Lecture 04

Design of Two-Way Slabs Without Beams (Flat Plates and Flat Slabs)

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- **Part – I Analysis and Design of two-way slab Systems without beams for Flexure**
  - Background
  - Direct Design Method (DDM)
  - Analysis Procedure using DDM
  - Example 5.1
  - Example 5.2
  - Other ACI Provisions for Flat Slabs

Lecture Contents

- **Part – II Analysis and Design of two-way slab Systems without beams for Shear**
  - Introduction
  - Design of Slabs for Punching Shear
  - Example 5.3
- References

Learning Outcomes

- **At the end of this lecture, students will be able to;**
  - ***Design*** flat slabs and flat plates for flexure using Direct Design Method (DDM)
  - ***Design*** flat slabs and flat plates for shear

Section – I

Flexural Design of Two-Way Slab System without Beams

(Flat Plates and Flat Slabs)

Direct Design Method

❑ Background

- In previous lectures, a 90′ x 60′ Hall (as shown below) was analyzed as a one-way slab system using ACI approximate coefficients.

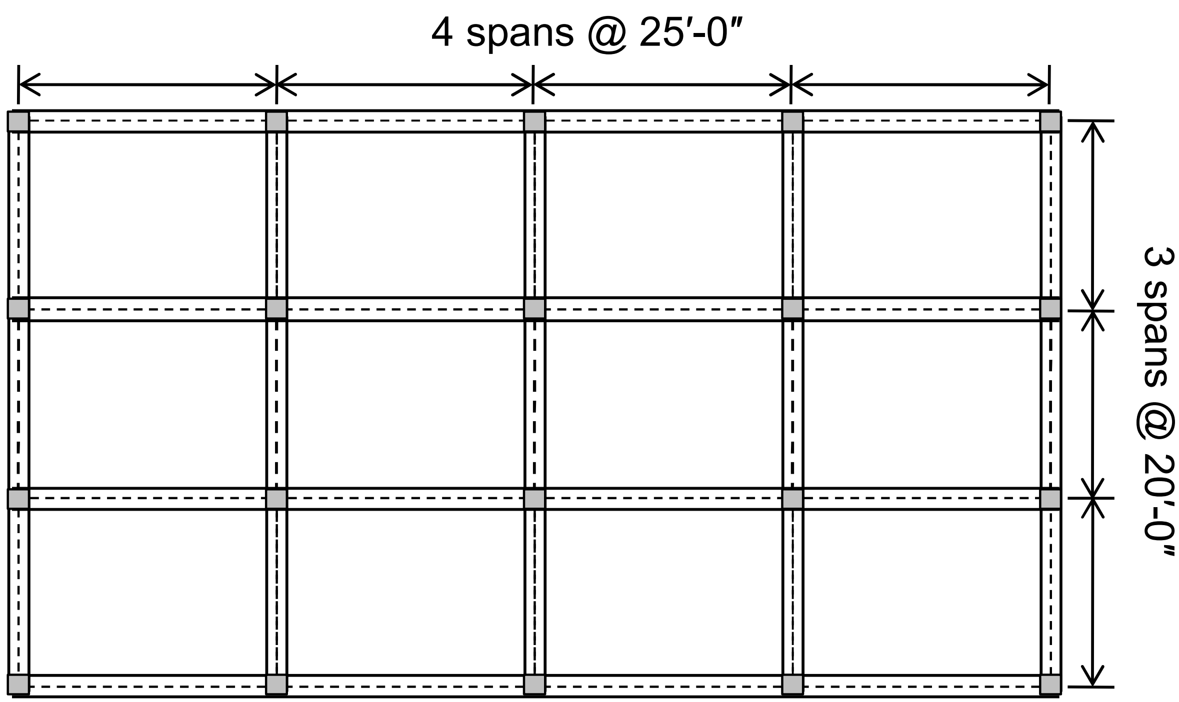
Option1

Option 2

Direct Design Method

❑ Background

- Also, in previous lecture, the slab of 100′ x 60′ commercial building was analyzed as slab with beams using Moment coefficient method.



Direct Design Method

❑ Background

- In the same 100′ x 60′ commercial building, if there are shallow or no beams, Moment Coefficient Method cannot be used.
- For such cases, the Direct Design Method (DDM) can be used.

Direct Design Method

❑ Introduction

- The Direct Design Method (DDM) consists of a set of rules for distributing moments to slab and beam sections to satisfy safety requirements and most serviceability requirements simultaneously.

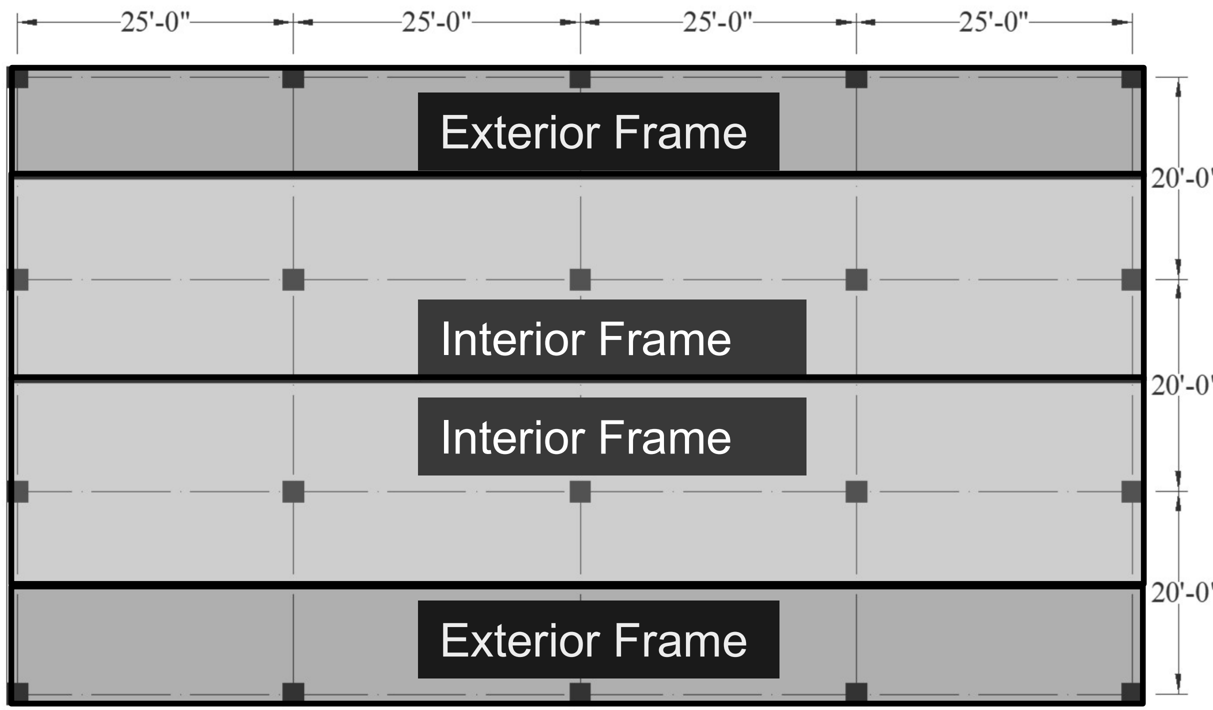
- The DDM is applicable to two-way slabs both with and without beams. However, it becomes comparatively challenging when dealing with slabs that have beams or walls. Hence, it will be only used for slabs without beams in this course.

- Although the provisions for DDM present in ACI 318-14 Section 8.10 have been deleted from the latest version (ACI 318-19), it can still be used as per ACI R6.2.4.1.

Direct Design Method

❑ Introduction

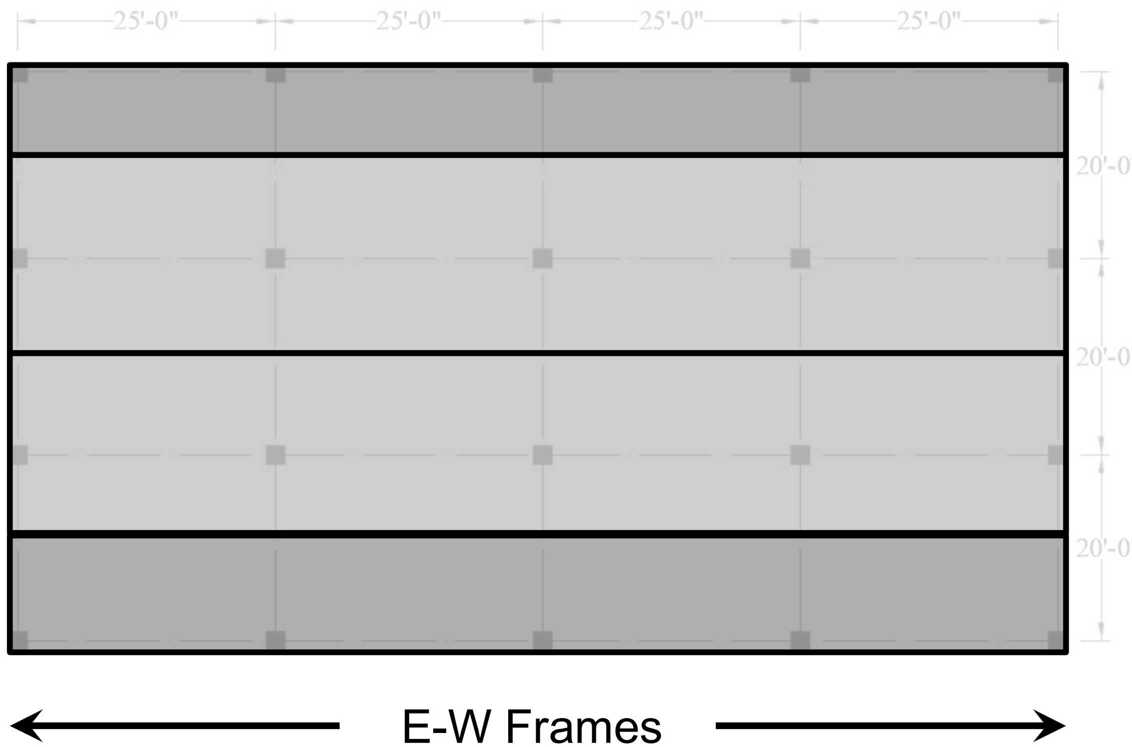
- In DDM, frames rather than panels are analyzed.



Direct Design Method

❑ Introduction

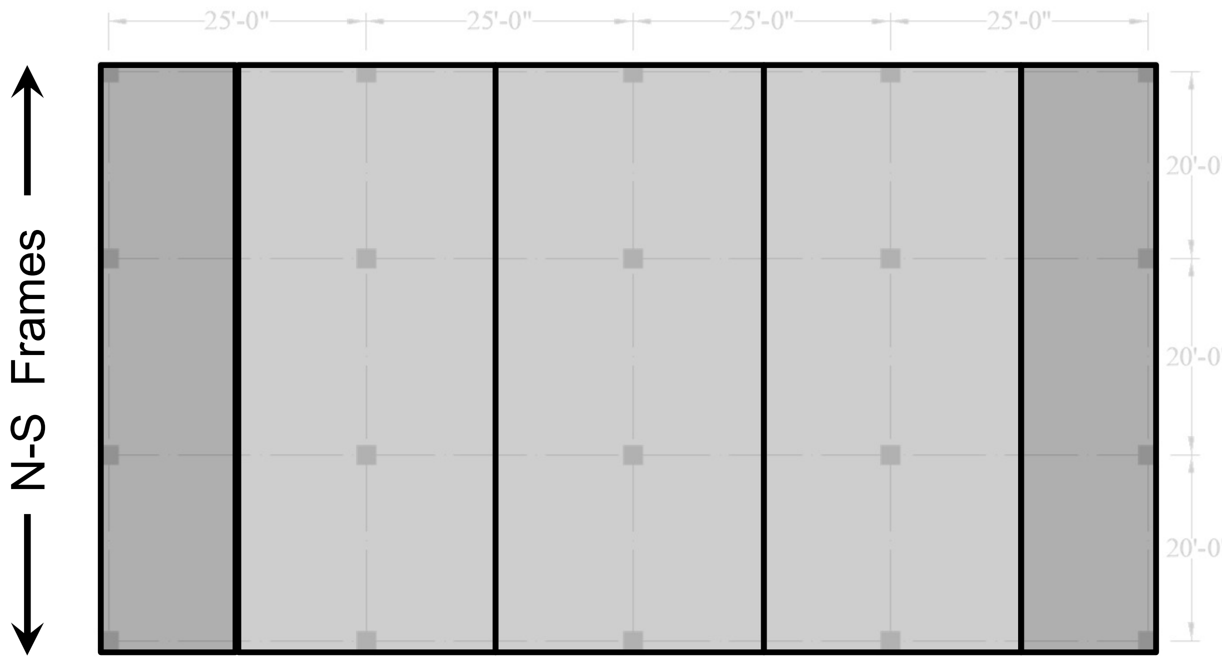
- For complete analysis of slab system, frames are analyzed in E-W and N-S directions.



Direct Design Method

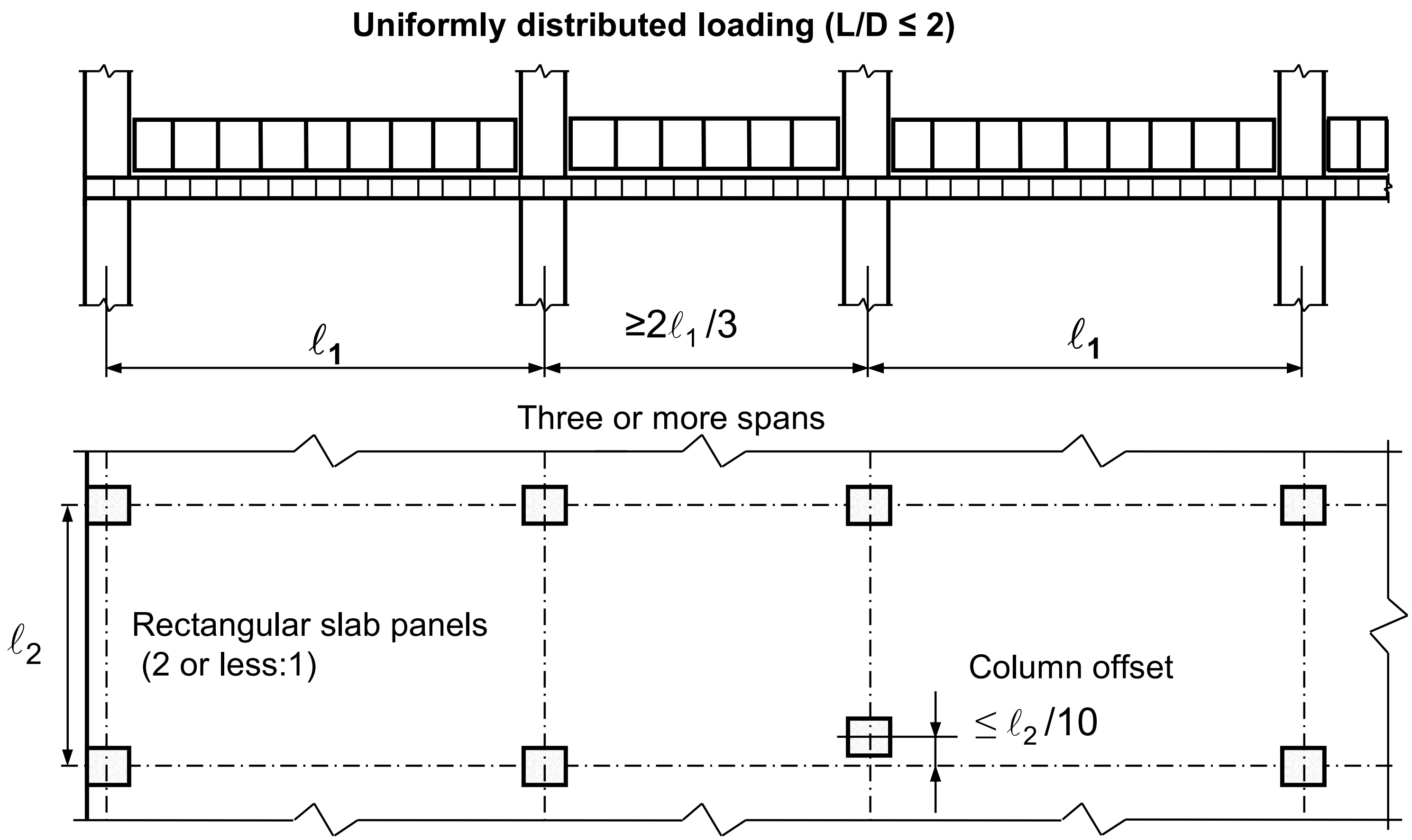
❑ Introduction

- For complete analysis of slab system, frames are analyzed in E-W and N-S directions.



Direct Design Method

❑ Limitations (ACI 8.10.2)



Analysis Procedure using DDM

❑ Step 1: Selection of Sizes

- ACI table 8.3.1.1 is used for finding the slab thickness.

Table 8.3.1.1 —Minimum thickness of nonprestressed two-way slabs without interior beams (in.)

f_y (psi)	Without drop panels			With drop panels		
	Exterior Panels		Interior panels	Exterior Panels		Interior panels
	Without edge beams	With edge beams		Without edge beams	With edge beams	
40,000	$l_n/33$	$l_n/36$	$l_n/36$	$l_n/36$	$l_n/40$	$l_n/40$
60,000	$l_n/30$	$l_n/33$	$l_n/33$	$l_n/33$	$l_n/36$	$l_n/36$
80,000	$l_n/27$	$l_n/30$	$l_n/30$	$l_n/30$	$l_n/33$	$l_n/33$

- l_n is the clear span in the long direction, measured face-to-face of supports (in.).
- $h_{min} = 5''$ for slabs without drop panels
- $h_{min} = 4''$ for slabs with drop panels

Analysis Procedure using DDM

❑ Step 2: Calculation of Loads

- The slab load is calculated in usual manner.

$$W_u = 1.2D + 1.6L$$

Where;

W_u = ultimate/ factored load

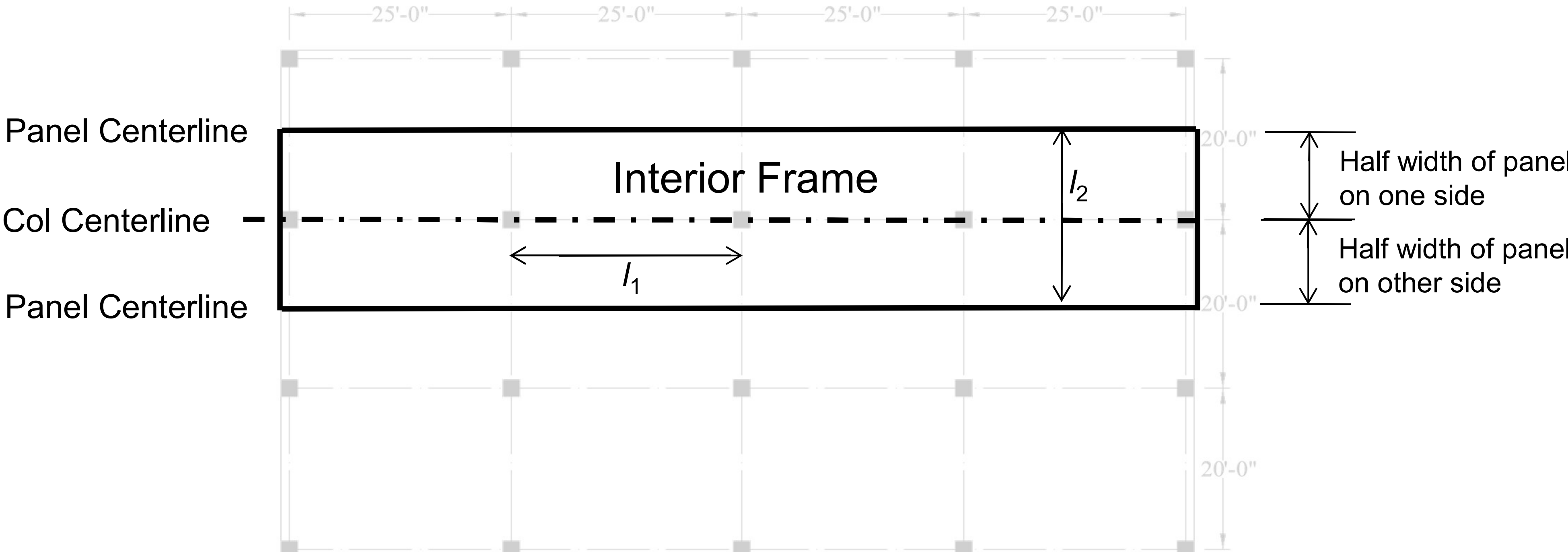
D = service dead load

L = service live load

Analysis Procedure using DDM

❑ Step 3: Analysis

❖ Step I: Marking E-W Frame (Interior)

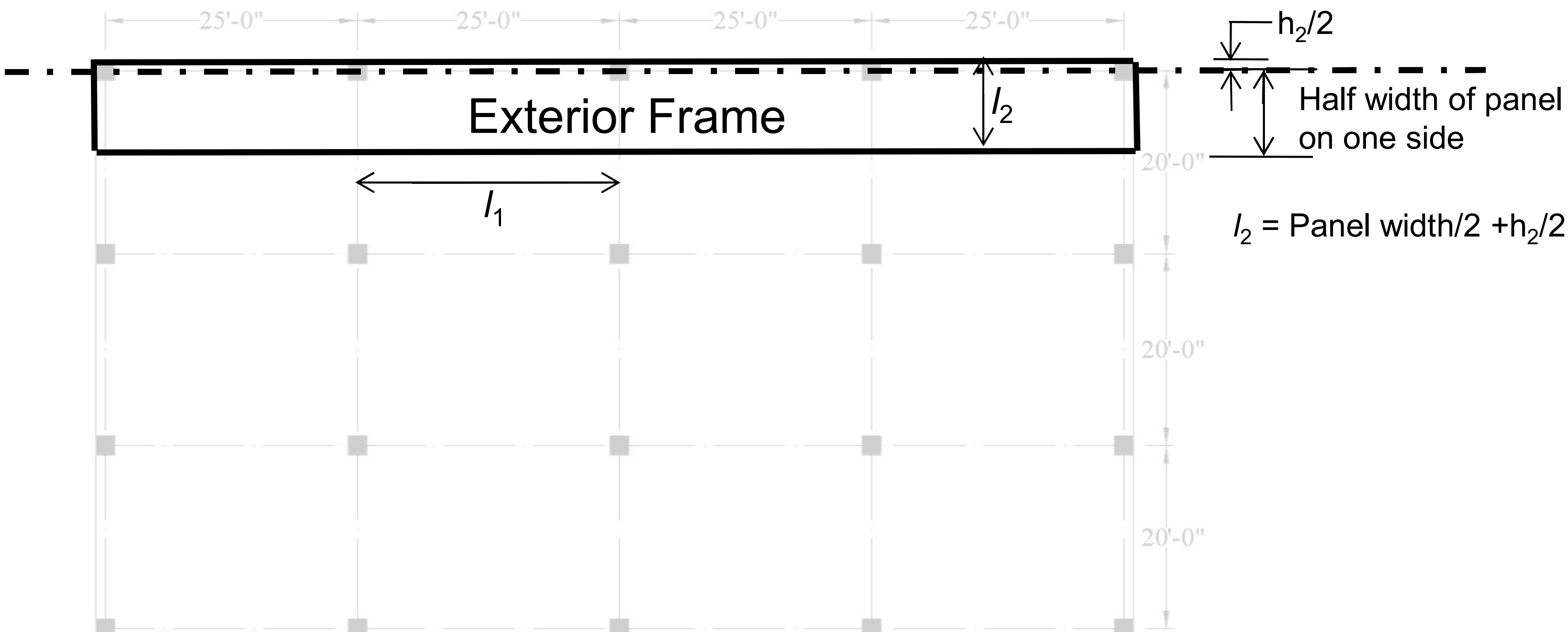


Where the transverse span of panels on either side of the centerline of supports varies, l_2 shall be taken as the average of adjacent transverse spans (ACI 318-14 Section 8.10.3.2.2).

Analysis Procedure using DDM

❑ Step 3: Analysis

❖ Step I: Marking E-W Frame (Exterior)



Analysis Procedure using DDM

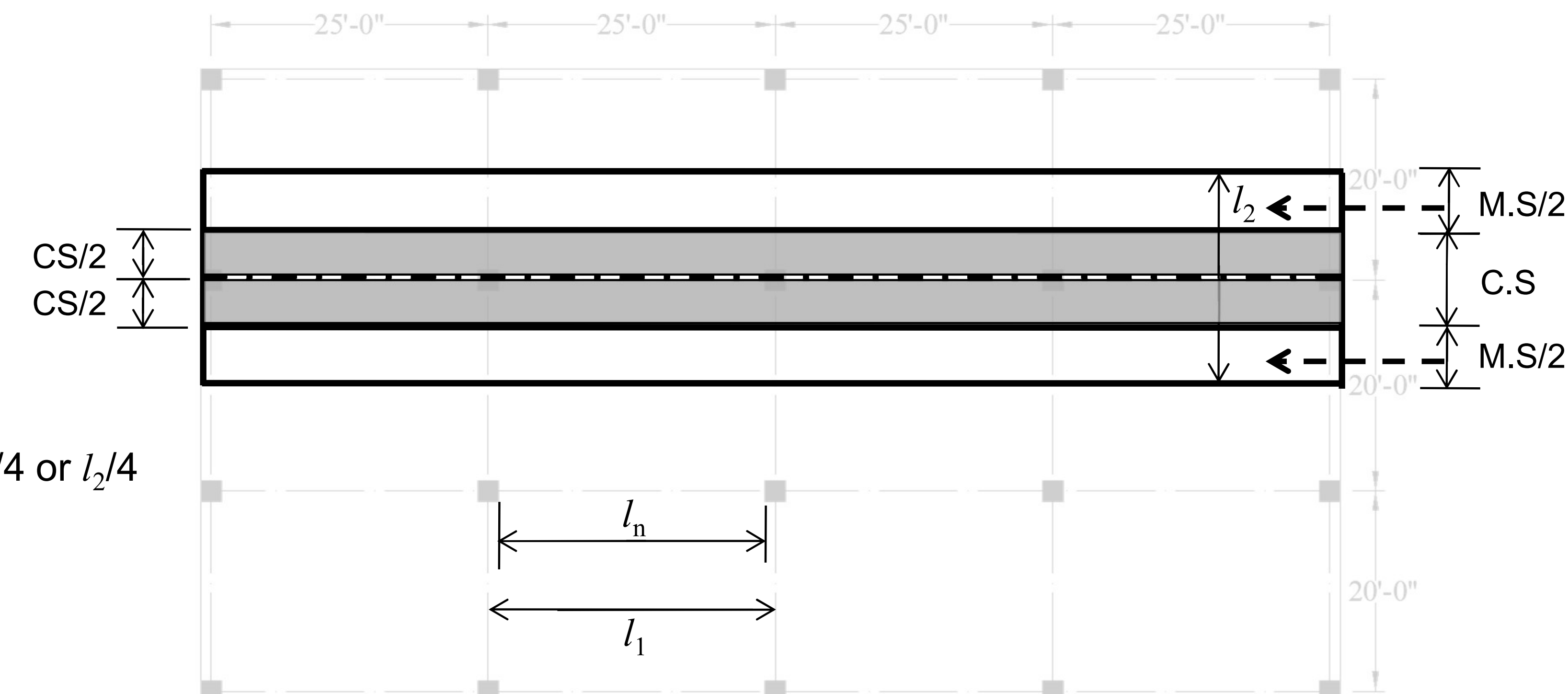
❑ Step 3: Analysis

❖ Step II: Marking Column and Middle strips

a) Marking Column Strip (For Interior Frame)

b) Marking Middle Strip (For Interior Frame)

Half Column strip



CS/2 = Least of $l_1/4$ or $l_2/4$

Analysis Procedure using DDM

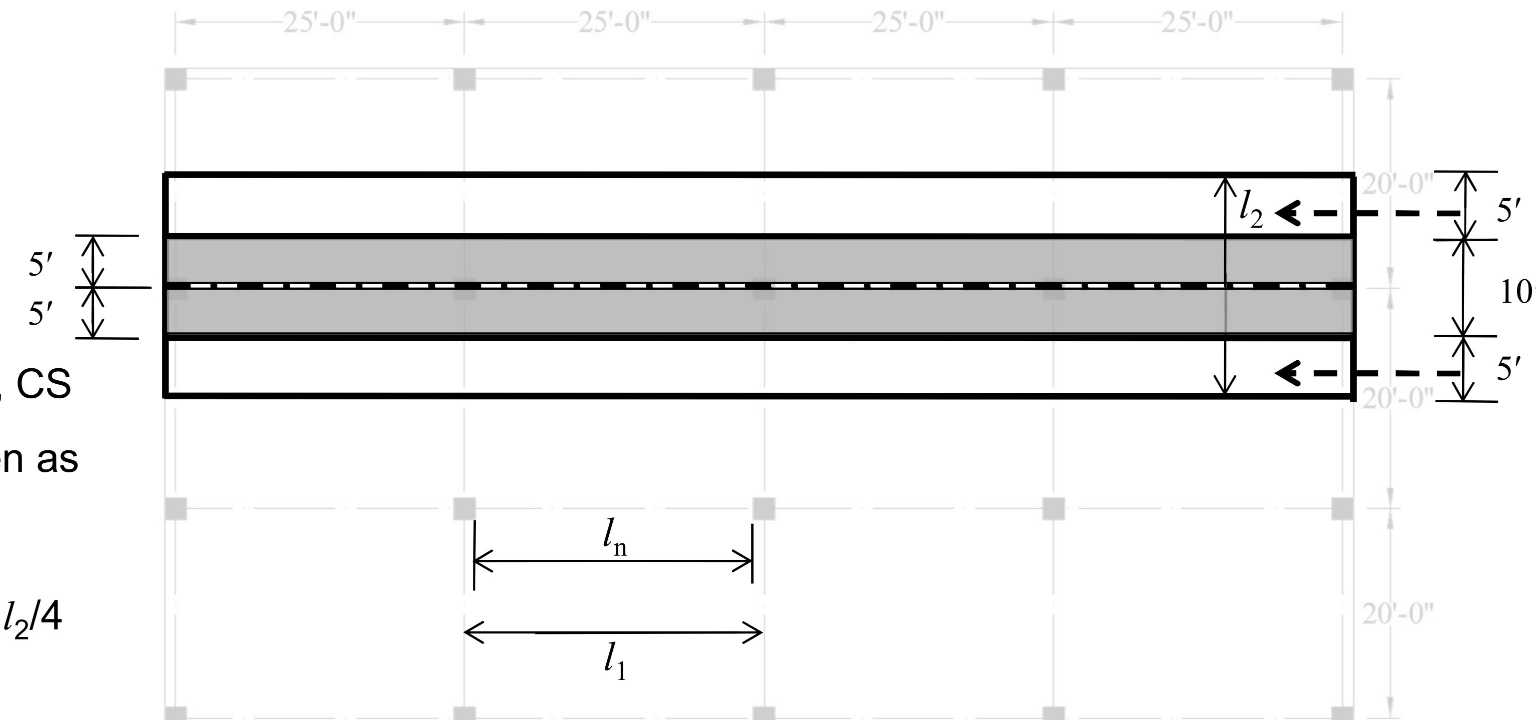
❑ Step 3: Analysis

❖ Step II: Marking Column and Middle strips

a) Marking Column Strip (For Interior Frame)

b) Marking Middle Strip (For Interior Frame)

Half Column Strip



For l_1 = 25′ and l_2 = 20′, CS and MS widths are given as follows:

CS/2 = Least of $l_1/4$ or $l_2/4$

$l_2/4$ = 20/4 = 5′

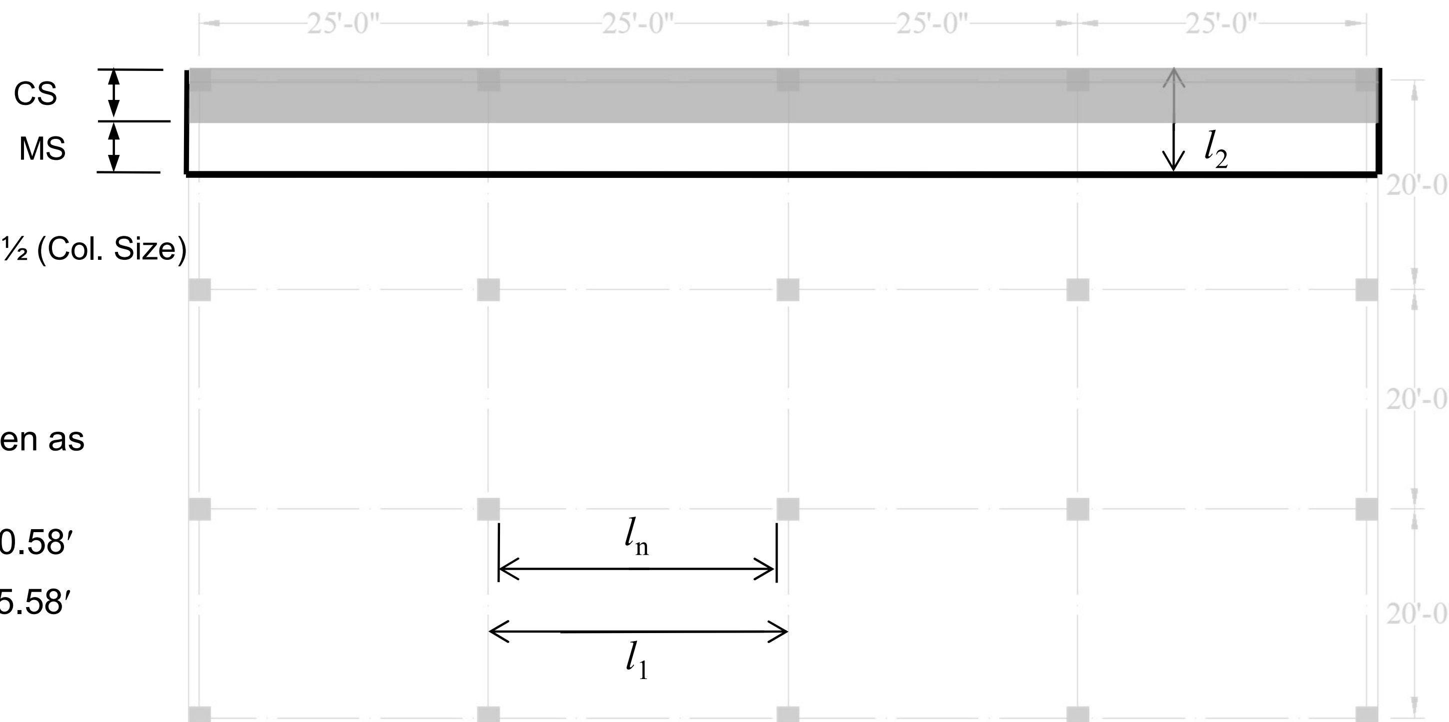
Analysis Procedure using DDM

❑ Step 3: Analysis

❖ Step II: Marking Column and Middle strips

c) Marking Column Strip (For Exterior Frame)

d) Marking Middle Strip (For Exterior Frame)



CS = Min. Panel Width[1]/4 + ½ (Col. Size)

MS = l_2 - CS

For Given Frame:

CS and MS widths are given as follows:

l_2 = ½ (20) + ½ (14/12) = 10.58′

CS = (20/4) + ½ (14/12) = 5.58′

MS = 10.58 – 5.58 = 5′

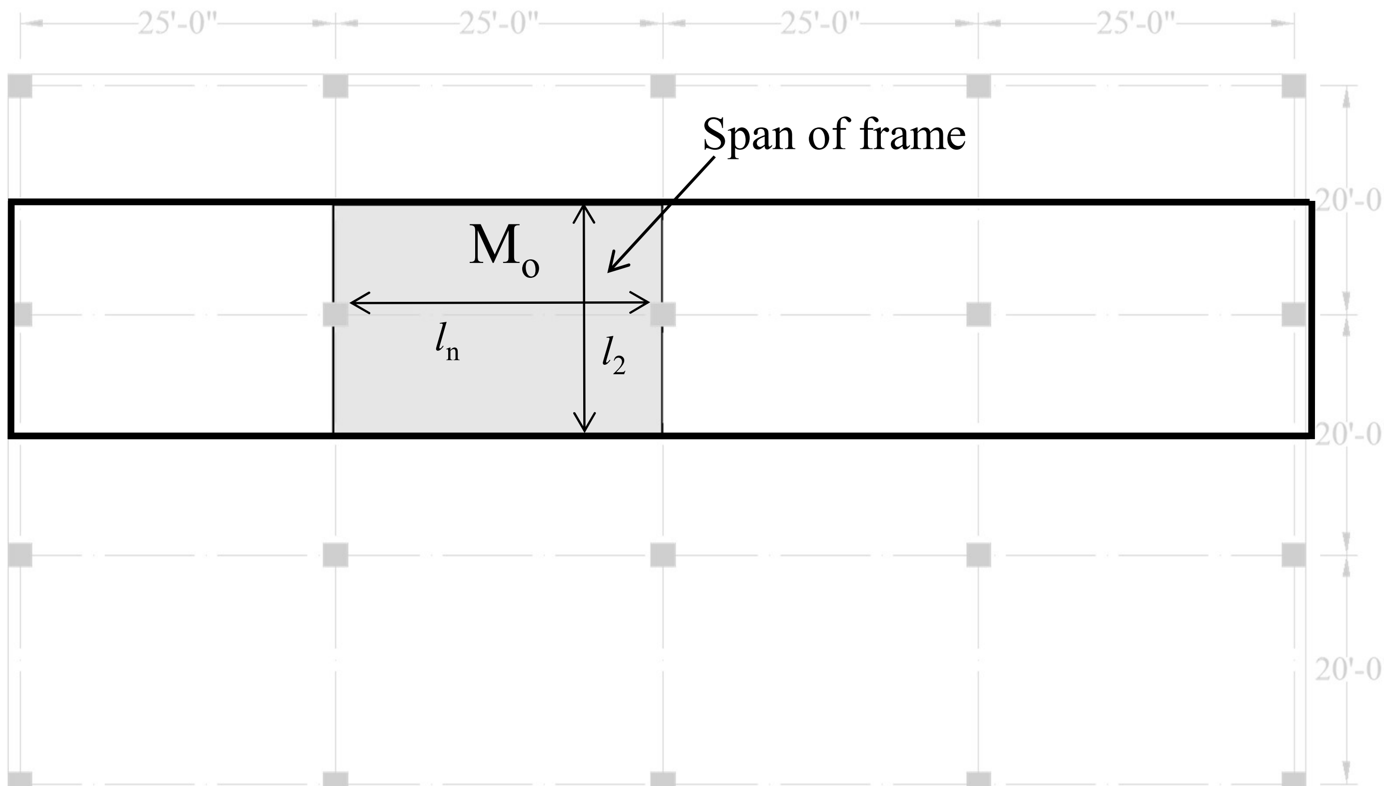
[1] For the exterior frame, the CS is not taken as l_1 or $l_2/4$,but rather as the lesser of the full panel widths adjacent to the exterior frame, which are 20 ft and 25 ft in this case.

Analysis Procedure using DDM

❑ Step 3: Analysis – Interior Span of Interior Frame

❖ Step III: Calculate Static Moment (M_o)

$$M_o = \frac{w_u l_2 \,(l_n)^2}{8}$$

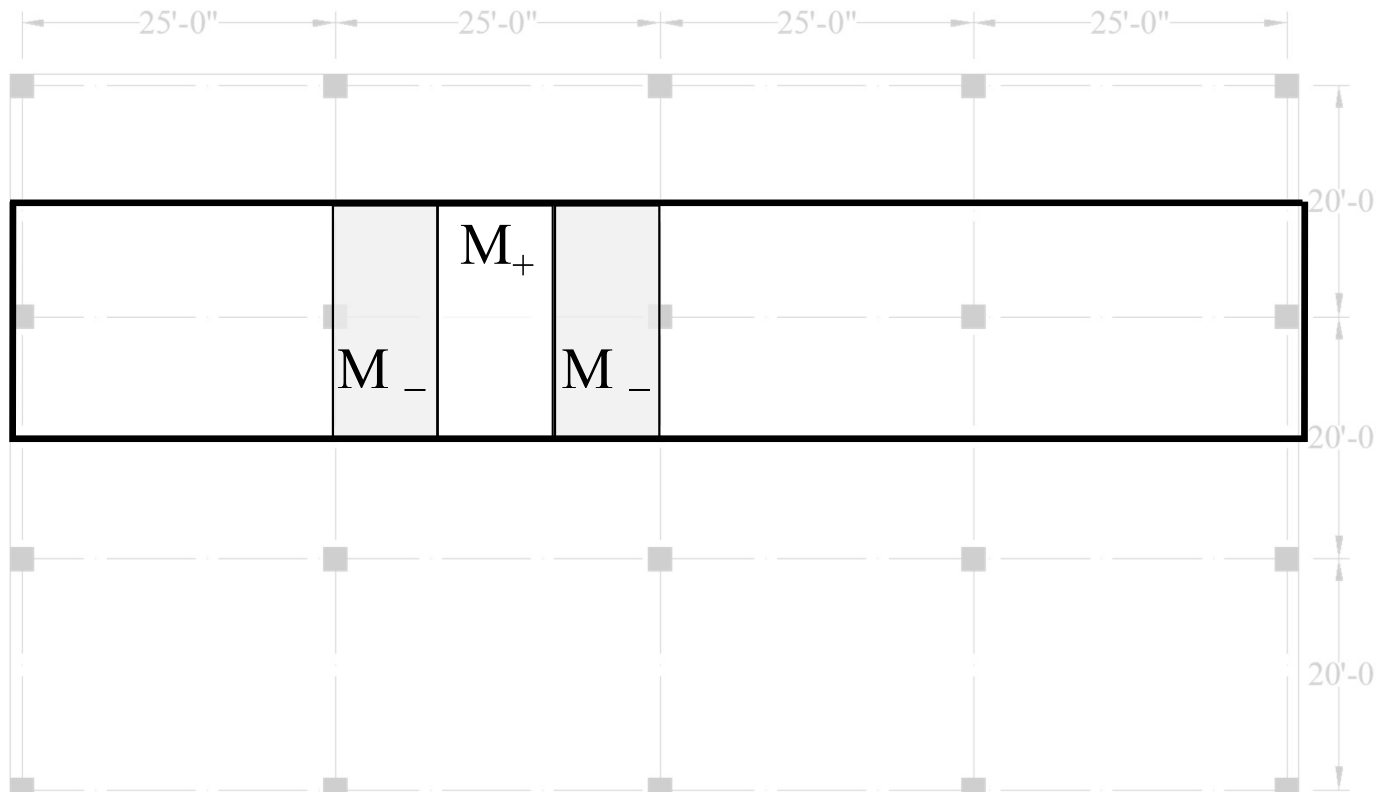


Analysis Procedure using DDM

- **Step 3: Analysis – Interior Span of Interior Frame**
  - **Step IV: Longitudinal Distribution of Static Moment (M_o)**

$M_- = 0.65M_o$

$M_+ = 0.35M_o$

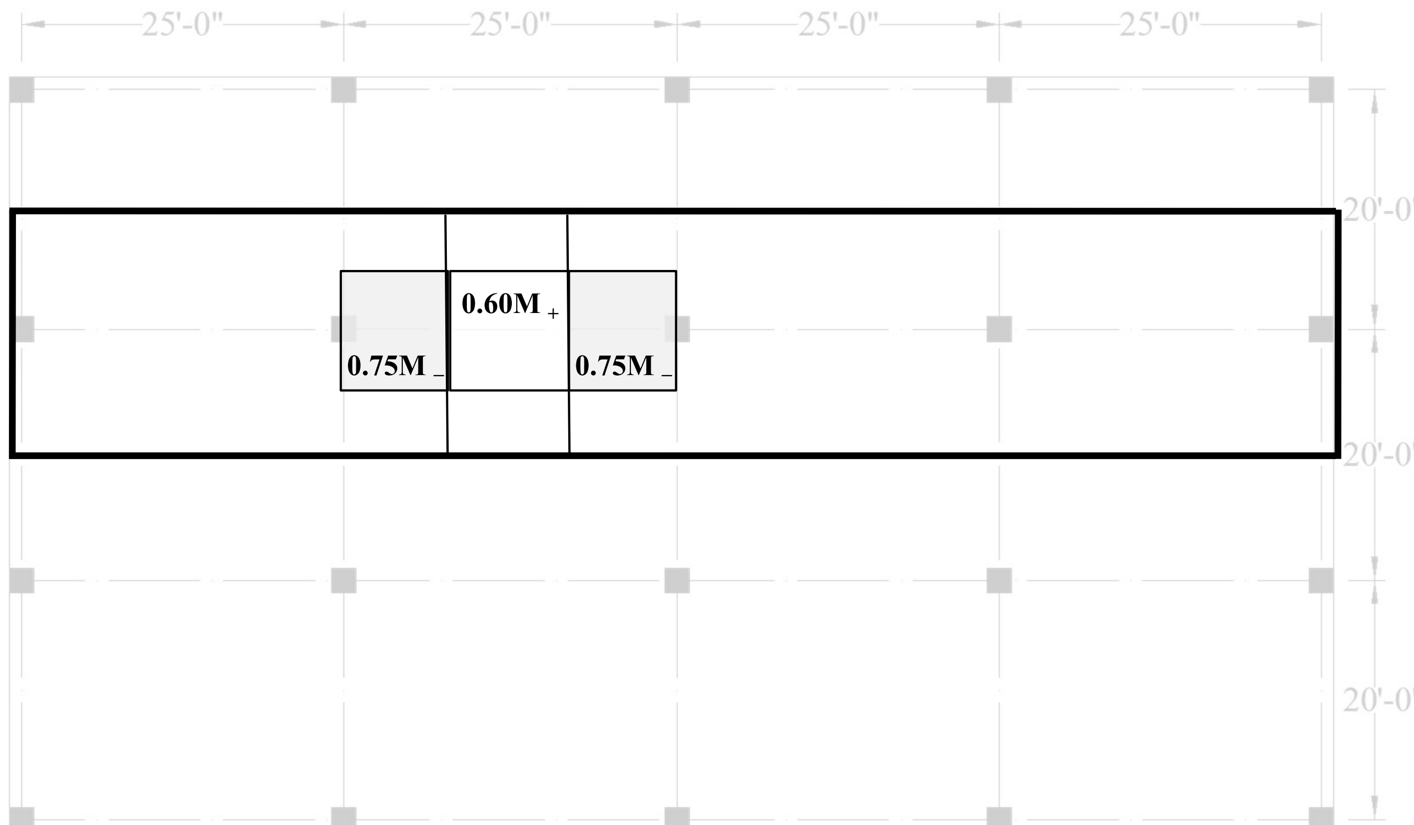


Analysis Procedure using DDM

- **Step 3: Analysis – Interior Span of Interior Frame**
  - **Step V: Lateral Distribution to column and middle strips**

$M_- = 0.65M_o$

$M_+ = 0.35M_o$



Analysis Procedure using DDM

❑ Step 3: Analysis – Exterior Span of Interior Frame

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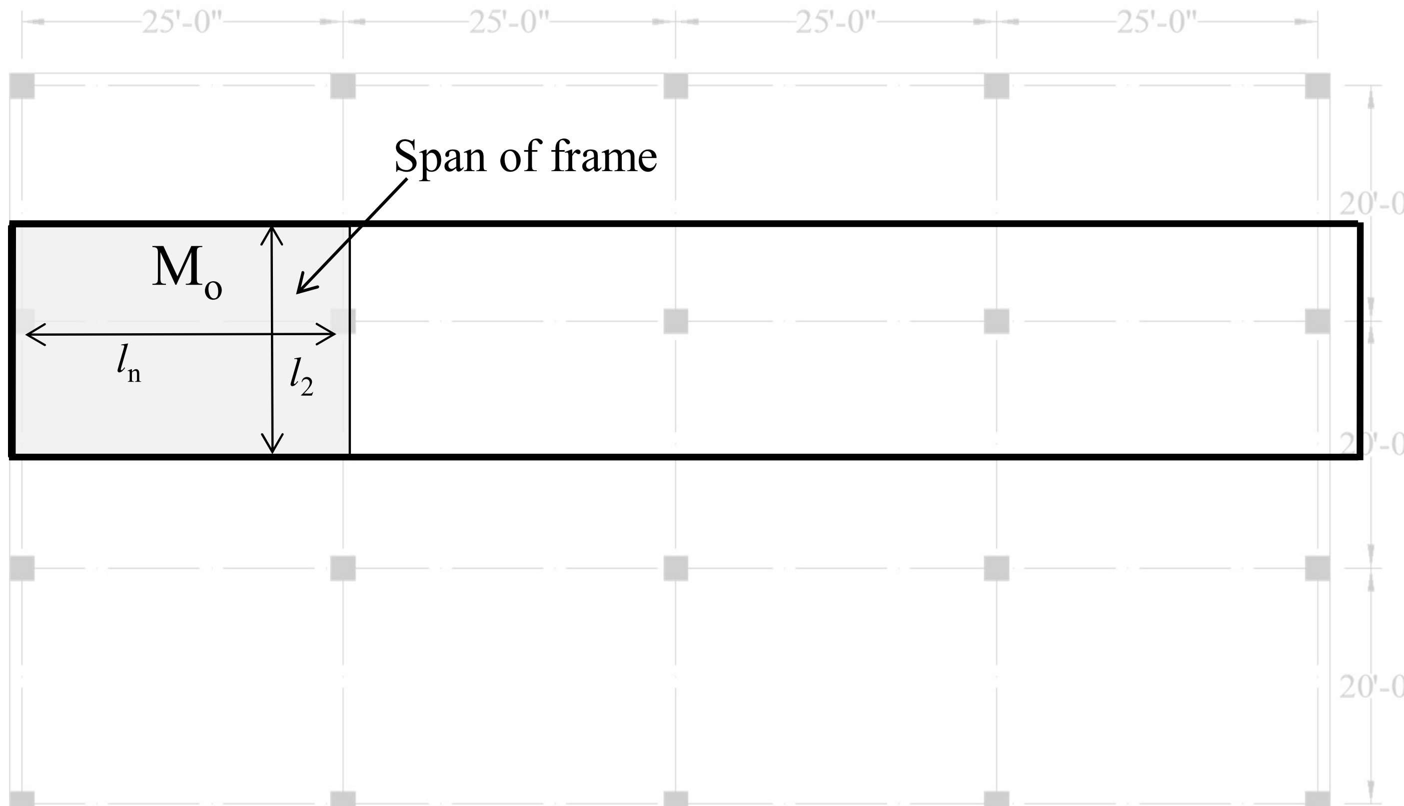

Analysis Procedure using DDM

❑ Step 3: Analysis – Exterior Span of Interior Frame

❖ Step IV: Longitudinal Distribution of Static Moment (M_o)

$M_{ext-} = 0.26M_o$

$M_{ext+} = 0.52M_o$

$M_{int-} = 0.70M_o$

Analysis Procedure using DDM

- Step 3: Analysis – Exterior Span of Interior Frame
  - Step V: Lateral Distribution to column and middle strips

$M_{ext-} = 0.26M_o$

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$M_{int-} = 0.70M_o$

Analysis Procedure using DDM

❑ Step 3: Analysis

❖ Summary of Static Moment Distribution

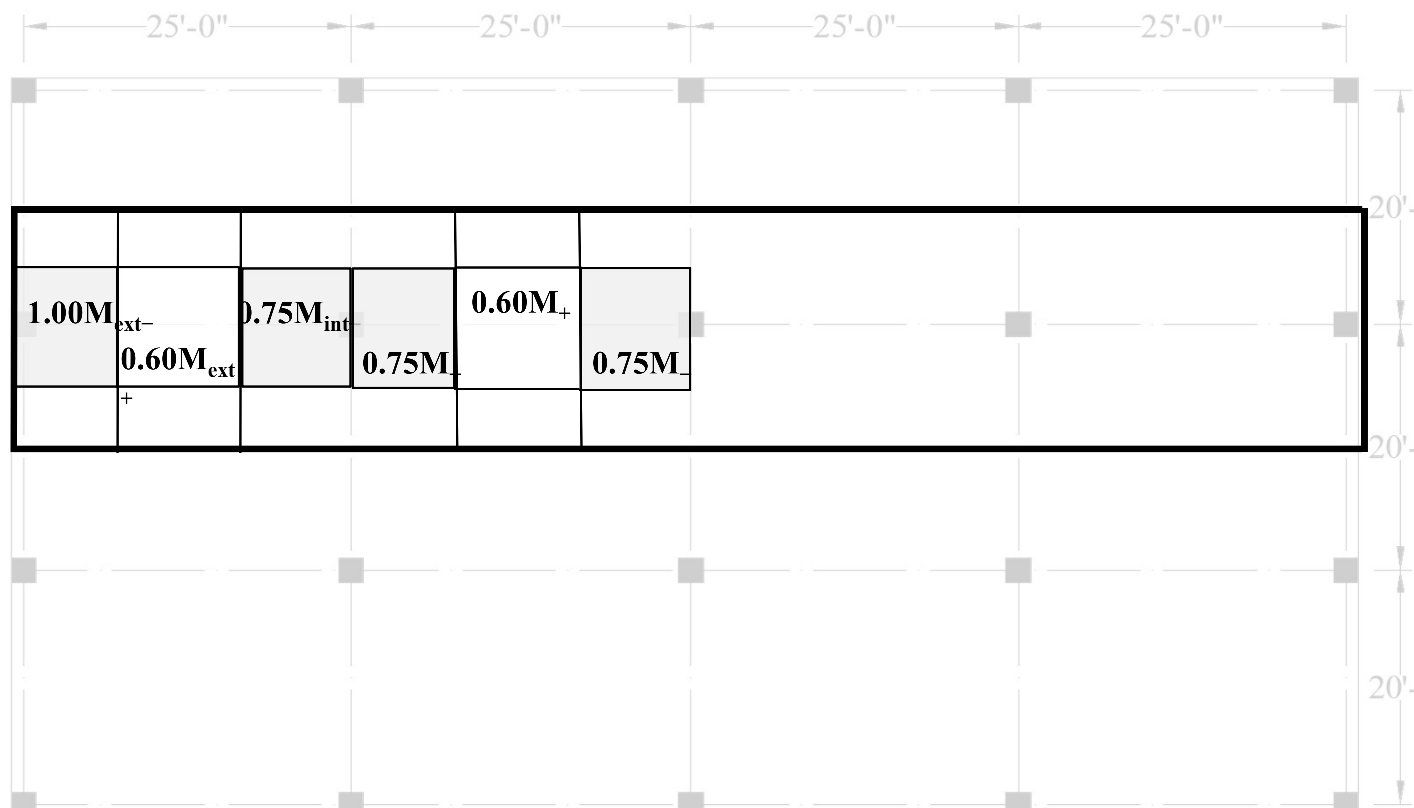
$M_{ext-} = 0.26M_o$

$M_{ext+} = 0.52M_o$

$M_{int-} = 0.70M_o$

$M_{-} = 0.65M_o$

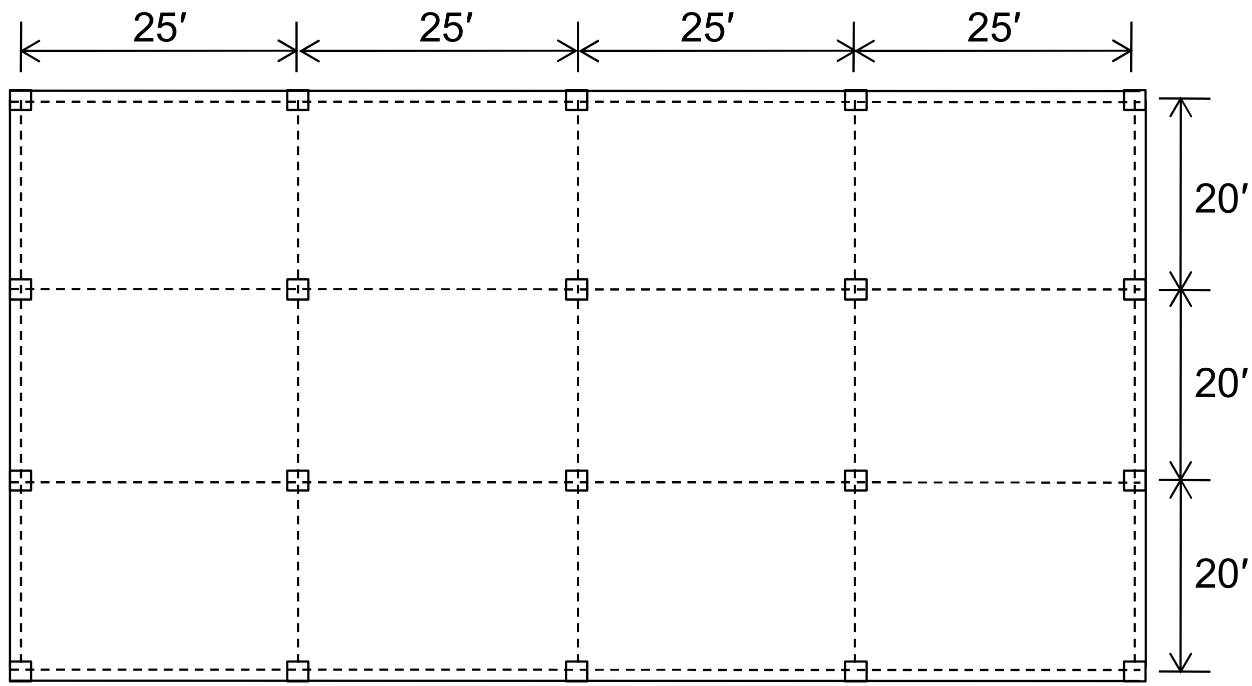
$M_{+} = 0.35M_o$



Example 5.1

❑ Problem Statement

- ***Analyze*** the flat plate shown below using DDM. The slab supports a uniformly distributed live load of 144 psf. All columns are 14″ square. Take $f_c' = 3$ ksi and $f_y = 60$ ksi.



Example 5.1

❑ Solution

➢ Step 1: Selection of sizes

- ACI table 8.3.1.1 is used for finding flat plate and flat slab thickness

Table 8.3.1.1 —Minimum thickness of nonprestressed two-way slabs without interior beams (in.)

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60,000	$l_n/30$	$l_n/33$	$l_n/33$	$l_n/33$	$l_n/36$	$l_n/36$
80,000	$l_n/27$	$l_n/30$	$l_n/30$	$l_n/30$	$l_n/33$	$l_n/33$

Example 5.1

Solution

Step 1: Selection of sizes

Exterior panel governs. Therefore;

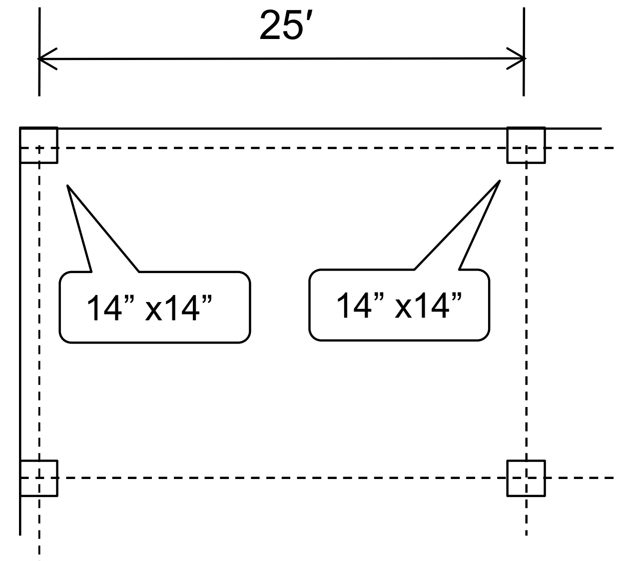
$$h_{min} = \frac{l_n}{30} = \frac{25 - 2\left(\frac{14}{2 \times 12}\right)}{30}$$

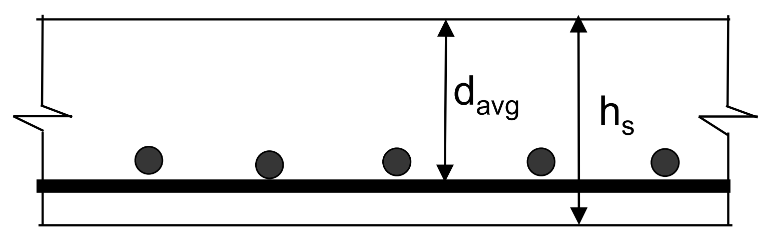
$$h_{min} = 0.794' \text{ or } 9.53''$$

Take $h_s = 10''$

Assuming #6 bar ;

$$d_{avg} = h_s - 0.75 - 0.75 = 8.5''$$





Slab

Example 5.1

❑ Solution

➢ Step 2: Calculation of Loads

Self-weight of plate = $h_s\gamma = (10/12) \times 0.150 = 0.125 \text{ ksf}$

Super imposed dead load = Nil

Service live load = $0.144\ ksf$

Now,

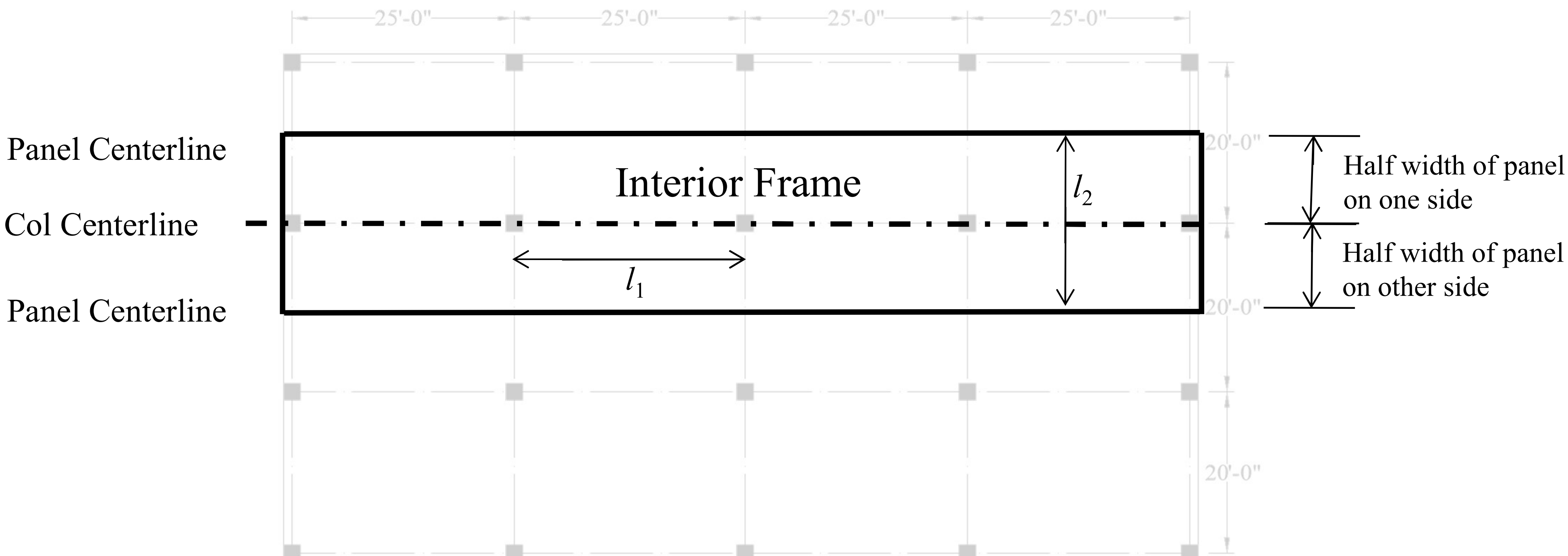
$W_u = 1.2W_u + 1.6L = 1.2(0.125) + 1.6(0.144) = 0.3804\ ksf$

Example 5.1

❑ Solution

➢ Step 3: Analysis (E-W Direction) – Interior Frame

❖ **Marking Frame**



Example 5.1

❑ Solution

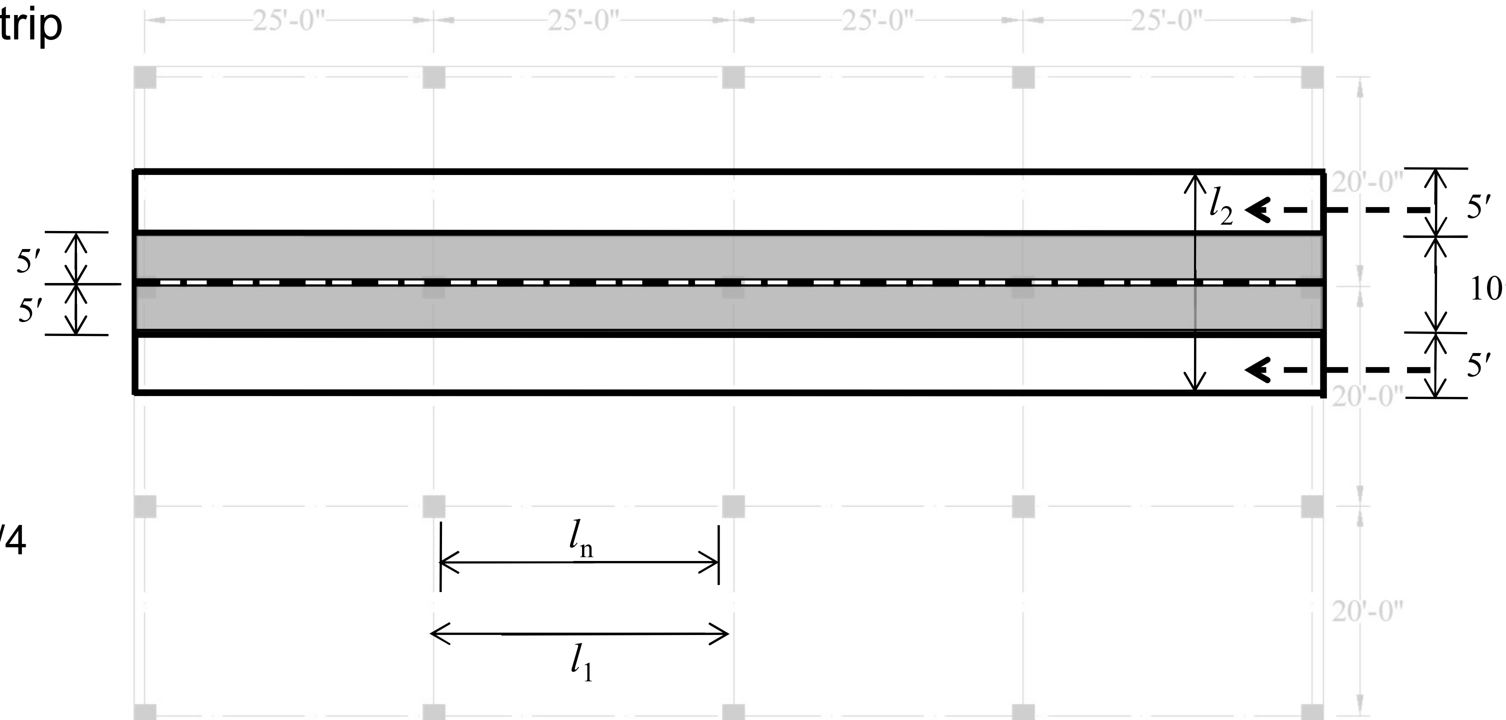
➢ Step 3: Analysis (E-W Direction)– Interior Frame

❖ Marking Column and Middle Strips

a) Marking Column Strip

b) Middle Strip

Half Column strip



CS/2 = Least of $l_1/4$ or $l_2/4$

$l_2/4 = 20/4 = 5'$

Example 5.1

❑ Solution

➢ Step 3: Analysis (E-W Direction) – Interior Frame

❖ Calculation of Total Static Moment

$M_o = w_u l_2 l_n^2/8$

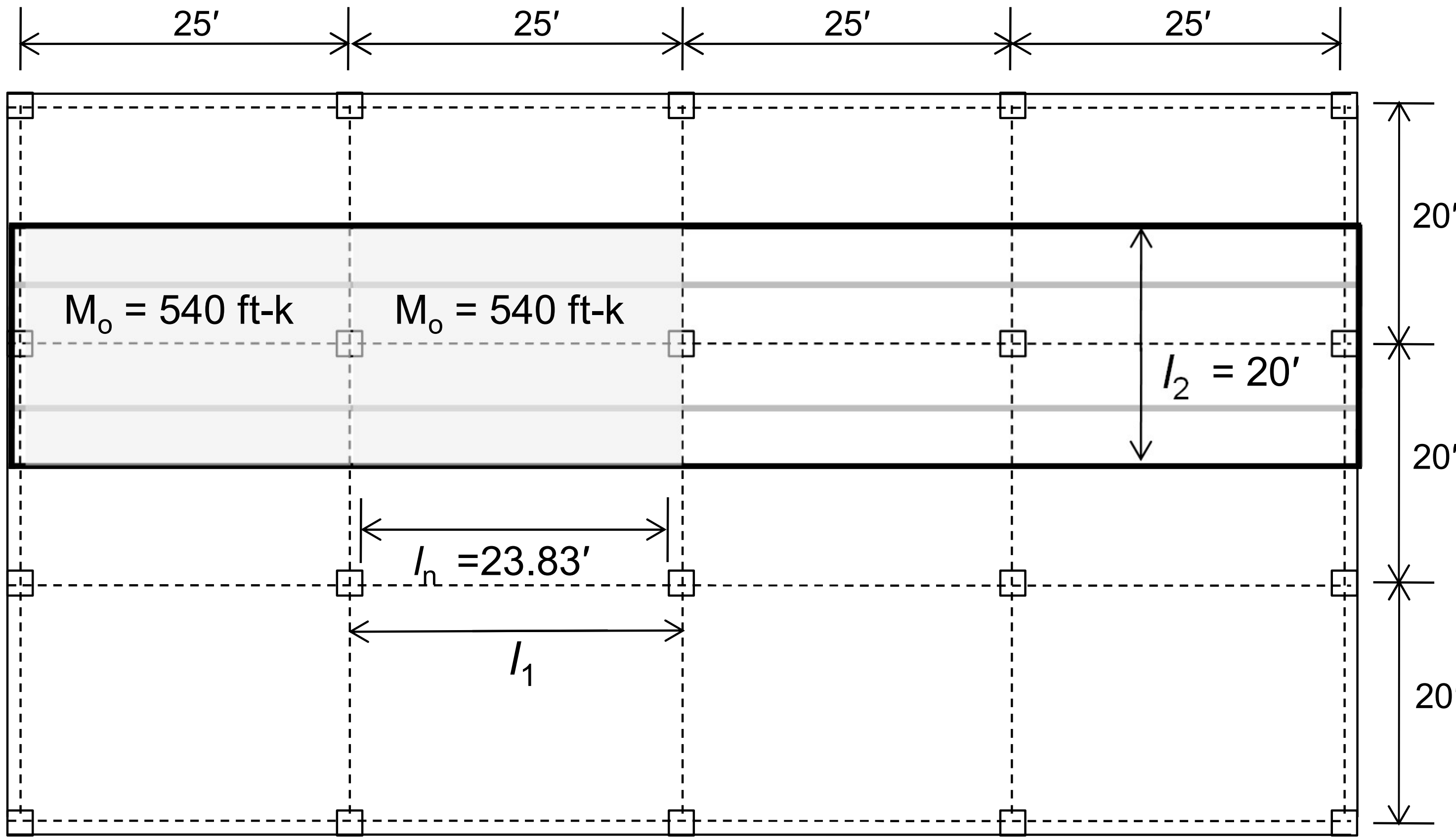
$w_u = 0.38$ psf

$l_2 = 20'$

$l_n = 23.83'$

On putting values,

$M_o \approx 540$ ft-kip



Example 5.1

❑ Solution

➢ Step 3: Analysis (E-W Direction) – Interior Frame

❖ Longitudinal Distribution of Total Static Moment

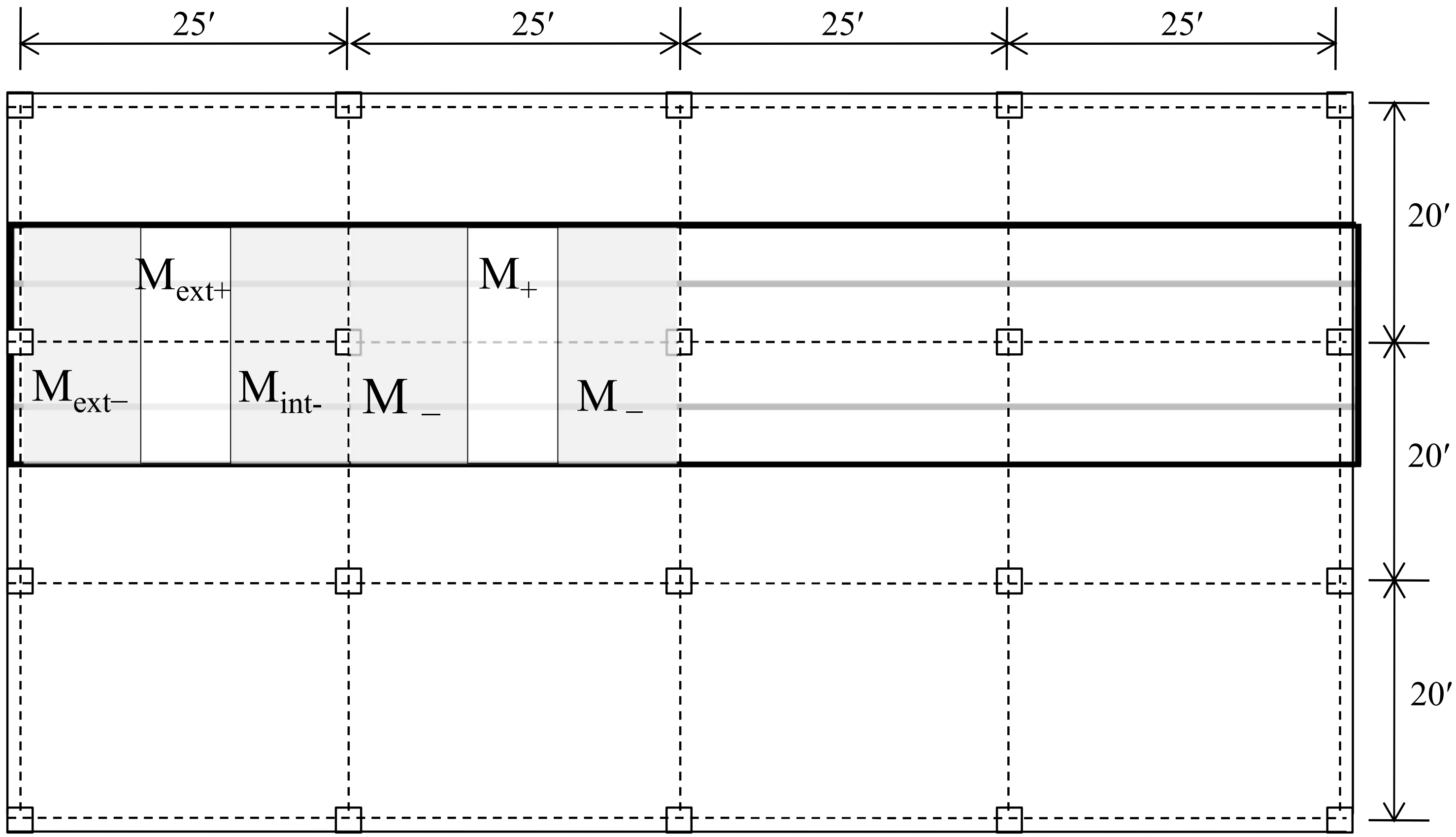
$M_{ext\,-} = 0.26M_o = 140$

$M_{ext\,+} = 0.52M_o = 281$

$M_{int\,-} = 0.70M_o = 378$

$M_{\,-} = 0.65M_o = 351$

$M_{\,+} = 0.35M_o = 189$



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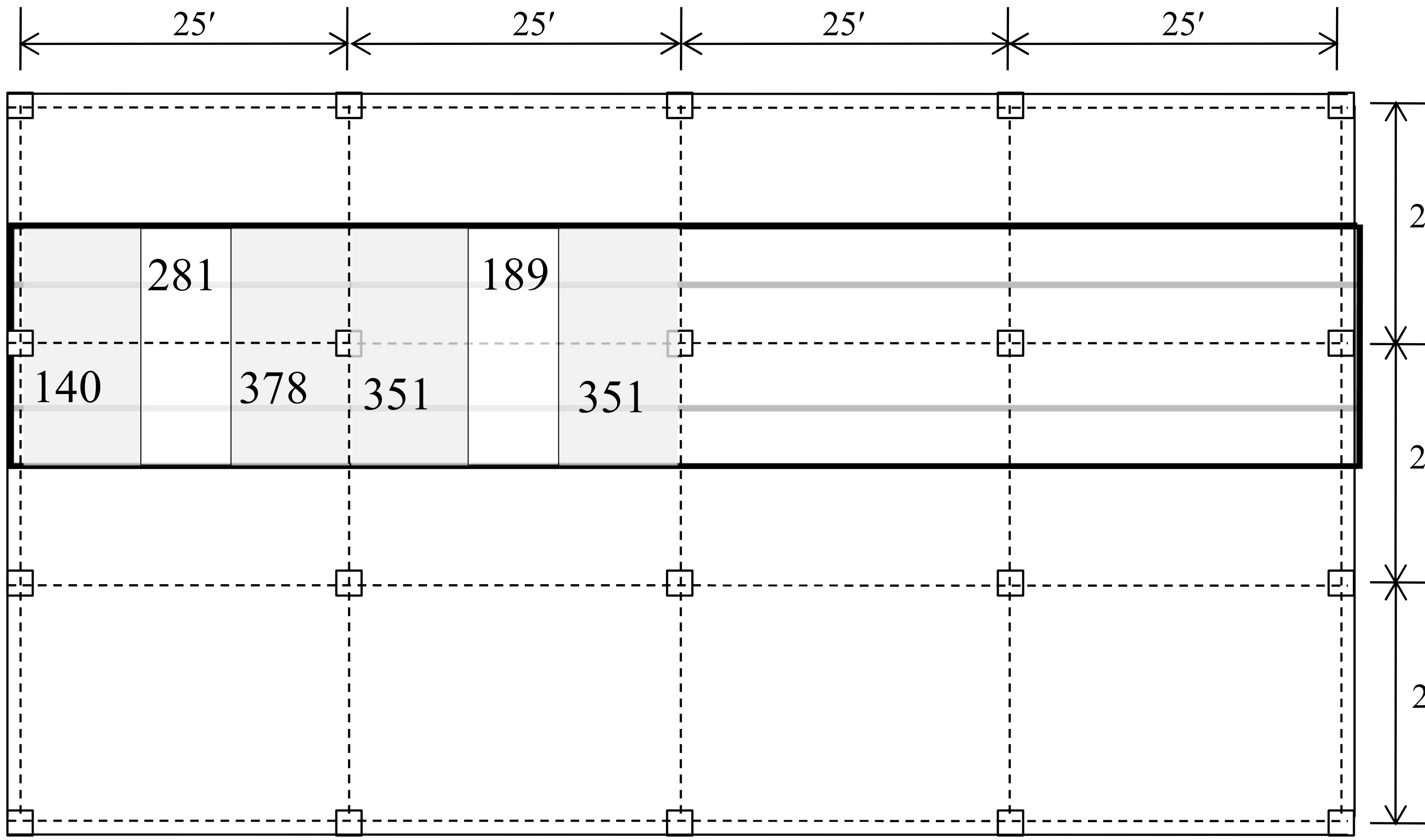

Example 5.1

❑ Solution

➢ Step 3: Analysis (E-W Direction) – Interior Frame

❖ Lateral Distribution to Column and Middle strips

$M_{ext-} = 0.26M_o = 140$

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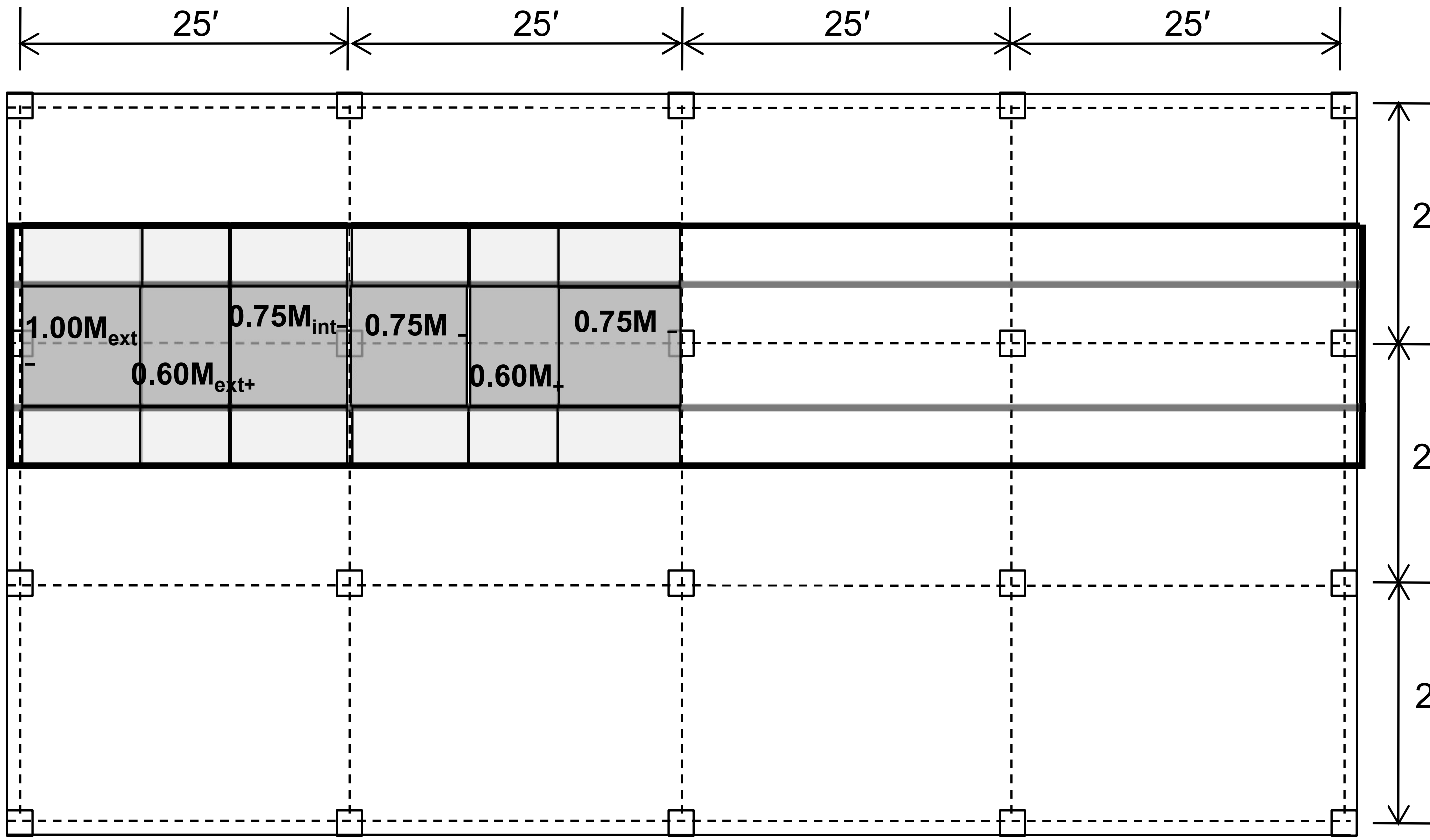
$M_{-} = 0.65M_o = 351$

$M_{+} = 0.35M_o = 189$



Example 5.1

❑ Solution

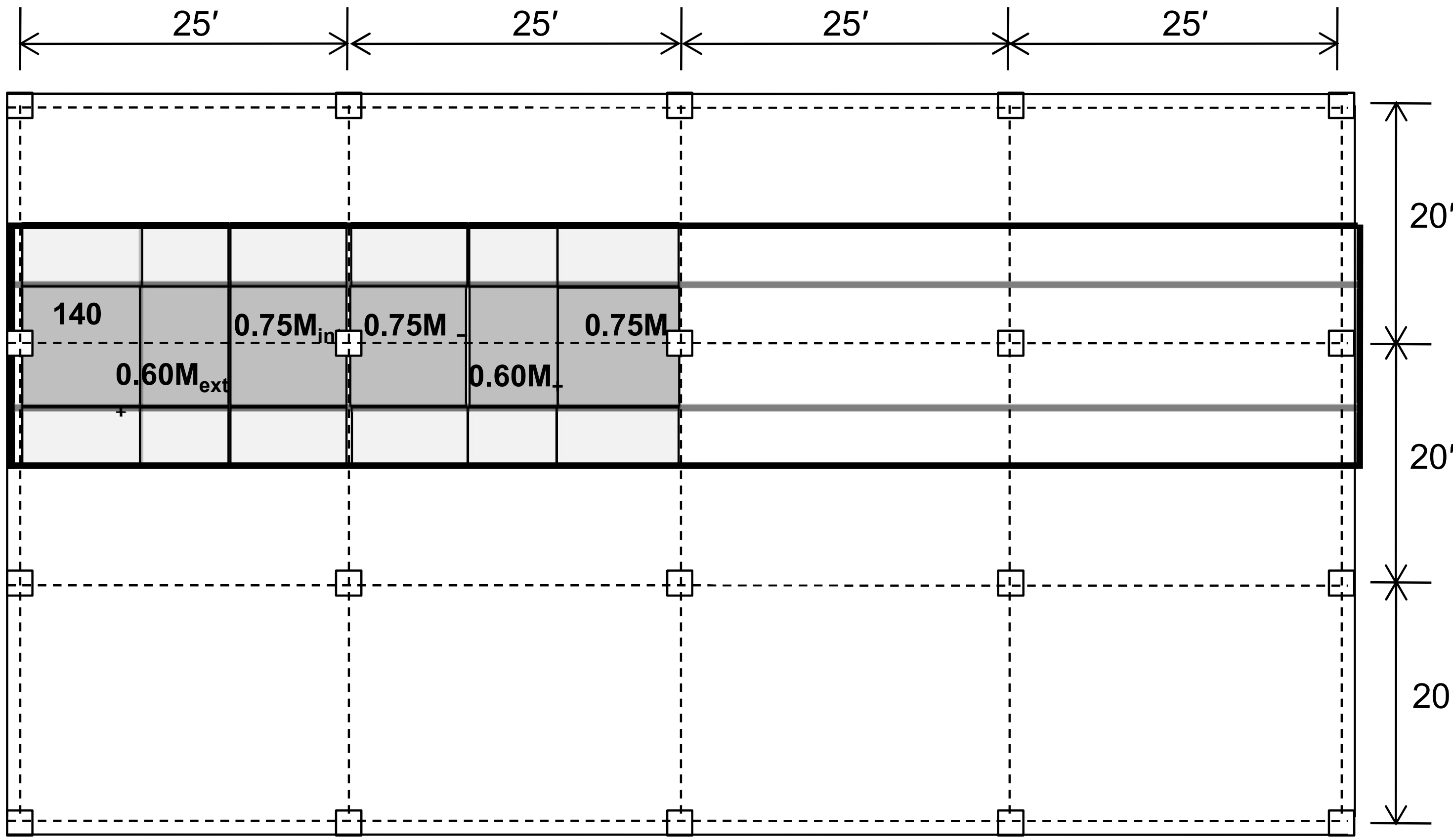
➢ Step 3: Analysis (E-W Direction) – Interior Frame

❖ Lateral Distribution to Column and Middle strips

$M_{ext-} = 0.26M_o = 140$
$M_{ext+} = 0.52M_o = 281$
$M_{int-} = 0.70M_o = 378$

$M_{-} = 0.65M_o = 351$
$M_{+} = 0.35M_o = 189$

100 % of M_{ext-} goes to column strip and remaining to middle strip



Example 5.1

❑ Solution

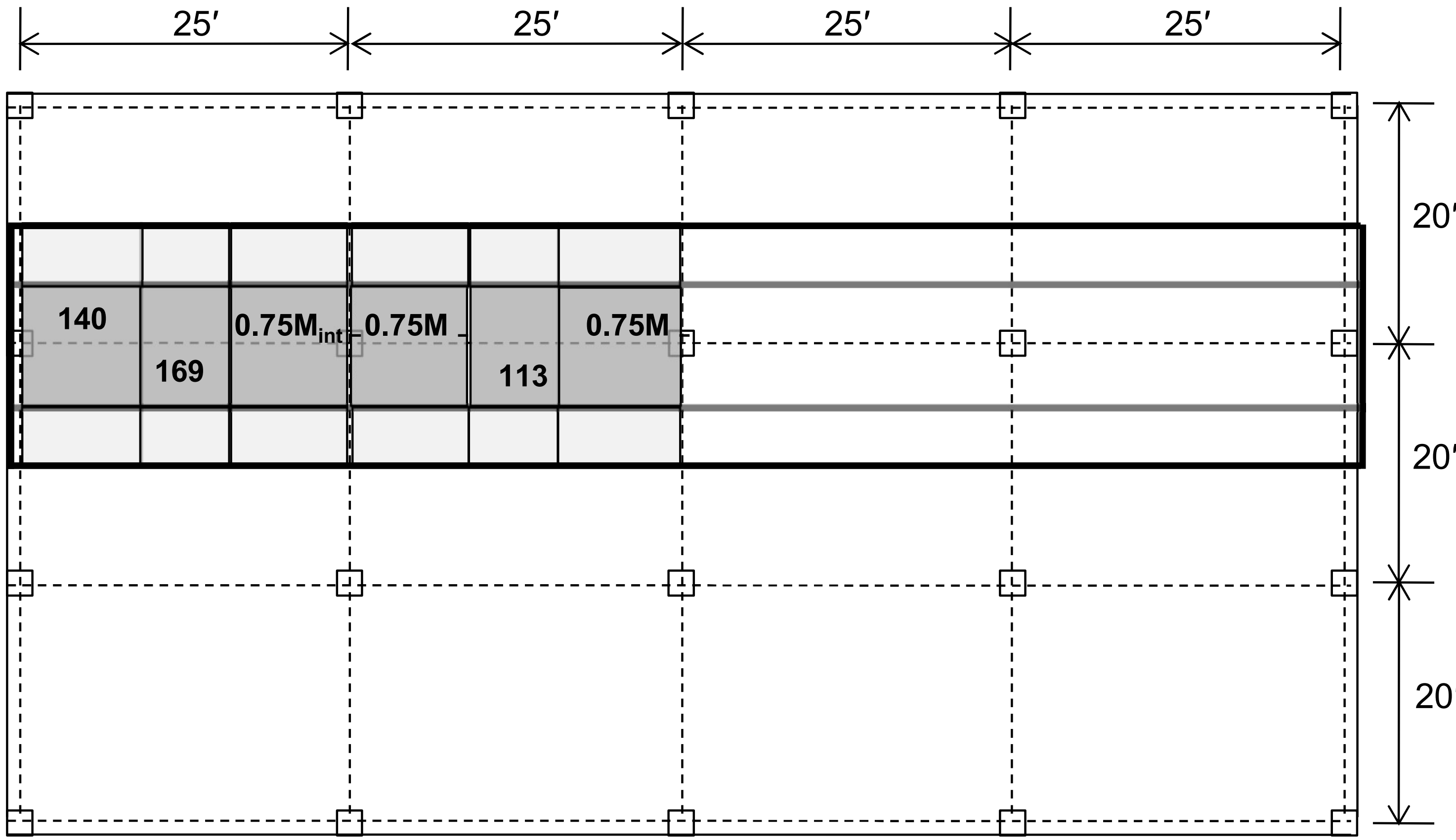
➢ Step 3: Analysis (E-W Direction) – Interior Frame

❖ Lateral Distribution to Column and Middle strips

$M_{ext-} = 0.26M_o = 140$
$M_{ext+} = 0.52M_o = 281$
$M_{int-} = 0.70M_o = 378$

$M_{-} = 0.65M_o = 351$
$M_{+} = 0.35M_o = 189$

60 % of M_{ext+} & M_{+} goes to column strip and remaining to middle strip



Example 5.1

Solution

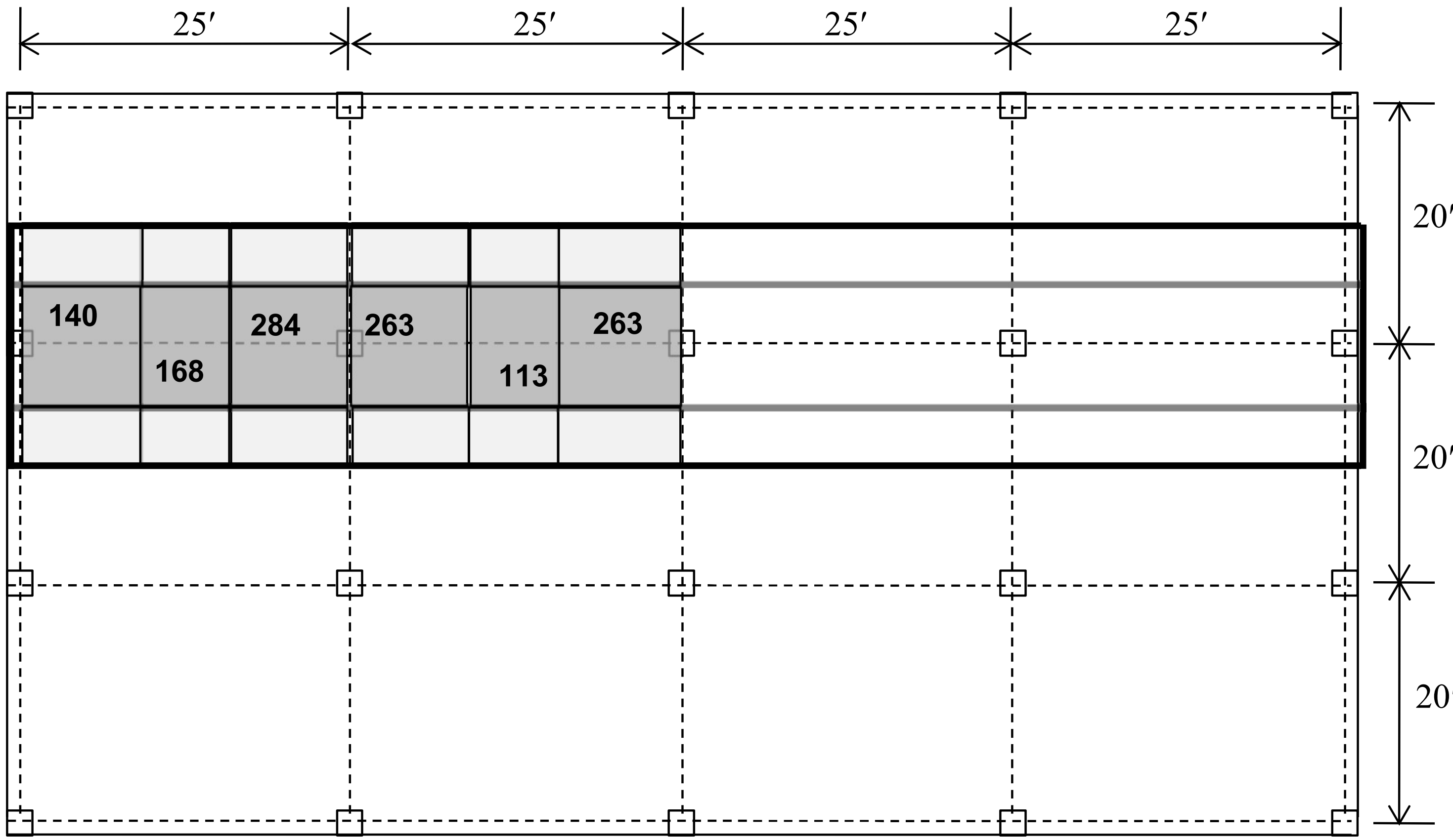
Step 3: Analysis (E-W Direction) – Interior Frame

Lateral Distribution to Column and Middle strips

$M_{ext-} = 0.26M_o = 140$
$M_{ext+} = 0.52M_o = 281$
$M_{int-} = 0.70M_o = 378$

$M_{-} = 0.65M_o = 351$
$M_{+} = 0.35M_o = 189$

75 % of M_{int-} goes to column strip and remaining to middle strip



Example 5.1

❑ Solution

➢ Step 3: Analysis (E-W Direction) – Interior Frame

❖ Lateral Distribution to Column and Middle strips

$M_{ext-} = 0.26M_o = 140$
$M_{ext+} = 0.52M_o = 281$
$M_{int-} = 0.70M_o = 378$

$M_{-} = 0.65M_o = 351$
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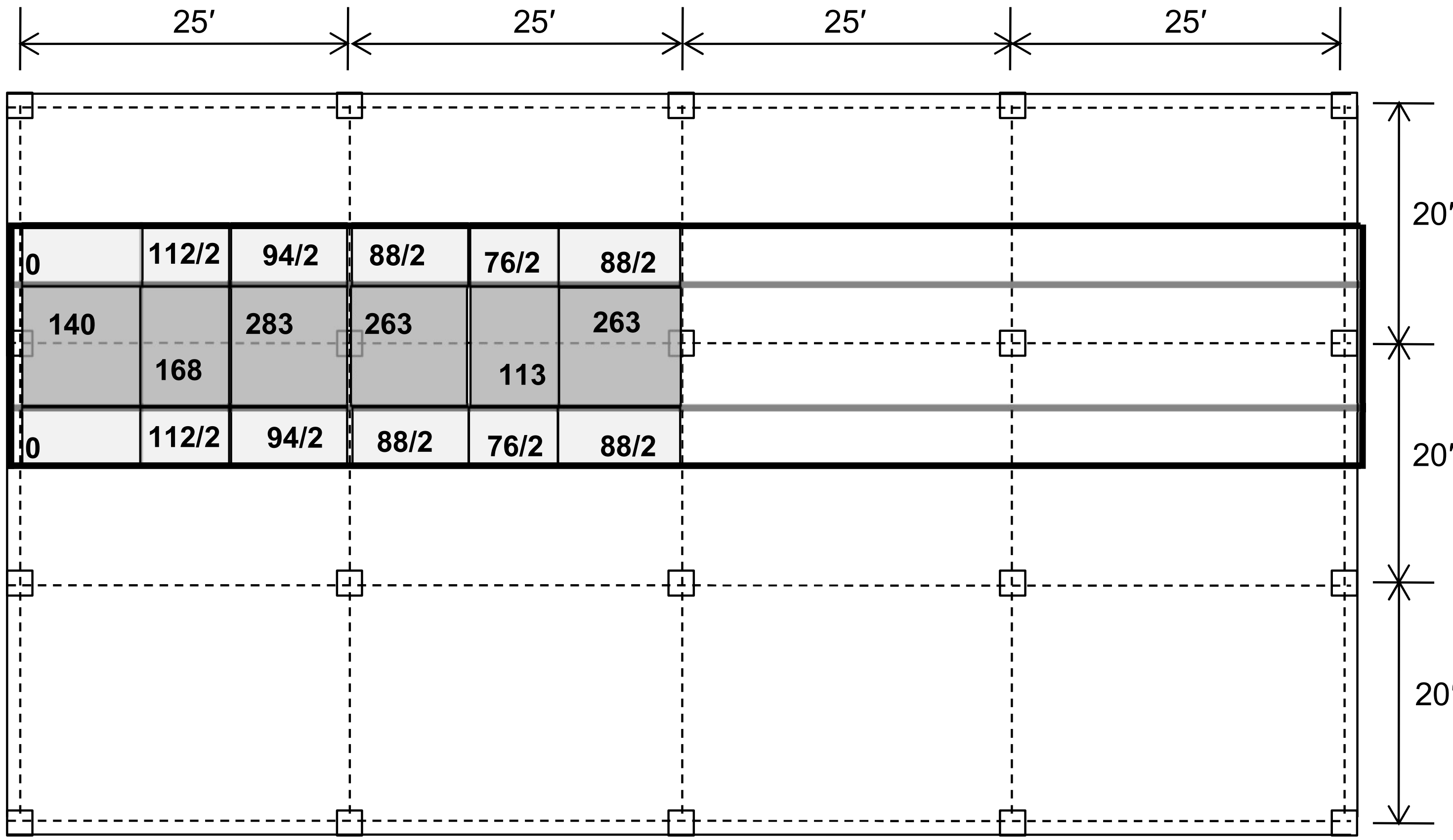


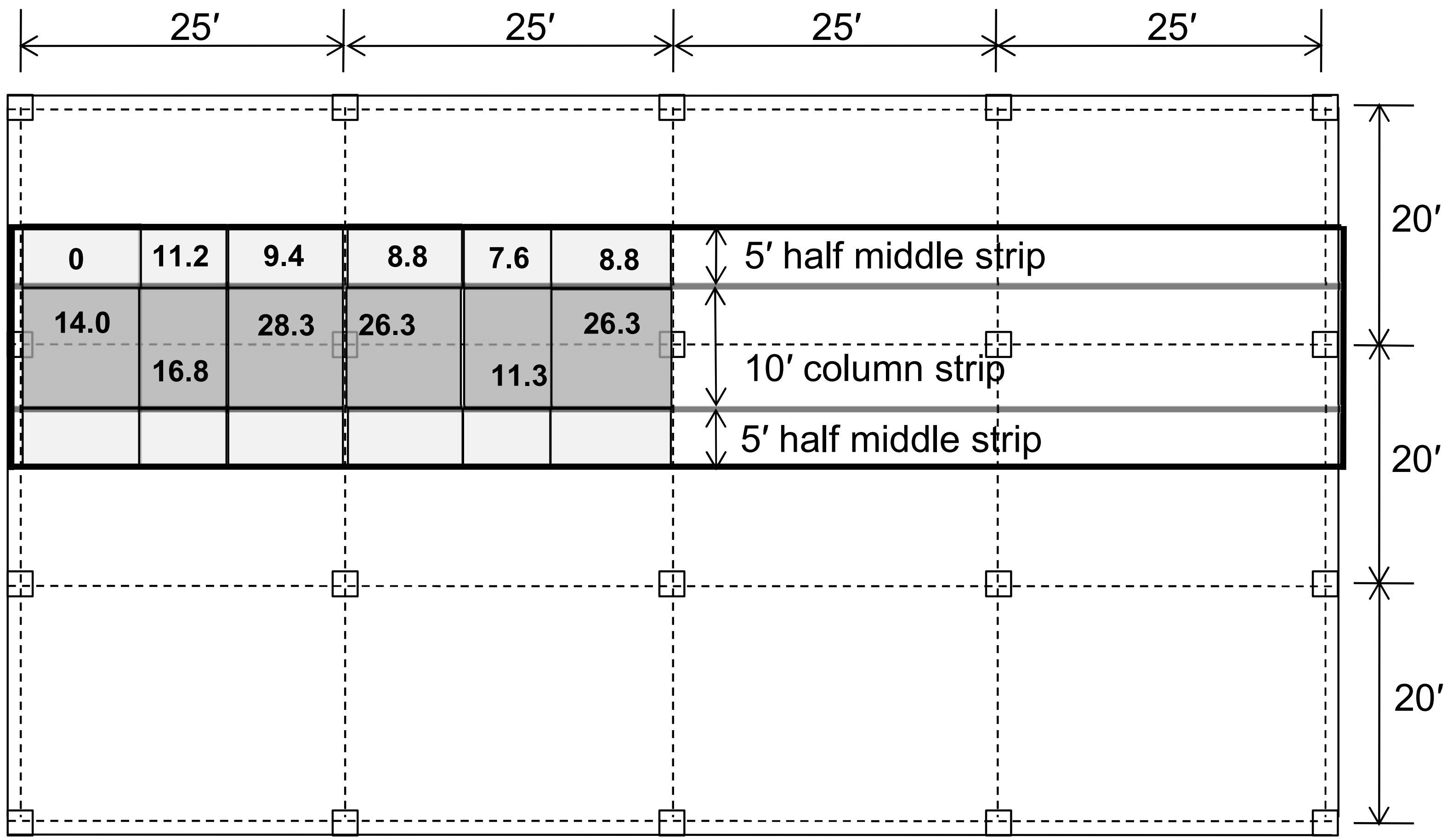
Example 5.1

❑ Solution

➢ Step 3: Analysis (E-W Direction) – Interior Frame

❖ Lateral Distribution to Column and Middle strips

M_u (per foot width)
= M / strip width

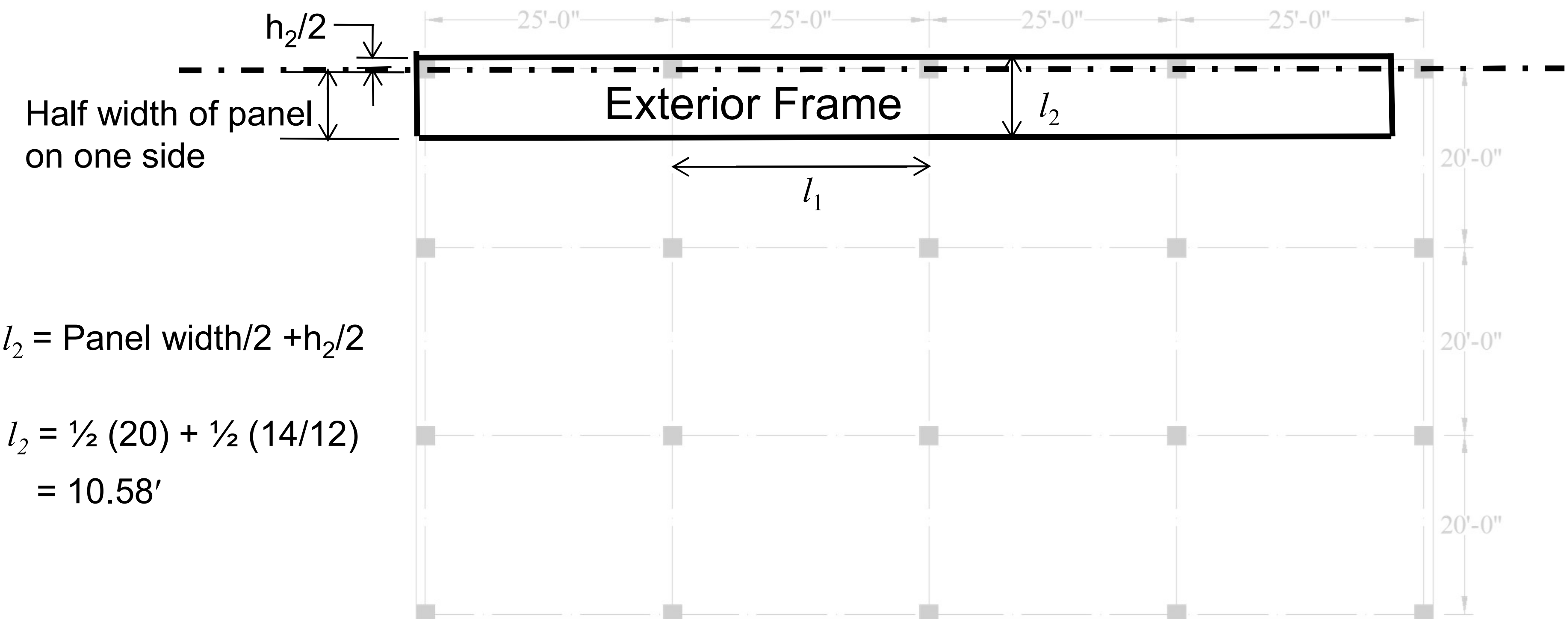


Example 5.1

❑ Solution

➢ Step 3: Analysis (E-W Direction) – Exterior Frame

❖ **Marking Frame**



l_2 = Panel width/2 + h_2/2

l_2 = ½ (20) + ½ (14/12)

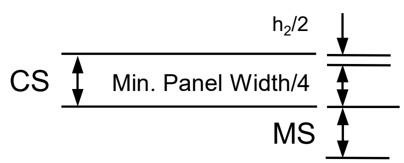
= 10.58′

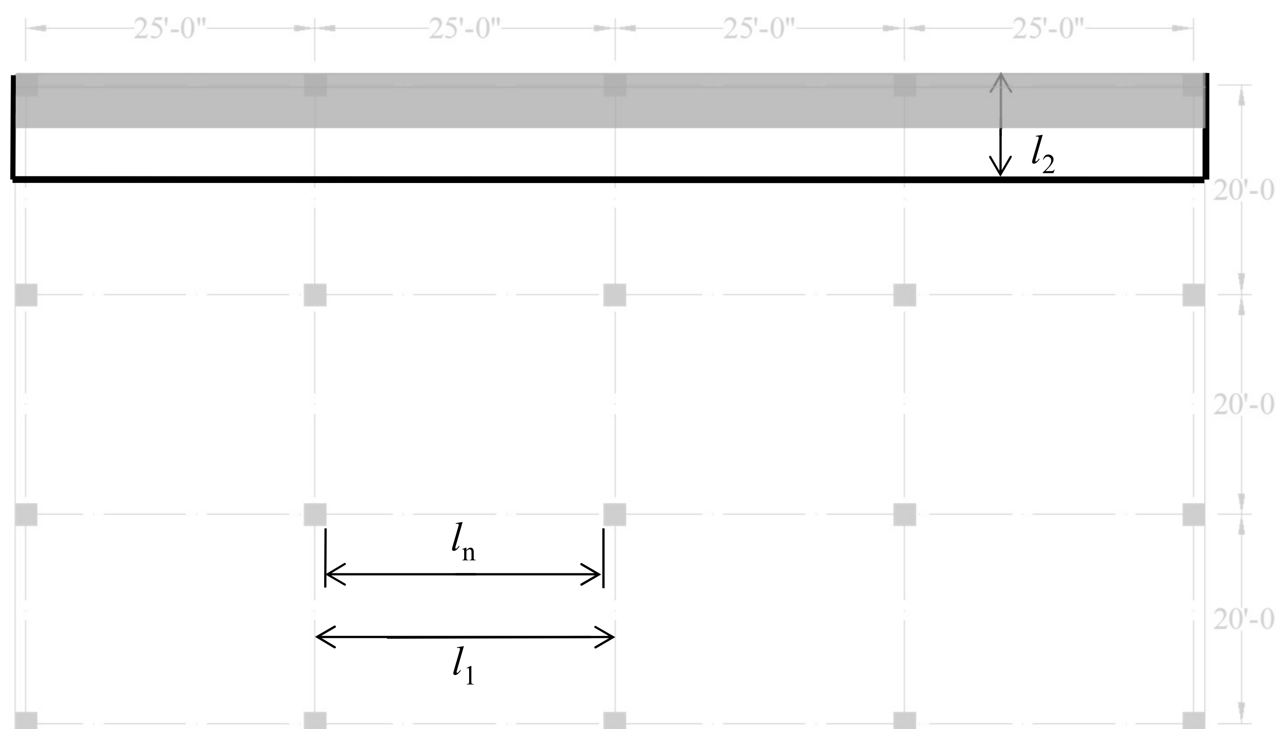
Example 5.1

Solution

Step 3: Analysis (E-W Direction) – Exterior Frame

Marking Column Strip and Middle Strips





CS = Min. Panel Width/4 + ½ (Col. Size)

MS = l_2 - CS

CS and MS widths are given as follows:

l_2 = ½ (20) + ½ (14/12) = 10.58′

CS = (20/4) + ½ (14/12) = 5.58′

MS = 10.58 – 5.58 = 5′

Example 5.1

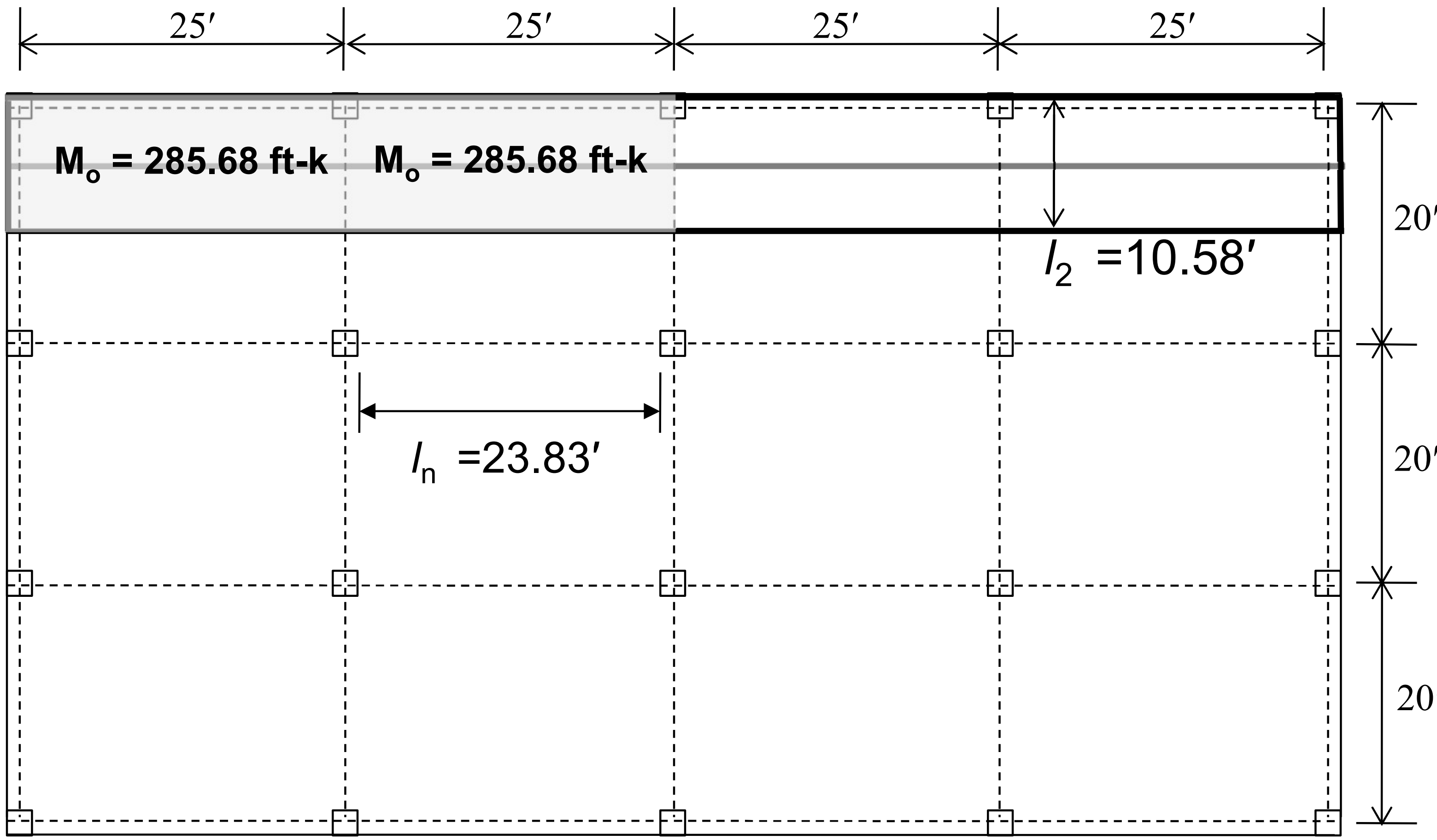
❑ Solution

➢ Step 3: Analysis (E-W Direction) – Exterior Frame

❖ Calculation of Total Static Moment

$M_o = w_u l_2 l_n^2/8$

$= 285.68$ ft-kip



Example 5.1

❑ Solution

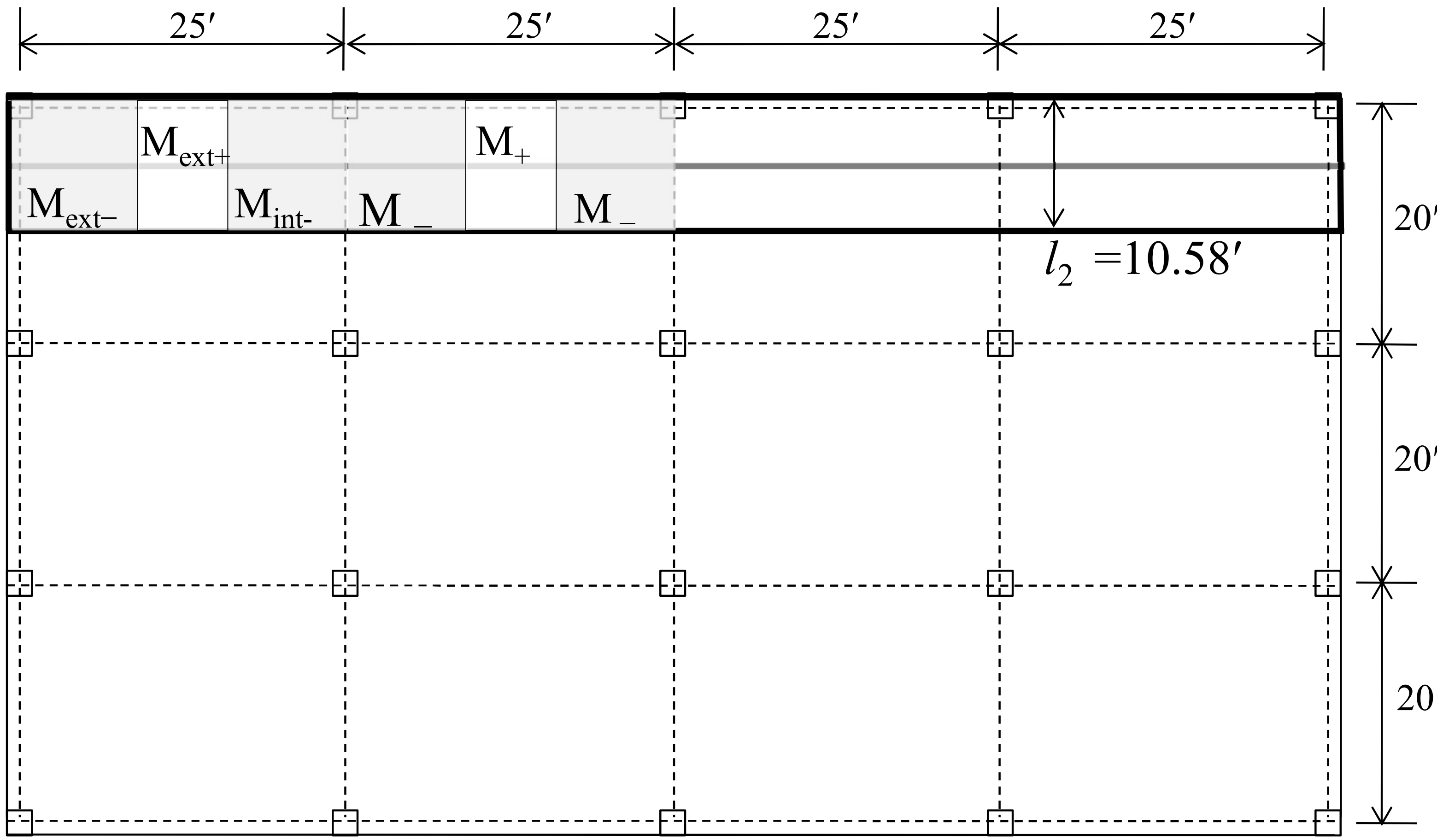
➢ Step 3: Analysis (E-W Direction) – Exterior Frame

❖ Longitudinal distribution to column and middle strips

M_o = 285.68 ft-kip

M_{ext-} = 0.26M_o = 74
M_{ext+} = 0.52M_o = 148
M_{int-} = 0.70M_o = 200

M_- = 0.65M_o = 186
M_+ = 0.35M_o = 100



Example 5.1

❑ Solution

➢ Step 3: Analysis (E-W Direction) – Exterior Frame

❖ **Longitudinal distribution to column and middle strips**

M_o = 285.68 ft-kip

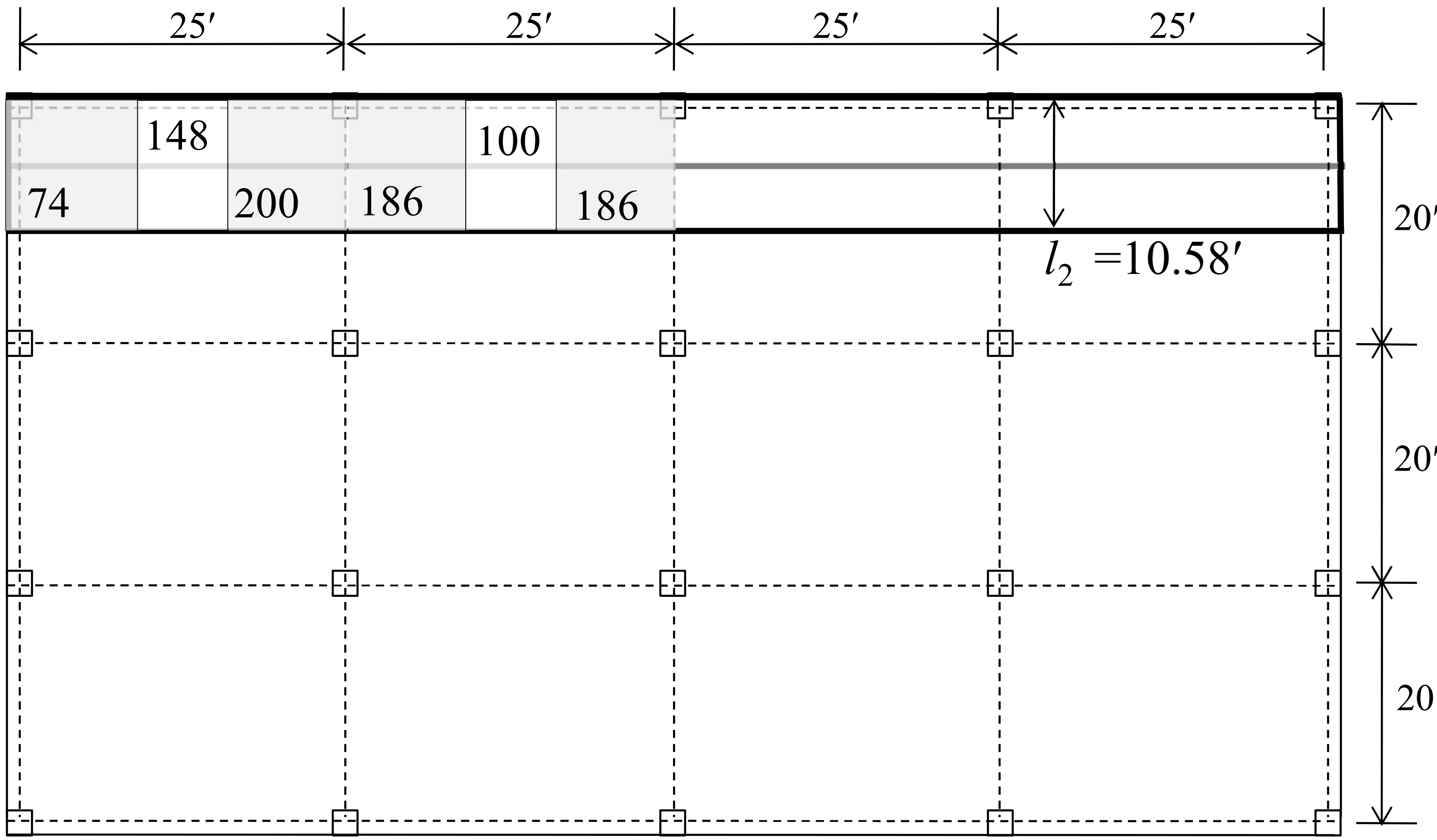
$M_{ext-} = 0.26M_o = 74$

$M_{ext+} = 0.52M_o = 148$

$M_{int-} = 0.70M_o = 200$

$M_- = 0.65M_o = 186$

$M_+ = 0.35M_o = 100$



Example 5.1

❑ Solution

➢ Step 3: Analysis (E-W Direction) – Exterior Frame

❖ Lateral distribution to column and middle strips

$M_o = 285.68$ ft-kip

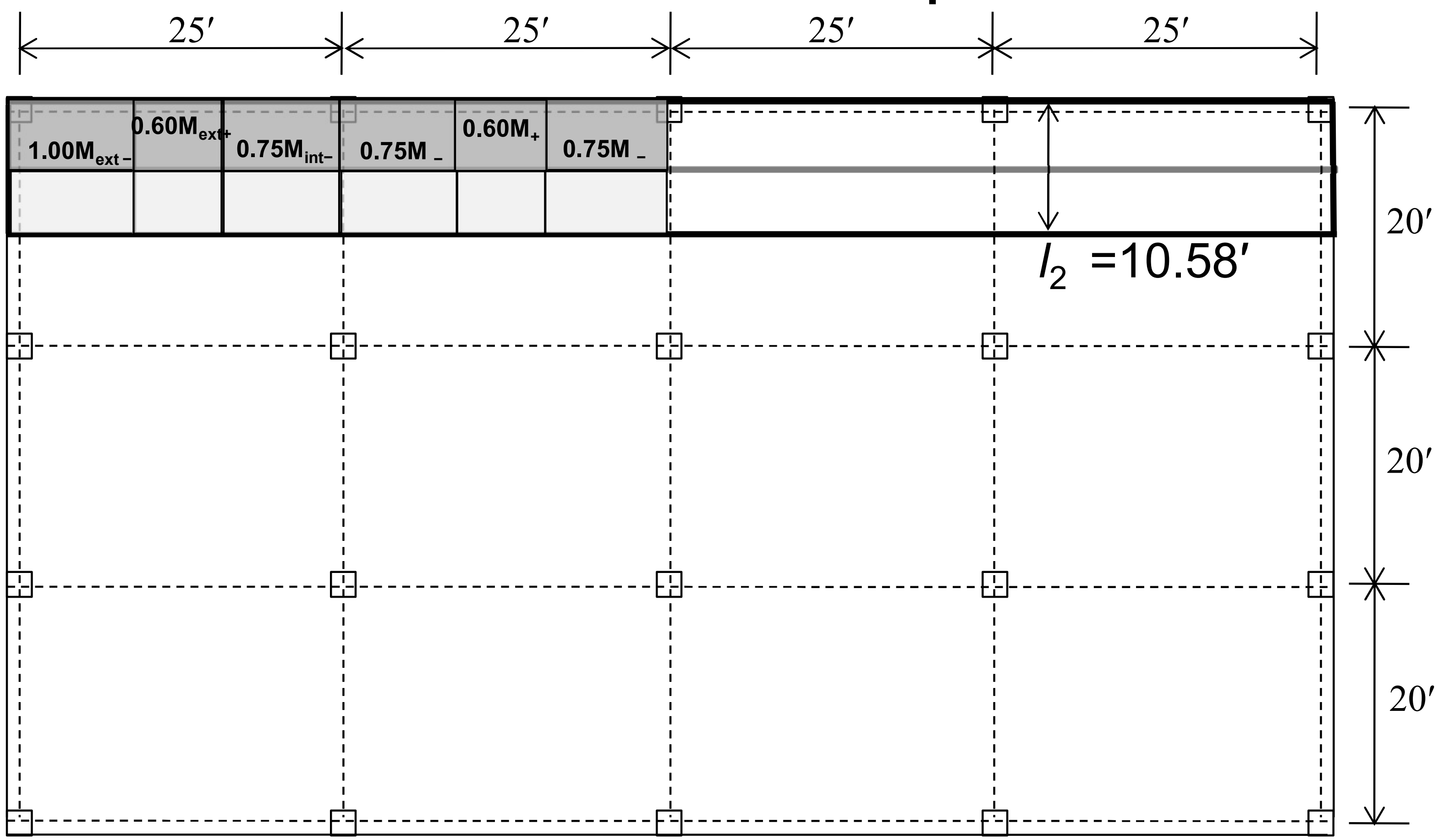
$M_{ext-} = 0.26M_o = 74$

$M_{ext+} = 0.52M_o = 148$

$M_{int-} = 0.70M_o = 200$

$M_{-} = 0.65M_o = 186$

$M_{+} = 0.35M_o = 100$



Example 5.1

❑ Solution

➢ Step 3: Analysis (E-W Direction) – Exterior Frame

❖ **Lateral distribution to column and middle strips**

$M_o = 285.68$ ft-kip

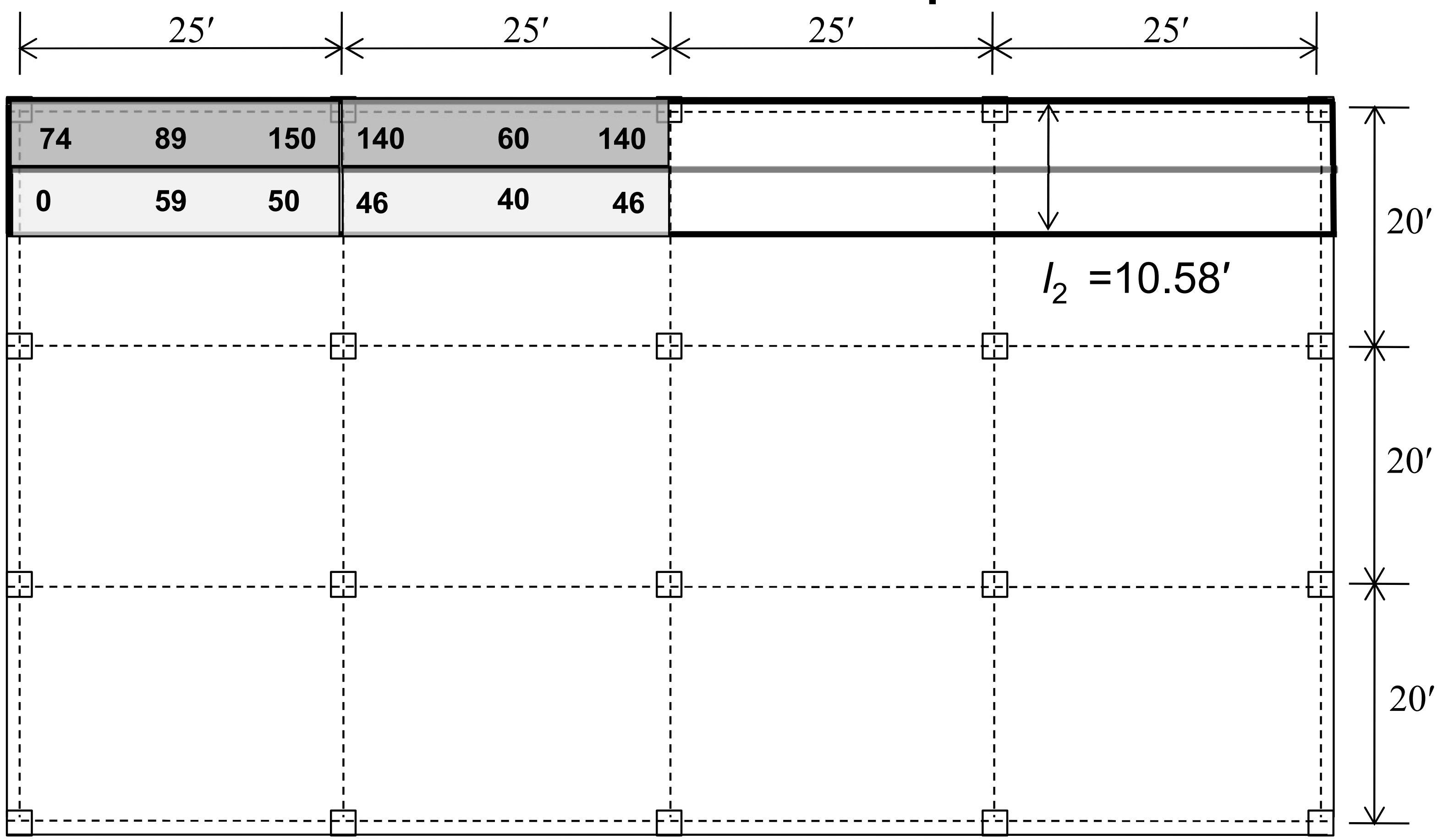
$M_{ext-} = 0.26M_o = 74$

$M_{ext+} = 0.52M_o = 148$

$M_{int-} = 0.70M_o = 200$

$M_{-} = 0.65M_o = 186$

$M_{+} = 0.35M_o = 100$



Example 5.1

❑ Solution

➢ Step 3: Analysis (E-W Direction) – Exterior Frame

❖ **Lateral distribution to column and middle strips (per strip width)**

M_o = 285.68 ft-kip

$M_{ext-} = 0.26M_o = 74$

$M_{ext+} = 0.52M_o = 148$

$M_{int-} = 0.70M_o = 200$

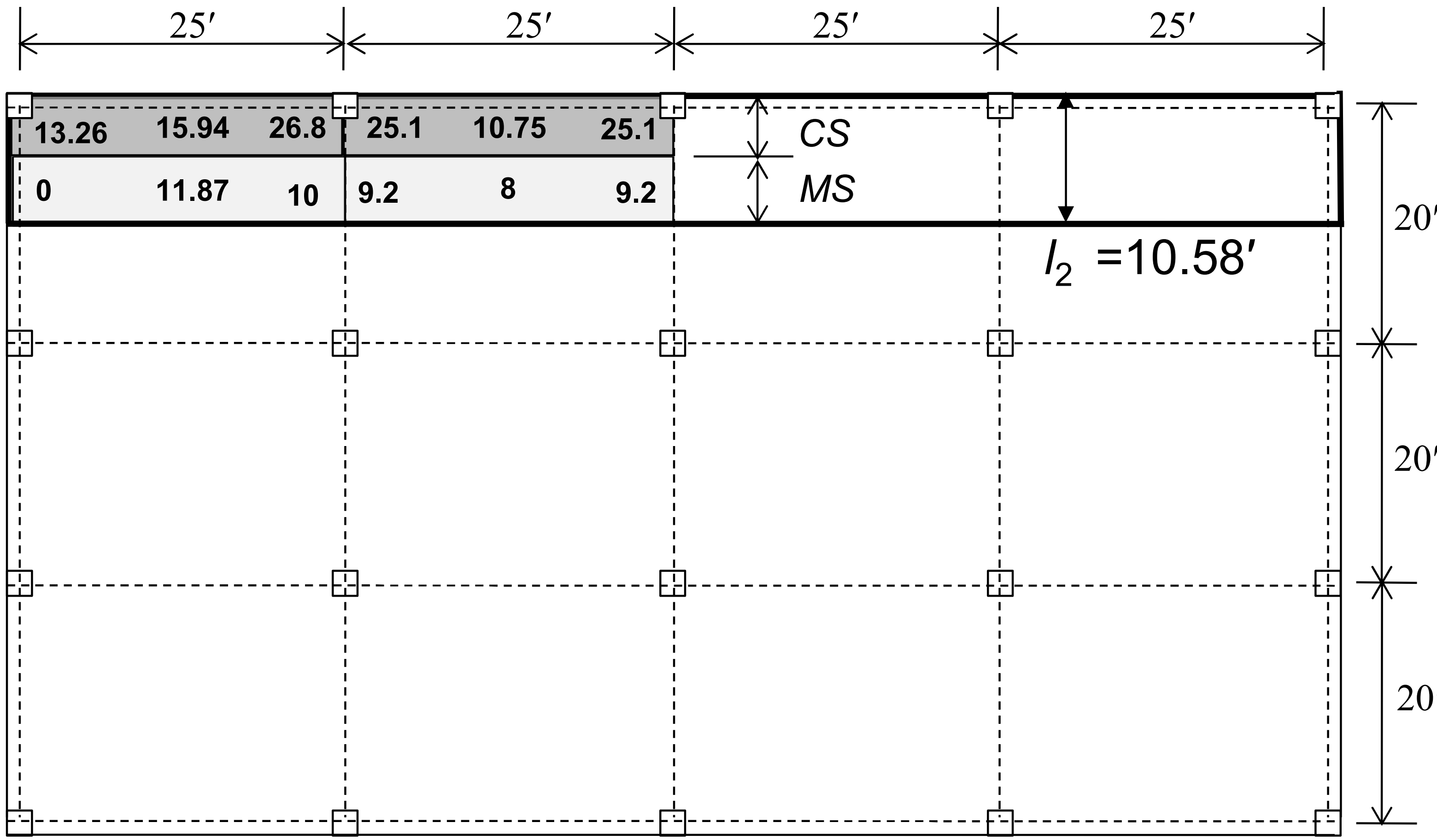
$M_{-} = 0.65M_o = 186$

$M_{+} = 0.35M_o = 100$

l_2 = ½ (20) + ½ (14/12) = 10.58′

CS = (20/4) + ½ (14/12) = 5.58′

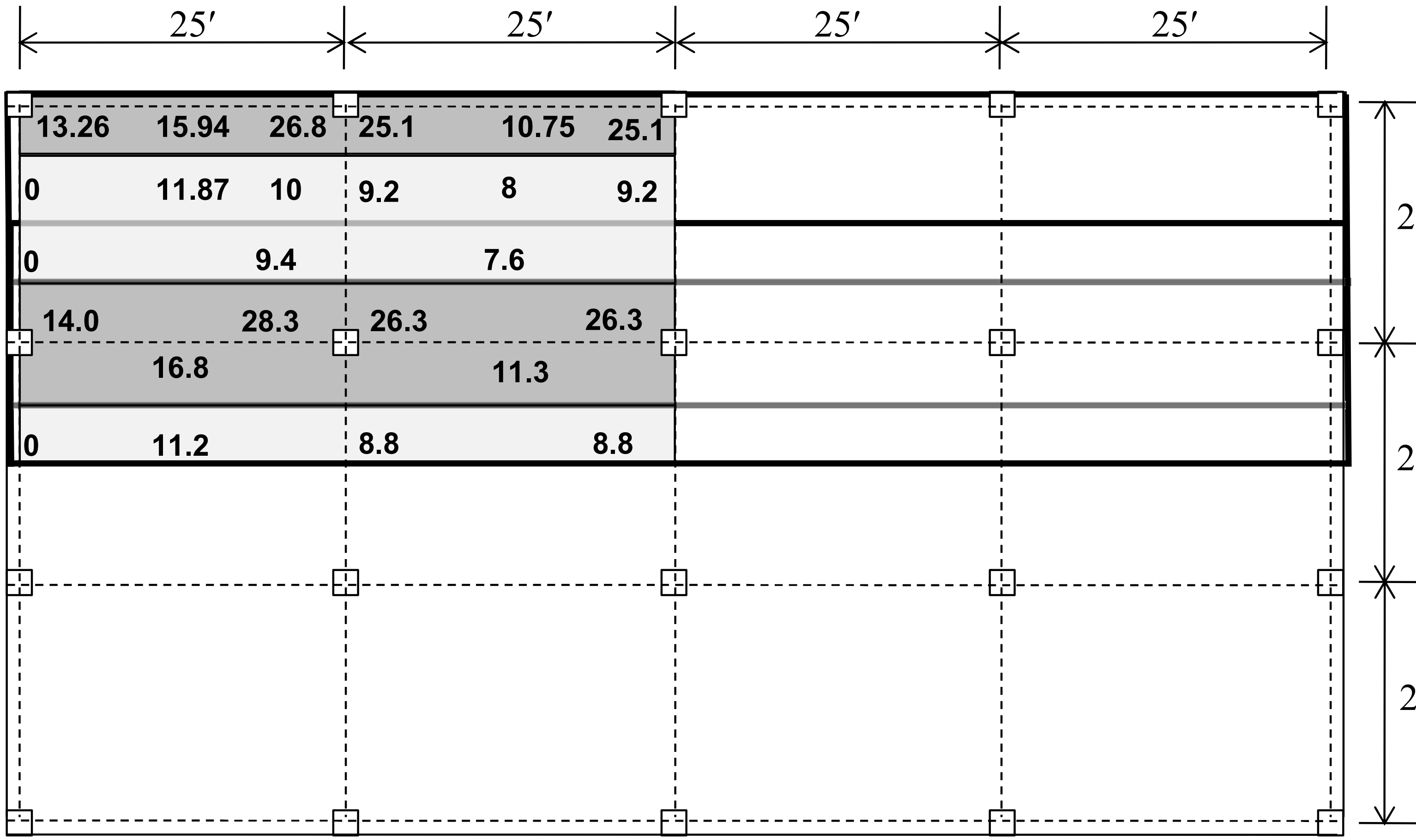
MS = 10.58 – 5.58 = 5′



Example 5.1

❑ Solution

➢ Step 3: Analysis (E-W Direction) – Summary of Moments

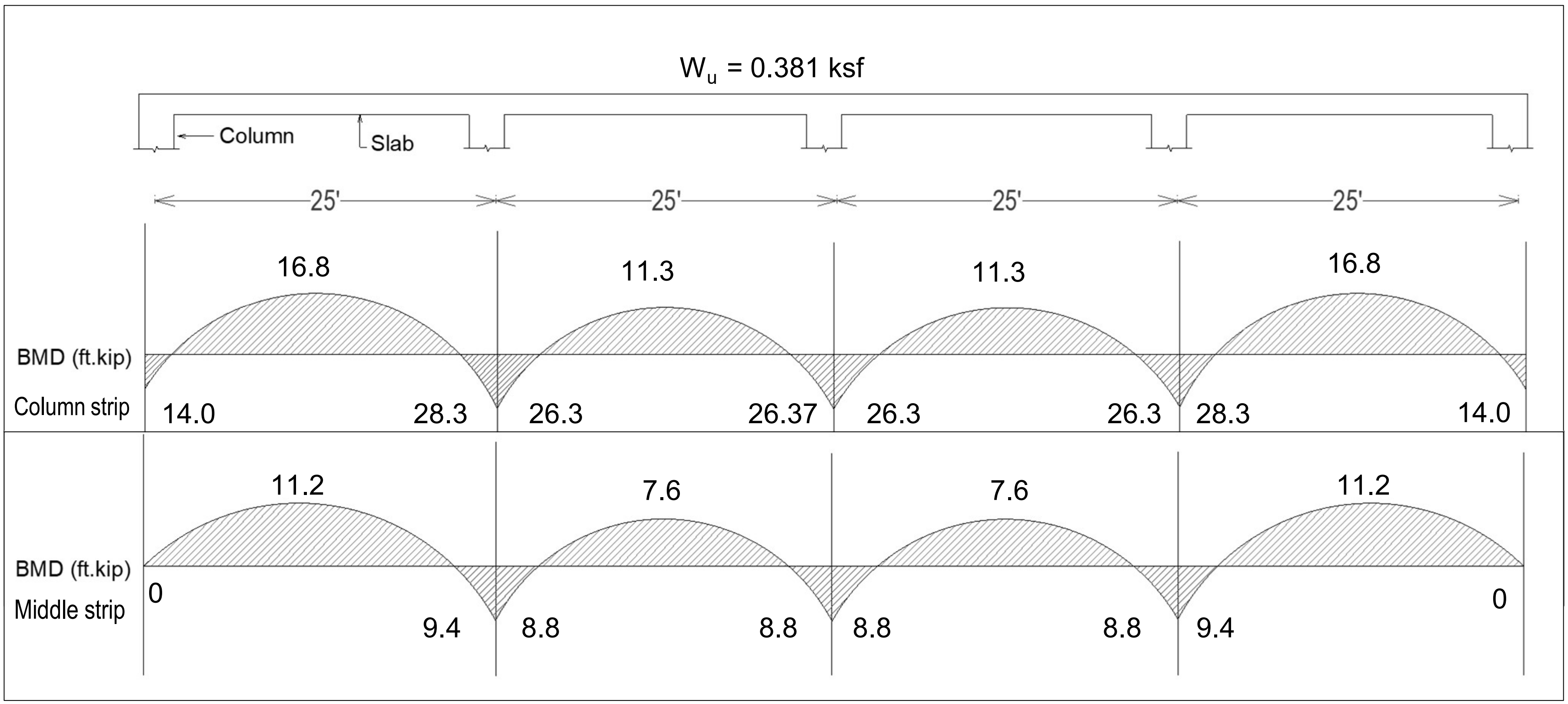


Example 5.1

❑ Solution

➢ Step 3: Analysis (E-W Direction)

❖ Bending Moment Diagrams (Interior Frame)

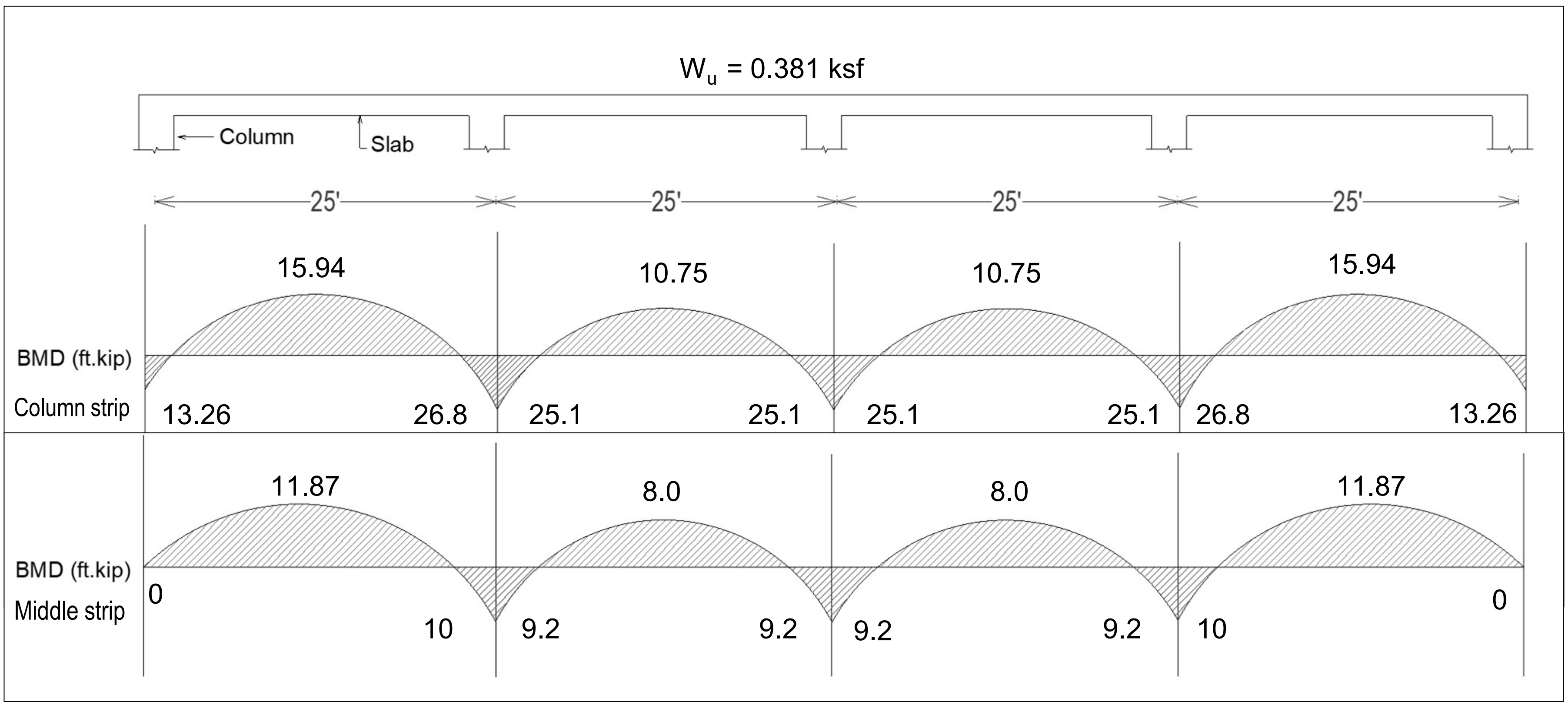


Example 5.1

❑ Solution

➢ Step 3: Analysis (E-W Direction)

❖ Bending Moment Diagrams (Exterior Frame)



Example 5.1

❑ Solution

➢ Step 3: Analysis (N-S Direction) – Interior Frame

❖ Lateral distribution to column and middle strips

$M_o = 421.5$ ft-kip

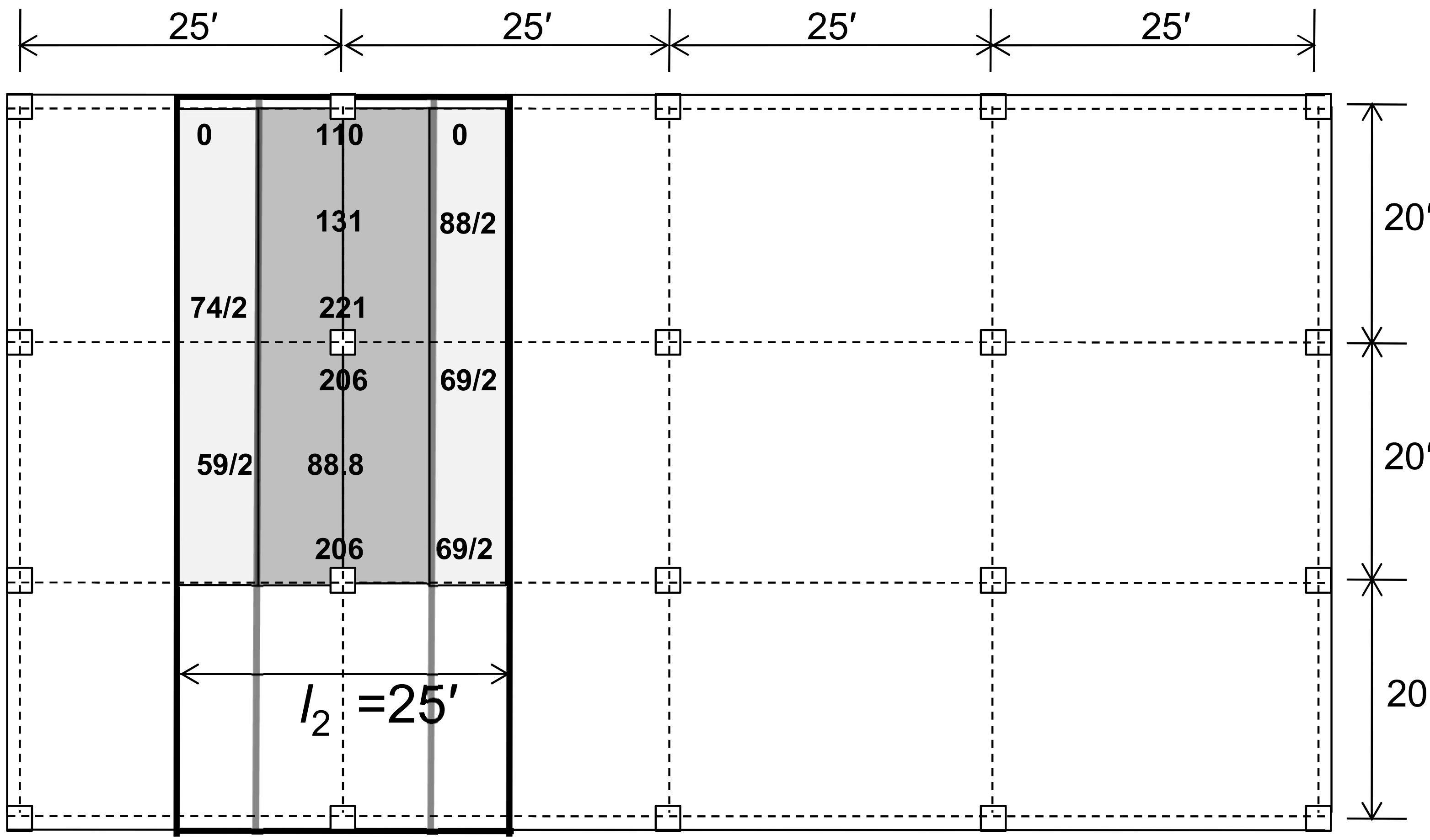
$M_{ext-} = 0.26M_o = 110$

$M_{ext+} = 0.52M_o = 219$

$M_{int-} = 0.70M_o = 295$

$M_{-} = 0.65M_o = 274$

$M_{+} = 0.35M_o = 148$



Example 5.1

❑ Solution

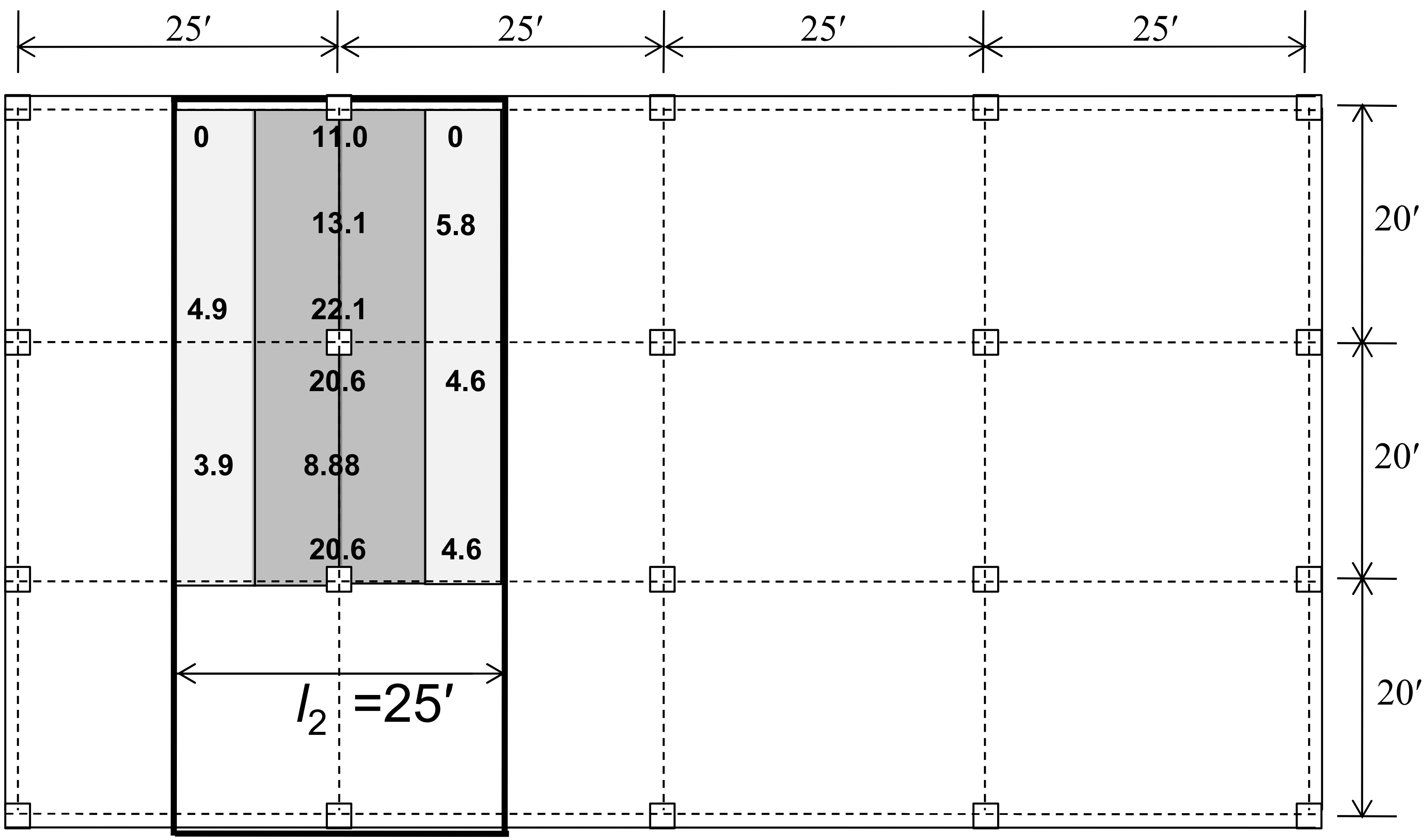
➢ Step 3: Analysis (N-S Direction) – Interior Frame

❖ **Lateral distribution to column and middle strips (per strip width)**

M_o = 421.5 ft-kip

M_{ext-} = 0.26M_o = 110
M_{ext+} = 0.52M_o = 219
M_{int-} = 0.70M_o = 295

M_- = 0.65M_o = 274
M_+ = 0.35M_o = 148



Example 5.1

- **Solution**
  - **Step 3: Analysis (N-S Direction) – Exterior Frame**
    - **Lateral distribution to column and middle strips**

$M_o = 220.5$ ft-kip

$M_{ext-} = 0.26M_o = 58$

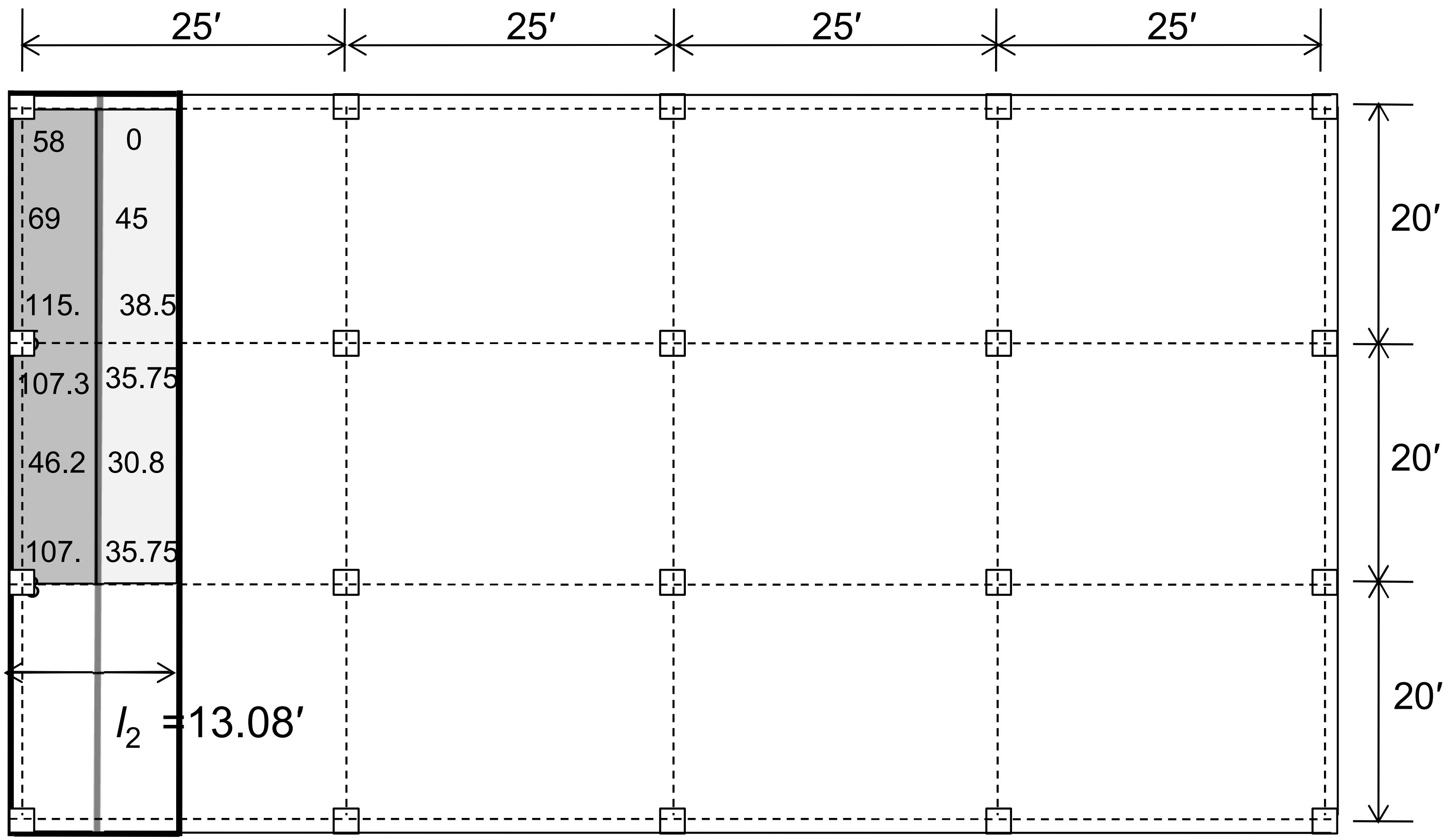
$M_{ext+} = 0.52M_o = 114$

$M_{int-} = 0.70M_o = 154$

$M_- = 0.65M_o = 143$

$M_+ = 0.35M_o = 77$

$l_2 = ½ (25) + ½ (14/12)$
$= 13.08′$



Example 5.1

❑ Solution

➢ Step 3: Analysis (N-S Direction) – Exterior Frame

❖ **Lateral distribution to column and middle strips (per strip width)**

M_o = 220.5 ft-kip

$M_{ext-} = 0.26M_o = 58$

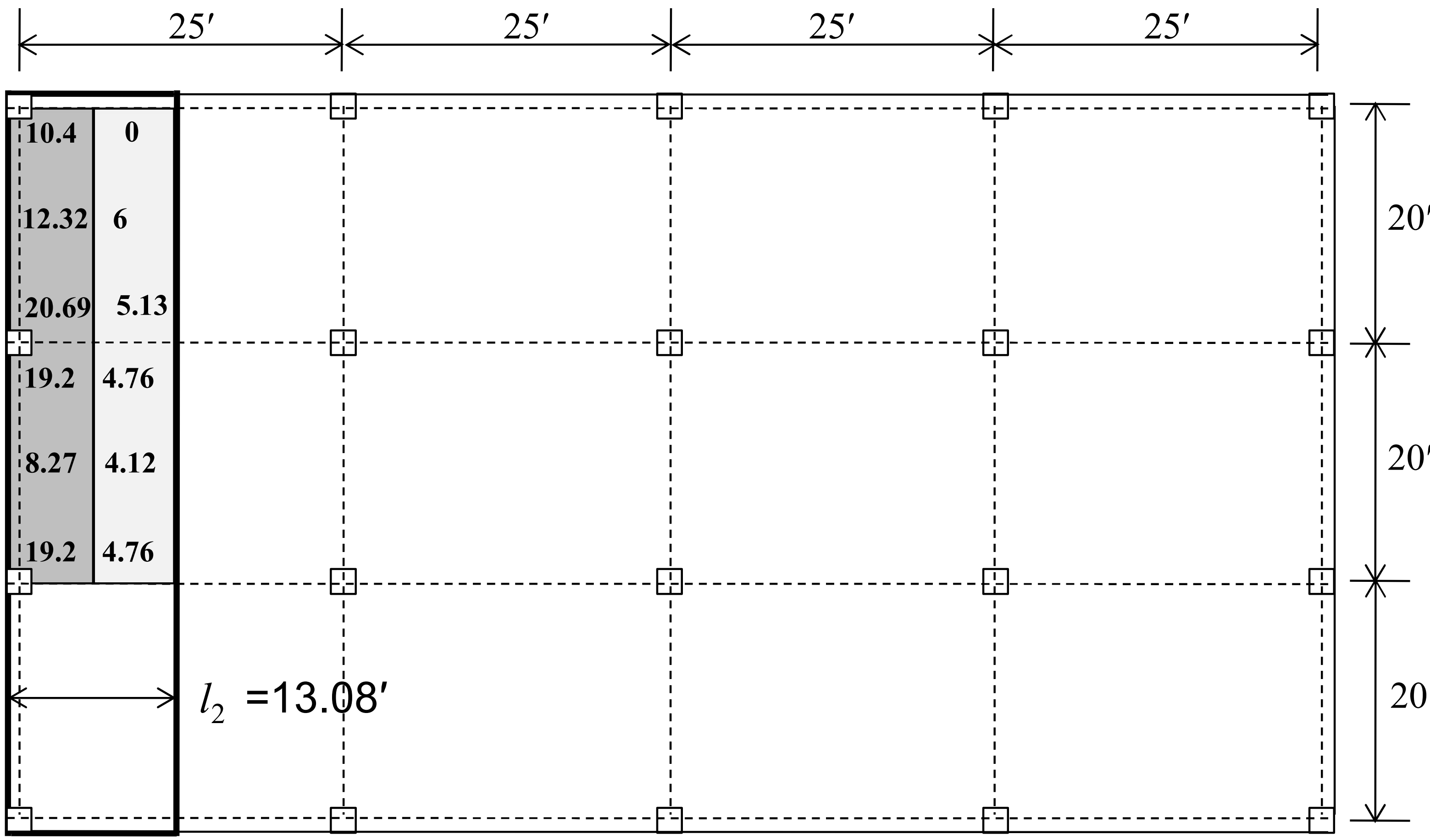
$M_{ext+} = 0.50M_o = 110$

$M_{int-} = 0.70M_o = 154$

$M_{-} = 0.65M_o = 143$

$M_{+} = 0.35M_o = 77$

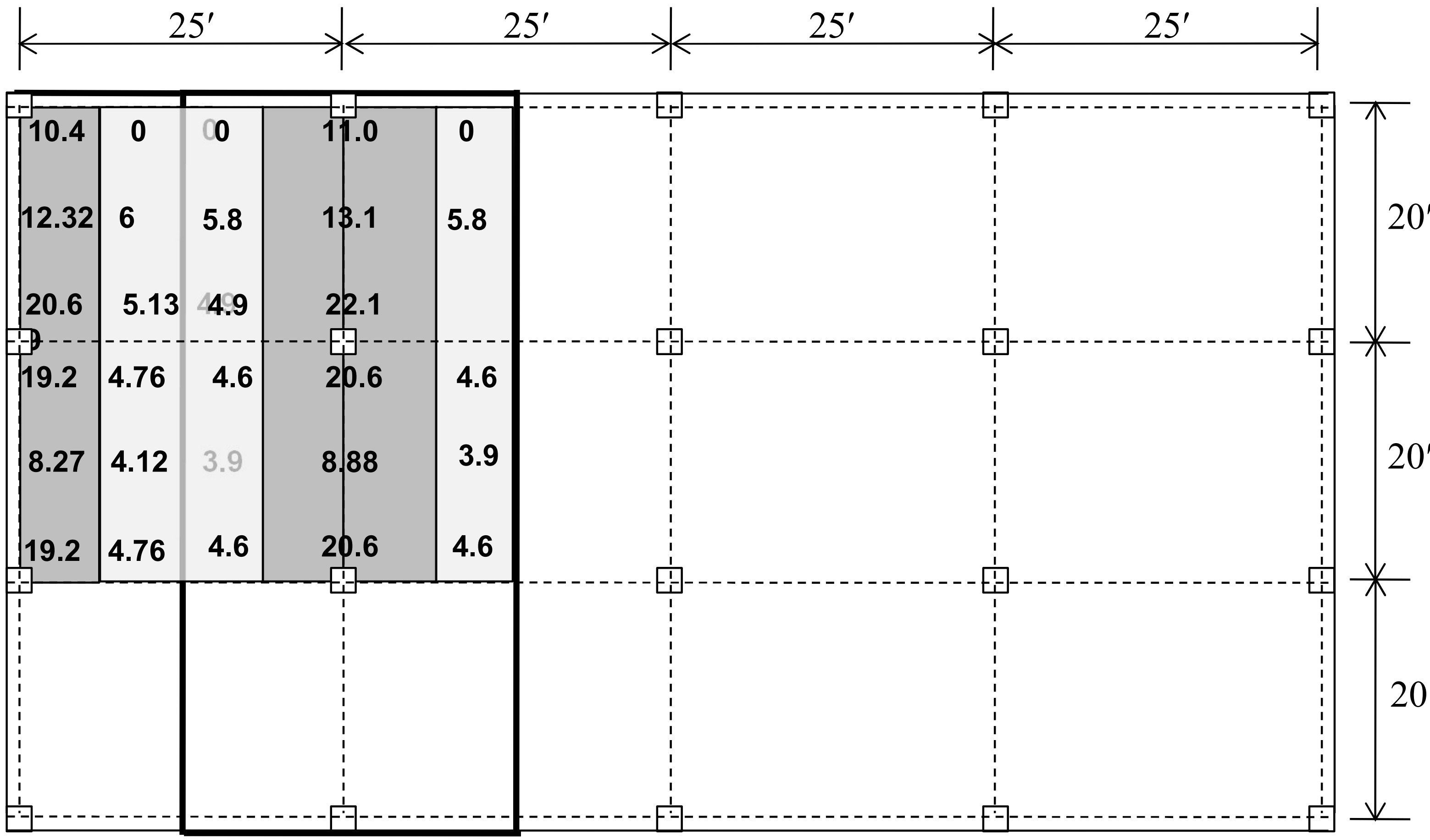
Column Strip Width
= (20/4) + ½ (14/12)
= 5.58′
MS = 13.08 – 5.58
= 7.5′



Example 5.1

❑ Solution

➢ Step 3: Analysis (N-S Direction) – Summary of Moments

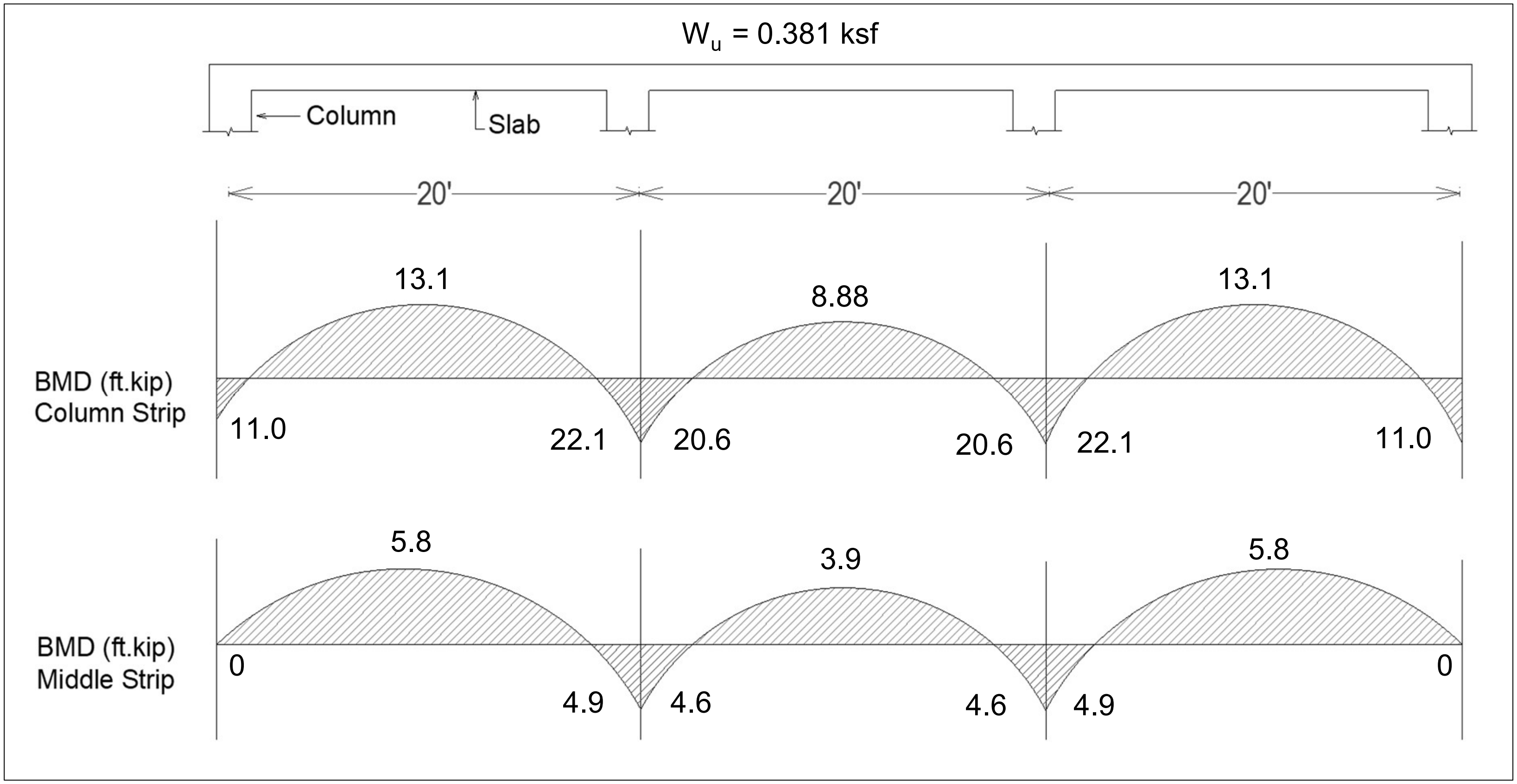


Example 5.1

❑ Solution

➢ Step 3: Analysis (N-S Direction)

❖ **Bending Moment Diagrams – Interior Frame**

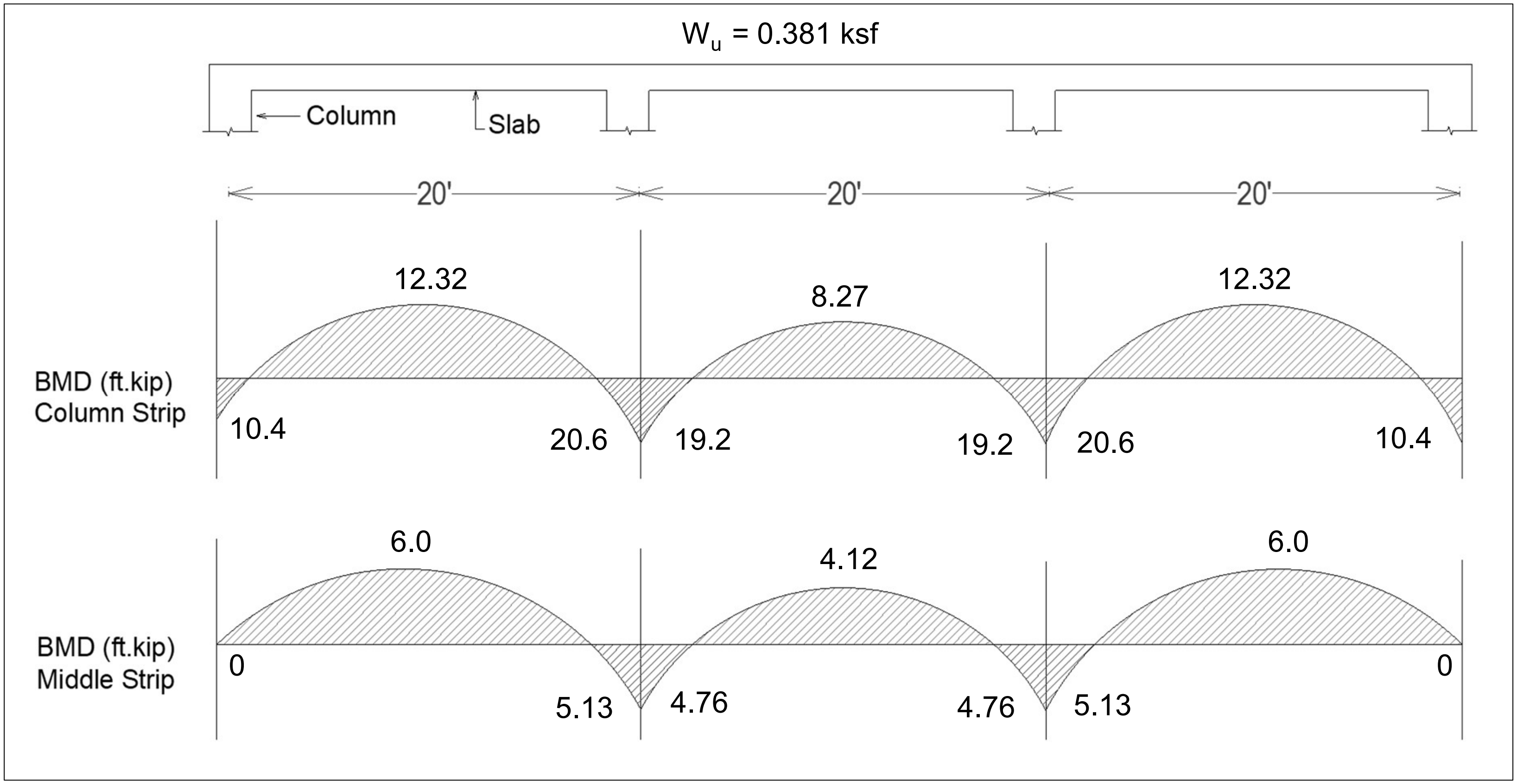


Example 5.1

❑ Solution

➢ Step 3: Analysis (N-S Direction)

❖ Bending Moment Diagrams – Exterior Frame

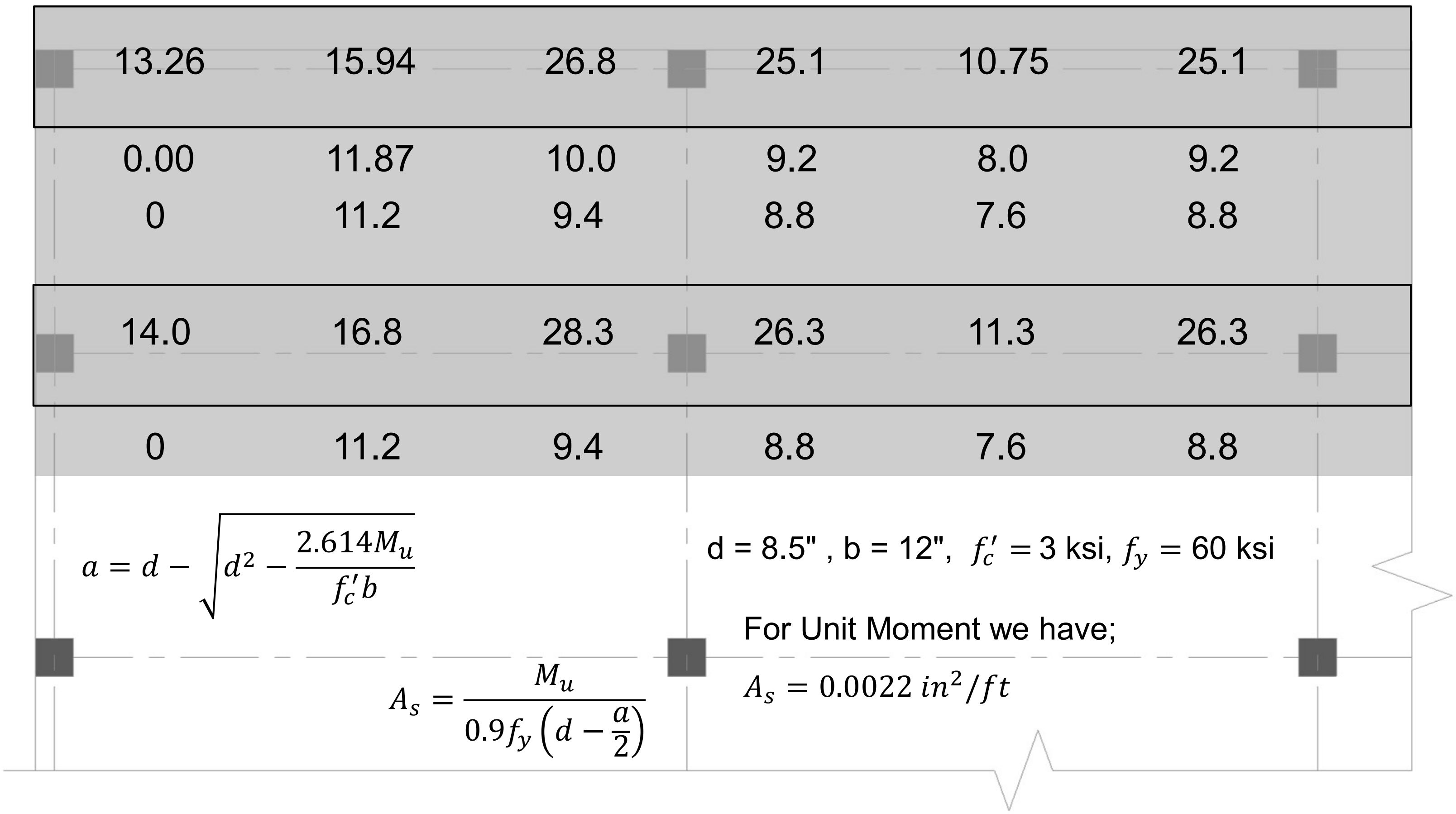


Example 5.1

❑ Solution

➢ Step 4: Determination of Reinforcement (E-W Direction)

❖ Summary of Moments for Interior Frame (per unit strip)



$$a = d - \sqrt{d^2 - \frac{2.614M_u}{f_c' b}}$$

$$A_s = \frac{M_u}{0.9f_y\left(d - \frac{a}{2}\right)}$$

d = 8.5" , b = 12", $f_c' = 3$ ksi, $f_y = 60$ ksi

For Unit Moment we have;

$A_s = 0.0022\ in^2/ft$

Example 5.1

❑ Solution

➢ Step 4: Determination of Reinforcement (E-W Direction)

❖ **Summary of Reinforcement (in²/ft)**

0.35	0.42	0.71	0.66	0.28	0.66
0.00	0.32	0.26	0.24	0.32	0.24
0	0.30	0.25	0.23	0.20	0.23
0.37	0.44	0.75	0.70	0.30	0.70
0	0.30	0.25	0.23	**0.20***	0.23

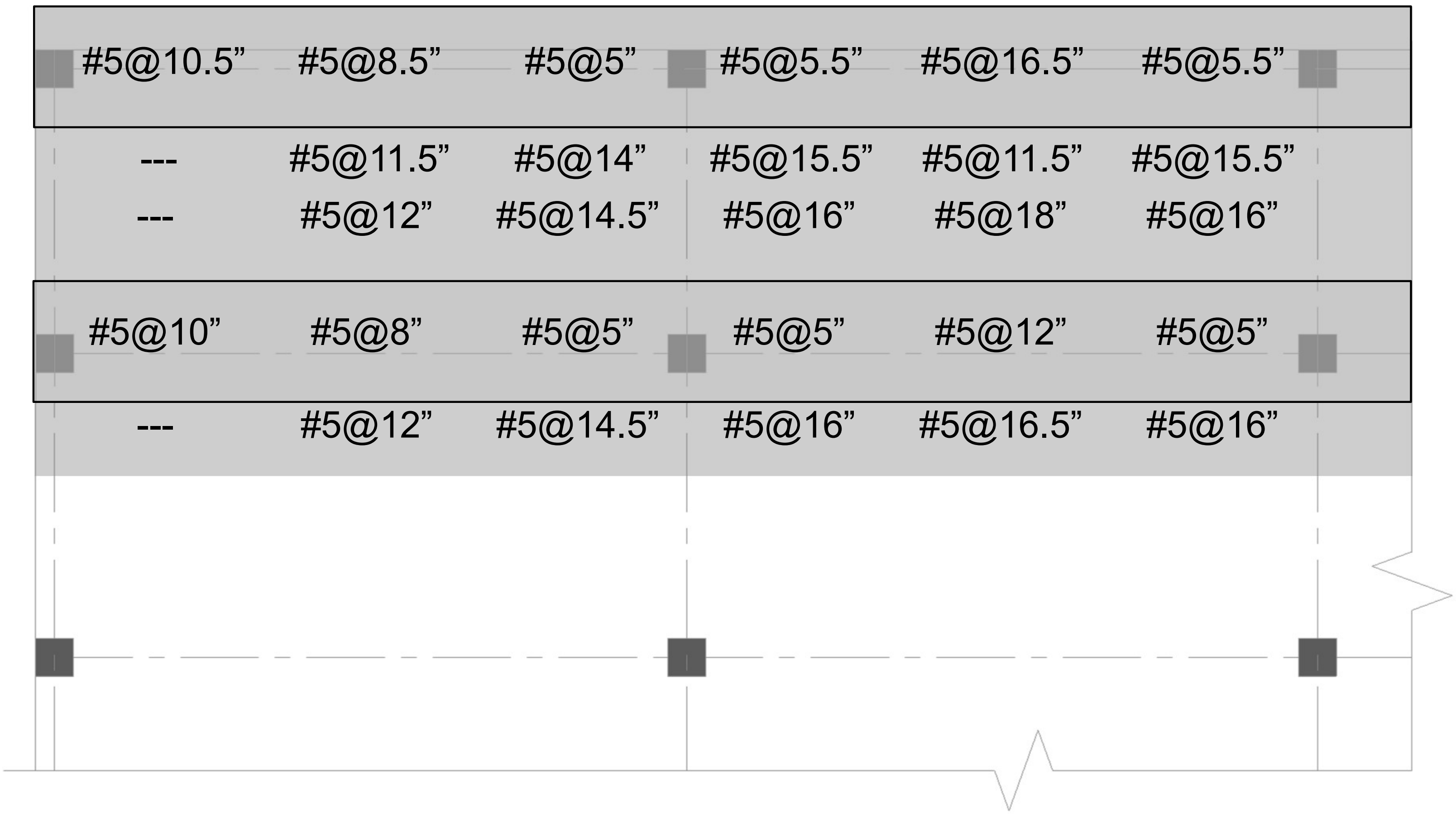
$A_s = 0.0018bh$
$= 0.0018(12)(10) = 0.22\ in^2/ft$

*Minimum reinforcement governs

Example 5.1

❑ Solution

➢ **Step 4: Determination of Reinforcement (E-W Direction)**

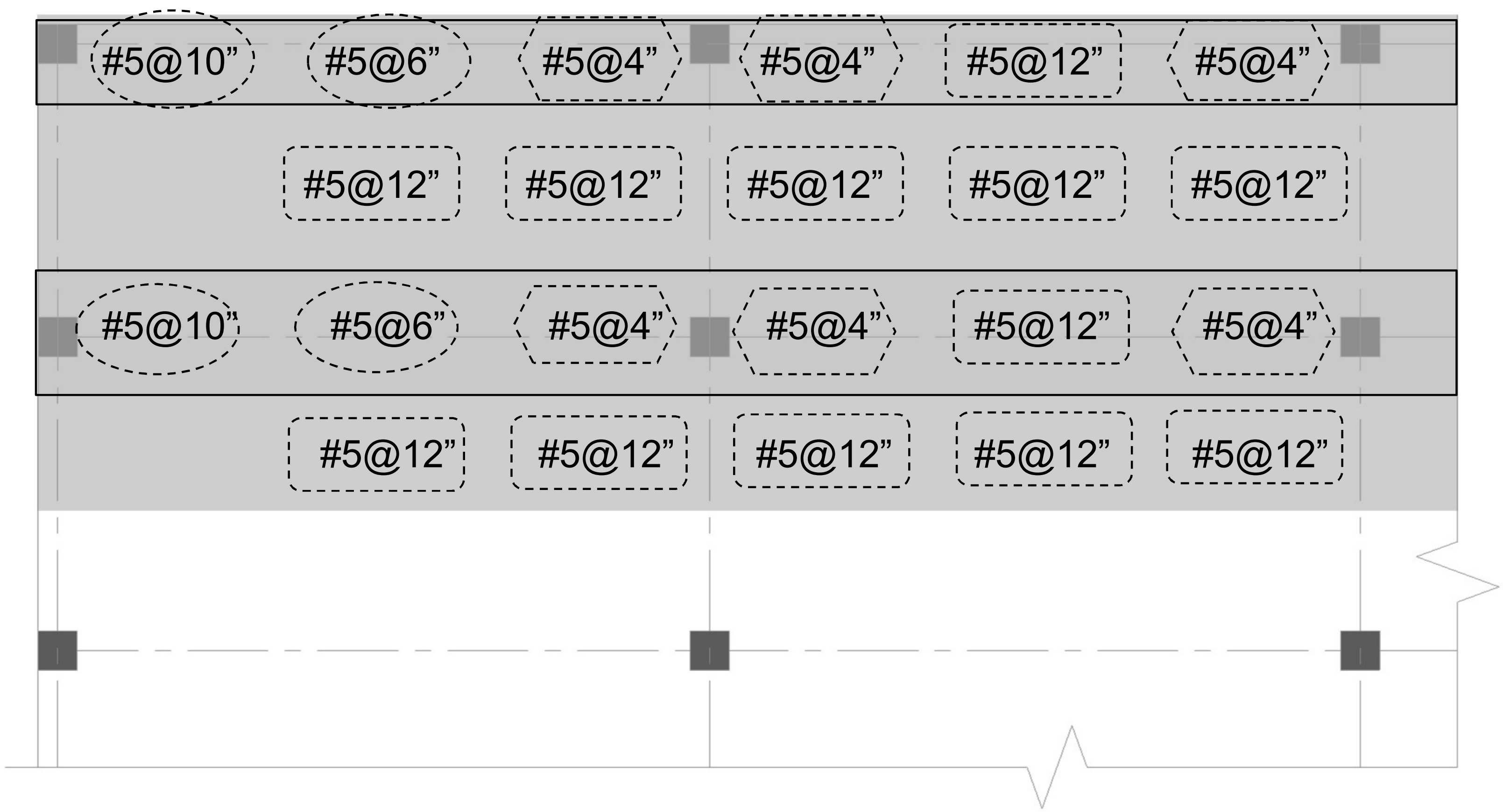


Example 5.1

❑ Solution

➢ Step 4: Determination of Reinforcement (E-W Direction)

❖ **Final Detailing (after re-grouping of spacing)**

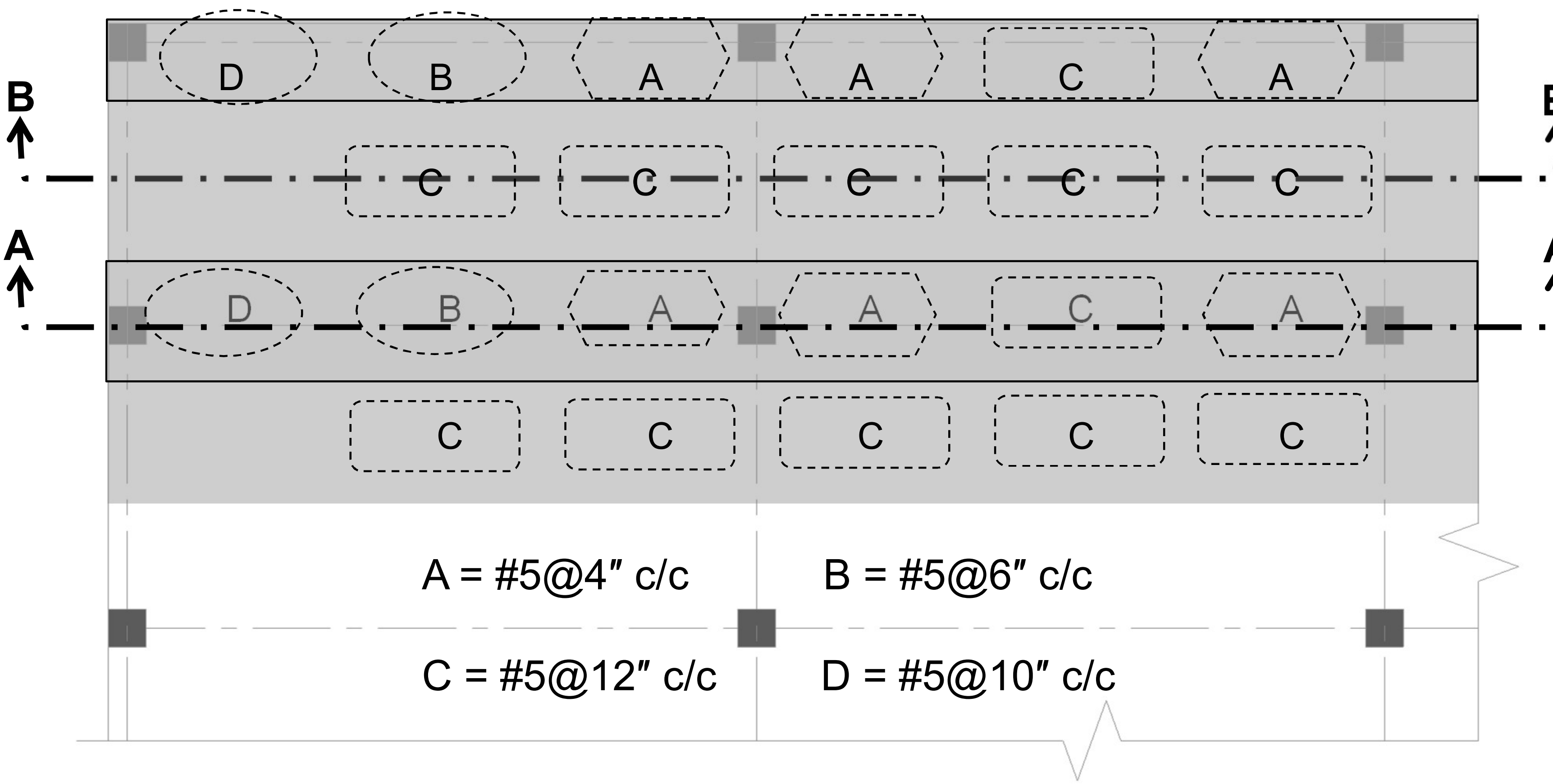


Example 5.1

❑ Solution

➢ Step 4: Determination of Reinforcement (E-W Direction)

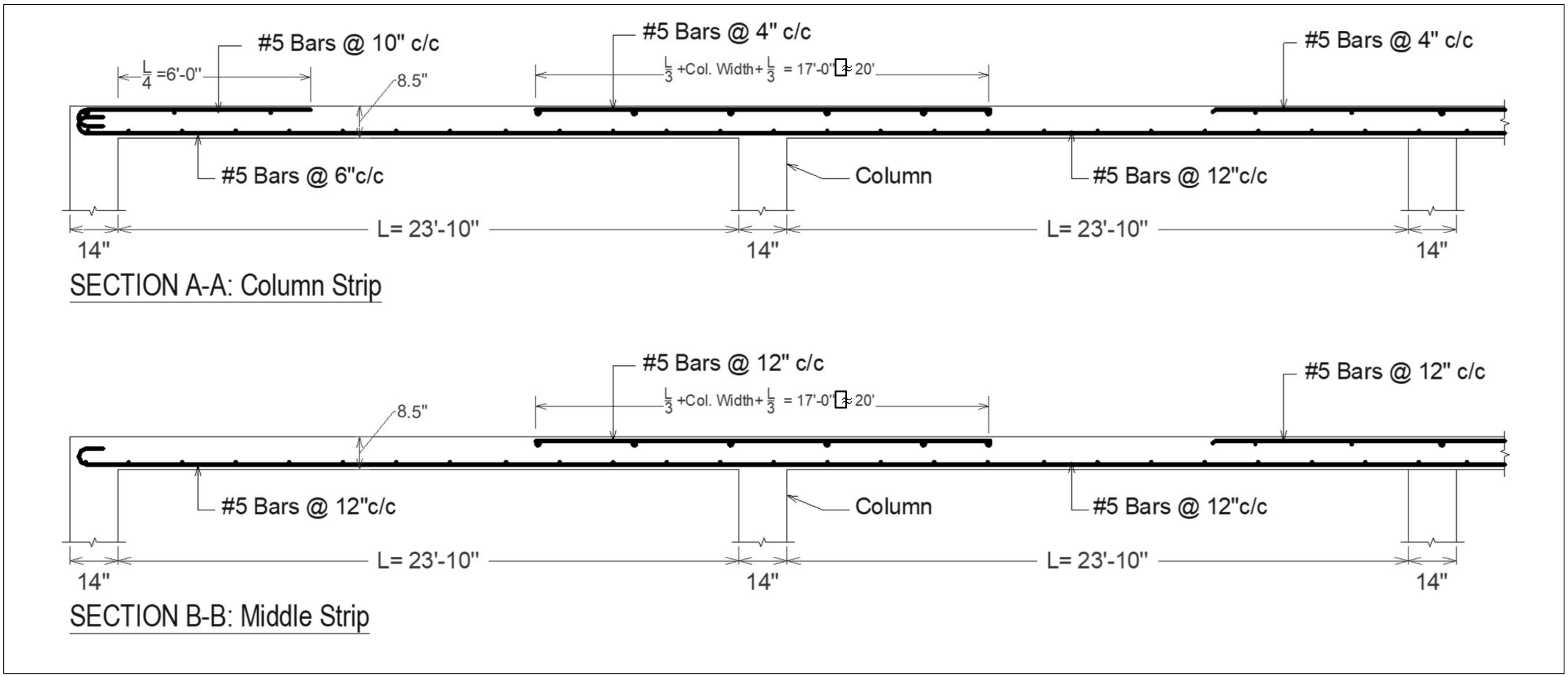
❖ **Final Detailing (after re-grouping of spacing)**



Example 5.1

❑ Solution

➢ Step 4: Determination of Reinforcement (E-W Direction)



Example 5.1

❑ Solution

➢ Step 4: Determination of Reinforcement (N-S Direction)

❖ **Summary of bending moments (ft.kip/ft)**

10.4	0.00	0	11.0	0
12.32	6.0	5.8	13.1	5.8
20.6	5.13	4.9	22.1	4.9
19.2	4.76	4.6	20.6	4.6
8.27	4.13	3.9	8.88	3.9
19.2	4.76	4.6	20.6	4.6

Example 5.1

- **Solution**
  - **Step 4: Determination of Reinforcement (N-S Direction)**
    - **Summary of bending moments (ft.kip/ft)**

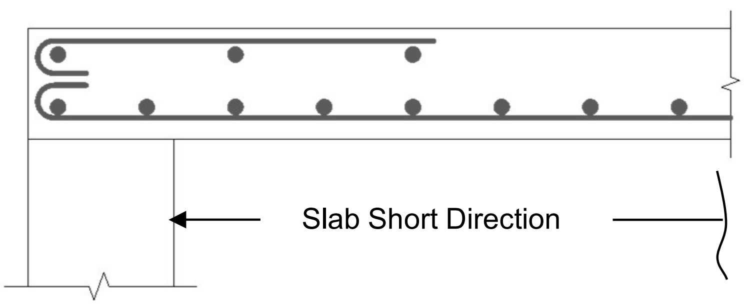
Design and Detail reinforcement yourself

for the given moments

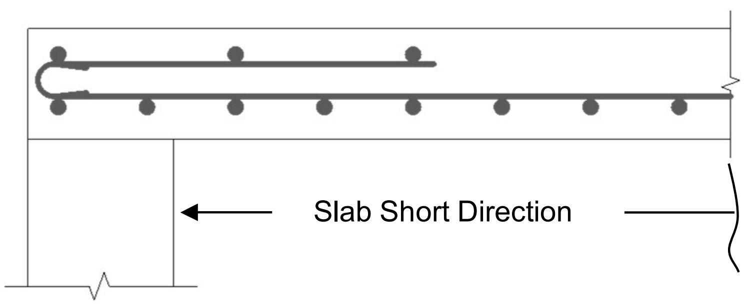
Other ACI Provisions for Flat Slabs

❑ Detailing of Flexural Reinforcement

- In two-way slabs with beam supports, bars in shorter direction are placed closer to the surface due to greater moments in the shorter direction.
- However, for flat plates, long-direction bars in middle and column strips are placed nearer the surface due to larger moments in the longer direction.



Slab with stiff beams



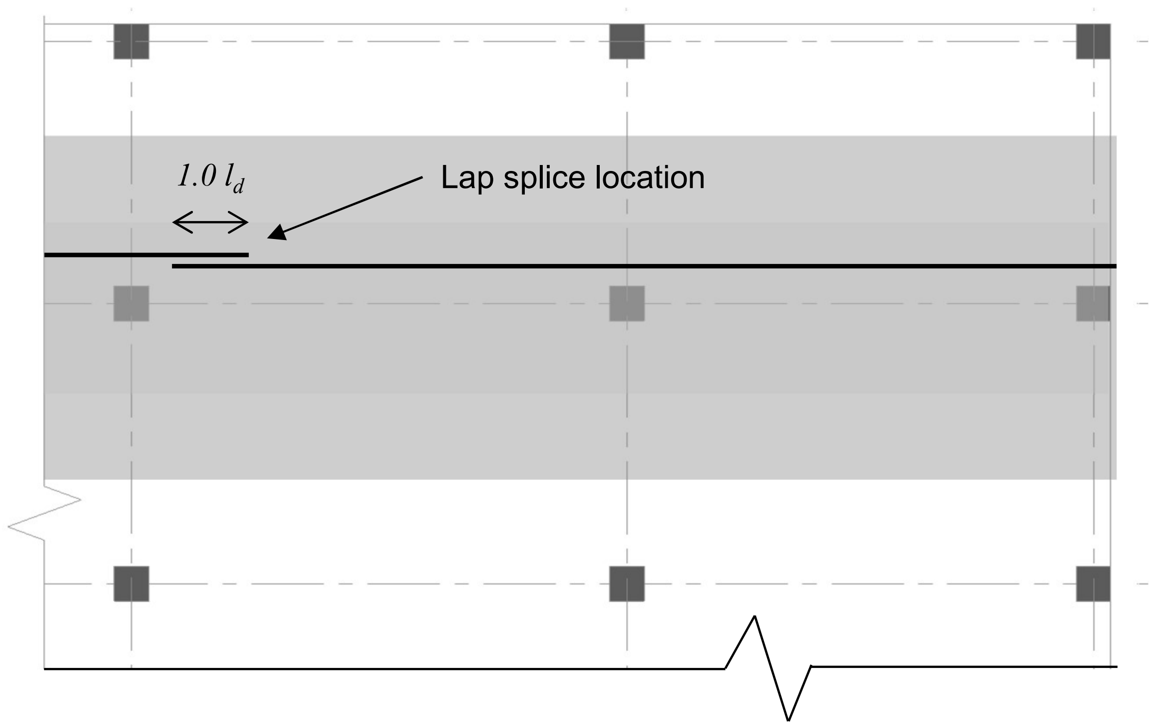
Flat Plate

Other ACI Provisions for Flat Slabs

Detailing of Flexural Reinforcement

Splicing

- ACI 8.7.4.2.1 requires that all bottom bars within the column strip in each direction be continuous or spliced with length equal to 1.0 l_d

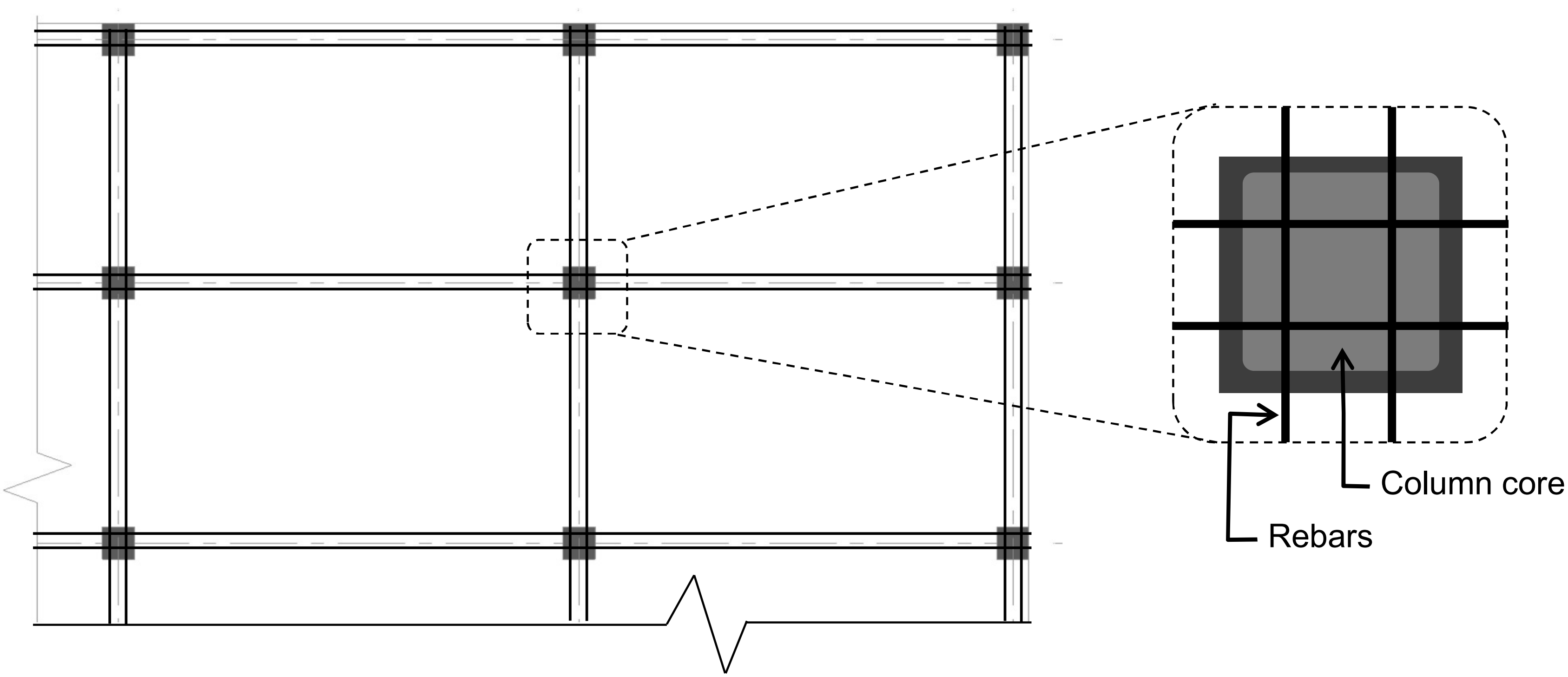


Other ACI Provisions for Flat Slabs

❑ Detailing of Flexural Reinforcement

❖ Continuity of Bars

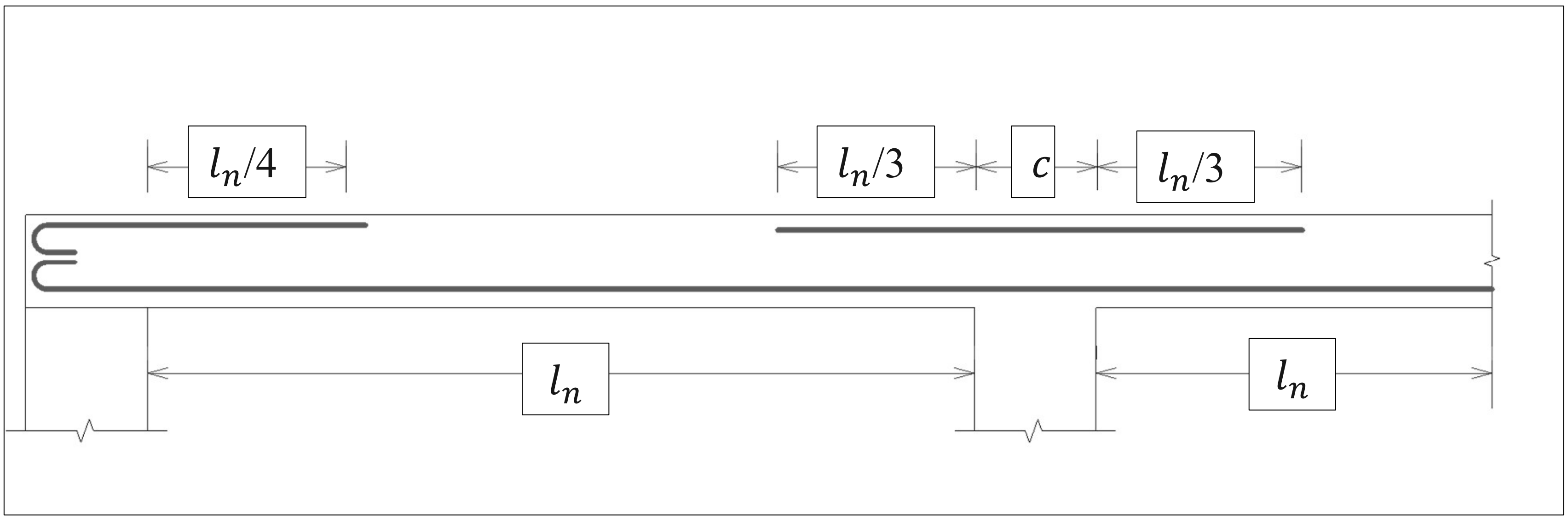
- ACI 8.7.4.2.2 requires that at least two of the column strip bottom bars in each direction must pass within the column core and must be anchored at exterior supports.



Other ACI Provisions for Flat Slabs

❑ Detailing of Flexural Reinforcement

- Standard Bar Cut off Points (Practical Recommendation) for column and middle strips both are shown below.

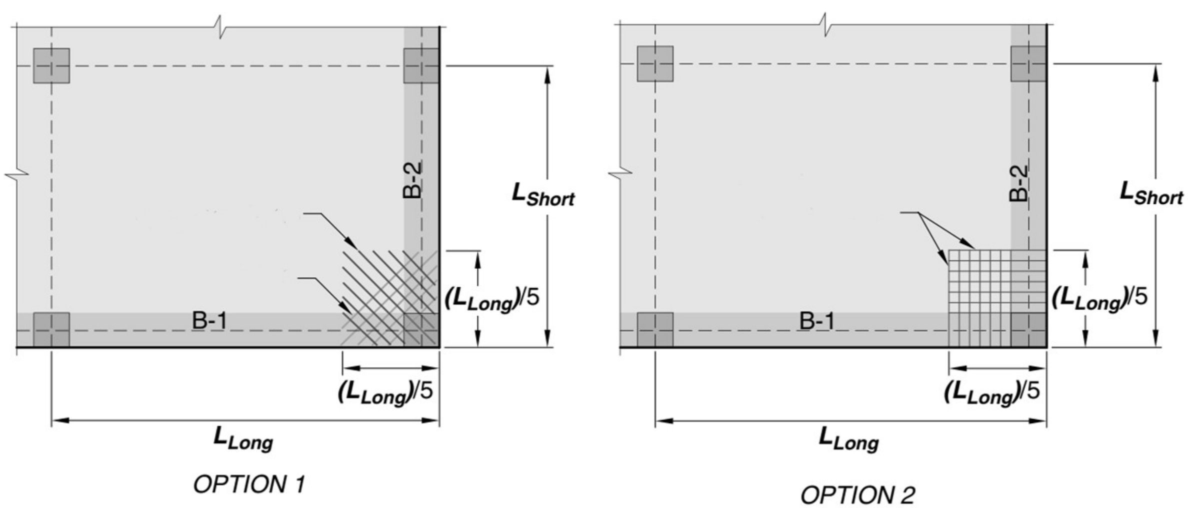


Other ACI Provisions for Flat Slabs

❑ Detailing of Flexural Reinforcement

❖ Reinforcement at Exterior Corners (ACI 8.7.3.1.2)

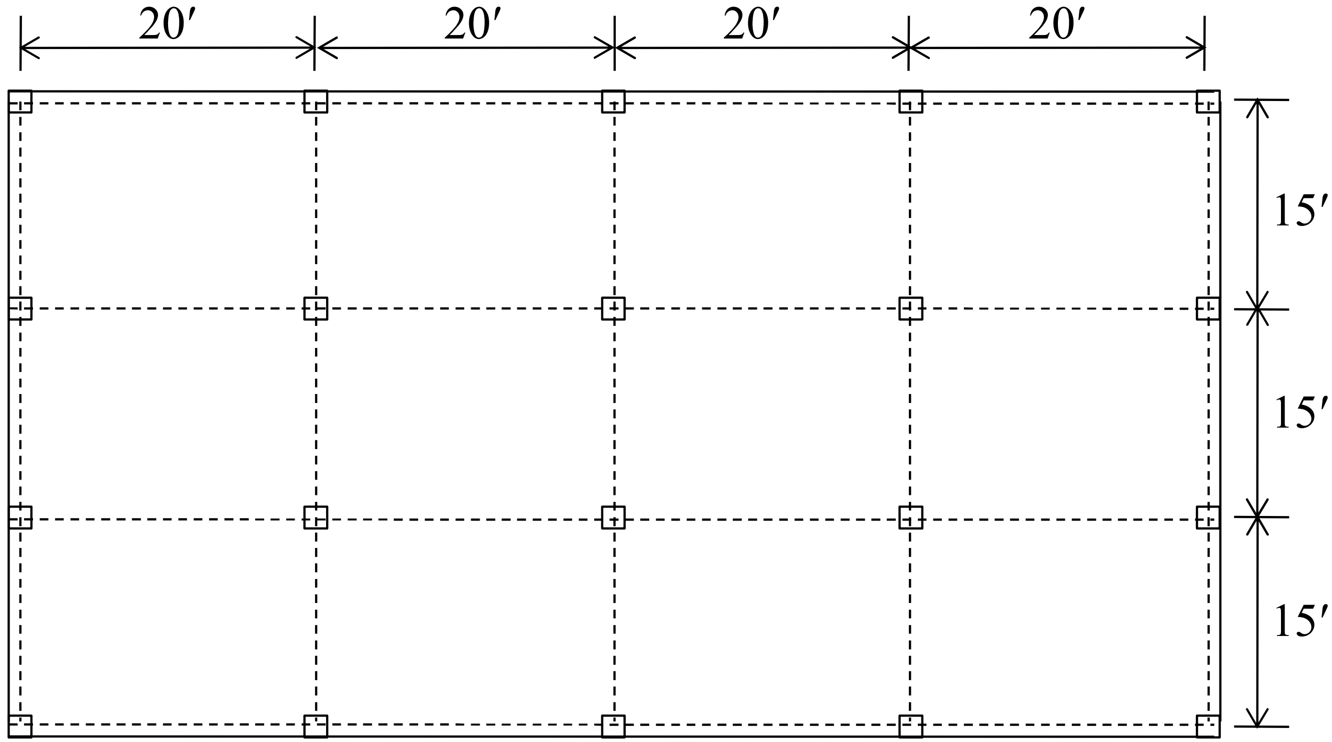
- Reinforcement of size and spacing equivalent to that required for maximum positive moments (per foot of width) in the panel shall be provided at exterior corners on the top and bottom of the slab.



Slab Corner Reinforcement (ACI Fig. R8.7.3.1)

Example 5.2

- **Homework:** Analysis results of the slab shown below using DDM are presented next. The slab supports a live load of 60 psf. Superimposed dead load is equal to 40 psf. All columns are 12″ square. Take $f_c' = 3$ ksi and $f_y = 60$ ksi.



Section – II

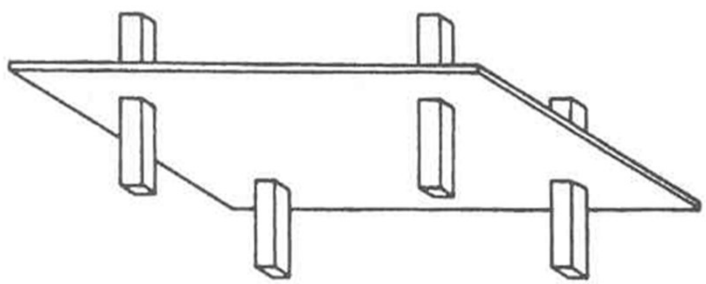
Shear Design of Two-Way Slab System without Beams

(Flat Plates and Flat Slabs)

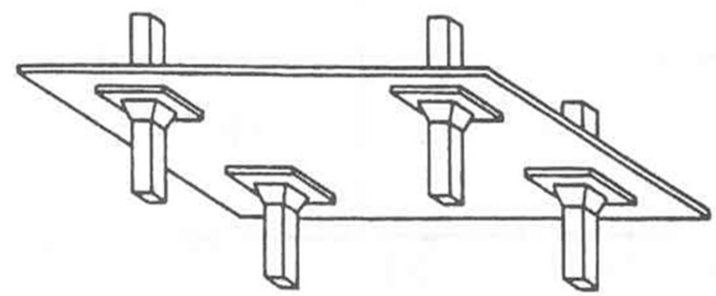
General

❑ Flat Plates and Flat Slabs

- These are the most common types of two-way slab system, which are commonly used in multi-story construction.
- They render low story heights and have easy construction and formwork.



Flat Plate



Flat Slab

General

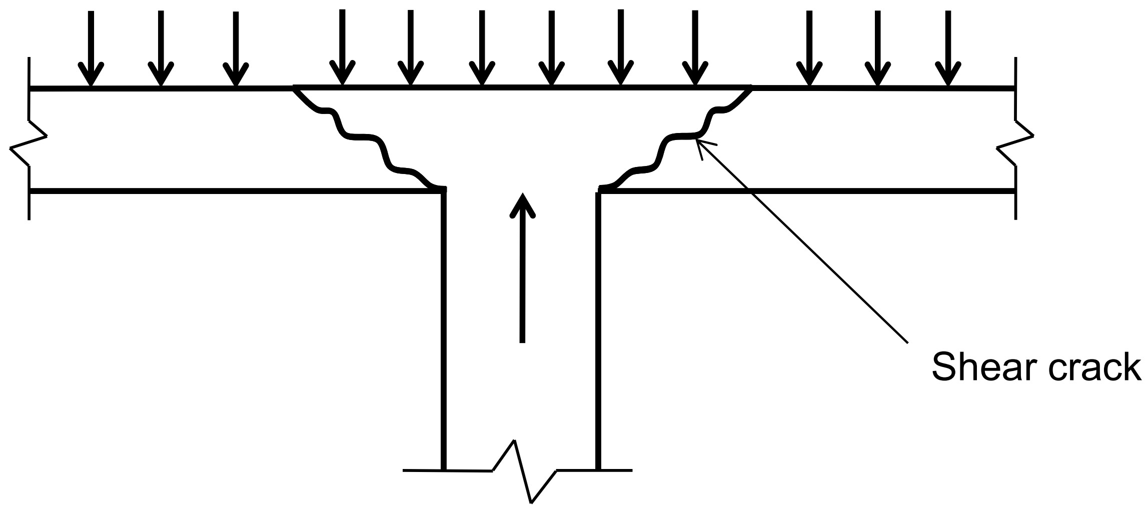
❑ Behavior

- These two-way slabs are directly supported by columns, shear near the column (punching shear) is of critical importance.
- Therefore, in addition to flexure, flat plates shall also be designed for two-way shear (punch out shear) stresses.

General

❑ Punching Shear in Flat Plates

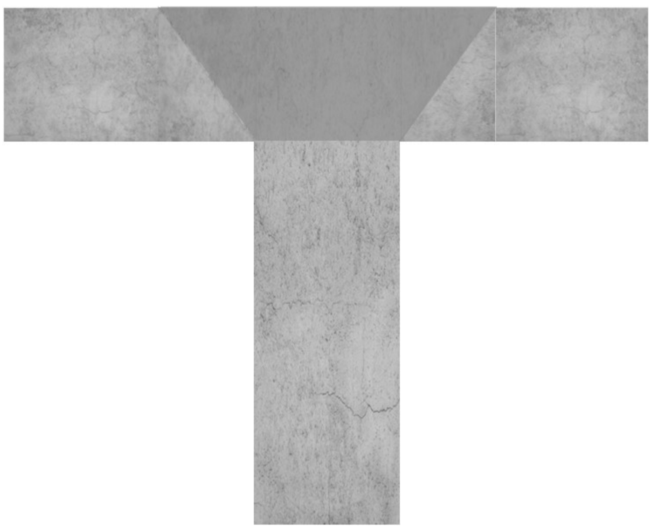
- Punching shear occurs at column support points in flat plates and flat slabs.

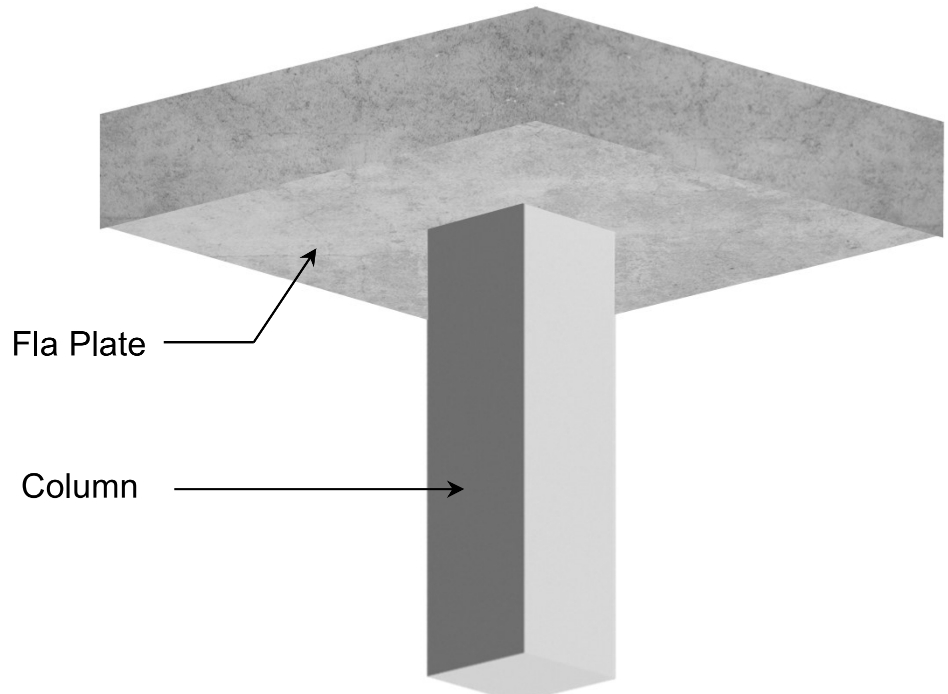


Punch Out

General

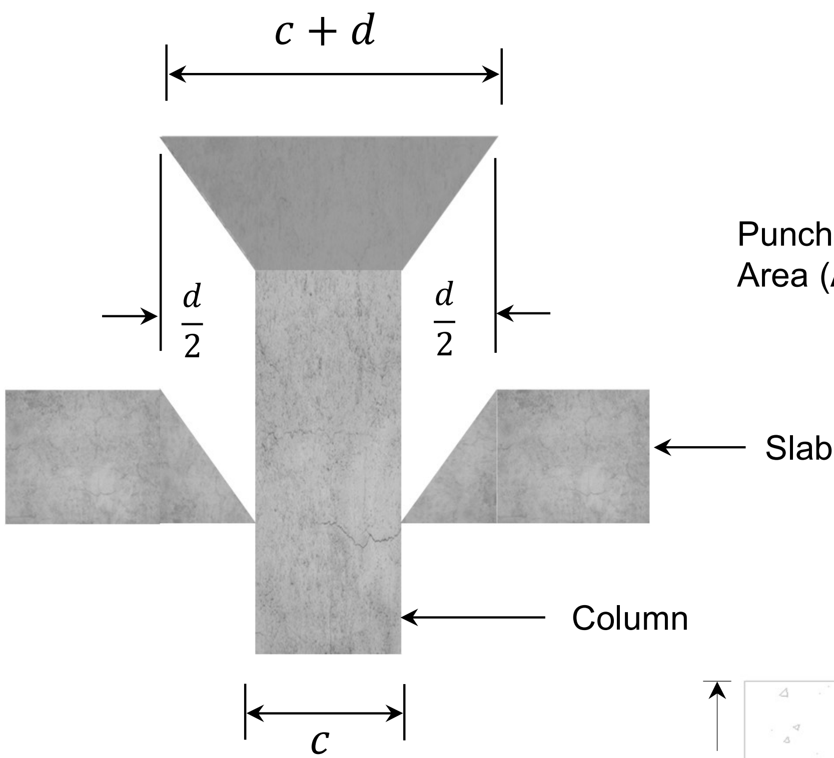
❑ Critical Section for Punching Shear

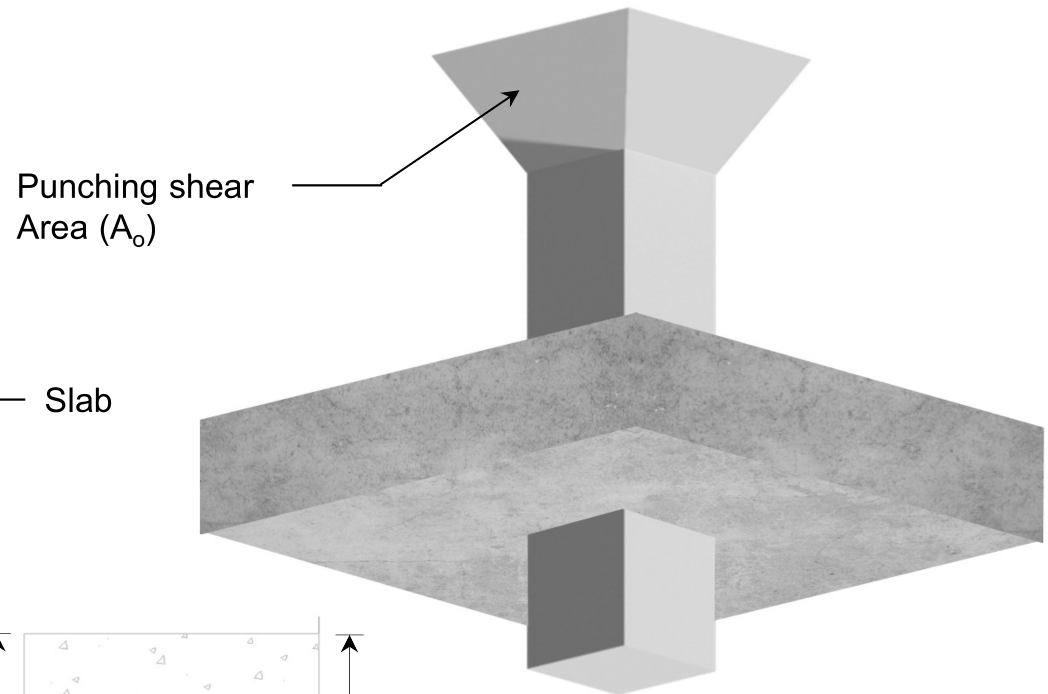


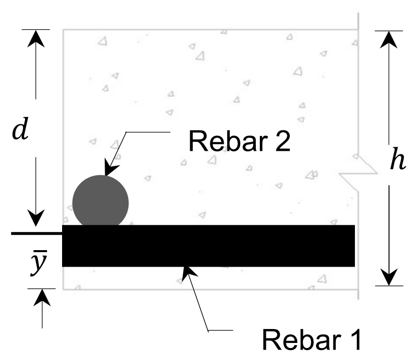


General

❑ Critical Section for Punching Shear



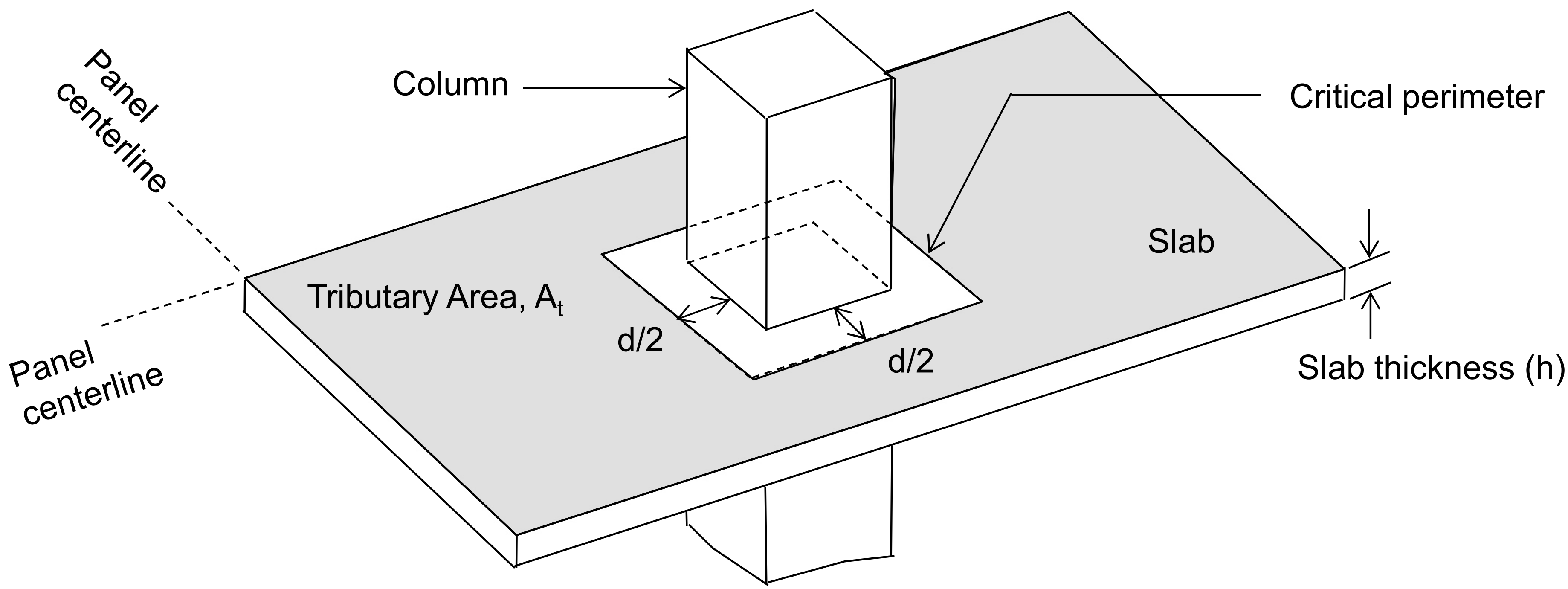




General

❑ Critical Section for Punching Shear

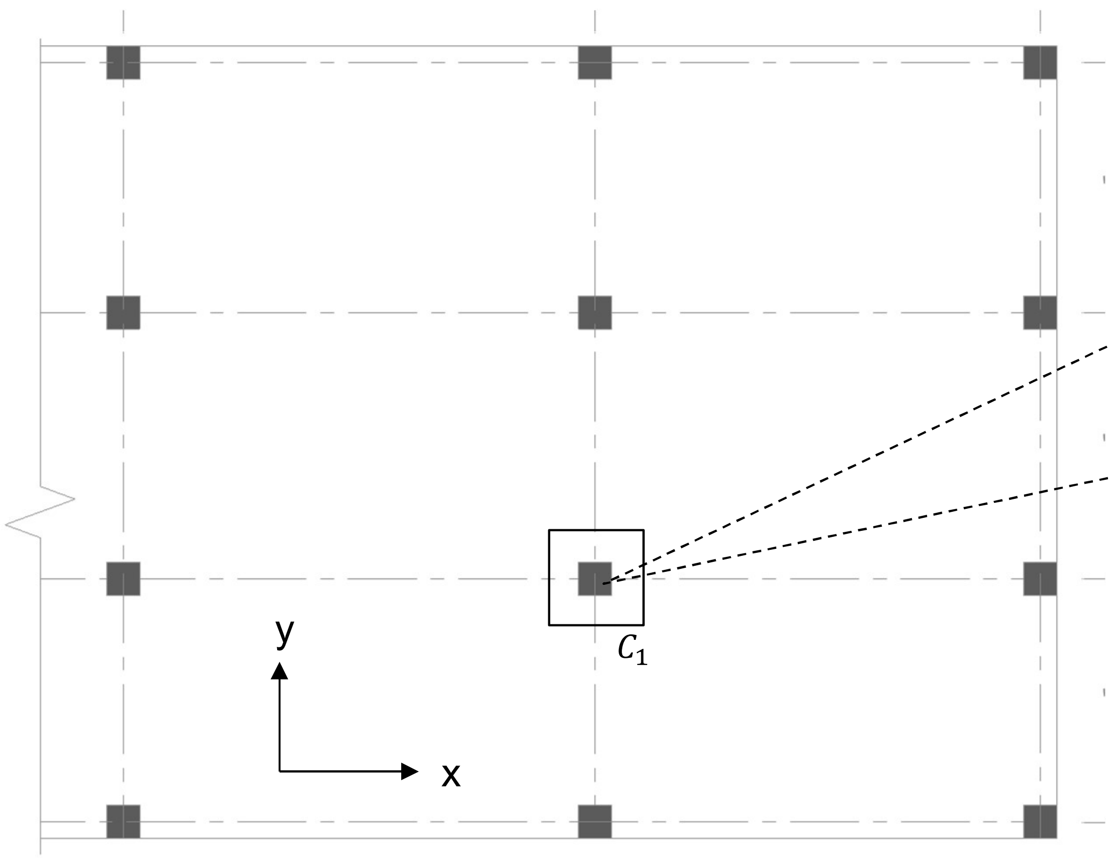
- In shear design of flat plates, the critical section is an area taken at a distance “d/2” from all face of the support.

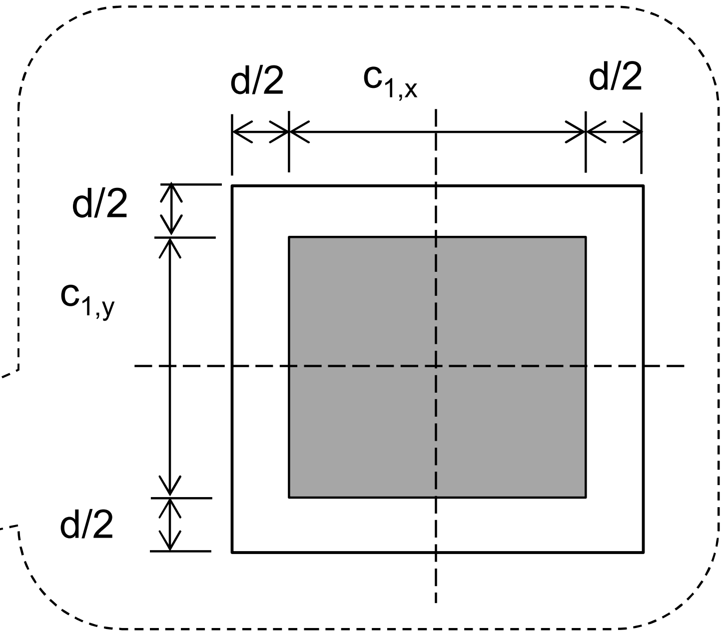


General

❑ Calculation of Critical Perimeter b_o

❖ Interior Columns



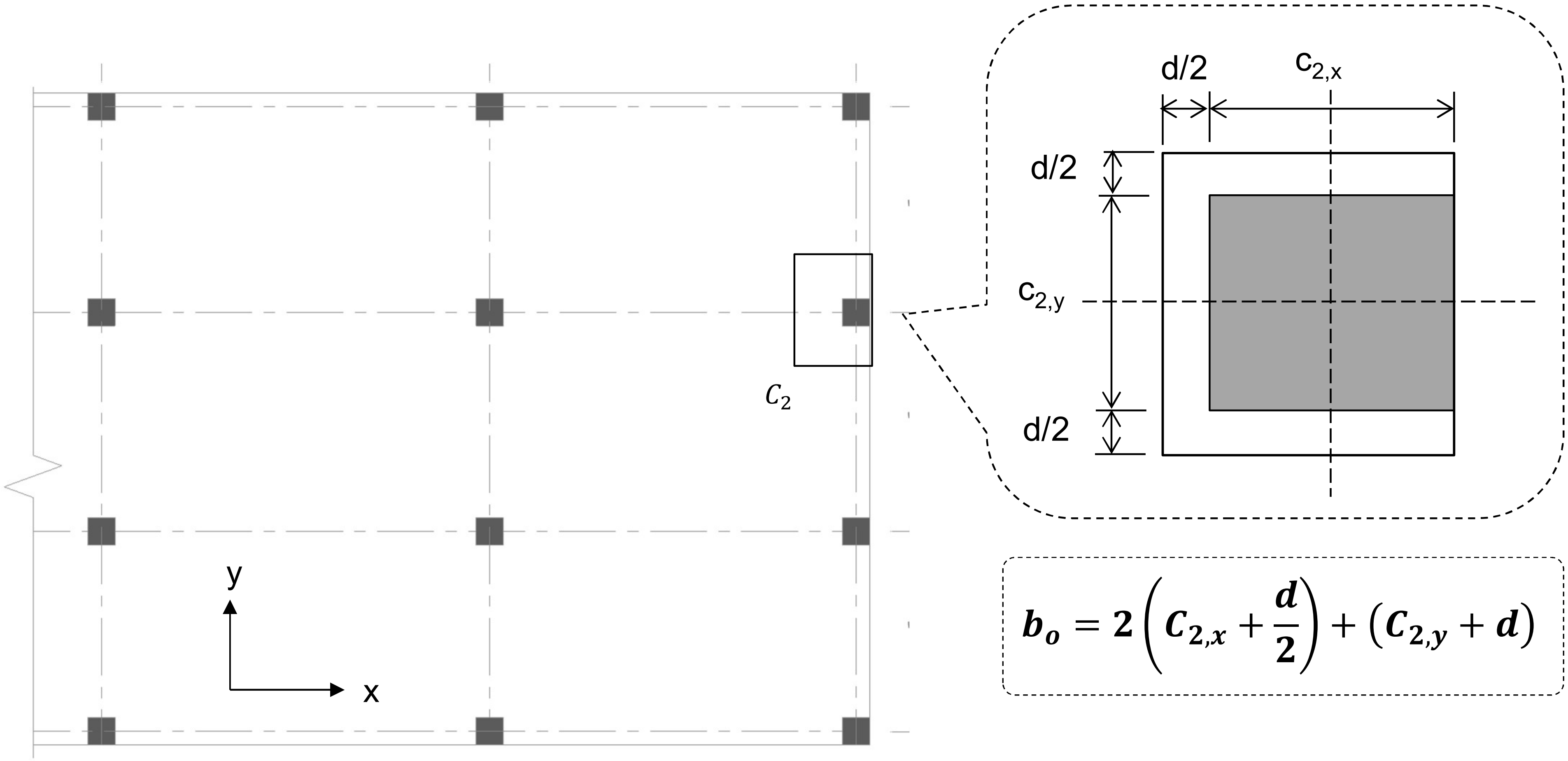


$$b_o = 2(C_{1,x} + d) + 2(C_{1,y} + d)$$

General

❑ Calculation of Critical Perimeter b_o

❖ Edge Columns

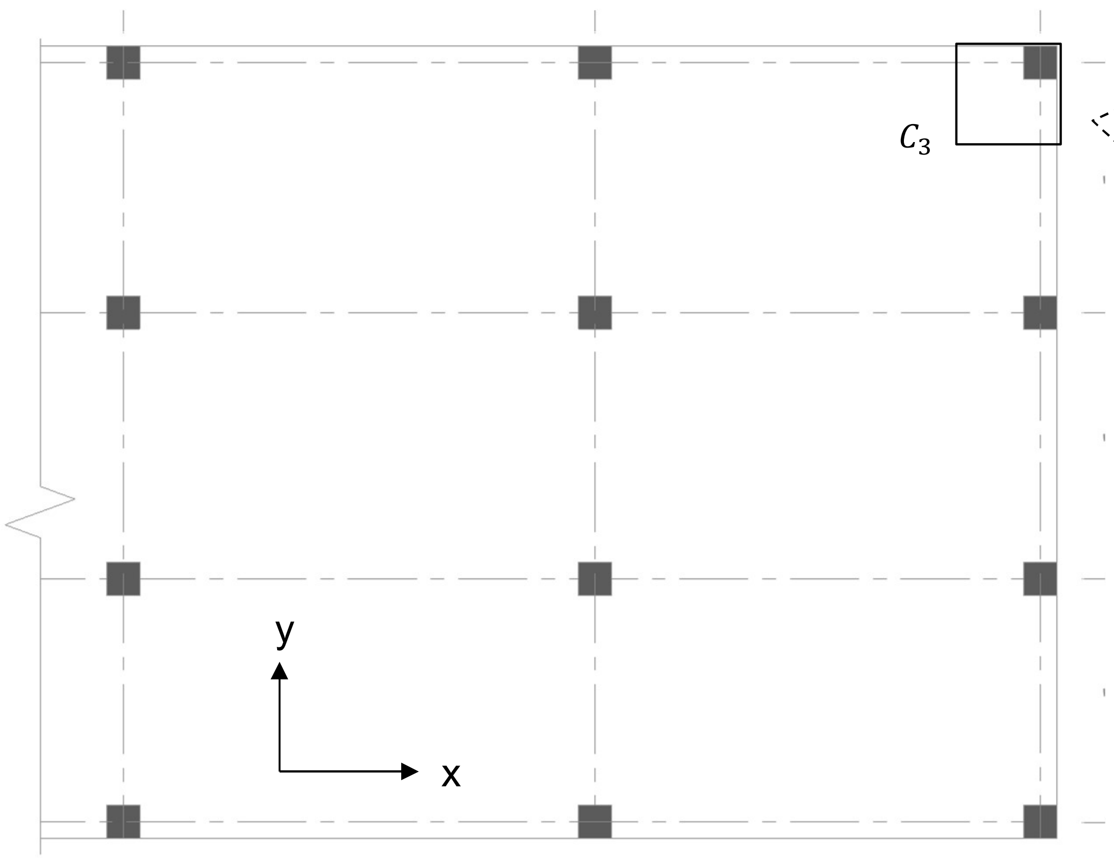


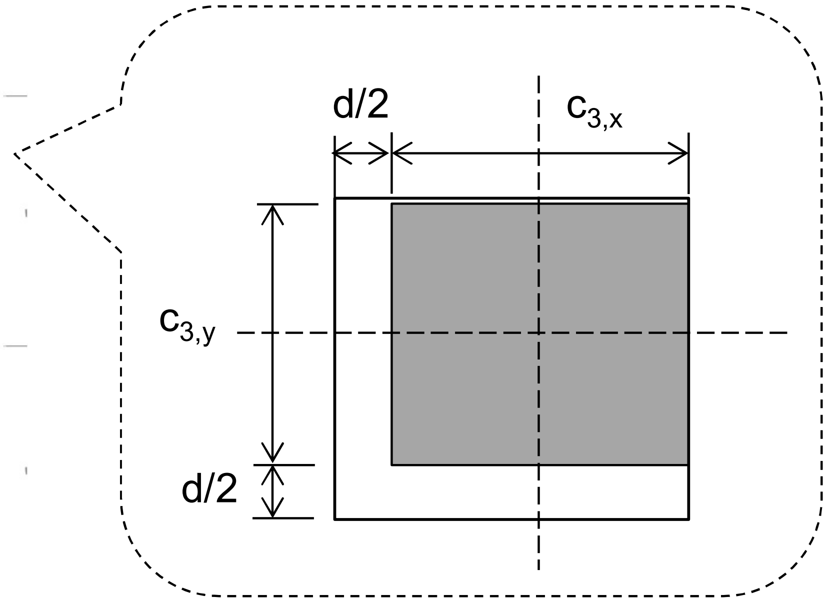
$$b_o = 2\left(C_{2,x} + \frac{d}{2}\right) + \left(C_{2,y} + d\right)$$

General

❑ Calculation of Critical Perimeter b_o

❖ Corner Columns





$$b_o = \left(C_{3,x} + \frac{d}{2}\right) + \left(C_{3,y} + \frac{d}{2}\right)$$

General

❑ Punching Shear Demand V_u for Square Columns

- Punching shear demand for square columns can be determined using the following steps.

1. **Calculate Tributary Area**

$$A_t = l_1 \times l_2 - A_o$$

where;

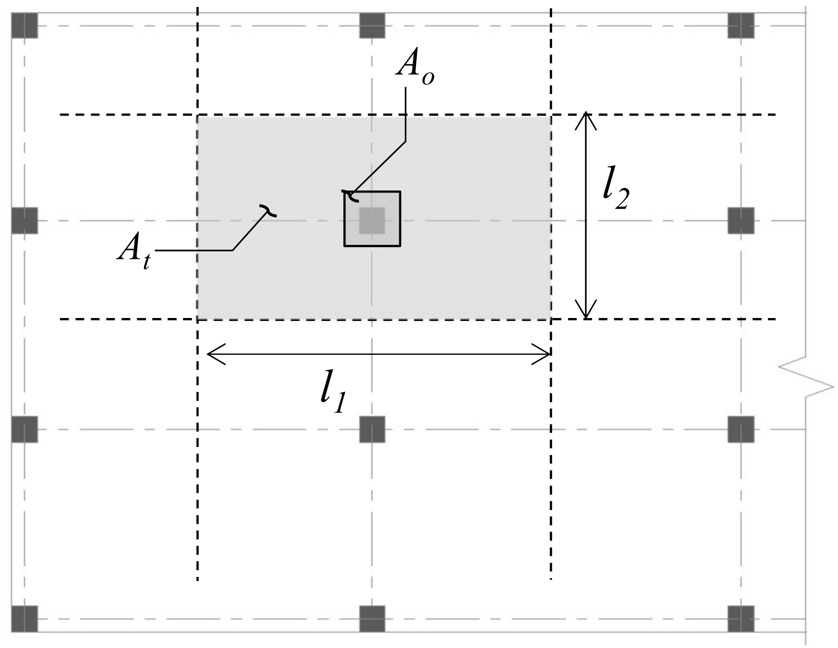
$$A_o = (c + d)^2 \rightarrow \text{for square columns}$$

$$A_t = l_1 l_2 - (c + d)^2/144$$

[l_1 & l_2 are in ft. and c & d are in inches]

2. **Calculate Demand V_u**

$$V_u = w_u A_t$$



General

Capacity of Slab in Punching Shear

The total punching shear capacity is given by

$$\emptyset V_n = \emptyset V_c + \emptyset V_s$$

$\emptyset V_c$ calculated from ACI Table 22.6.5.2 , is the least of:

$$\emptyset V_c = min\left(4, 2 + \frac{4}{\beta_c}, \frac{\alpha_s d}{b_o} + 2\right)\emptyset\lambda_s\sqrt{f_c'}b_o d$$

Where;

β_c = longer side of column/shorter side of column

α_s = 40 for interior column, 30 for edge column, 20 for corner columns

λ_s = size effect factor that can be determined as per 22.5.5.1.3

$$\lambda_s = \sqrt{\frac{2}{1 + d/10}} \leq 1$$

Design of Slabs for Punching Shear

❑ Various Design Options

- When $\Phi V_c \geq V_u$ ($\Phi = 0.75$) → nothing is required.
- When $\Phi V_c < V_u$ → it can be increased by either of the following ways.

  i. Increasing d, depth of slab: This can be done by increasing the slab depth as a whole or in the vicinity of column (Drop Panel)

  ii. Increasing b_o critical shear perimeter: This can be done by increasing column size as a whole or by increasing size of column head (Column capital)

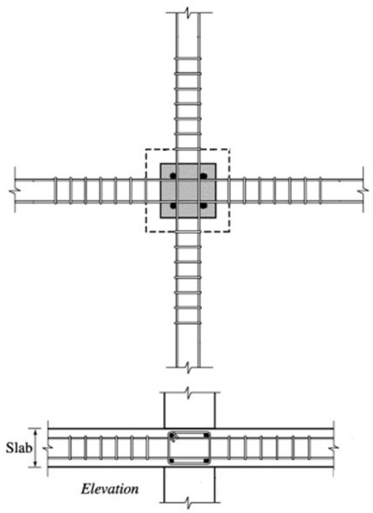
  iii. Increasing fc′ (high Strength Concrete)

- If it is not possible, provide shear reinforcement.

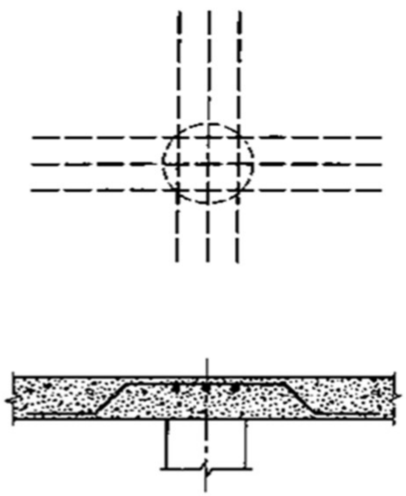
Design of Slabs for Punching Shear

❑ Various Design Options

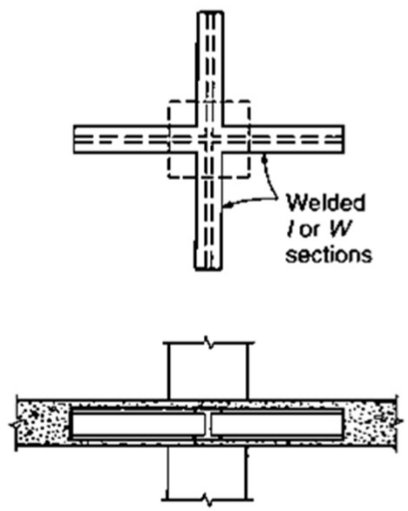
- The shear reinforcement can be provided by any of the following methods.



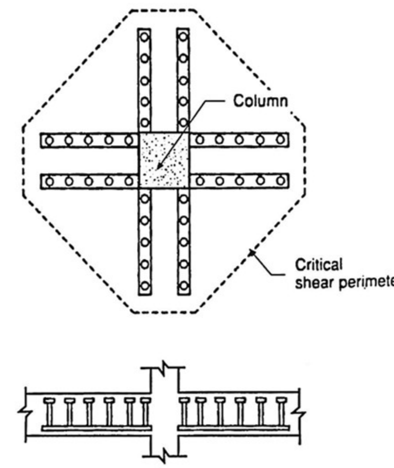
Integral Beams



Bent bars



Shear heads



Shear studs

Design of Slabs for Punching Shear

❑ Various Design Options

❖ Drop Panel

- A drop panel is the projection of slab provided at the vicinity of column to minimize punching shear demand.

Design of Slabs for Punching Shear

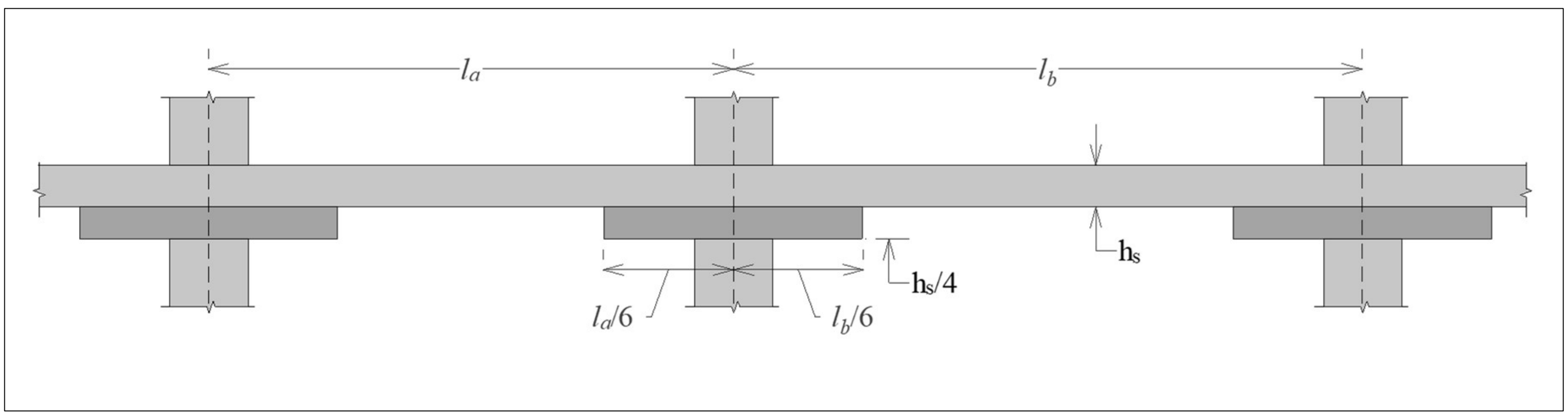
❑ Various Design Options

❖ Drop Panel

- ACI Section 8.2.4 requires that drop panel shall:

a) Project below the slab at least one-fourth of the adjacent slab thickness.

b) Extend in each direction from the centerline of support a distance not less than one-sixth the span length measured from center-to-center of supports in that direction.



Design of Slabs for Punching Shear

❑ Various Design Options

❖ Column Capital

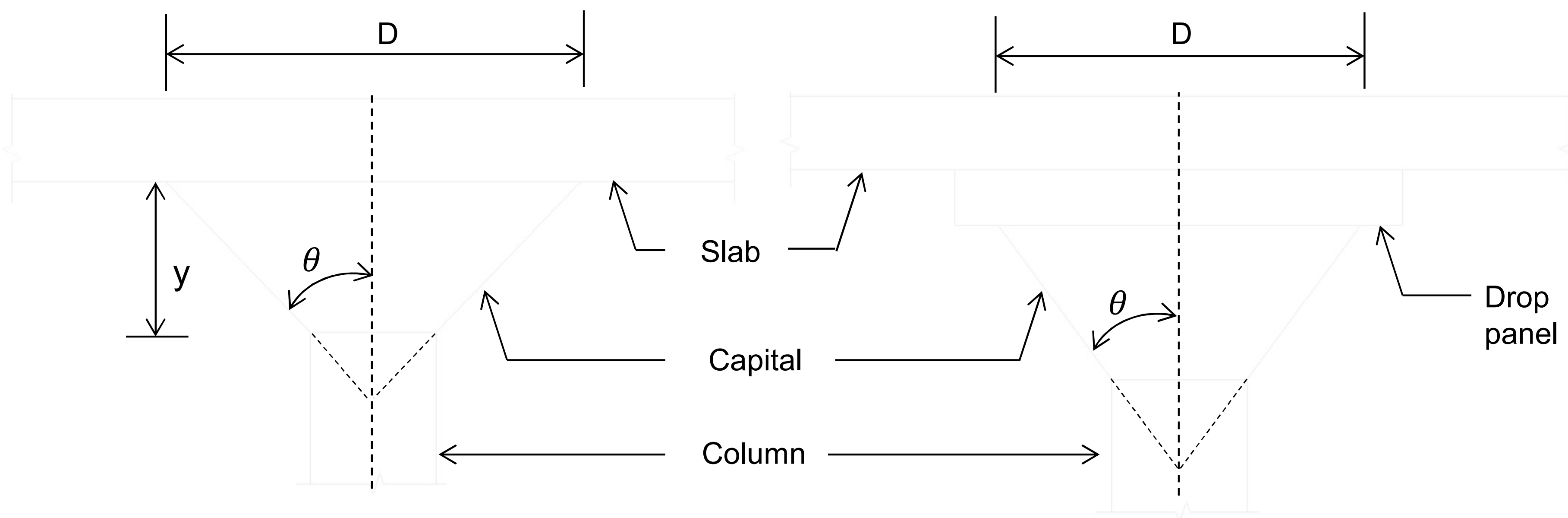
- Column capital is the enlargement of the top of a concrete column located directly below the slab or drop panel that is cast monolithically with the column (ACI 2.2).

Design of Slabs for Punching Shear

❑ Various Design Options

❖ Column Capital

- ACI Section 8.4.1.4 requires the column capital should be oriented no greater than 45º to the axis of the column ($\theta \leq 45^o$).



Design of Slabs for Punching Shear

❑ Various Design Options

❖ Column Capital

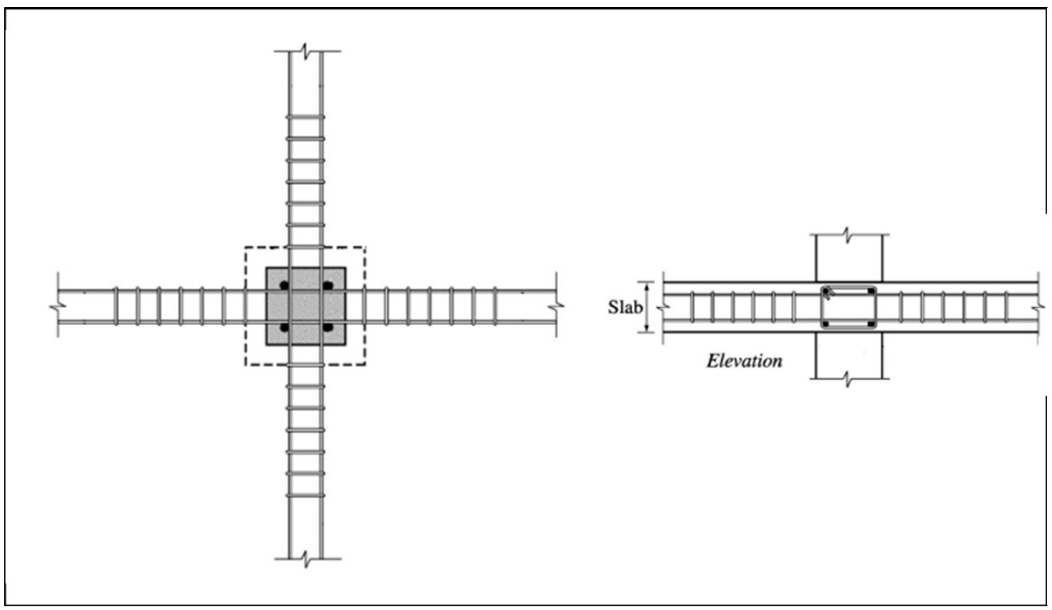
- ACI Section 26.5.7 (d) requires that the concrete be placed at the same time as the slab concrete. As a result, the floor forming becomes considerably more complicated and expensive.
- The increased perimeter can be computed by equating V_u to $\emptyset V_c$ and simplifying the resulting equation for b_o.

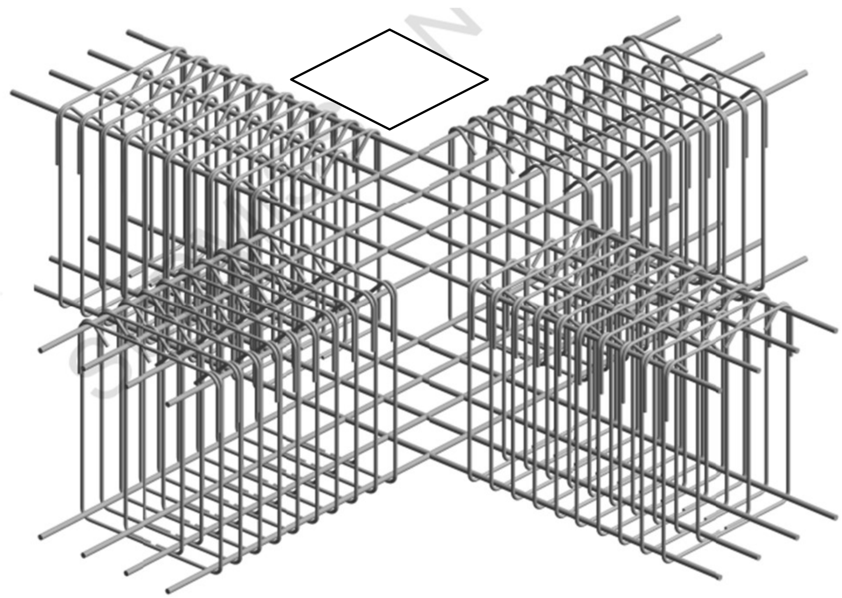
Design of Slabs for Punching Shear

❑ Various Design Options

❖ Integral Beams

- An integral beam is a concealed or flush beam formed within the slab thickness, designed to act compositely with the slab.



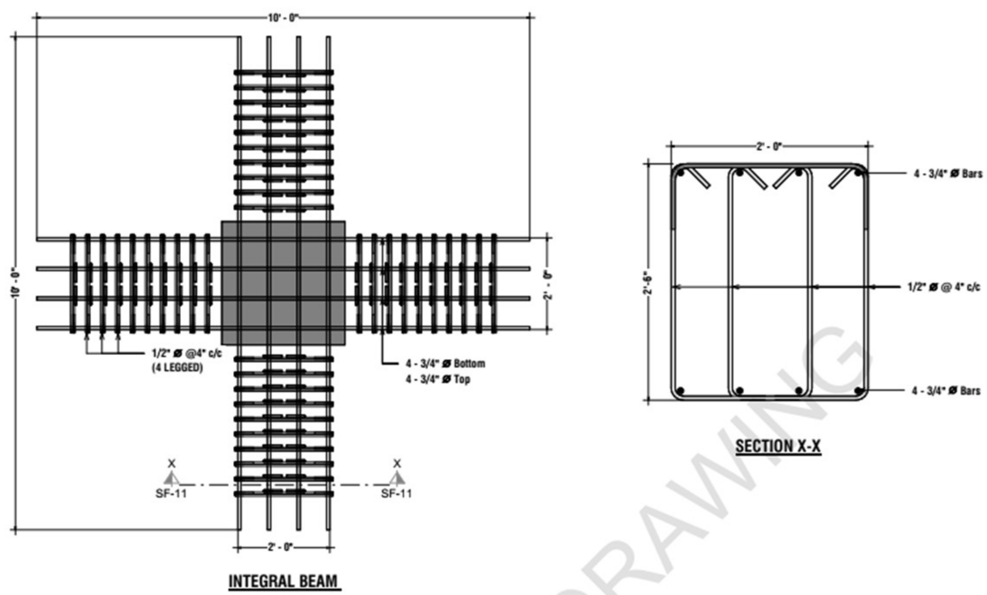


Design of Slabs for Punching Shear

❑ Various Design Options

❖ Integral Beams

- Integral Beams require the design of two main components:
  1. Vertical stirrups
  2. Horizontal bars radiating outward from column faces.



Design of Slabs for Punching Shear

Various Design Options

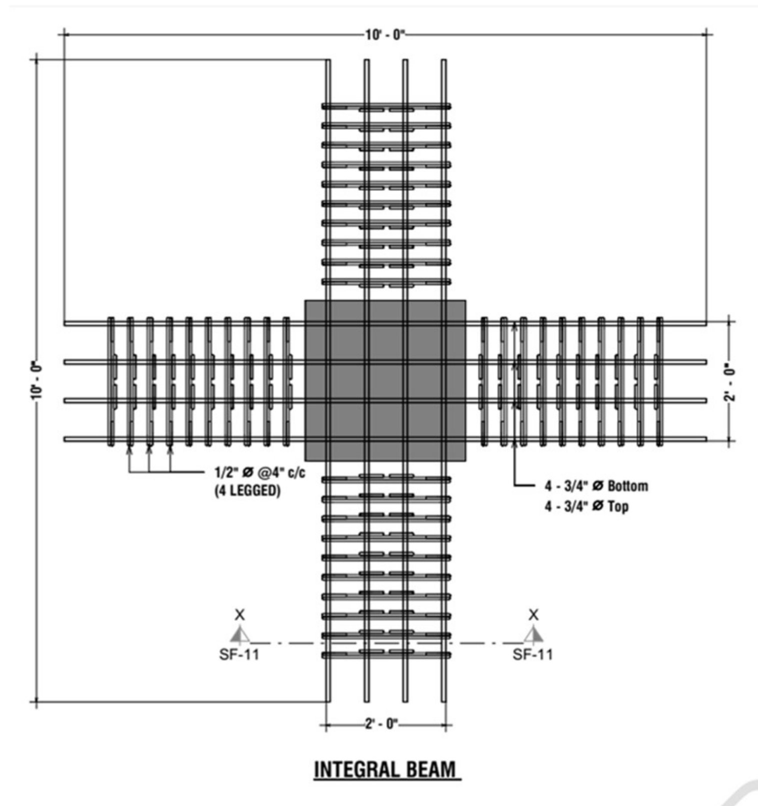
Integral Beams

1. Vertical Stirrups

- The required spacing of stirrups can be calculated as

$$s = \frac{\emptyset A_v f_y d}{V_u - \emptyset V_c}$$

- The calculated spacing shall not exceed d/2.
- First stirrup should be provided at s/2 from the face of support.



Design of Slabs for Punching Shear

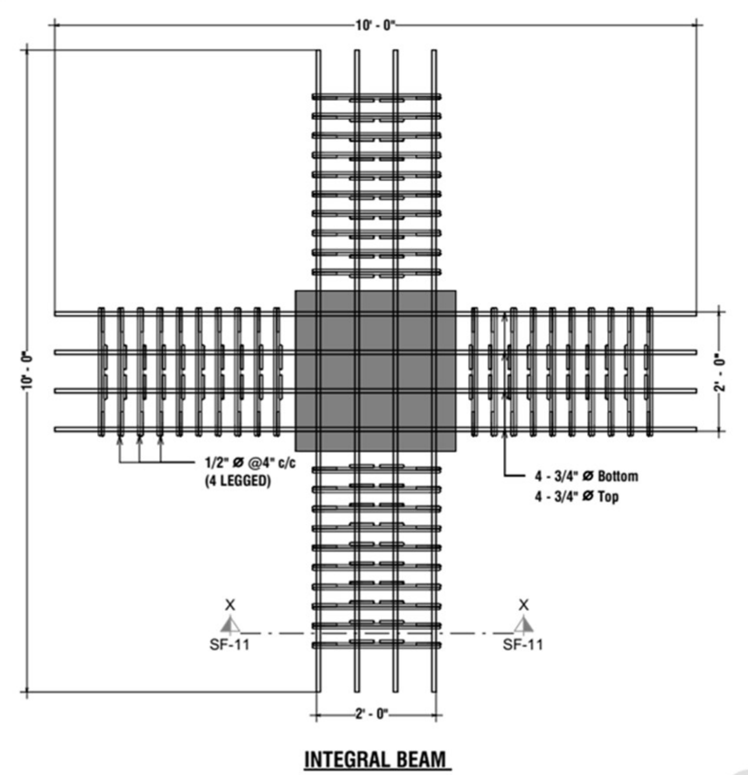
❑ Various Design Options

❖ Integral Beams

1. Vertical Stirrups

- As stirrups are placed vertically on all four sides, resulting in a total stirrup area that equals four times the area of each individual two-legged stirrup, the area of shear reinforcement A_v is given as:

$$A_v = 4(2A_b) = 8A_b$$



Design of Slabs for Punching Shear

❑ Various Design Options

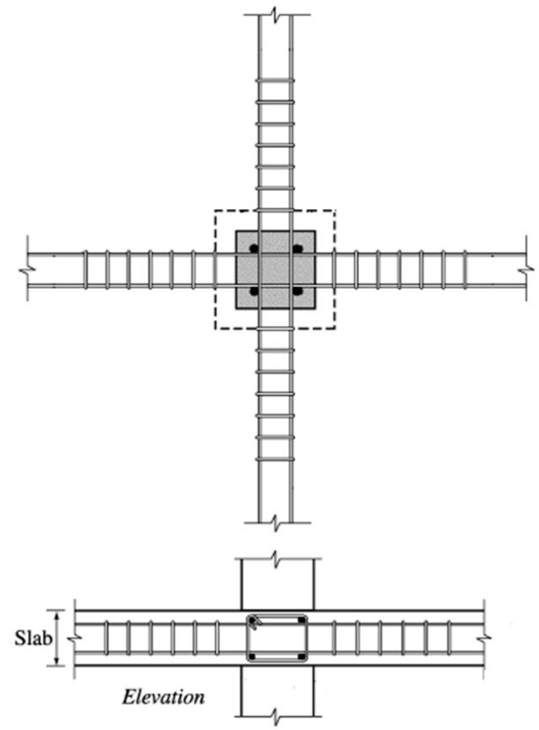
❖ Integral Beams

1. Vertical Stirrups

- Additionally, when integral beams are to be used:

i. the slab effective depth d to be at least 6 in., but not less than 16 times the diameter of the shear reinforcement (ACI 26.6.7.1), and

ii. Reduce ΦV_c by 2 for calculation of vertical stirrup spacing (ACI R22.6.6.1).



Design of Slabs for Punching Shear

❑ Various Design Options

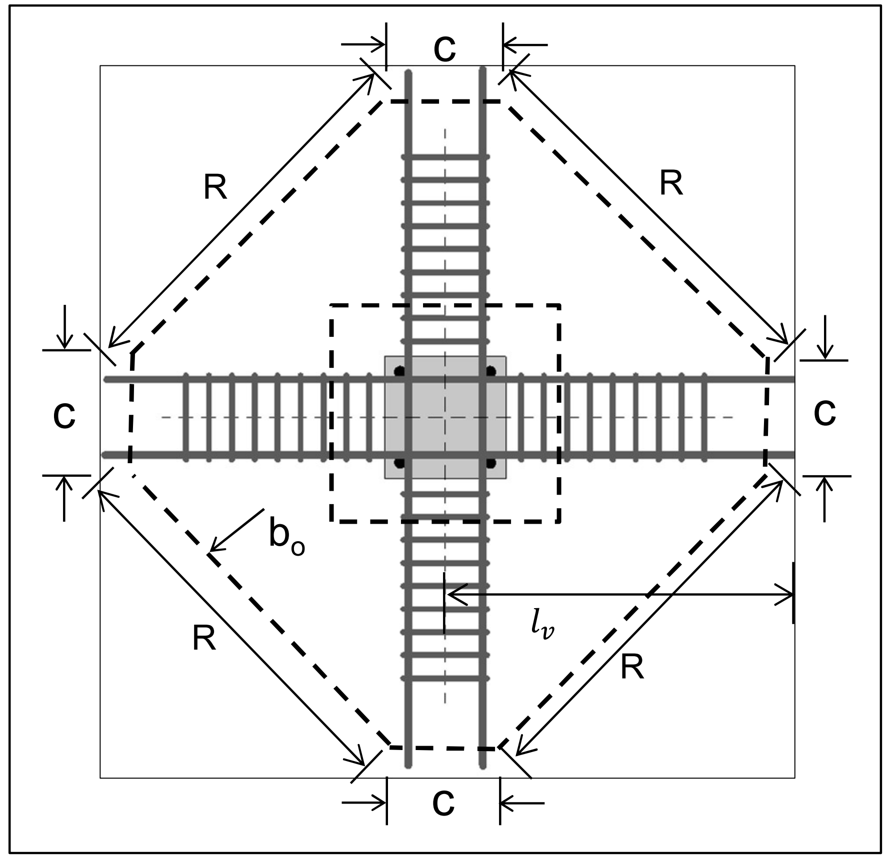
❖ Integral Beams

2. Horizontal Bars

- The length of horizontal bars l_v from the center of column can be determined using critical perimeter b_o which is shifted outward after providing integral beam.

- For square column of size "c", we have

$b_o = 4R + 4c$



Design of Slabs for Punching Shear

❑ Various Design Options

❖ Integral Beams

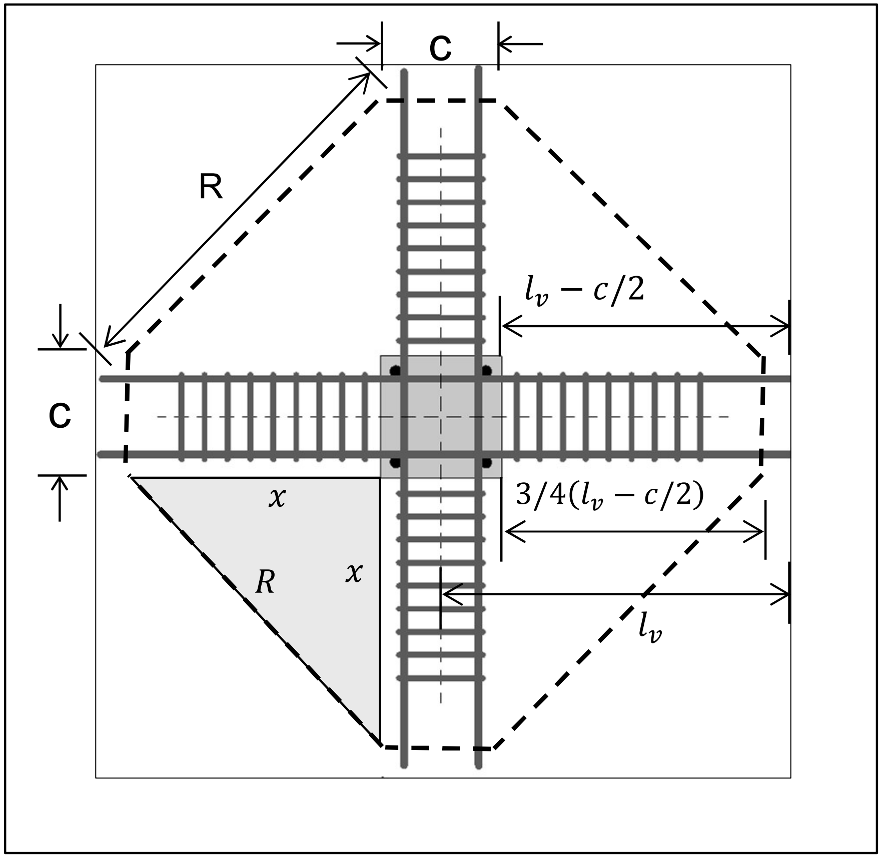
2. Horizontal Bars

From the shaded triangle

$$R = \sqrt{x^2 + x^2} = \sqrt{2}\,x$$

- According to the ACI Code, the projection from the face of column to the boundary of critical perimeter x is:

$$x = \frac{3}{4}\left(l_v - \frac{c}{2}\right)$$



Design of Slabs for Punching Shear

❑ Various Design Options

❖ Integral Beams

2. **Horizontal Bars**

$$R = \sqrt{2}\,x = \sqrt{2}\left[\frac{3}{4}\left(l_v - \frac{c}{2}\right)\right]$$

$$b_o = 4R + 4c = 4\left[\sqrt{2} \times \frac{3}{4}\left(l_v - \frac{c}{2}\right)\right] + 4c$$

$$b_o = 4.24 l_v + 1.88c$$

Solving for l_v, we get

$$l_v = \frac{b_o - 1.88c}{4.24}$$

Design of Slabs for Punching Shear

Various Design Options

Integral Beams

2. Horizontal Bars

b_o can be obtained by equating V_u to $\emptyset V_c$ and solving the resulting equation for b_o as given below.

$$l_v = \frac{\boxed{b_o} - 1.88c}{4.24}$$

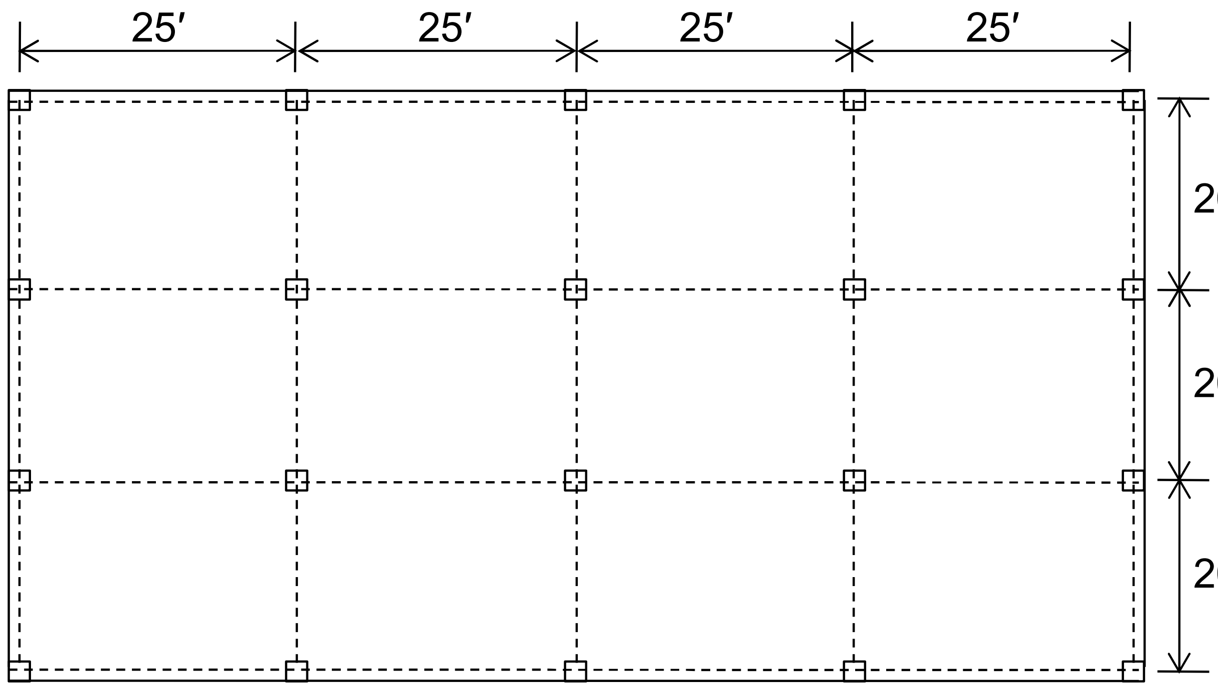
$$V_u = \emptyset V_c = min\left(4, 2 + \frac{4}{\beta_c}, \frac{\alpha_s d}{b_o} + 2\right)\emptyset\lambda_s\sqrt{f_c'}\,\boxed{b_o}\,d$$

This is the required equation that can be used for determining the length up to which the horizontal bars should be extended beyond the face of column.

Example 5.3

❑ Problem Statement

- A 10 in-thick flat plate shown below supports a uniformly distributed factored load (including self weight of plate) of 0.381 ksf. ***Design*** the plate for punching shear using various options. Take $f_c' = 4$ ksi and $f_y = 60$ ksi.



All columns are 14″ square

Example 5.3

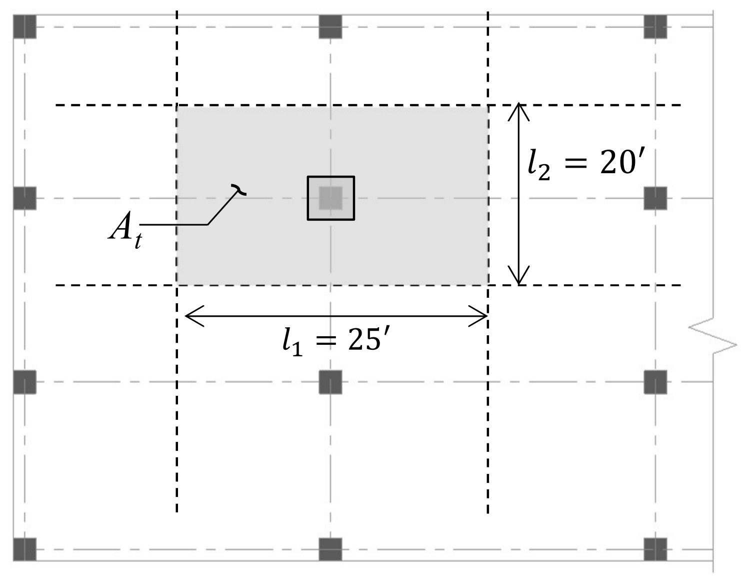
❑ Solution

➢ Step 1: Calculation of Punching Shear Demand V_u

The punching shear demand V_u can be calculated as follows

$$V_u = w_u \times A_t$$

$$A_t = l_1 l_2 - \frac{(c+d)^2}{144}$$



Assuming #6 bar ;

$$d = h_s - 0.75 - 0.75 = 8.5''$$

$$A_t = 25 \times 20 - \frac{(14+8.5)^2}{144} = 496.48 \text{ ft}^2$$

$$V_u = w_u \times A_t = 0.381 \times 496.48 = \mathbf{189.16\ kip}$$

Example 5.3

- **Solution**

- **Step 2: Calculation of Punching Shear Capacity**

The design punching shear capacity of concrete $\emptyset V_c$ can be calculated as follows

$$V_c = min\left(4, 2 + \frac{4}{\beta_c}, \frac{\alpha_s d}{b_o} + 2\right)\emptyset\lambda_s\sqrt{f_c'}b_o d$$

Where;

$$b_o = 4(c + d) = 4(14 + 8.5) = 90''$$

$$\beta_c = \frac{longer\ side\ of\ column}{shorter\ side\ of\ column} = \frac{14}{14} = 1$$

$\alpha_s = 40$ for interior column

Example 5.3

❑ Solution

➢ Step 2: Calculation of Punching Shear Capacity

$$\lambda_s = \sqrt{\frac{2}{1 + d/10}} = \sqrt{\frac{2}{1 + 8.5/10}} = 1.04 > 1 \rightarrow \text{Take } \lambda_s = 1$$

Now, substituting all values, we get

$$V_c = min\left(4, 2 + \frac{4}{1}, \frac{40 \times 8.5}{90} + 2\right) \times 0.75 \times 1\sqrt{4000} \times 90 \times 8.5$$

$$V_c = min(4, 6, 5.8) \times 36287.136 = 4 \times 36287.136 = 145148.54\ lb$$

$$\boldsymbol{\emptyset V_c = 145.15\ \text{kip}}$$

$\emptyset V_c = 145.15\ \text{kip} < \text{V}_\text{u} = 189.16\ \text{kip} \rightarrow$ Shear design is required

Example 5.3

❑ Solution

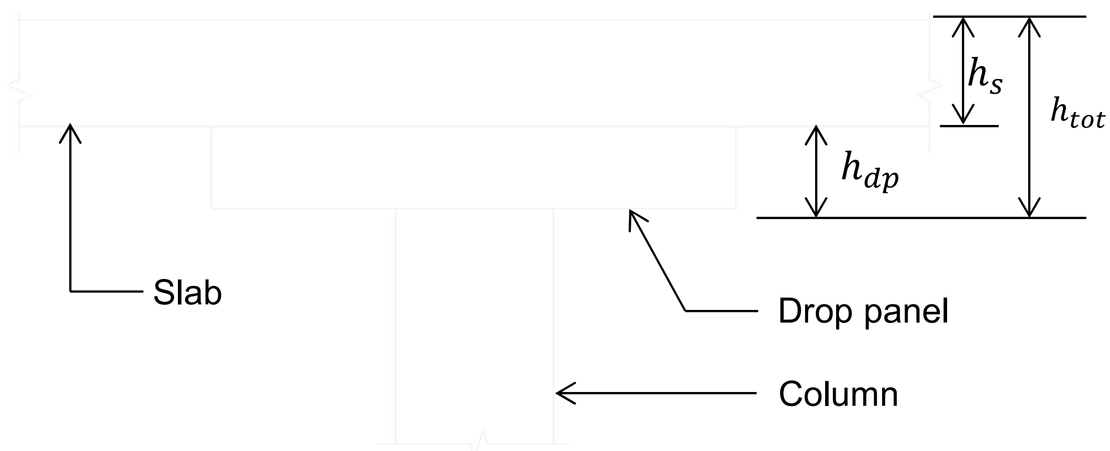
➢ Step 3: Design for Shear

❖ Option 1: Drop Panels

The thickness of drop panel can be computed by equating V_u to $\varnothing V_c$ and simplifying the resulting equation for "d" to calculate total depth.

$$h_{tot} = h_s + h_{dp}$$

$$h_{dp} = h_{tot} - h_s$$



Example 5.3

Solution

Step 3: Design for Shear

Option 1: Drop Panels

Setting $\phi V_c = V_u \Rightarrow 4\phi\sqrt{f_c'}\, b_o d \Rightarrow 4\phi\sqrt{f_c'}\, 4(c+d)d = 189.16$

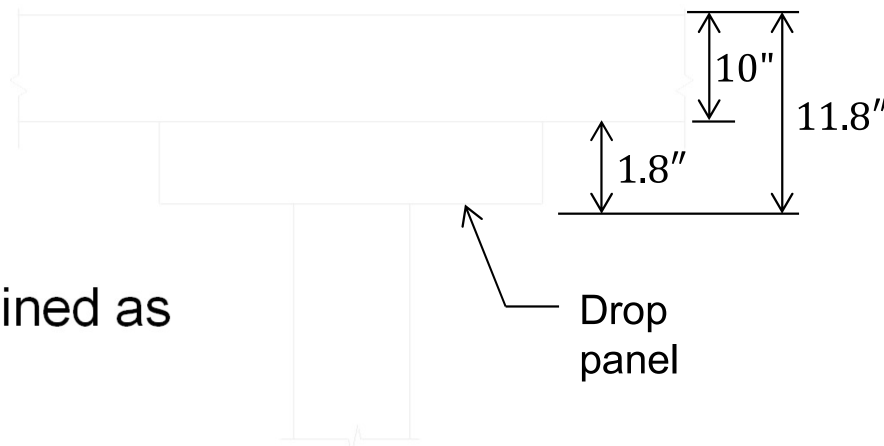
$$4 \times 0.75 \times \sqrt{4000} \times 4(14 + d) \times d = 189.16 \times 1000 \Rightarrow d = 10.27''$$

Assuming #6 bar;

$$h_{total} = 10.27 + (0.75 + 6/8) = 11.8''$$

Now, drop panel depth can be determined as

$$h_{dp} = h_{tot} - h_s = 11.8 - 10.0 = 1.8''$$



Example 5.3

❑ Solution

➢ Step 3: Design for Shear

❖ Option 1: Drop Panels

Minimum thickness of drop panel is given by

$$h_{dp,min} = \frac{h_s}{4} = \frac{10}{4} = 2.5''$$

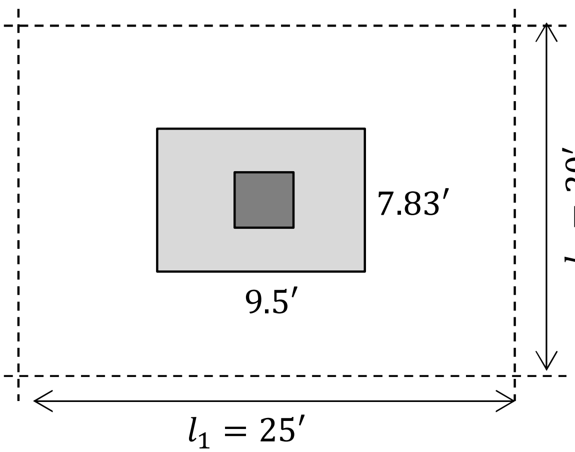
→ Calculated depth of 1.8″ is Not OK.

Increase thickness to **2.5 in**

Width of drop panel in each direction is

$$B_1 = l_1/3 + c = 25/3 + 14/12 = 9.5'$$

$$B_2 = l_2/3 + c = 20/3 + 14/12 = 7.83'$$



Hence, provide 2.5″ thick 9′- 6″ x 7′- 8″ drop panels.

Example 5.3

❑ Solution

➢ Step 3: Design for Shear

❖ Option 2: Column Capitals

Setting $\emptyset V_c = V_u \;\Rightarrow\; 4\emptyset\sqrt{f_c'}\, b_o d = 189.16$ and calculate b_o

$$4 \times 0.75 \times \sqrt{4000} \times b_o \times 8.5 = 189.16 \times 1000 \;\Rightarrow b_o = 117.29''$$

As $b_o = 4(c + d) \Rightarrow \; 117.29 = 4(c + 8.5)$ which gives

$$c = \frac{117.29}{4} - 8.5 = 20.82'' \approx 21''$$

Take width of capital, D = 21″

Example 5.3

❑ Solution

➢ Step 3: Design for Shear

❖ Option 2: Column Capitals

From figure,

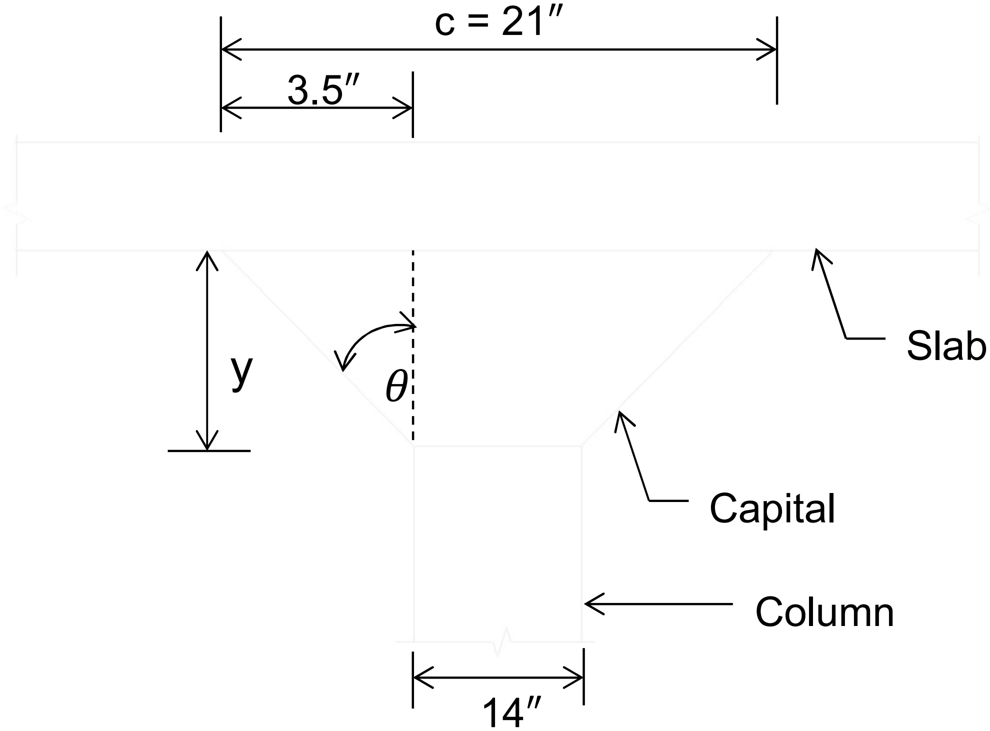
$$tan\theta = \frac{3.5}{y}$$

According to ACI, $\theta \leq 45^o$

$y = 6.0'' \quad for\ \theta = 30^o$

$y = 9.6'' \quad for\ \theta = 20^o$

Provide y = 6.0 in



Example 5.3

❑ Solution

➢ Step 3: Design for Shear

❖ Option 3: Integral Beams

1. Vertical Stirrups

The required spacing of stirrups can be calculated as

$$s = \frac{\emptyset A_v f_y d}{V_u - \emptyset V_c}$$

Using 2-legged #4 bars on all four sides, $A_v = 4(2 \times 0.20) = 1.6 \text{ in}^2$

$$\emptyset V_c = \frac{145.16}{2} = 72.58 \text{ kip}$$

(Shear capacity of concrete shall be reduced by 2 as per ACI R22.6.6.1)

Example 5.3

❑ Solution

➢ Step 3: Design for Shear

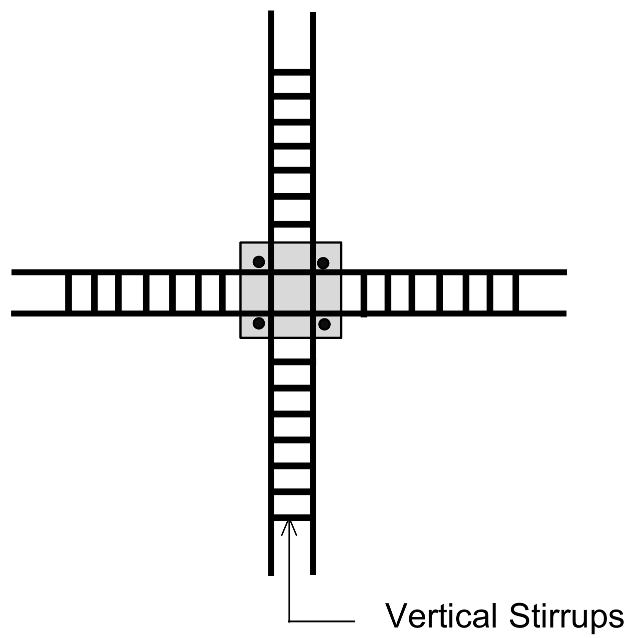
❖ Option 3: Integral Beams

1. Vertical Stirrups

Substituting the values, we get

$$s = \frac{\emptyset A_v f_y d}{V_u - \emptyset V_c} = \frac{0.75 \times 1.6 \times 60 \times 8.5}{189.16 - 72.58} = 5.25''$$

$$s_{max} = \frac{d}{2} = \frac{8.5}{2} = 4.25'' < 5.25'' \rightarrow \text{Not OK}$$



Finally provide #4 @ 4″ c/c on all 4 sides.

First stirrup is provided at s/2 = 2 ″ from the face of support.

Example 5.3

- **Solution**

- **Step 3: Design for Shear**

  - **Option 3: Integral Beams**

2. Horizontal Bars

The distance of horizontal bars from center of column to each direction is given by

$$l_v = \frac{b_o - 1.88c}{4.24}$$

Setting $\emptyset V_c = V_u$ and solving for b_o

$$4 \times 0.75 \times \sqrt{4000} \times b_o \times 8.5 = 189.16 \times 1000 \Rightarrow b_o = 117.29''$$

Example 5.3

❑ Solution

➢ Step 3: Design for Shear

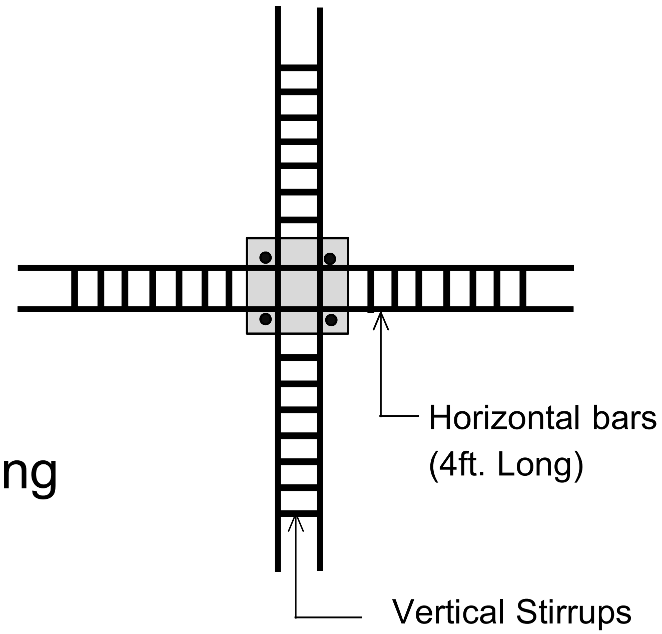
❖ Option 3: Integral Beams

2. Horizontal Bars

$$l_v = \frac{b_o - 1.88c}{4.24} = \frac{117.29 - 1.88(14)}{4.24}$$

$$l_v = 21.5'' \approx 24''$$

Hence, provide 2 - #5 bars each measuring 4 feet in length for both directions.

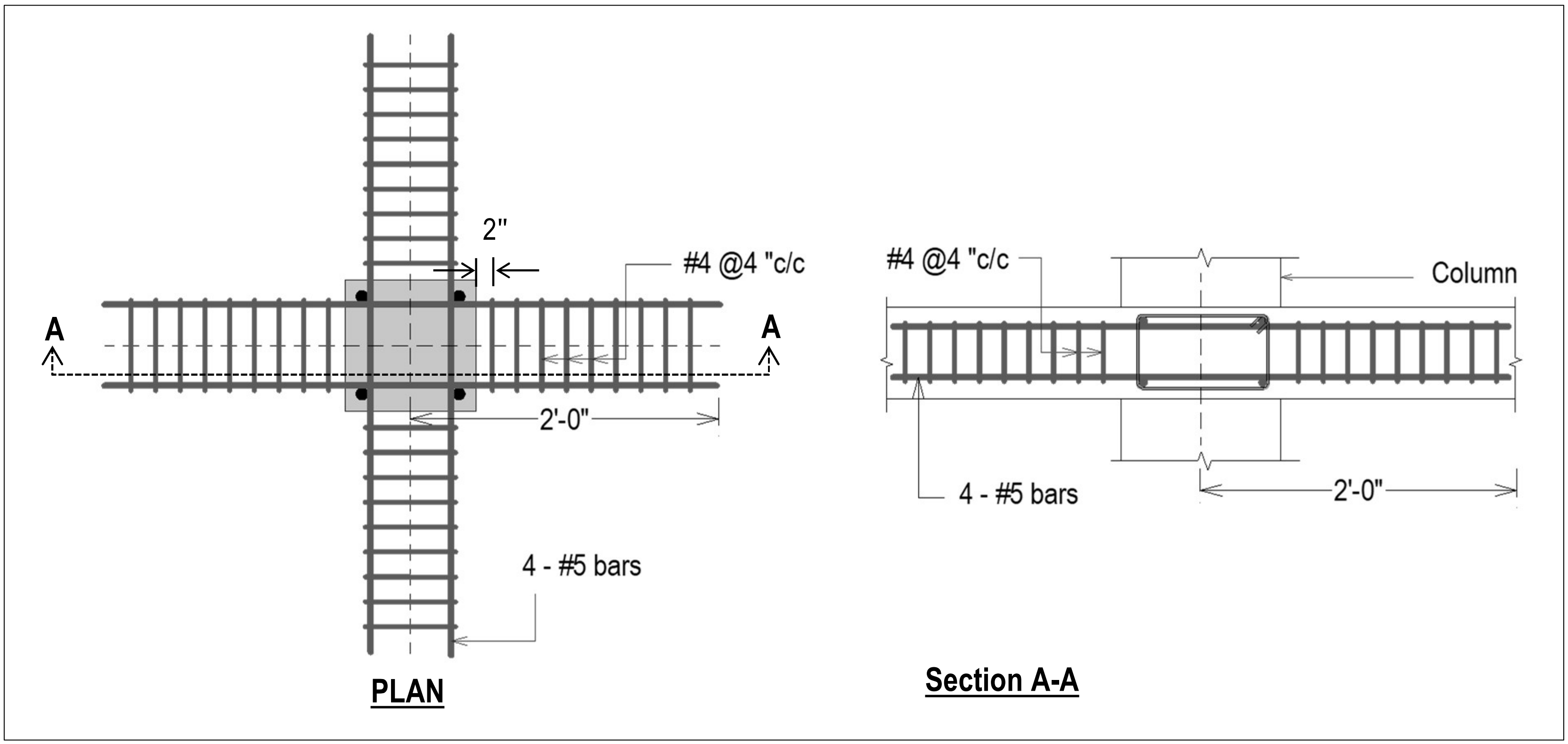


Example 5.4

Solution

Step 3: Design for Shear

Option 3: Integral Beams



Summary

- For a particular span of frame, find static moment ($M_o = w_u l_2 l_n^2/8$).
- Find longitudinal distribution of static moment:
  - Exterior span ($M_{ext -} = 0.26M_o$; $M_{ext +} = 0.52M_o$; $M_{int -} = 0.70M_o$)
  - Interior span ($M_{int -} = 0.65M_o$; $M_{int +} = 0.35M_o$)
- Find lateral Distribution of each longitudinal moment (column strip):
  - 100 % of $M_{ext -}$ goes to column strip
  - 60 % of $M_{ext +}$ and M_{int+} goes to column strip
  - 75 % of $M_{int -}$ goes to column strip
- Max spacing requirements ($s_{max} = 2h_f$ or 18 whichever is less)

Summary

- Find Critical Perimeter, b_o:

  $b_o = 4(c + d)$

- Area Contributing to Load (Excluding Area of b_o), A_t :

  $A_t = (l_1 \times l_2) - (c + d)^2 / 144$; l_1 & l_2 are in ft. units and c & d are in inches.

- Punching Shear Demand:

  $V_u = w_u \times A_t$

- Find ΦV_c

$$\emptyset V_c = min\left(4, 2 + \frac{4}{\beta_c}, \frac{\alpha_s d}{b_o} + 2\right)\emptyset \lambda_s \sqrt{f_c'} b_o d$$

References

- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-19)