000	0.065	0.000	0.071	0.083	0.086	0.000	0.055	0.075
0.85	0.000	0.060	0.000	0.066	0.082	0.083	0.000	0.049	0.072
0.90	0.000	0.055	0.000	0.060	0.080	0.079	0.000	0.043	0.068
0.95	0.000	0.050	0.000	0.055	0.079	0.075	0.000	0.038	0.065
1.00	0.000	0.045	0.000	0.050	0.075	0.071	0.000	0.033	0.061



- □ Solution
 - Slab Design
 - Step 3: Analysis (two-way slab)

$$C_{a,neg} = 0.076$$
0.076

$$C_{b,neg} = 0.024$$
0.076

	Table A2: Coefficients (C _{b, Negative}) For Negative Moment in Slab along Short Direction									
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	
0.50	0.000	0.006	0.022	0.006	0.000	0.000	0.014	0.010	0.003	
0.55	0.000	0.007	0.028	0.008	0.000	0.000	0.019	0.014	0.005	
0.60	0.000	0.010	0.035	0.011	0.000	0.000	0.024	0.018	0.006	
0.65	0.000	0.014	0.043	0.015	0.000	0.000	0.031	0.024	0.008	
0.70	0.000	0.017	0.050	0.019	0.000	0.000	0.038	0.029	0.011	
0.75	0.000	0.022	0.056 →	0.024	0.000	0.000	0.044	0.036	0.014	
0.80	0.000	0.027	0.061	0.029	0.000	0.000	0.051	0.041	0.017	
0.85	0.000	0.031	0.065	0.034	0.000	0.000	0.057	0.046	0.021	
0.90	0.000	0.037	0.070	0.040	0.000	0.000	0.062	0.052	0.025	
0.95	0.000	0.041	0.072	0.045	0.000	0.000	0.067	0.056	0.029	
1.00	0.000	0.045	0.076	0.050	0.000	0.000	0.071	0.061	0.033	



- □ Solution
 - Slab Design
 - Step 3: Analysis (two-way slab)

$$C_{a,neg} = 0.076$$
0.076

$$C_{b,neg} = 0.024$$
0.076

$$C_{a,pos,dl} = 0.043$$
0.076

Та	able A3: Co	efficients ($C_{a,\mathit{dl}}$) For D	ead Load F	ositive Mo	ment in Sla	ab along Sh	ort Directi	on
		.111111			.111111				
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.095	0.037	0.080	0.059	0.039	0.061	0.089	0.056	0.023
0.55	0.088	0.035	0.071	0.056	0.038	0.058	0.081	0.052	0.024
0.60	0.081	0.034	0.062	0.053	0.037	0.056	0.073	0.048	0.026
0.65	0.074	0.032	0.054	0.050	0.036	0.054	0.065	0.044	0.028
0.70	0.068	0.030	0.046	0.046	0.035	0.051	0.058	0.040	0.029
0.75	0.061	0.028	0.040 ▶	0.043	0.033	0.048	0.051	0.036	0.031
0.80	0.056	0.026	0.034	0.039	0.032	0.045	0.045	0.032	0.029
0.85	0.050	0.024	0.029	0.036	0.031	0.042	0.040	0.029	0.028
0.90	0.045	0.022	0.025	0.033	0.029	0.039	0.035	0.025	0.026
0.95	0.040	0.020	0.021	0.030	0.028	0.036	0.031	0.022	0.024
1.00	0.036	0.018	0.018	0.027	0.027	0.033	0.027	0.020	0.023



- □ Solution
 - Slab Design
 - Step 3: Analysis (two-way slab)

$$C_{a,neg} = 0.076$$
0.076

$$C_{b,neg} = 0.024$$
0.076

$$C_{a,pos,dl} = 0.043$$
0.076

$$C_{a,pos,ll} = 0.052$$
0.076

T	able A4: Co	pefficients	$(C_{a, ll})$ For L	ive Load P	ositive Mor	nent in Sla	b along Sh	ort Directio	n
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.095	0.066	0.088	0.077	0.067	0.078	0.092	0.076	0.067
0.55	0.088	0.062	0.080	0.072	0.063	0.073	0.085	0.070	0.063
0.60	0.081	0.058	0.071	0.067	0.059	0.068	0.077	0.065	0.059
0.65	0.074	0.053	0.064	0.062	0.055	0.064	0.070	0.059	0.054
0.70	0.068	0.049	0.057	0.057	0.051	0.060	0.063	0.054	0.050
0.75	0.061	0.045	0.051 ▶	0.052	0.047	0.055	0.056	0.049	0.046
0.80	0.056	0.041	0.045	0.048	0.044	0.051	0.051	0.044	0.042
0.85	0.050	0.037	0.040	0.043	0.041	0.046	0.045	0.040	0.039
0.90	0.045	0.034	0.035	0.039	0.037	0.042	0.040	0.035	0.036
0.95	0.040	0.030	0.031	0.035	0.034	0.038	0.036	0.031	0.032
1.00	0.036	0.027	0.027	0.032	0.032	0.035	0.032	0.028	0.030



□ Solution

- Slab Design
- Step 3: Analysis (two-way slab)

Moment Coefficients

$$C_{a,neg} = 0.076$$
0.076

$$C_{b,neg} = 0.024$$
0.076

$$C_{a,pos,dl} = 0.043$$
0.076

$$C_{a,pos,ll} = 0.052$$
0.076

$$C_{b,pos,dl} = 0.013$$
0.076

Ta	ible A5: Co	efficients ($C_{b,\mathit{dl}}$) For D	ead Load F	Positive Mo	ment in Sla	ab along Lo	ong Direction	on
		.11111							
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.006	0.002	0.007	0.004	0.001	0.003	0.007	0.004	0.002
0.55	0.008	0.003	0.009	0.005	0.002	0.004	0.009	0.005	0.003
0.60	0.010	0.004	0.011	0.007	0.003	0.006	0.012	0.007	0.004
0.65	0.013	0.006	0.014	0.009	0.004	0.007	0.014	0.009	0.005
0.70	0.016	0.007	0.016	0.011	0.005	0.009	0.017	0.011	0.006
0.75	0.019	0.009	0.018 ▶	0.013	0.007	0.013	0.020	0.013	0.007
0.80	0.023	0.011	0.020	0.016	0.009	0.015	0.022	0.015	0.010
0.85	0.026	0.012	0.022	0.019	0.011	0.017	0.025	0.017	0.013
0.90	0.029	0.014	0.024	0.022	0.013	0.021	0.028	0.019	0.015
0.95	0.033	0.016	0.025	0.024	0.015	0.024	0.031	0.021	0.017
1.00	0.036	0.018	0.027	0.027	0.018	0.027	0.033	0.023	0.020



□ Solution

- Slab Design
- Step 3: Analysis (two-way slab)

$$C_{a,neg} = 0.076$$
0.076

$$C_{b,neg} = 0.024$$
0.076

$$C_{a,pos,dl} = 0.043$$
0.076

$$C_{a,pos,ll} = 0.013$$
0.076

$$C_{b,pos,dl} = 0.052$$
0.076

$$C_{b,pos,ll} = 0.016$$
0.076

Т	able A6: Co	oefficients	(C _{b, //}) For L	ive Load P	ositive Mo	ment in Sla	b along Lo	ng Directio	n
		.111111			.111111				
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.006	0.004	0.007	0.005	0.004	0.005	0.007	0.005	0.007
0.55	0.008	0.006	0.009	0.007	0.005	0.006	0.009	0.007	0.006
0.60	0.010	0.007	0.011	0.009	0.007	0.008	0.011	0.009	0.007
0.65	0.013	0.010	0.014	0.011	0.009	0.010	0.014	0.011	0.009
0.70	0.016	0.012	0.016	0.014	0.011	0.013	0.017	0.014	0.011
0.75	0.019	0.014	0.019 ▶	0.016	0.013	0.016	0.020	0.016	0.013
0.80	0.023	0.017	0.022	0.020	0.016	0.019	0.023	0.019	0.017
0.85	0.026	0.019	0.024	0.023	0.019	0.022	0.026	0.022	0.020
0.90	0.029	0.022	0.027	0.026	0.021	0.025	0.029	0.024	0.022
0.95	0.033	0.025	0.029	0.029	0.024	0.029	0.032	0.027	0.025
1.00	0.036	0.027	0.032	0.032	0.027	0.032	0.035	0.030	0.08



□ Solution

- Slab Design
- Step 3: Analysis (two-way slab)
 - Calculate bending Moments

$$w_{u,dl} = 0.147 ksf$$
 , $w_{u,dl} = 0.064 ksf$, $w_u = 0.211 ksf$, $\, l_a = 12'$ and $\, l_b = 16'$

Coefficients	Moment formulae	Moment Values (in.kip)
$C_{a,neg} = 0.076$	$M_{a,neg} = C_{a,neg} w_u l_a^2$	27.7
$C_{b,neg} = 0.024$	$M_{b,neg} = C_{b,neg} w_u l_b^2$	15.6
$C_{a,pos,dl} = 0.043$	M = C $m = 12 + C$ $m = 12$	16.7
$C_{a,pos,ll} = 0.052$	$M_{a,pos} = C_{a,pos,dl} w_{u,dl} l_a^2 + C_{a,pos,ll} w_{u,ll} l_a^2$	10.7
$C_{b,pos,dl} = 0.013$	M = C $m = 12 + C$ $m = 12$	9.0
$C_{b,pos,ll} = 0.016$	$M_{b,pos} = C_{b,pos,dl} w_{u,dl} l_b^2 + C_{b,pos,ll} w_{u,ll} l_b^2$	9.0



☐ Solution

- Slab Design
- Step 4: Analysis (one-way slab)

$$M_{ver,int(-)} = \frac{w_u l_n^2}{9} = 18.0 in. kip/ft$$

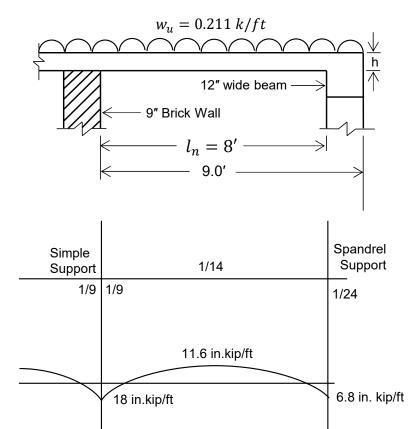
$$M_{ver(+)} = \frac{w_u l_n^2}{14} = 11.6 \text{ in. kip/ft}$$

$$M_{ver,ext(-)} = \frac{w_u l_n^2}{24} = 6.8 \text{ in. kip/ft}$$

Note:

For negative moment above the long wall common to rooms and veranda, maximum moment will be picked from both analyses.

Moment of 27.72 from two-way slab analysis is more than 18, therefore we will design for 27.72.





- □ Solution
 - Slab Design
 - Step 3: Analysis (comparison of results)

Analysis	Two – w		loments (in-kip/ft)	One-way Slab Moment (in-kip/ft) (Verandah)			
Method	$M_{a(+)}$	$M_{b(+)}$	$M_{a(-)}$	$M_{b(-)}$	$M_{ver,int(-)}$	$M_{ver(+)}$	$M_{ver,ext(-)}$	
FEA (SAFE)	19.0	14.0	25.2	20.0	25.2	13.2	4.6	
Manual	16.7	9.0	27.7	15.6	18.0	11.6	6.8	

- Analysis results from both approaches are almost similar.
- Hence the intelligent use of manual analysis yields reasonable results in most cases.



□ Solution

- Slab Design
- > Step 4: Determination of Flexural Steel Area

The minimum reinforcement is given by

$$A_{s,min} = 0.0018bh = 0.0018(12)(5) = 0.108 in^2/ft$$

Using #3 bars with $A_b = 0.11 in^2$

$$S = \frac{12A_b}{A_s} = \frac{12 \times 0.11}{0.108} = 12.2''c/c$$

Calculated spacing shall not exceed S_{max} which is given by

$$S_{max} = min(2h, 18") \Rightarrow min(2 \times 5, 18") = 10"$$

Calculated spacing of 12.2" exceeds 10". Finally Provide #3@10" c/c.



□ Solution

- Slab Design
- > Step 4: Determination of Flexural Steel Area

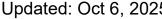
With #3@10" c/c, calculate moment capacity

$$A_{s,min} = \frac{12A_b}{S} = \frac{12(0.11)}{10} = 0.132 \text{ in}^2/\text{ft}$$

$$a = \frac{A_{s,min}f_y}{0.85f_c'b} = \frac{0.132 \times 60}{0.85 \times 3 \times 12} = 0.26 in$$

Now,

$$\emptyset M_n = 0.9 \times 0.132 \times 60 \left(4 - \frac{0.26}{2} \right) = 27.6 \text{ in. kip/ft}$$





Solution

- Slab Design
- > Step 4: Determination of Flexural Steel Area

The flexural design summary is provided below.

Location	Moments (in.kip/ft)	M _{n,min} (in. kip/ft)	A_s (in^2)	S using #3 bar (in)
$M_{a,neg}$	27.71		$pprox A_{s,min}$ governs	10
$M_{b,neg}$	15.56		$A_{s,min}$ governs	10
$M_{a,pos}$	16.67	27.60	$A_{s,min}$ governs	10
$M_{b,pos}$	9.02		$A_{s,min}$ governs	10
$M_{+,Ver}$	11.52		$A_{s,min}$ governs	10



□ Solution

- Slab Design
- > Step 5: Determination of temperature/shrinkage reinforcement

For one-way slab, the temperature reinforcement is given by

$$A_{s+T} = A_{min} = 0.108 \text{ in}^2/\text{ft}$$

$$S = \frac{12A_b}{A_s} = \frac{12 \times 0.11}{0.108} = 12.2''c/c$$

Maximum spacing for shrinkage reinforcement is given by

$$s_{max} = min[5(5) \ or \ 18''] = 18'' \rightarrow OK!$$

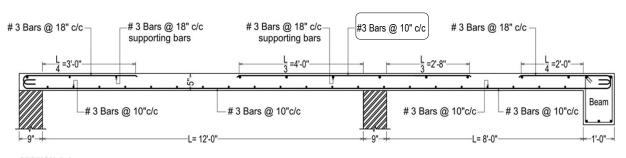
Finally, provide #3 @10 in. c/c



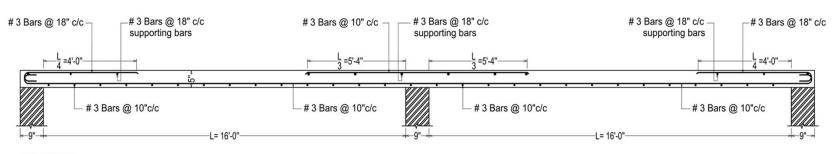
- **□** Solution
 - Slab Design
 - > Step 5: Determination of temperature/shrinkage reinforcement
 - Reinforcement at discontinuous ends
 - Reinforcement at discontinuous ends in a two way slab is 1/3 of the positive reinforcement.
 - Positive reinforcement at midspan in this case is #3 @ 10" c/c. Therefore,
 reinforcement at discontinuous end may be provided @ 30" c/c.
 - However, in field practice, the spacing of reinforcement at discontinuous ends seldom exceeds 18" c/c. The same is provided here as well.



- □ Solution
 - Slab Design
 - > Step 6: Drafting



SECTION A-A



SECTION B-B



Solution

- **Beam Design**
- **Step 1: Selection of sizes**

Assume $b_w = 12$ "

$$h_{min,40} = \frac{16.75}{18.5} \times 12 = 10.9$$
" $\rightarrow Take h = 18$ "

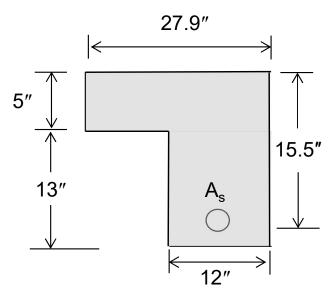
Now,

Now,
$$b_{w} + 6h_{f} = 12 + 16 \times 5 = 92$$

$$b_{w} + \frac{S_{w}}{2} = Not \ applicable$$

$$b_{w} + \frac{l_{n}}{12} = 12 + \frac{15.875}{12} \times 12 = 27.9$$

$$b_{f,L} = 27.9$$
"





☐ Solution

- Beam Design
- > Step 2: Calculation of loads

Self weight of beam is given by

$$SW = \frac{12(18-5)}{144} \times 0.150 = 0.163 \text{ k/ft}$$

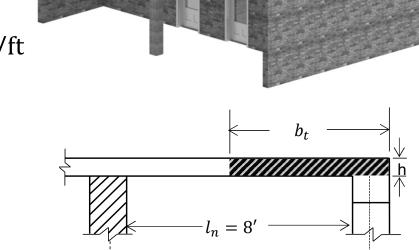


$$W_{u,beam} = w_{u,slab} \times b_t + 1.2SW$$

By putting values

$$W_{u,beam} = 0.211 \times 5 + 1.2(0.163)$$

$$W_{u,beam} = 1.25 \, k/ft$$

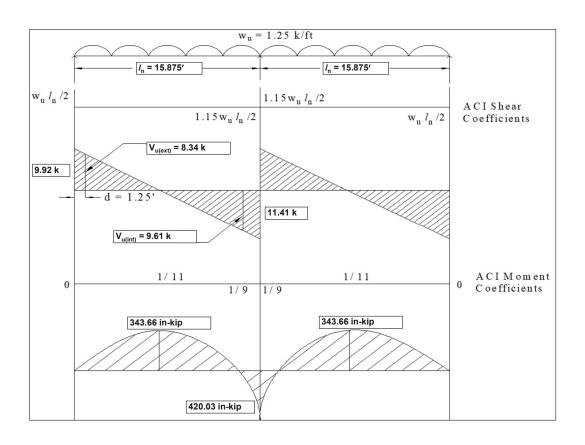


$$b_t = \frac{l_c}{2} + \frac{b_w}{2} = \frac{8.875'}{2} + \frac{1'}{2} = 4.9 \approx 5'$$

 $l_{c/c} = 8.875' \longrightarrow$



- □ Solution
 - Beam Design
 - > Step 3: Analysis





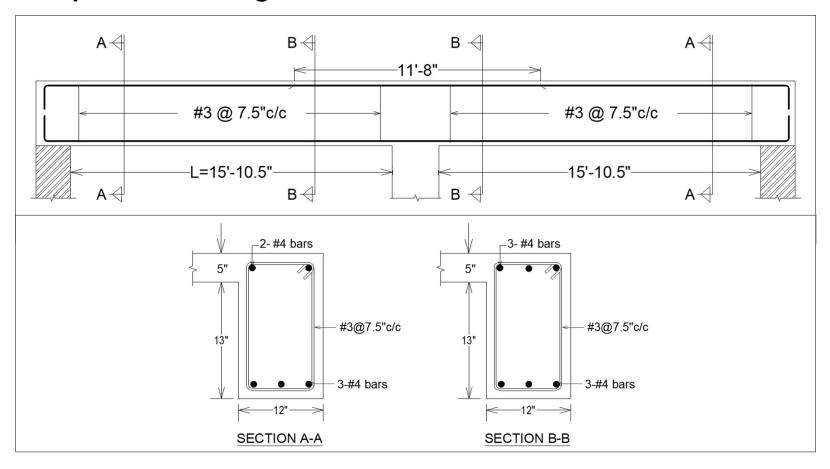
- □ Solution
 - Beam Design
 - > Step 4: Determination of Flexural Reinforcement

Flexural Design Summary										
M _u (in-kip)	d (in.)	b (in.)	A _s (in²)	A _{smin} (in²)	A _{smax} (in²)	A _s	Detailing			
343.66 (+)	15	27.875	0.43	0.60	2.42	0.60	3 - #4			
420.03 (-)	15	12	0.44	0.60	2.42	0.60	3 - #4			

Shear Design Summary									
Location	V _u (@ d)(kip)	ΦV _c (kips)	S _{max} (in)	Detailing					
Exterior	8.34	14.78	7.5"	2-legged #3 @7.5" c/c					
Interior	9.61	14.78	7.5"	2-legged #3 @7.5" c/c					



- □ Solution
 - Beam Design
 - > Step No 5: Drafting





□ Solution

- * Column Design
- > Step 1: Selection of Sizes

Assume column size = 12" × 12"

> Step 2: Calculation of Loads

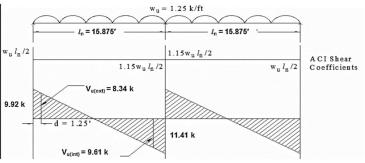
$$P_{II} = 11.41 \times 2 = 22.82 \text{ kip}$$

> Step 3: Longitudinal Reinforcement

Assuming the column as concentric, the axial capacity is given by:

$$\alpha \emptyset P_n = 0.8 \times 0.65 [0.85 f_c' (A_g - A_{st}) - A_{st} f_y]$$







□ Solution

- Column Design
- > Step 3: Longitudinal Reinforcement

Assuming
$$A_{st} = 0.01A_g = 0.01 \times 144 = 1.44 in^2$$

 $\alpha \emptyset P_n = 0.8 \times 0.65[0.85 \times 3(144 - 1.44) - 1.44 \times 60]$
 $= 144.10 > P_u = 22.82 \text{ kip} \rightarrow \text{OK}$

Using #4 bar with $A_b = 0.20 \text{ in}^2$

No. of bars = $1.44/0.20 = 7.2 \approx 8$

Hence, Provide 8-#4 bars.



□ Solution

- Column Design
- > Step 4: Determination of Spacing for Shear Reinforcement

Using #3 bar with $A_b = 0.11 in^2$, S_{max} is the least of:

i.
$$\frac{A_v f_y}{50b}$$
 = 0.22 x 60,000/ (50x12) = 22.0"

ii.
$$\frac{A_v f_y}{0.75 \sqrt{f_c'} b} = 0.22 \times 60,000 / (0.75 \sqrt{3000} \times 12) = 26.8"$$

iii.
$$16d_b$$
 of longitudinal bar = $16 \times 4/8 = 8$ "

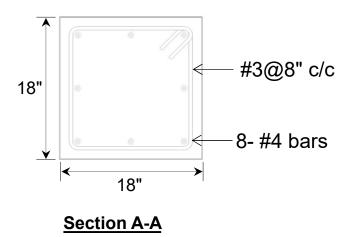
iv.
$$48 d_b$$
 of tie bar = $48 \times 3/8 = 18$ "

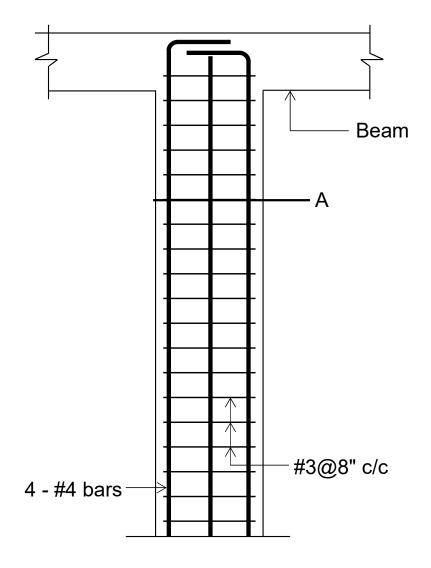
v. Smallest dimension of member = 12"

$$S_{max}$$
 = 8". Provide #3 ties @ 8" c/c



- □ Solution
 - Column Design
 - > Step 5: Drafting







☐ Crack in slab in village house due to absence of negative reinforcement







Design Example 4.2

Design of Three-story Commercial Building

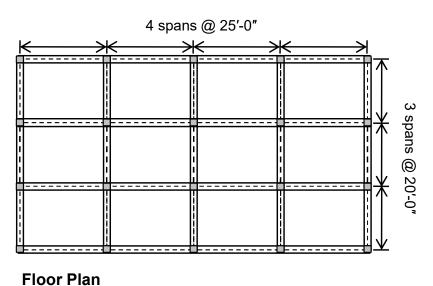


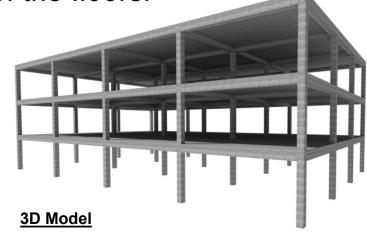


Problem Statement

 A 100' x 60' three-story 4 by 3 bay commercial building is shown below. The 7 in-thick floors are subjected to uniform service live load of 144psf. Taking $f_c' = 3ksi$ and $f_v = 40 ksi$.

Design slab and beams of one of the floors.





- All beams are 14" x 20"
- All columns are 14" x 14"



☐ Given Data

Dimensions of floor: 100' x 60' (center – to – center)

Story height, h = 12'

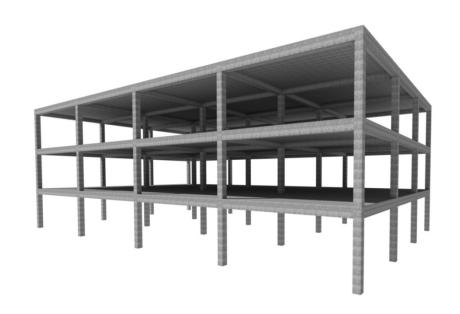
All beams are 14" x 20"

All columns are 14" x 14"

Superimposed Dead load: Nil

Live load: 144 psf

 $f_c' = 3 \text{ ksi } \& f_y = 40 \text{ ksi}$



□ Required Data

Design slab and beams of one of the floors



□ Solution

- Slab Design
- > Step 1 and 2: Selection of Structural Configuration and Sizes

Structural configuration and slab thickness are given, $h_f = 7$ "

> Step 2: Calculation of Loads

Self weight of slab =
$$\frac{7}{12} \times 0.150 = 0.0875 \, ksf$$

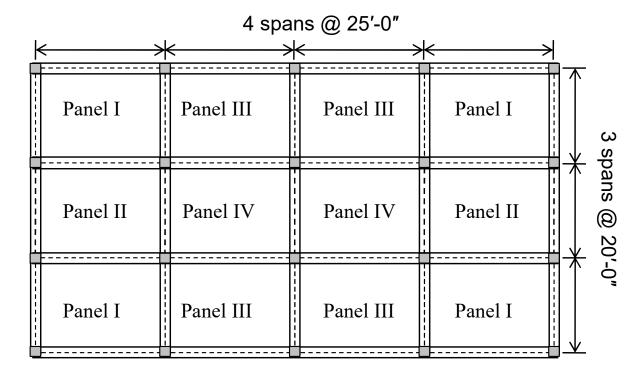
$$w_{u,dl} = 1.2(0.0875) = 0.105 \, ksf$$

$$w_{u,ll} = 1.6(0.144) = 0.230 \, ksf$$

$$w_u = 0.105 + 0.2304 = 0.335 \, ksf$$

- **Solution**
 - Slab Design
 - **Step 3: Analysis**

Complete analysis of the slab is done by analyzing four panels





- □ Solution
 - Slab Design
 - > Step 3: Analysis
 - Moment Coefficients

$$C_{a,neg} = 0.0710$$
.076

.076

Panel - I

	Table A1: Coefficients (C _{a, Negative}) For Negative Moment in Slab along Short Direction									
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	
0.50	0.000	0.086	0.000	0.094	0.090	0.097	0.000	0.089	0.088	
0.55	0.000	0.084	0.000	0.092	0.089	0.096	0.000	0.085	0.086	
0.60	0.000	0.081	0.000	0.089	0.088	0.095	0.000	0.080	0.085	
0.65	0.000	0.077	0.000	0.085	0.087	0.093	0.000	0.074	0.083	
0.70	0.000	0.074	0.000	0.081	0.086	0.091	0.000	0.068	0.081	
0.75	0.000	0.069	0.000	0.076	0.085	0.088	0.000	0.061	0.078	
0.80	0.000	0.005	0.000 ▶	0.071	0.083	0.086	0.000	0.055	0.075	
0.85	0.000	0.060	0.000	0.066	0.082	0.083	0.000	0.049	0.072	
0.90	0.000	0.055	0.000	0.060	0.080	0.079	0.000	0.043	0.068	
0.95	0.000	0.050	0.000	0.055	0.079	0.075	0.000	0.038	0.065	
1.00	0.000	0.045	0.000	0.050	0.075	0.071	0.000	0.033	0.061	

$$m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$$

$$w_{u,dl} = 0.105 \, ksf$$
, $w_{u,ll} = 0.230 \, ksf$ and $w_u = 0.335 \, ksf$



□ Solution

- Slab Design
- > Step 3: Analysis
 - Moment Coefficients

$$C_{a,neg} = 0.071$$
0.076

$$C_{b,neg} = 0.029$$
0.076

Panel - I

Table A2: Coefficients (C _{b, Negative}) For Negative Moment in Slab along Short Direction									
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.000	0.006	0.022	0.006	0.000	0.000	0.014	0.010	0.003
0.55	0.000	0.007	0.028	0.008	0.000	0.000	0.019	0.014	0.005
0.60	0.000	0.010	0.035	0.011	0.000	0.000	0.024	0.018	0.006
0.65	0.000	0.014	0.043	0.015	0.000	0.000	0.031	0.024	0.008
0.70	0.000	0.017	0.050	0.019	0.000	0.000	0.038	0.029	0.011
0.75	0.000	0.022	0.056	0.024	0.000	0.000	0.044	0.036	0.014
0.80	0.000	0.027	0.061 ▶	0.029	0.000	0.000	0.051	0.041	0.017
0.85	0.000	0.031	0.065	0.034	0.000	0.000	0.057	0.046	0.021
0.90	0.000	0.037	0.070	0.040	0.000	0.000	0.062	0.052	0.025
0.95	0.000	0.041	0.072	0.045	0.000	0.000	0.067	0.056	0.029
1.00	0.000	0.045	0.076	0.050	0.000	0.000	0.071	0.061	0.033

$$m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$$

$$w_{u,dl} = 0.105 \, ksf$$
, $w_{u,ll} = 0.230 \, ksf$ and $w_u = 0.335 \, ksf$



□ Solution

- Slab Design
- > Step 3: Analysis
 - Moment Coefficients

$$C_{a,neg} = 0.076$$
0.076

$$C_{b,neg} = 0.029$$
0.076

$$C_{a,pos,dl} = 0.039$$
0.076

Panel – I

Table A3: Coefficients ($C_{a,dl}$) For Dead Load Positive Moment in Slab along Short Direction									
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.095	0.037	0.080	0.059	0.039	0.061	0.089	0.056	0.023
0.55	0.088	0.035	0.071	0.056	0.038	0.058	0.081	0.052	0.024
0.60	0.081	0.034	0.062	0.053	0.037	0.056	0.073	0.048	0.026
0.65	0.074	0.032	0.054	0.050	0.036	0.054	0.065	0.044	0.028
0.70	0.068	0.030	0.046	0.046	0.035	0.051	0.058	0.040	0.029
0.75	0.061	0.028	0.040	0.043	0.033	0.048	0.051	0.036	0.031
0.80	0.056	0.026	0.034 ▶	0.039	0.032	0.045	0.045	0.032	0.029
0.85	0.050	0.024	0.029	0.036	0.031	0.042	0.040	0.029	0.028
0.90	0.045	0.022	0.025	0.033	0.029	0.039	0.035	0.025	0.026
0.95	0.040	0.020	0.021	0.030	0.028	0.036	0.031	0.022	0.024
1.00	0.036	0.018	0.018	0.027	0.027	0.033	0.027	0.020	0.023

$$m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$$

$$w_{u,dl} = 0.105 \, ksf$$
, $w_{u,ll} = 0.230 \, ksf$ and $w_u = 0.335 \, ksf$



□ Solution

- Slab Design
- > Step 3: Analysis
 - Moment Coefficients

$$C_{a,neg} = 0.071$$
0.076

$$C_{b,neg} = 0.029$$
0.076

$$C_{a,pos,dl} = 0.039$$
0.076

$$C_{a,pos,ll} = 0.0480.076$$

Panel - I

Table A4: Coefficients (C _{a, il}) For Live Load Positive Moment in Slab along Short Direction									
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.095	0.066	0.088	0.077	0.067	0.078	0.092	0.076	0.067
0.55	0.088	0.062	0.080	0.072	0.063	0.073	0.085	0.070	0.063
0.60	0.081	0.058	0.071	0.067	0.059	0.068	0.077	0.065	0.059
0.65	0.074	0.053	0.064	0.062	0.055	0.064	0.070	0.059	0.054
0.70	0.068	0.049	0.057	0.057	0.051	0.060	0.063	0.054	0.050
0.75	0.061	0.045	0.051	0.052	0.047	0.055	0.056	0.049	0.046
0.80	0.056	0.041	0.045 ▶	0.048	0.044	0.051	0.051	0.044	0.042
0.85	0.050	0.037	0.040	0.043	0.041	0.046	0.045	0.040	0.039
0.90	0.045	0.034	0.035	0.039	0.037	0.042	0.040	0.035	0.036
0.95	0.040	0.030	0.031	0.035	0.034	0.038	0.036	0.031	0.032
1.00	0.036	0.027	0.027	0.032	0.032	0.035	0.032	0.028	0.030

$$m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$$

$$w_{u,dl} = 0.105 \, ksf$$
, $w_{u,ll} = 0.230 \, ksf$ and $w_u = 0.335 \, ksf$



□ Solution

- Slab Design
- > Step 3: Analysis
 - Moment Coefficients

$$C_{a,neg} = 0.071$$
0.076

$$C_{b,neg} = 0.029$$
0.076

$$C_{a,pos,dl} = 0.039$$
0.076

$$C_{a,pos,ll} = 0.048$$
0.076

$$C_{b,pos,dl} = 0.016$$
 0.076

Panel - I

Та	Table A5: Coefficients ($C_{b, dl}$) For Dead Load Positive Moment in Slab along Long Direction							on	
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.006	0.002	0.007	0.004	0.001	0.003	0.007	0.004	0.002
0.55	0.008	0.003	0.009	0.005	0.002	0.004	0.009	0.005	0.003
0.60	0.010	0.004	0.011	0.007	0.003	0.006	0.012	0.007	0.004
0.65	0.013	0.006	0.014	0.009	0.004	0.007	0.014	0.009	0.005
0.70	0.016	0.007	0.016	0.011	0.005	0.009	0.017	0.011	0.006
0.75	0.019	0.009	0.018	0.013	0.007	0.013	0.020	0.013	0.007
0.80	0.023	0.011	0.020 ▶	0.016	0.009	0.015	0.022	0.015	0.010
0.85	0.026	0.012	0.022	0.019	0.011	0.017	0.025	0.017	0.013
0.90	0.029	0.014	0.024	0.022	0.013	0.021	0.028	0.019	0.015
0.95	0.033	0.016	0.025	0.024	0.015	0.024	0.031	0.021	0.017
1.00	0.036	0.018	0.027	0.027	0.018	0.027	0.033	0.023	0.020

$$m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$$

$$w_{u,dl} = 0.105 \, ksf$$
, $w_{u,ll} = 0.230 \, ksf$ and $w_u = 0.335 \, ksf$

Prof. Dr. Qaisar Ali



□ Solution

- Slab Design
- > Step 3: Analysis
 - Moment Coefficients

$$C_{a,neg} = 0.071$$
0.076

$$C_{b,neg} = 0.029$$
0.076

$$C_{a,pos,dl} = 0.039$$
0.076

$$C_{a,pos,ll} = 0.048$$
0.076

$$C_{b,pos,dl} = 0.016$$
0.076

$$C_{b,pos,ll} = 0.020$$
0.076

Panel - I

Т	Table A6: Coefficients (C _{b, //}) For Live Load Positive Moment in Slab along Long Direction							on	
									.11111
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
0.50	0.006	0.004	0.007	0.005	0.004	0.005	0.007	0.005	0.007
0.55	0.008	0.006	0.009	0.007	0.005	0.006	0.009	0.007	0.006
0.60	0.010	0.007	0.011	0.009	0.007	0.008	0.011	0.009	0.007
0.65	0.013	0.010	0.014	0.011	0.009	0.010	0.014	0.011	0.009
0.70	0.016	0.012	0.016	0.014	0.011	0.013	0.017	0.014	0.011
0.75	0.019	0.014	0.019	0.016	0.013	0.016	0.020	0.016	0.013
0.80	0.023	0.017	0.022 ▶	0.020	0.016	0.019	0.023	0.019	0.017
0.85	0.026	0.019	0.024	0.023	0.019	0.022	0.026	0.022	0.020
0.90	0.029	0.022	0.027	0.026	0.021	0.025	0.029	0.024	0.022
0.95	0.033	0.025	0.029	0.029	0.024	0.029	0.032	0.027	0.025
1.00	0.036	0.027	0.032	0.032	0.027	0.032	0.035	0.030	0.08

$$m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$$

Slab Case = 4

$$w_{u,dl} = 0.105 \, ksf$$
, $w_{u,ll} = 0.230 \, ksf$ and $w_u = 0.335 \, ksf$

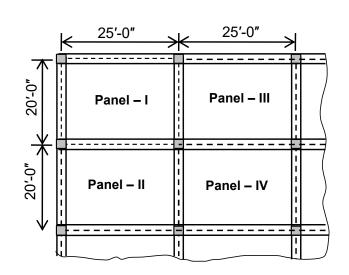
Prof. Dr. Qaisar Ali



□ Solution

- Slab Design
- ➤ Step 3: Analysis (Panel I)

$$m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$$



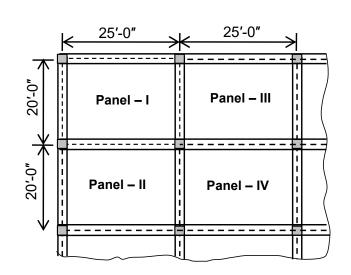
Coefficients	Moment formulae	Moment Values (in.kip)	
$C_{a,neg} = 0.071$	$M_{a,neg} = C_{a,neg} w_u l_a^2$	101.32	
$C_{b,neg} = 0.029$	$M_{b,neg} = C_{b,neg} w_u l_b^2$	66.28	
$C_{a,pos,dl} = 0.039$	$M_{a,pos} = C_{a,pos,dl} w_{u,dl} l_a^2 + C_{a,pos,ll} w_{u,ll} l_a^2$	64.48	
$C_{a,pos,ll} = 0.048$	$M_{a,pos} = C_{a,pos,dl} W_{u,dl} l_a + C_{a,pos,ll} W_{u,ll} l_a$	04.40	
$C_{b,pos,dl} = 0.016$	$M_{b,pos} = C_{b,pos,dl} w_{u,dl} l_b^2 + C_{b,pos,ll} w_{u,ll} l_b^2$	42.85	
$C_{b,pos,ll} = 0.02$	$M_{b,pos} - C_{b,pos,dl} w_{u,dl} t_b + C_{b,pos,ll} w_{u,ll} t_b$	42.03	



□ Solution

- Slab Design
- ➤ Step 3: Analysis (Panel II)

$$m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$$



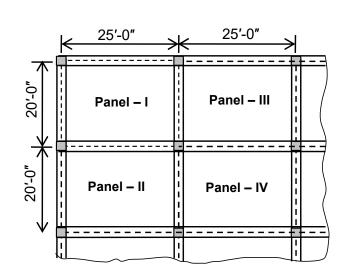
Coefficients	Moment formulae	Moment Values (in.kip)	
$C_{a,neg} = 0.075$	$M_{a,neg} = C_{a,neg} w_u l_a^2$	107.03	
$C_{b,neg} = 0.017$	$M_{b,neg} = C_{b,neg} w_u l_b^2$	38.85	
$C_{a,pos,dl} = 0.029$	$M_{a,pos} = C_{a,pos,dl} w_{u,dl} l_a^2 + C_{a,pos,ll} w_{u,ll} l_a^2$	54.13	
$C_{a,pos,ll} = 0.042$	$M_{a,pos} - C_{a,pos,dl} w_{u,dl} t_a + C_{a,pos,ll} w_{u,ll} t_a$	34.13	
$C_{b,pos,dl} = 0.01$	$M_{b,pos} = C_{b,pos,dl} w_{u,dl} l_b^2 + C_{b,pos,ll} w_{u,ll} l_b^2$	33.85	
$C_{b,pos,ll} = 0.017$	$M_{b,pos} = C_{b,pos,dl} w_{u,dl} b + C_{b,pos,ll} w_{u,ll} b$	33.63	



□ Solution

- Slab Design
- Step 3: Analysis (Panel III)

$$m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$$



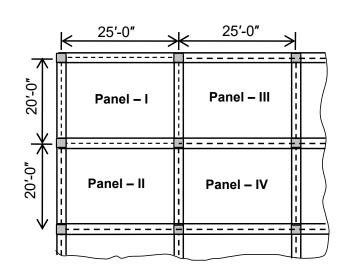
Coefficients	Coefficients Moment formulae		
$C_{a,neg} = 0.055$	$M_{a,neg} = C_{a,neg} w_u l_a^2$	78.49	
$C_{b,neg} = 0.041$	$M_{b,neg} = C_{b,neg} w_u l_b^2$	93.71	
$C_{a,pos,dl} = 0.032$	$M_{a,pos} = C_{a,pos,dl} w_{u,dl} l_a^2 + C_{a,pos,ll} w_{u,ll} l_a^2$	57.43	
$C_{a,pos,ll} = 0.044$	$M_{a,pos} - C_{a,pos,dl} W_{u,dl} t_a + C_{a,pos,ll} W_{u,ll} t_a$	37.43	
$C_{b,pos,dl} = 0.015$	$M_{b,pos} = C_{b,pos,dl} w_{u,dl} l_b^2 + C_{b,pos,ll} w_{u,ll} l_b^2$	40.56	
$C_{b,pos,ll} = 0.019$	$W_{b,pos} = C_{b,pos,dl} w_{u,dl} t_b + C_{b,pos,ll} w_{u,ll} t_b$	40.30	



□ Solution

- Slab Design
- Step 3: Analysis (Panel IV)

$$m = l_a/l_b = 18.83/23.83 = 0.78 \approx 0.80$$



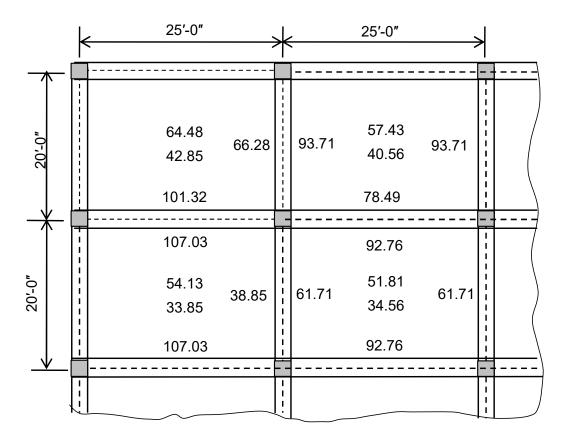
Coefficients	Moment formulae	Moment Values (in.kip)	
$C_{a,neg} = 0.065$	$M_{a,neg} = C_{a,neg} w_u l_a^2$	92.76	
$C_{b,neg} = 0.027$	$M_{b,neg} = C_{b,neg} w_u l_b^2$	61.71	
$C_{a,pos,dl} = 0.026$	$M_{a,pos} = C_{a,pos,dl} w_{u,dl} l_a^2 + C_{a,pos,ll} w_{u,ll} l_a^2$	51.81	
$C_{a,pos,ll} = 0.041$	$M_{a,pos} - C_{a,pos,dl} W_{u,dl} t_a + C_{a,pos,ll} W_{u,ll} t_a$	31.01	
$C_{b,pos,dl} = 0.011$	$M_{b,pos} = C_{b,pos,dl} w_{u,dl} l_b^2 + C_{b,pos,ll} w_{u,ll} l_b^2$	34.56	
$C_{b,pos,ll} = 0.017$	$M_{b,pos} - C_{b,pos,dl} w_{u,dl} t_b + C_{b,pos,ll} w_{u,ll} t_b$	34.30	

- □ Solution
 - Slab Design
 - Step 3: Analysis

Summary of Analysis							
Location	Moment Values (in. kip/ft)						
	Panel – I	Panel - II	Panel - III	Panel - IV			
$M_{a,neg}$	101.32	107.03	78.49	92.76			
$M_{b,neg}$	66.28	38.85	93.71	61.71			
$M_{a,pos}$	64.48	54.13	57.43	51.81			
$M_{b,pos}$	42.85	33.85	40.56	34.56			



- □ Solution
 - Slab Design
 - Step 3: Analysis



NOTE:

- All values are in in.kip/ft
- · White values: Long Direction Moments
- · Yellow values: Short Direction Moments

- □ Solution
 - Slab Design
 - > Step 4: Determination of Steel Area
 - As there are several bending moments, making it exceedingly timeconsuming and lengthy to calculate the steel area for each one.
 - As a result, we shall use the "Unity Rule" in this situation.
 - Calculate the area of steel for a unit moment and multiply it by the actual moments to obtain the necessary area of steel as described on the next slide.



- □ Solution
 - Slab Design
 - > Step 4: Determination of Steel Area
 - For $M_u = 1$ in. kip

$$a = d - \sqrt{d^2 - \frac{2.614M_u}{f_c'b}} = 6 - \sqrt{6^2 - \frac{2.614 \times (1)}{3 \times 12}} = 6.05 \times 10^{-3} in.$$

Now,

$$A_s = \frac{M_u}{0.9 f_y \left(d - \frac{a}{2}\right)} = \frac{1}{0.9 \times 40 \left(6 - \frac{6.05 \times 10^{-3}}{2}\right)} = 0.005 \ in^2 / ft$$



Solution

Slab Design

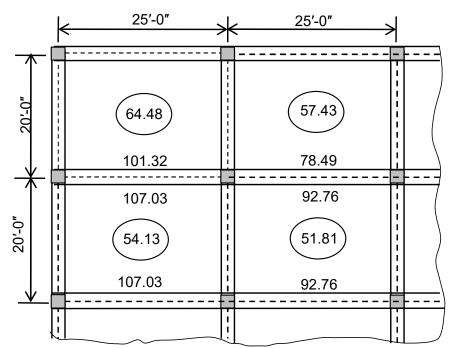
Step 4: Determination of Steel Area

For Positive Moments in Short Directions

$$A_{s,min} = 0.0018(12)(7) = 0.151in^2/ft$$

 $S_{max} = 2h \text{ or } 18" = 14"c/c$

M_u (in.kip/ft)	$A_s = 0.005 M_u$ (in^2/ft)	S _{req} using #4	Final S
64.48	$0.32 > A_{s,min}$	7.5"	7"
54.13	$0.27 > A_{s,min}$	8.9"	7"
57.43	$0.29 > A_{s,min}$	8.3"	7"
51.81	$0.26 > A_{s,min}$	9.2"	7"





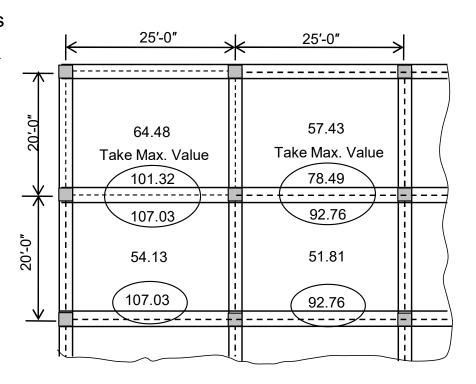
□ Solution

- Slab Design
- Step 4: Determination of Steel Area
 - For Negative Moments in Short Directions

$$A_{s,min} = 0.0018(12)(7) = 0.151in^2/ft$$

 $S_{max} = 2h \text{ or } 18" = 14"c/c$

M_u (in.kip/ft)	$A_s = 0.005 M_u$ (in^2/ft)	S _{req} using #4	Final S
107.03	$0.54 > A_{s,min}$	4.4"	4"
92.76	$0.46 > A_{s,min}$	5.2"	4"





□ Solution

Slab Design

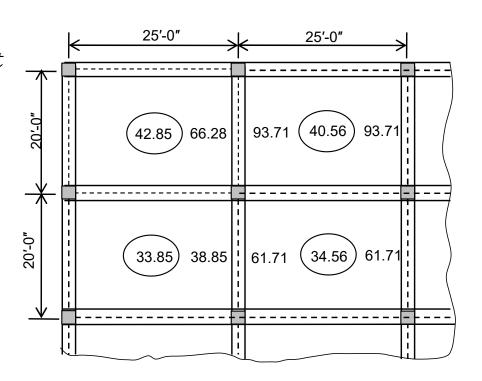
Step 4: Determination of Steel Area

For Positive Moments in Long Directions

$$A_{s,min} = 0.0018(12)(7) = 0.151in^2/ft$$

 $S_{max} = 2h \text{ or } 18" = 14"c/c$

M_u (in.kip/ft)	$A_s = 0.005 M_u$ (in^2/ft)	S _{req} using #4	Final S
42.85	$0.21 > A_{s,min}$	11.4"	10"
40.56	$0.20 > A_{s,min}$	11.88"	10"
33.85	$0.17 > A_{s,min}$	14.1"	10"
34.56	$0.17 > A_{s,min}$	14.1"	10"





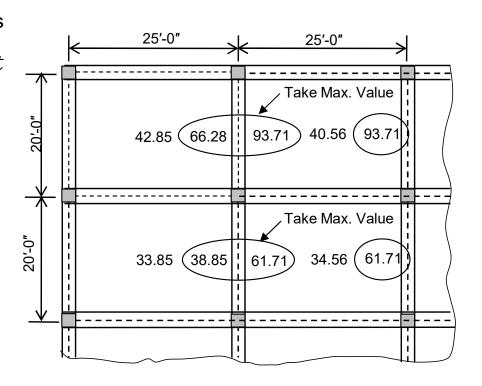
□ Solution

- Slab Design
- Step 4: Determination of Steel Area
 - For Negative Moments in Long Directions

$$A_{s,min} = 0.0018(12)(7) = 0.151in^2/ft$$

 $S_{max} = 2h \text{ or } 18" = 14"c/c$

M_u (in. kip/ft)	$A_s = 0.005 M_u$ (in^2/ft)	S _{req} using #4	Final S
93.73	$0.47 > A_{s,min}$	5.1"	4"
61.71	$0.31 > A_{s,min}$	8.1"	7"





□ Solution

Slab Design

Step 5: Reinforcement Detailing

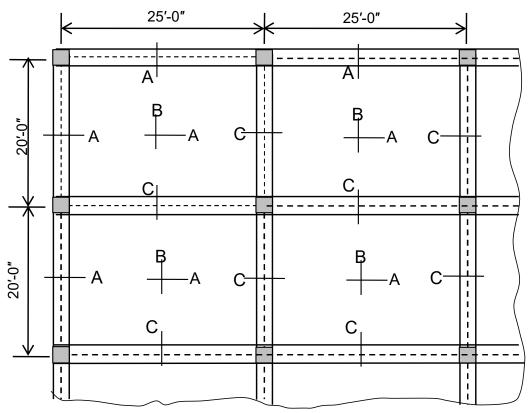
Finally, three set of spacings have been provided as shown.

$$A = #4 @ 10"c/c$$

 $B = #4 @ 7"c/c$

$$C = #4 @ 4"c/c$$

- Yellow values: Short Direction
- White values : Long direction





□ Solution

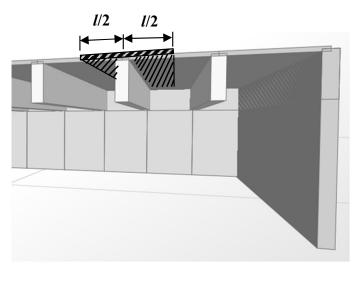
Beam Design

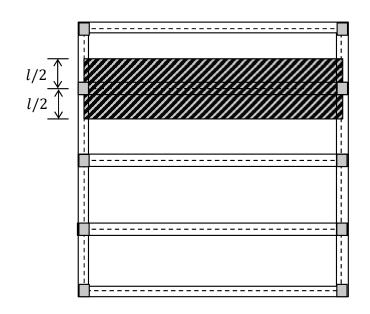
In the following session, only the mechanism of load transfer from slabs to beams will be discussed; the analysis and design portions can be completed as usual.

Solution

Beam Design

Load Transfer Mechanism





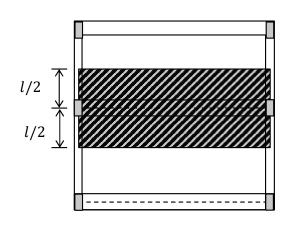
- In case of one-way slab system, the entire slab load is transferred in short direction.
- Load transfer in short direction = $(w_u \times \ell/2 \times 1) + (w_u \times \ell/2 \times 1)$
- Load transfer in long direction = $w_{11} \times \ell / 2 \times \mathbf{0}$



* Beam Design

Load Transfer Mechanism

In case of two-way slab system, the slab load is transferred in both directions as follows:



Load transfer in short direction =
$$(wu_{\times} l/2 \times W_a) + (wu \times l/2 \times W_a)$$

Load transfer in long direction =
$$(wu_{\times} l/2 \times W_b) + (wu \times l/2 \times W_b)$$

Where;

 W_a is taken from the ACI coefficient Tables and

$$W_b = 1 - W_a$$

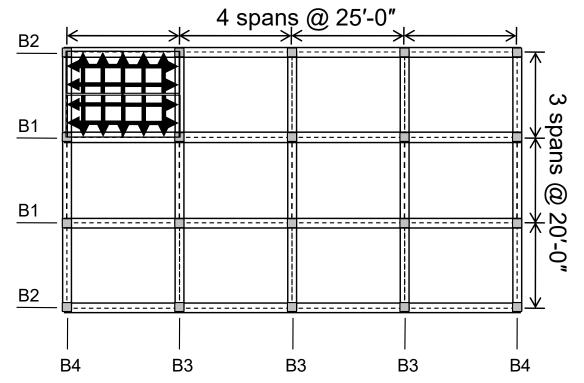
Solution

Beam Design

Load on Beams from Slab Panels

Panel I

Table: Load on beam in Panel I, using Coefficients (w _u = 0.336 ksf)							
Beam	Length (ft)	Width (b _s) of slab panel supported by beam	W_a	W_b	Load due to slab, Ww _u b _s (k/ft)		
B1	25	10	0.71	1	2.39		
B2	25	10	0.71	ı	2.39		
В3	20	12.5	-	0.29	1.22		
B4	B4 20 12		-	0.29	1.22		





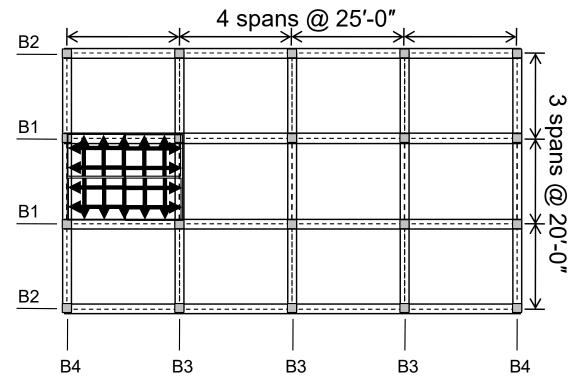
Solution

Beam Design

Load on Beams from Slab Panels

Panel II

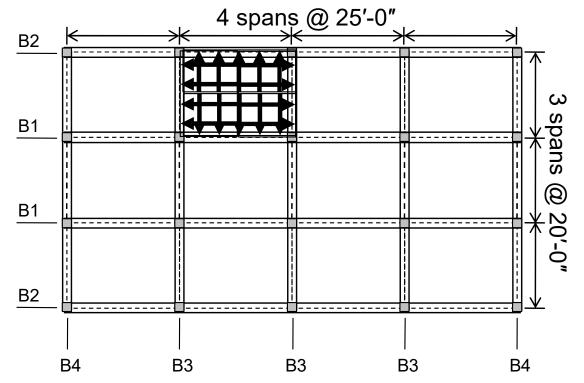
Table: Load on beam in Panel I, using Coefficients (w _u = 0.336 ksf)							
Beam	Length (ft)	Width (b _s) of slab panel supported by beam	W _a	W_b	Load due to slab, Ww _u b _s (k/ft)		
B1	25	10	0.83	ı	2.78		
В3	20	12.5	-	0.17	0.714		
B4	20	12.5	-	0.17	0.714		



- □ Solution
 - Beam Design
 - Load on Beams from Slab Panels

Panel III

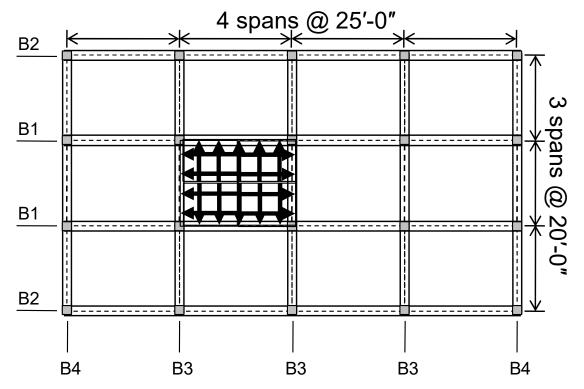
Table: Load on beam in Panel I, using Coefficients (w _u = 0.336 ksf)							
Beam	Length (ft)	Width (b _s) of slab panel supported by beam	Wa	W_b	Load due to slab, Ww _u b _s (k/ft)		
B1	25	10	0.55	ı	1.84		
B2	25	10	0.55	-	1.84		
В3	B3 ₂₀		_	0.45	1.89		



- □ Solution
 - Beam Design
 - Load on Beams from Slab Panels

Panel IV

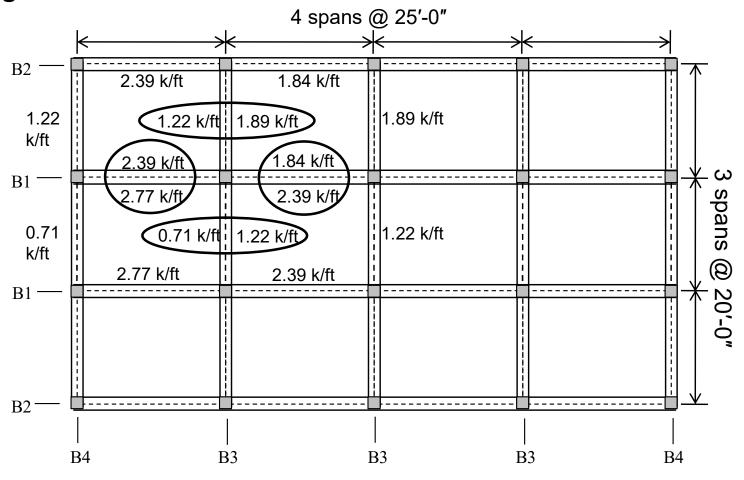
Table: Load on beam in Panel I, using Coefficients (w _u = 0.336 ksf)							
Beam	Length (ft)	Width (b _s) of slab panel supported by beam	Wa	W_b	Load due to slab, Ww _u b _s (k/ft)		
B1	25	10	0.55	ı	1.84		
B2	25	10	0.55	-	1.84		
В3	B3 ₂₀		_	0.45	1.89		





□ Solution

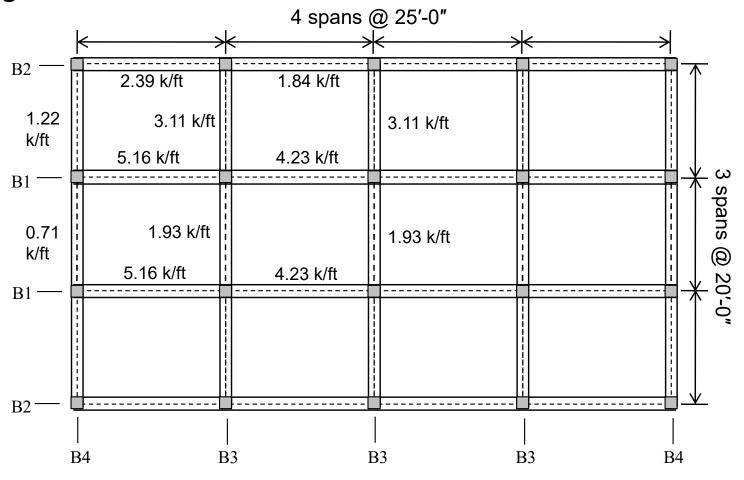
Beam Design





□ Solution

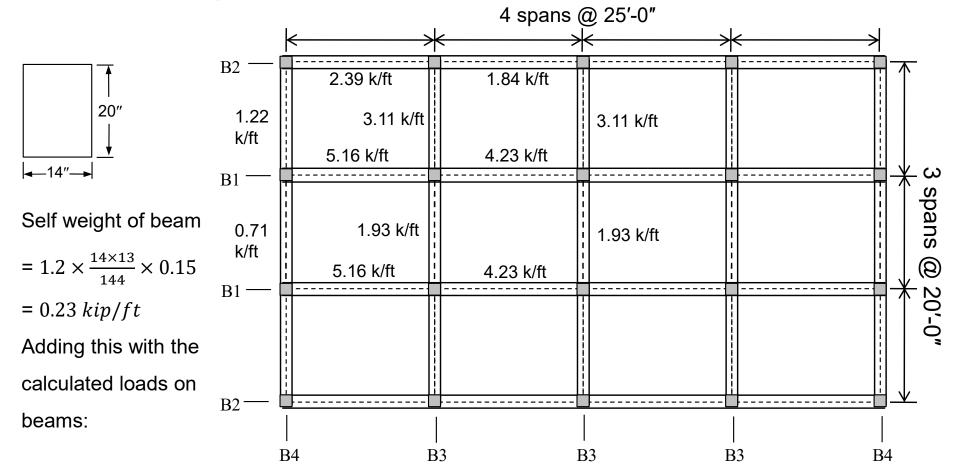
Beam Design





□ Solution

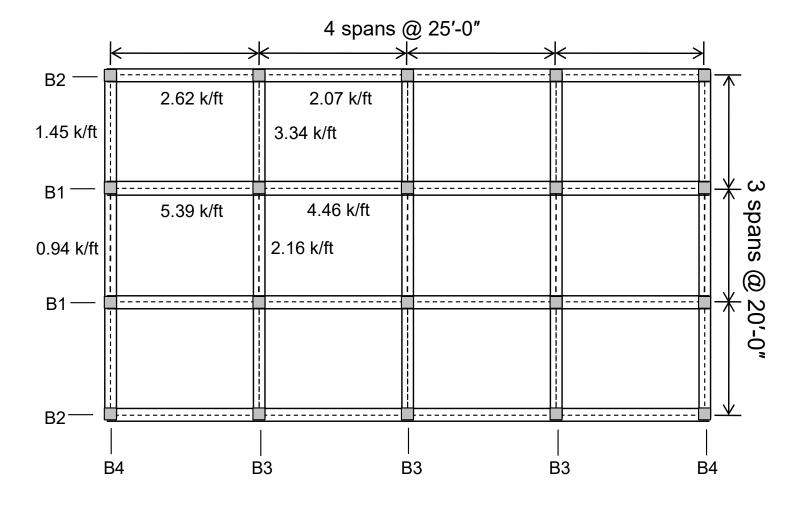
Beam Design





Solution

Beam Design (after including self-weight)



□ Pictures of a Multi-story Commercial Building











Homework

☐ Example 4.3

Design the given slab system using the data provided.

Slab thickness, $h_f = 6in$.

SDL = 60 psf

LL = 40 psf $f_c' = 3 ksi$ $f_y = 60 ksi$

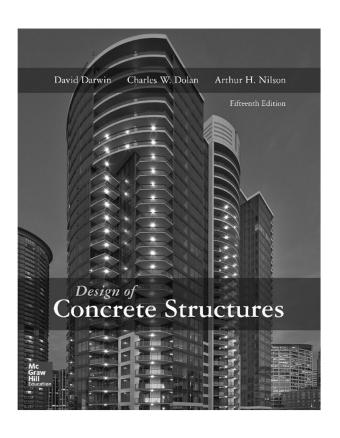
6	oln.	4 spans @ 25′-0″					
\ 		 			∤ ∏		
	Panel I	Panel III	Panel III	Panel I	ω w		
	Panel II	Panel IV	Panel IV	Panel II	3 spans @ 20′-0″		
	Panel I	Panel III	Panel III	Panel I	-'0"		

- All beams are 18" x 24"
- All columns are 18" x 18"



References

- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-19)







Appendix

Table 8.3.1.1 —Minimum thickness of nonprestressed two-way slabs without interior beams (in.)								
	Without drop panels			With drop panels				
f_y	Exterior	Panels	Interior	Exterior Panels				
(psi)	Without edge beams	With edge beams	panels	Without edge beams	With edge beams	Interior panels		
40,000	$l_n/33$	$l_{n}/36$	$l_n/36$	$l_{n}/36$	$l_{n}/40$	$l_n/40$		
60,000	$l_{n}/30$	$l_{n}/33$	$l_{n}/33$	$l_n/33$	$l_n/36$	$l_n/36$		
80,000	$l_{n}/27$	$l_{n}/30$	$l_n/30$	$l_{n}/30$	$l_n/33$	$l_n/33$		

Go Back