



Lecture 06

Design of Reinforced Concrete Retaining Walls

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- General
- RC cantilever retaining wall
- Stability evaluation of retaining wall
- Earth pressure and Soil parameters
- Design of RC cantilever retaining wall
- Example 6.1
- Practical examples
- References



Learning Outcomes

□ **At the end of this lecture, students will be able to;**

- **Identify** different types of retaining walls
- **Explain** failure mechanism of retaining walls
- **Analyze** and **Design** RC Cantilever Wall



General

□ Introduction

- A retaining wall is a structure that holds or retains soil masses of earth or other loose material behind it.
- Used in the construction of railways, highways, bridges, canals, basement walls in buildings, walls of underground reservoirs, swimming pools etc.





General

□ Introduction





General

□ Introduction





General

□ Introduction





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□ Introduction





General

❑ Introduction





General

□ Types of Retaining Walls

- Retaining walls are generally classified based on the method of attaining stability against the lateral load imposed by the retained earth.



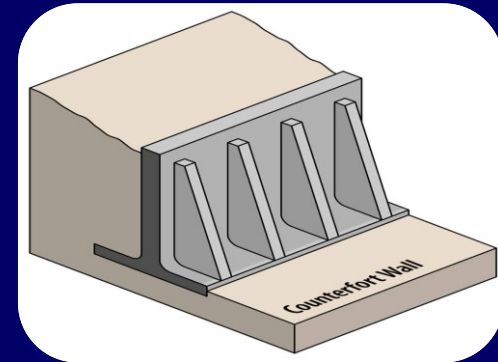
Gravity Wall

Uses its own weight and that of the retained soil for stability and are either lightly reinforced or contain no reinforcement.



Cantilever Wall

Weight of the soil on the heel of the footing provides the primary contribution to overall stability.



Counterfort Wall

Similar to Cantilever wall but the stem is stiffened by buttresses, to achieve more strength



General

❑ Selection of Suitable Type of Retaining Wall

- Gravity walls are economical only for relatively low walls; possibly up to about 10 ft.
- Cantilever Retaining Walls are generally economical up to a height of approximately 20ft.
- For greater heights or for the conditions where the backfill pressure is unusually high, Counterfort or Buttress Retaining Walls are recommended.
- This lecture is only focused on the structural analysis and design of Cantilever retaining wall.



RC Cantilever Retaining Wall

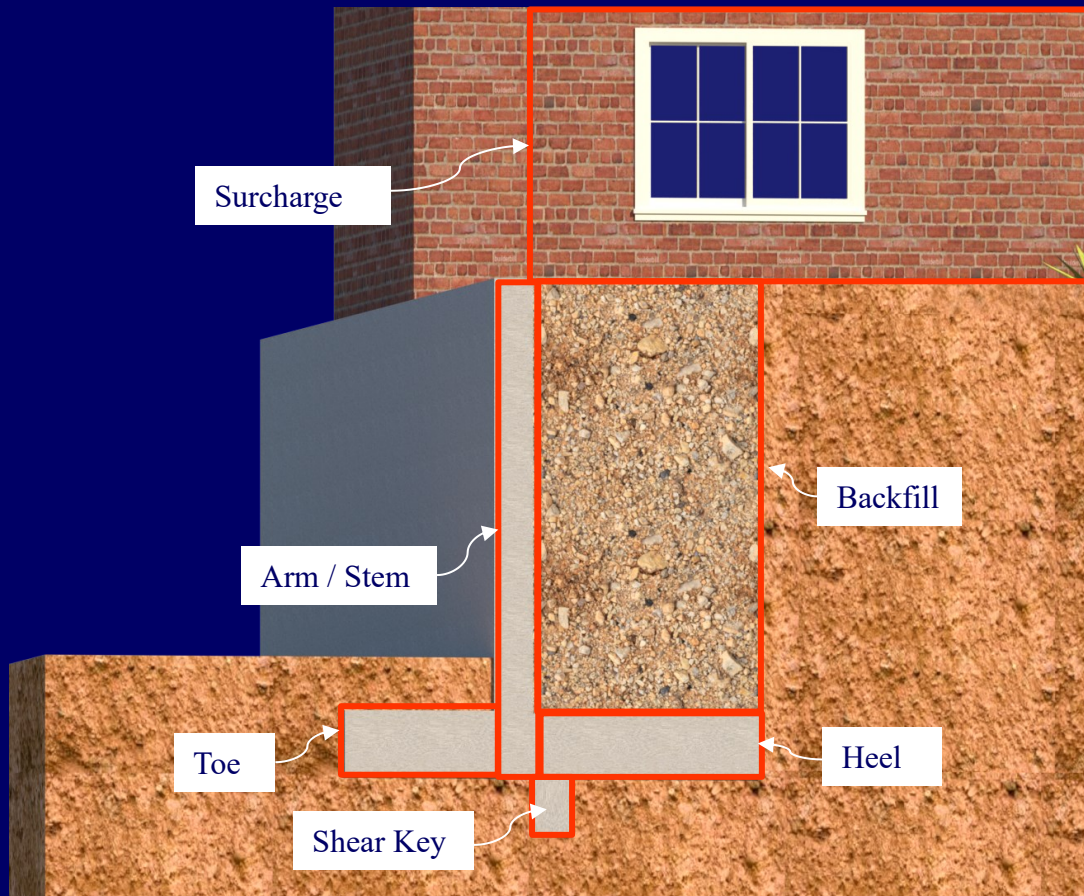
□ Different Terms Related to Retaining Wall

- A typical Cantilever retaining wall is an assembly of the following components
 1. **Stem:** Stem is a Vertical arm that provides horizontal resistance against the overturning force of the soil
 2. **Base:** It is a horizontal footing that is typically divided into two parts, the Toe and the Heel.
 3. **Key:** A key is basically a small vertical element constructed below the footing (base) to increase sliding resistance.
- Illustration of these components have been shown on the next slide.



RC Cantilever Retaining Wall

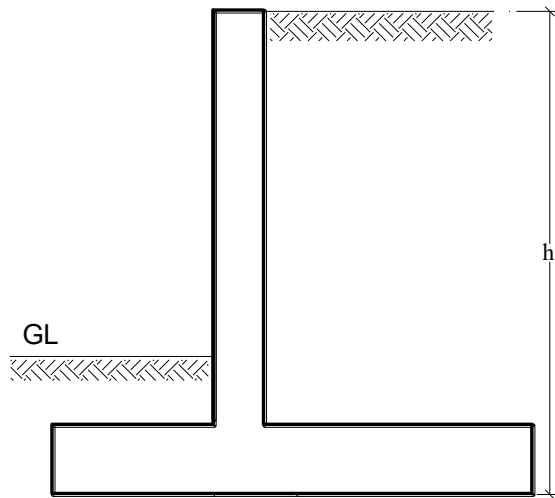
❑ Different Terms Related to Retaining Wall



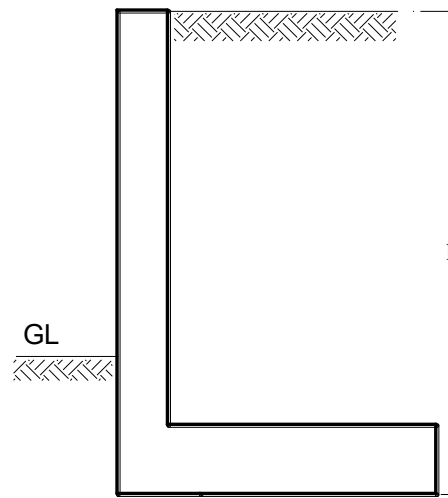


RC Cantilever Retaining Wall

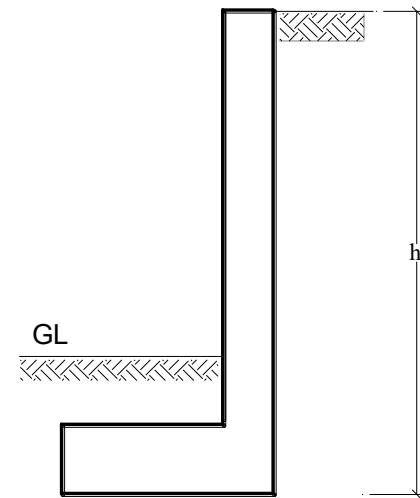
□ Types of cantilever Retaining Walls



(a) Typical



(b) with heel only



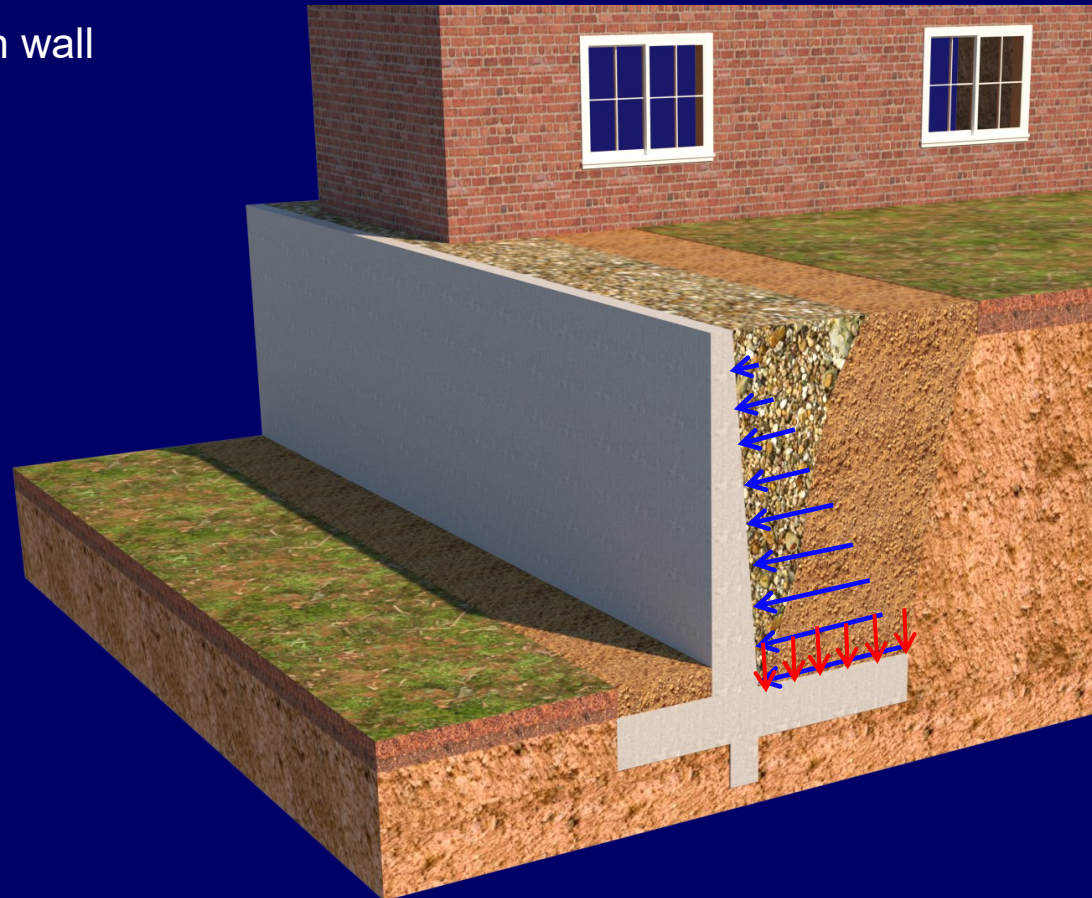
(c) with toe only



RC Cantilever Retaining Wall

❑ Behavior of Cantilever Retaining Wall

- Applied loading on wall

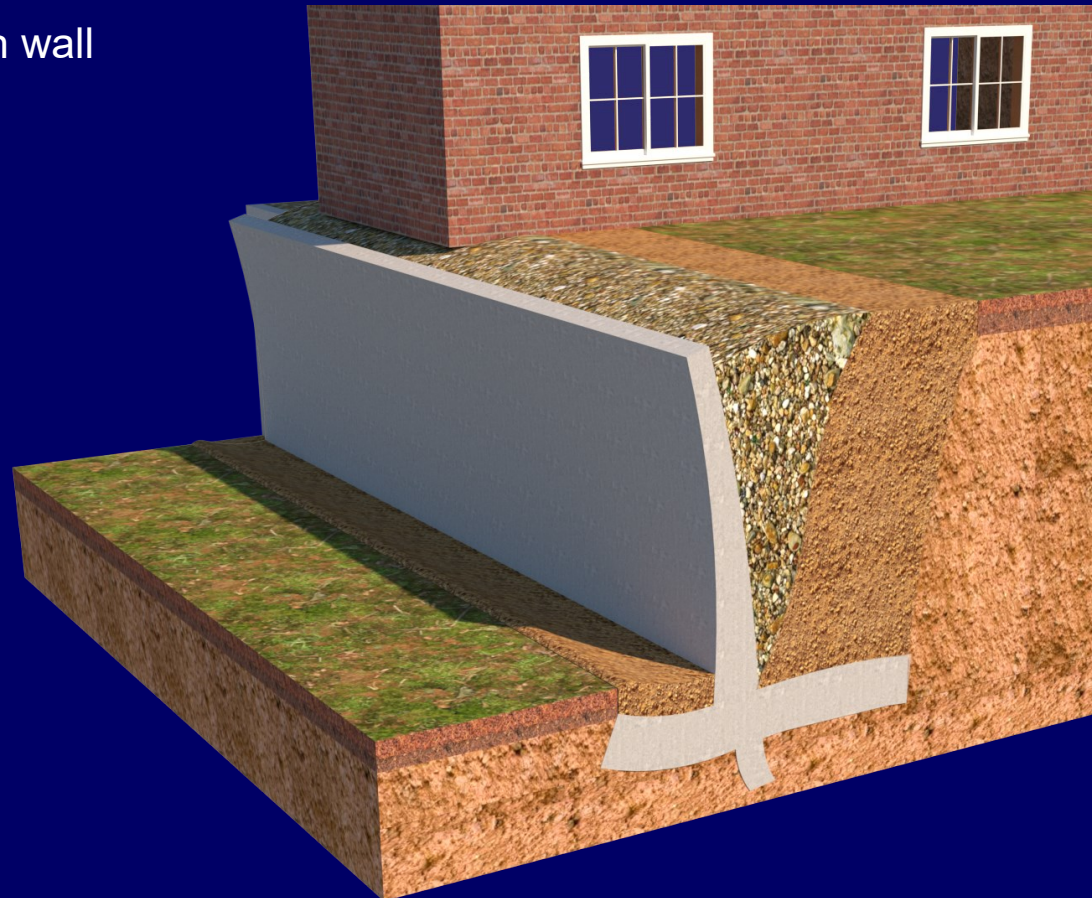




RC Cantilever Retaining Wall

❑ Behavior of Cantilever Retaining Wall

- Applied loading on wall
- Deflected shape

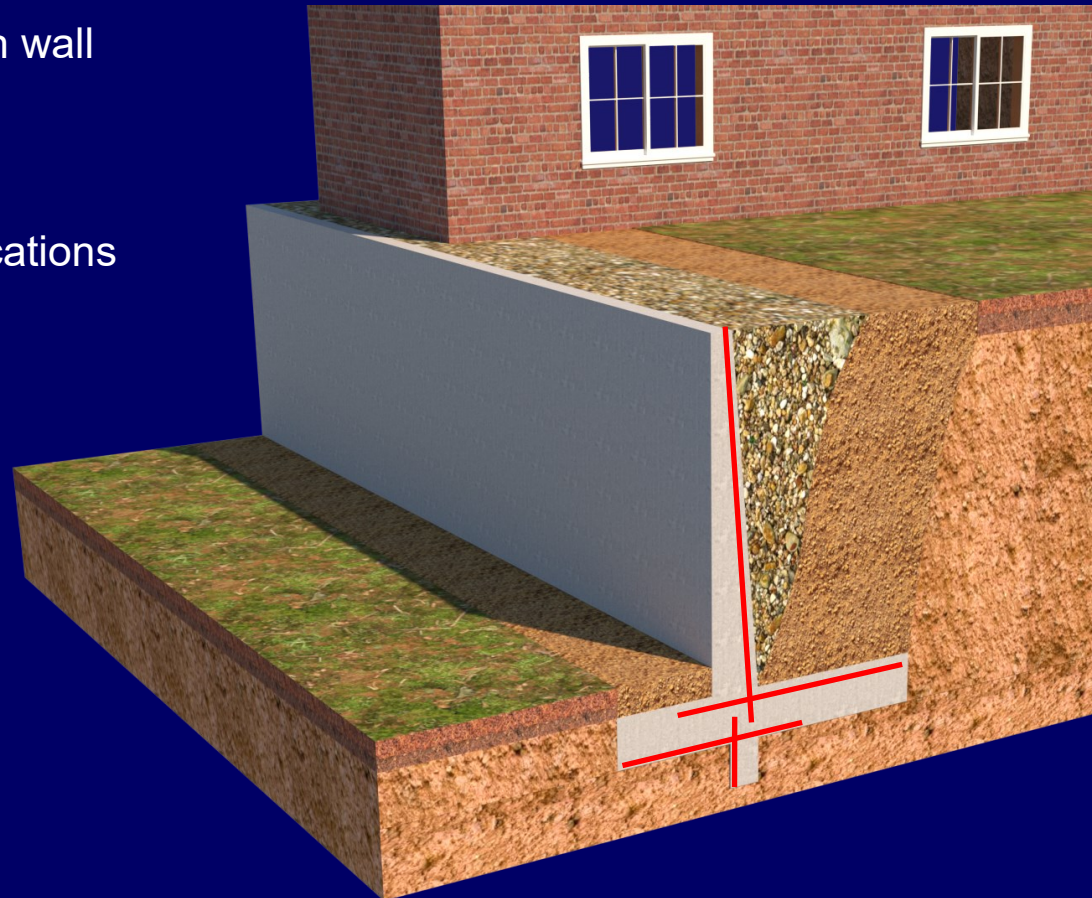




RC Cantilever Retaining Wall

❑ Behavior of Cantilever Retaining Wall

- Applied loading on wall
- Deflected shape
- Reinforcement locations





RC Cantilever Retaining Wall

❑ Various Failure Modes

- RC retaining wall may fail in two different ways:

1. Stability Failure

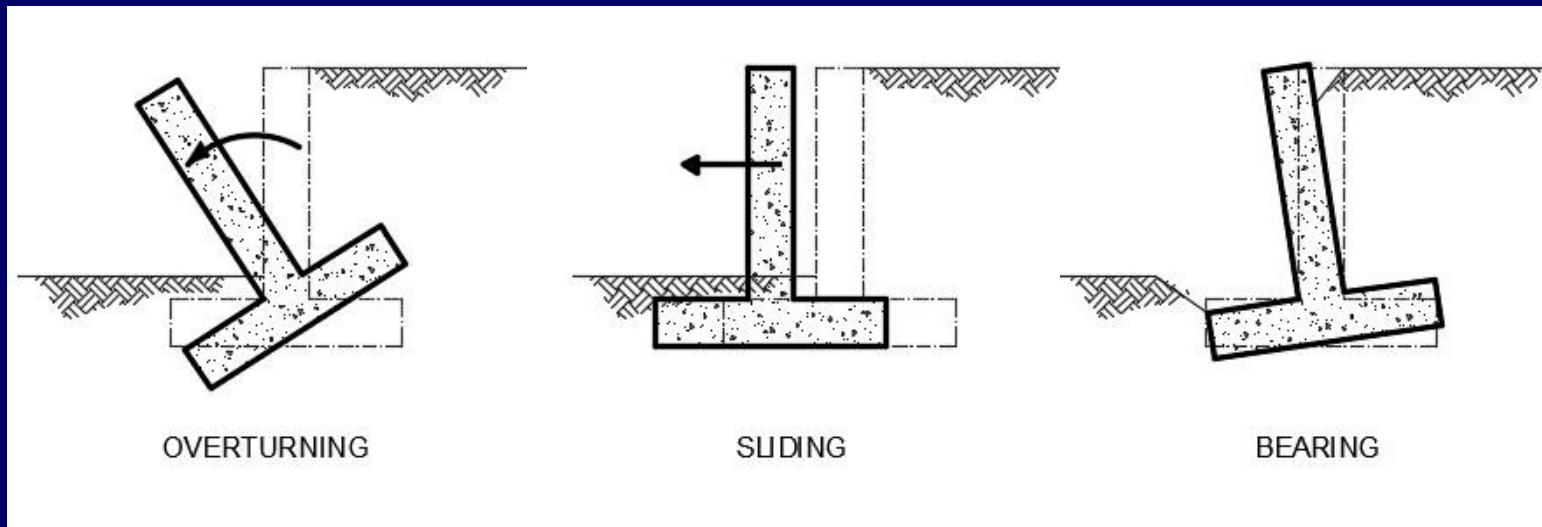
- The wall as a whole may be bodily displaced by the earth pressure, without breaking up internally. This can be
 - Overturning**
 - Sliding**
 - Bearing / Settlement**



RC Cantilever Retaining Wall

❑ Various Failure Modes

1. Stability Failure



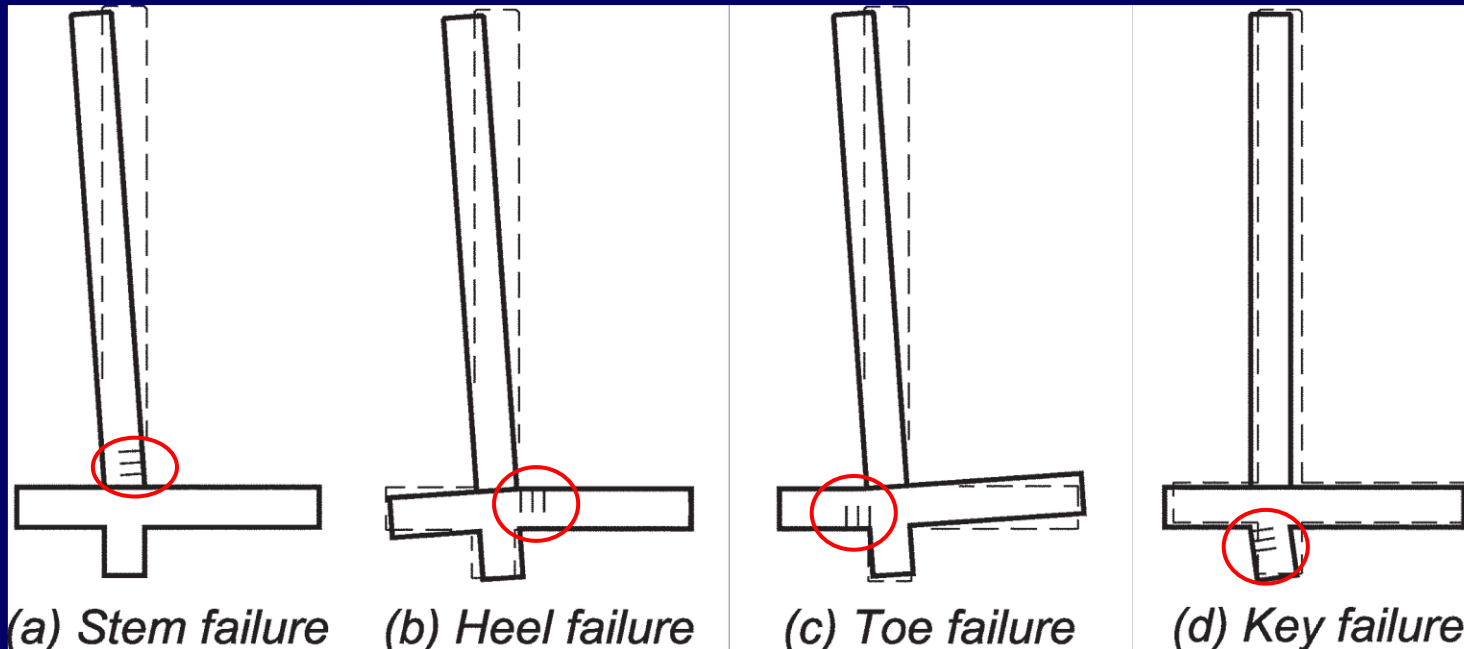


RC Cantilever Retaining Wall

❑ Various Failure Modes

2. Body Failure

- The individual structural parts (stem, toe, heel) of the wall may not be strong enough to resist the acting forces.





RC Cantilever Retaining Wall

❑ Various Failure Modes



Animation explaining behavior of Cantilever retaining wall under lateral pressure



Earth Pressure and Soil Parameters

□ Various Conditions of Loading

1. Horizontal surface of fill at the top

- From Rankine's formula, we have

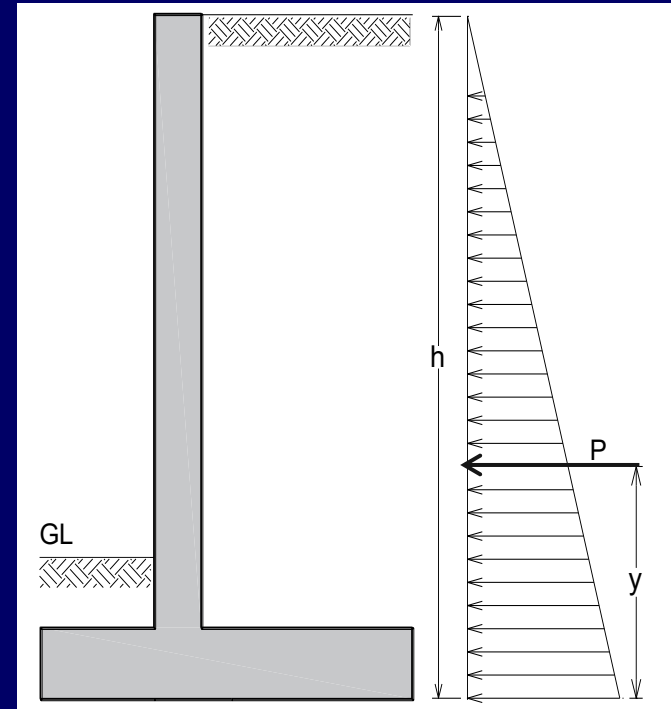
$$p_a = K_a \gamma_s h \quad \text{Where, } K_a = \frac{1 - \sin(\phi)}{1 + \sin(\phi)}$$

The resultant of this pressure is:

$$P = \frac{1}{2} (p_a)(h) = \frac{1}{2} K_a \gamma_s h^2$$

and the location of "P" from the base is:

$$y = \frac{h}{3}$$



ϕ = angle of internal friction of soil
 γ_s = unit weight of soil



Earth Pressure and Soil Parameters

□ Various Conditions of Loading

2. Inclined surface of fill at top

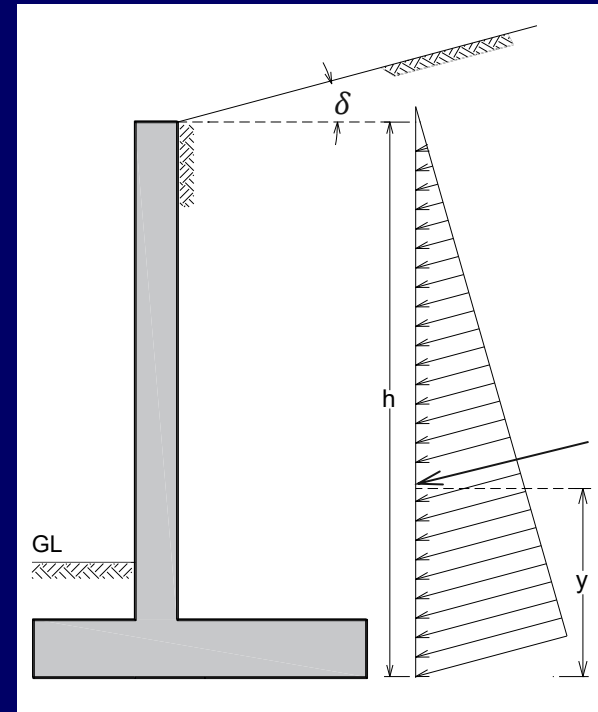
- The resultant of this pressure is

$$P = \frac{1}{2} (p_a)(h) = \frac{1}{2} K_a \gamma_s h^2$$

and the location of “P” is given by

$$y = \frac{h}{3}$$

$$\text{For } \delta = \phi, K_a = \cos \phi$$





Earth Pressure and Soil Parameters

□ Various Conditions of Loading

3. Horizontal surface of fill carrying UDL surcharge

- The increase in pressure caused by uniform surcharge “S” is computed by converting its load into an equivalent imaginary height of earth h_s above the top of the wall such that,

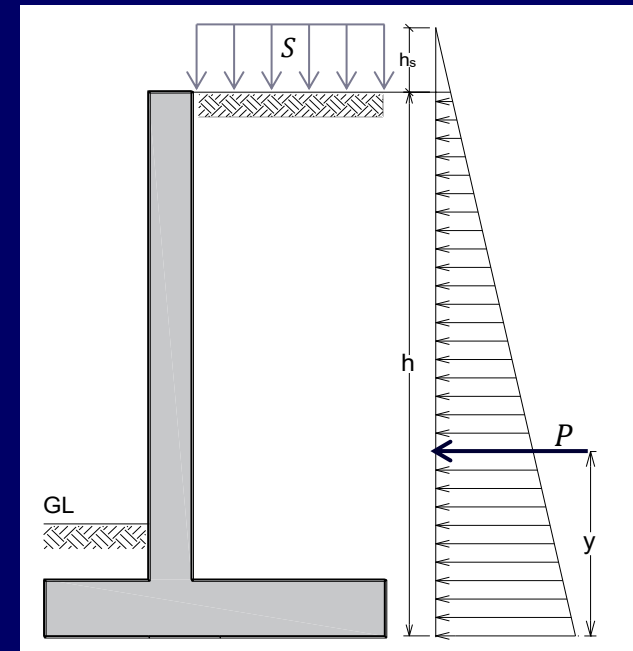
$$h_s = S/\gamma$$

Then resultant of pressure is

$$P = \frac{1}{2} K_a \gamma_s h(h + 2h_s)$$

and the location of “P” from the base is

$$y = \frac{h^2 + 3hh_s}{3(h + 2h_s)}$$





Earth Pressure and Soil Parameters

□ Soil Parameters

- The following table gives representative values of typical soil parameters, often used in engineering practice.

Table: Unit weight (γ_s), effective angles of internal friction (ϕ'), and the coefficient of friction with concrete (μ)					
S. No	Type of Soil	Typical parameters			Remarks
		Unit Weight γ_s (pcf)	Internal friction angle ϕ' (degrees) ^[1]	Coefficient of friction μ ^[2]	
1	Sand or gravel without fine particles, highly permeable	110 to 120	30 to 40	0.5 to 0.6	Should be used as backfill for retaining walls wherever possible
2	Sand or gravel with silt mixture, low permeability	120 to 130	25 to 35	0.4 to 0.5	
3	Silty sand, sand and gravel with high clay content	110 to 120	25 to 30	0.3 to 0.4	The value of Φ may be un-conservative under saturated conditions
4	Medium or stiff clay	100 to 120	25 to 35	0.2 to 0.4	
5	Soft clay, silt	90 to 110	20 to 35	0.2 to 0.3	

[1] The ϕ values do not account for probable additional pressures due to pore water, seepage, frost, etc.

[2] Coefficient of friction between concrete and various soils.



Stability Evaluation of Retaining Wall

□ Stability checks

- To evaluate the stability of retaining wall against overturning, sliding and bearing pressure, the following three factors of safety must be computed and compared with recommended values suggested in ACI Reinforced Concrete Design Handbook Volume-2_Special Topics, MNL-17(21).
 1. **Factor of safety against overturning (FS_{OT})**
 2. **Factor of safety against sliding (FS_{SL})**
 3. **Factor of safety against bearing (FS_{BR})**



Stability Evaluation of Retaining Wall

□ Stability Checks

1. Factor of safety against overturning

- The factor of safety against overturning about the tip of toe is given by

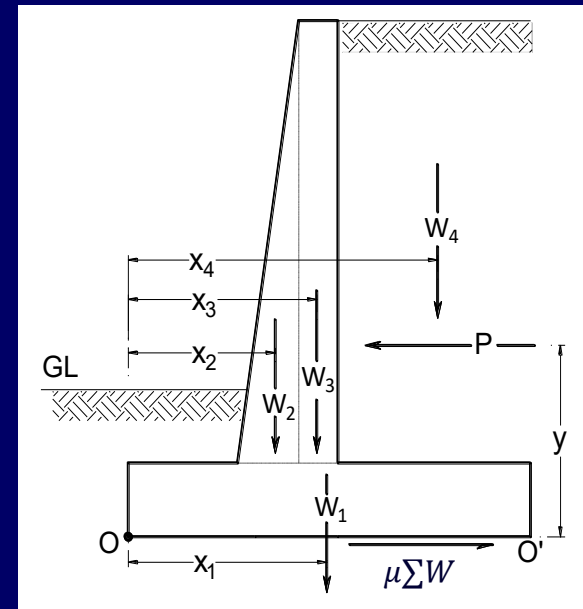
$$FS_{OT} = \frac{\text{Restoring Moment}}{\text{Overturning moment}} = \frac{M_R}{M_{OT}}$$

Where;

$$M_R = \sum(Wx) = W_1x_1 + W_2x_2 + W_3x_3 + W_4x_4$$

$$M_{OTM} = P y$$

- General recommended value is ≥ 2.0 .





Stability Evaluation of Retaining Wall

□ Stability Checks

2. Factor of safety against sliding

- The factor of safety against sliding at base is given by;

$$FS_{SL} = \frac{\mu \sum W}{P}$$

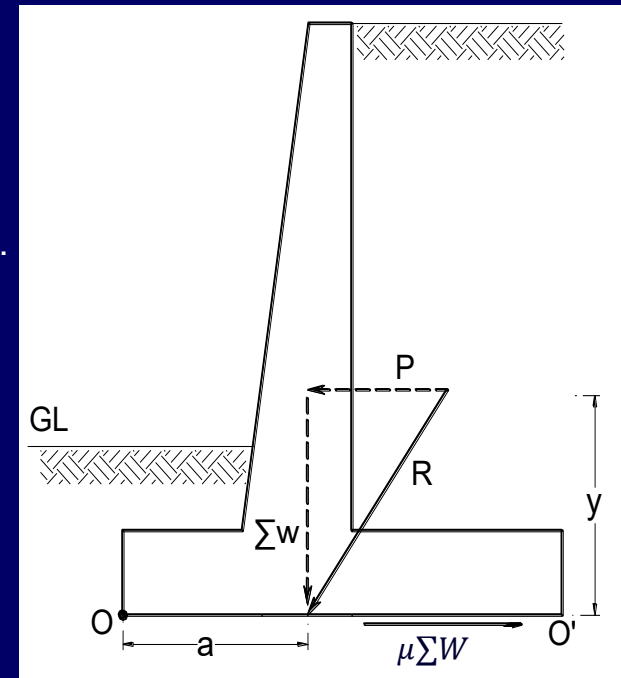
Where;

μ = Coefficient of friction b/w soil and concrete.

$\sum W$ = Total weight of retaining wall including front fill and backfill.

P = Active soil pressure.

- General recommended value is ≥ 1.5 .





Stability Evaluation of Retaining Wall

□ Stability Checks

3. Factor of safety against bearing

- The factor of safety against bearing can be determined as;

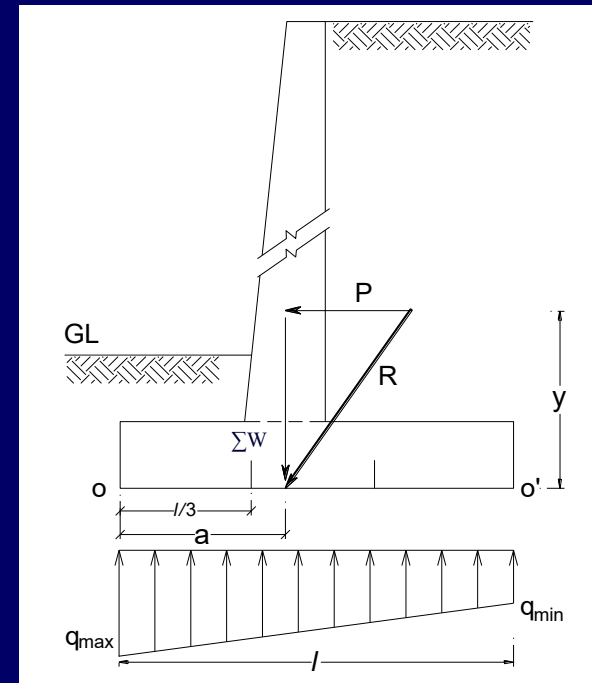
$$FS_{BR} = \frac{q_a}{q_{max}}$$

Where;

q_a = Allowable bearing capacity of soil

q_{max} = Maximum soil bearing pressure

- General recommended value is ≥ 3 .



❑ Stability Checks

■ Calculation of bearing pressure

Therefore, solving for a gives;



Stability Evaluation of Retaining Wall

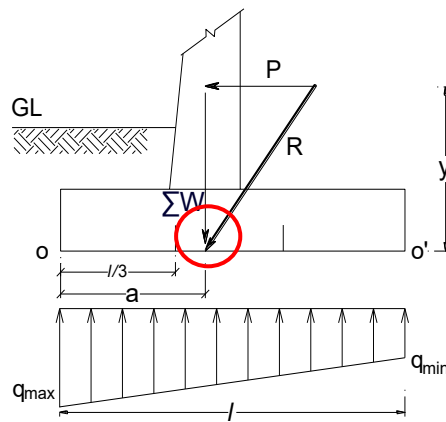
□ Stability Checks

3. Factor of safety against bearing

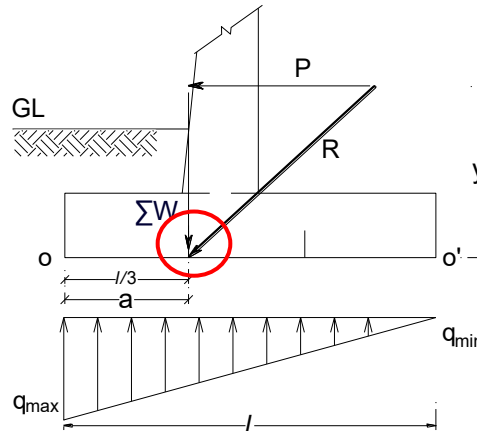
□ Calculation of bearing pressure

- The bearing pressure diagram of soil below the base, depends on the location of resultant as shown below.

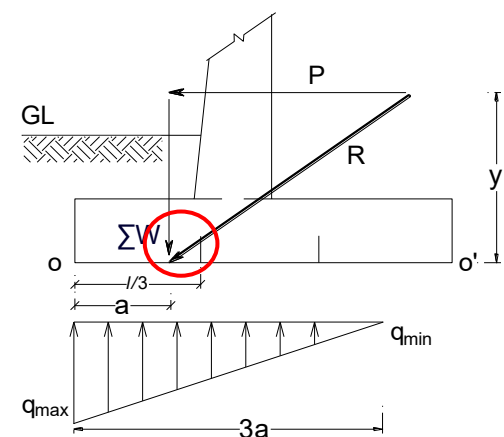
Case 1: $a > l/3$



Case 2: $a = l/3$



Case 3: $a < l/3$





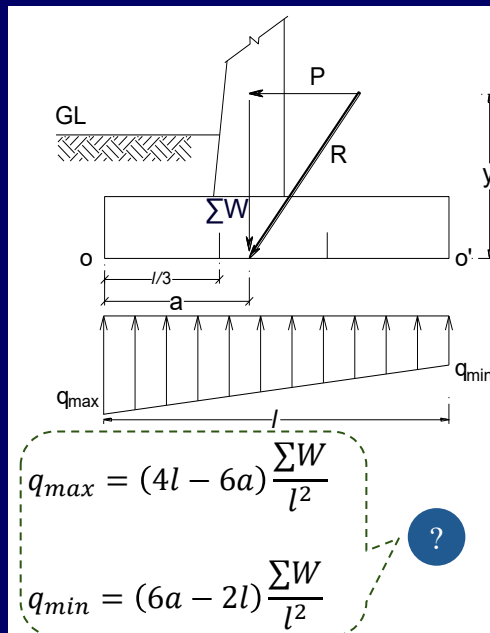
Stability Evaluation of Retaining Wall

□ Stability Checks

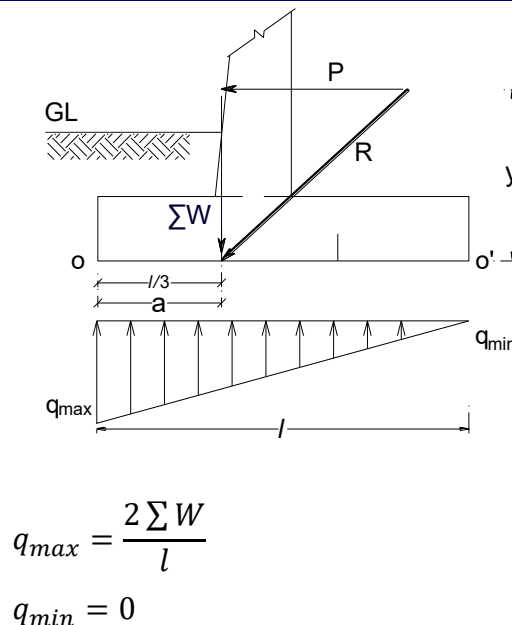
3. Factor of safety against bearing

□ Calculation of bearing pressure

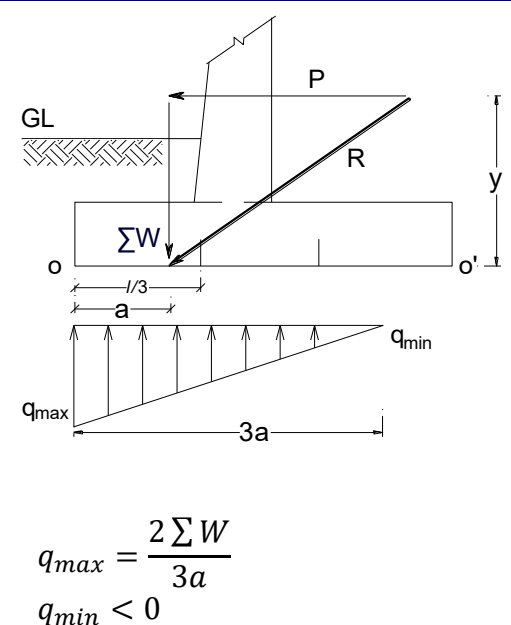
Case 1: $a > l/3$



Case 2: $a = l/3$



Case 3: $a < l/3$





Design of RC Cantilever Retaining Wall

□ Design Criteria

- The stem of a cantilever retaining wall shall be designed as a **one-way slab** in accordance with the applicable provisions of Chapter 7 (one - way slabs). (ACI 318 -19, section 13.3.6.1)
- The base is designed as **one-way shallow foundation** using applicable provisions of Chapter 7 (one - way slabs) and Chapter 9 (Beams). (ACI 318 -19, section 13.3.2.1)



Design of RC Cantilever Retaining Wall

□ Load Combinations

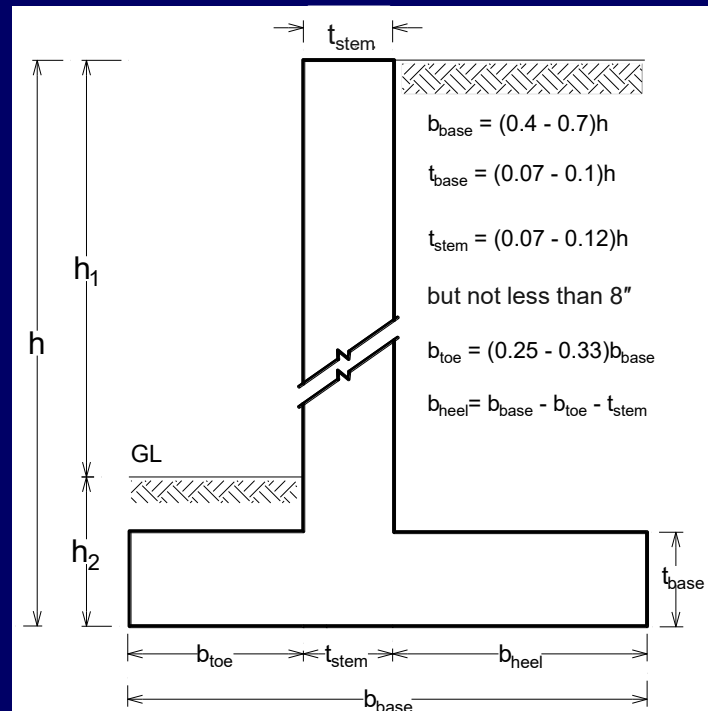
- Load combination relating to structural design of retaining walls shall be in accordance with **ACI 318 -19, section 5.3.1.**
 - $U = 1.2D + 1.6L + 1.6H$

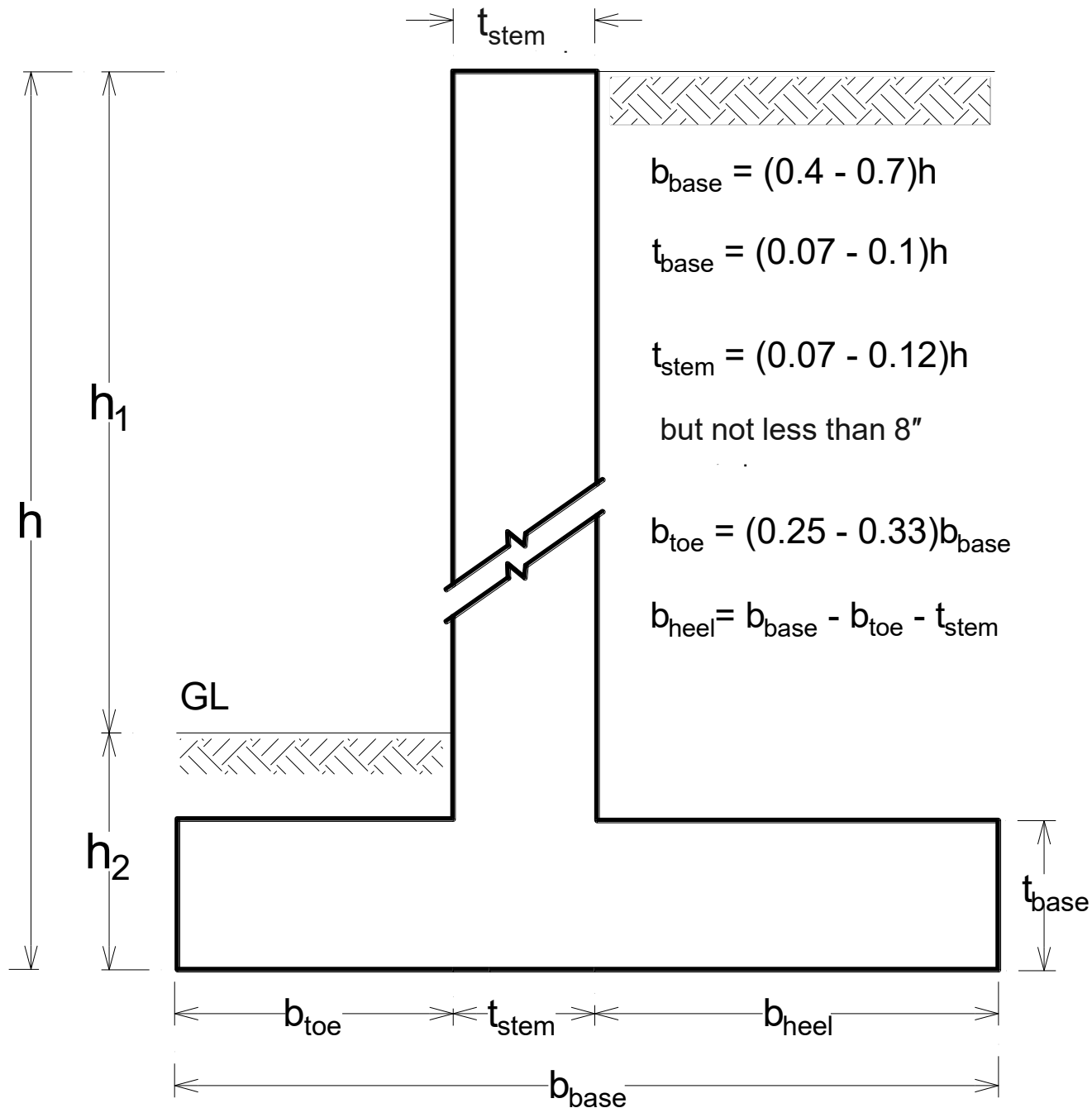


Design of RC Cantilever Retaining Wall

□ Preliminary Sizes

- ACI recommends the dimensions from the guidelines presented by “Bowels” in the fifth edition of *“Foundations Analysis and Design”*, for preliminary calculations.



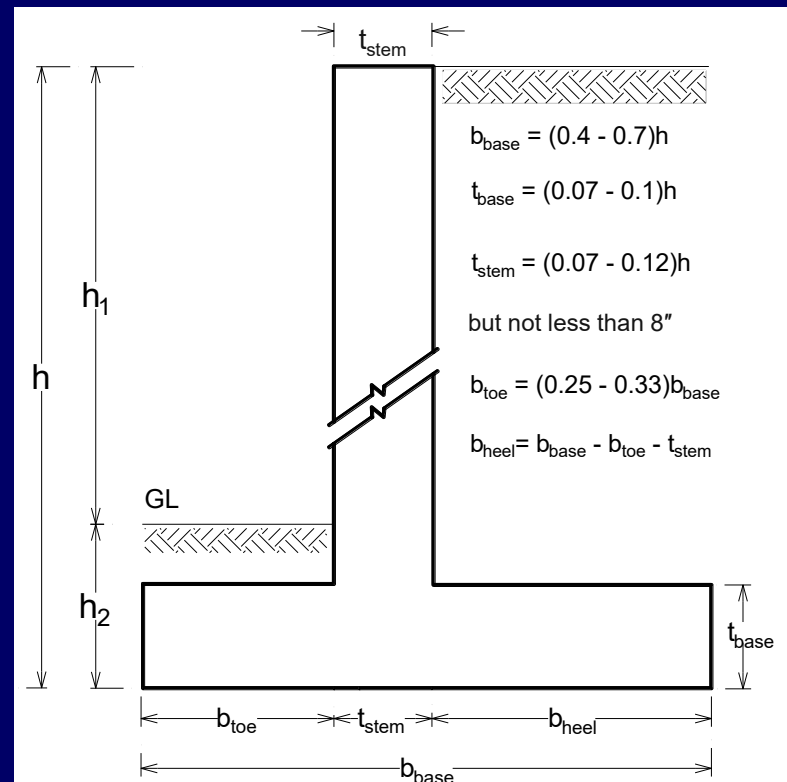




Design of RC Cantilever Retaining Wall

□ Preliminary Sizes

- In case of surcharge, “h” is replaced by $h_{eq} = h + h_s$ where ; h_s = height of surcharge





Design of RC Cantilever Retaining Wall

□ Preliminary Sizes

- The estimated sizes must be checked with the minimum thickness requirements for one-way cantilever slabs provided in ACI 318-19 Table 7.3.1.1

Table 7.3.1.1— Minimum thickness of solid nonprestressed one-way slabs	
Support condition	Minimum h ^[1]
Simply supported	$l/20$
One end continuous	$l/24$
Both ends continuous	$l/28$
Cantilever	$l/10$

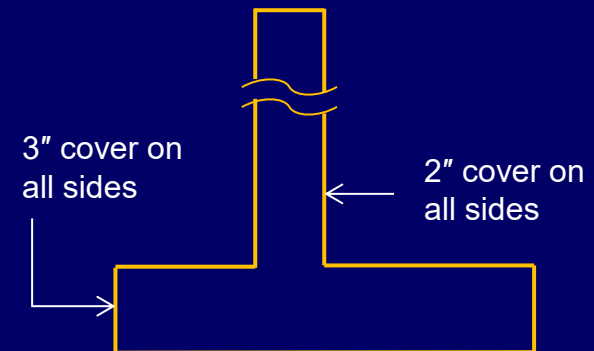
- l = Span length (Center to center length for interior spans, clear projection for cantilevers) (Section 2.2)
- [1] For f_y other than 60,000 psi, the expressions in the table shall be multiplied by $(0.4 + f_y / 100,000)$



Design of RC Cantilever Retaining Wall

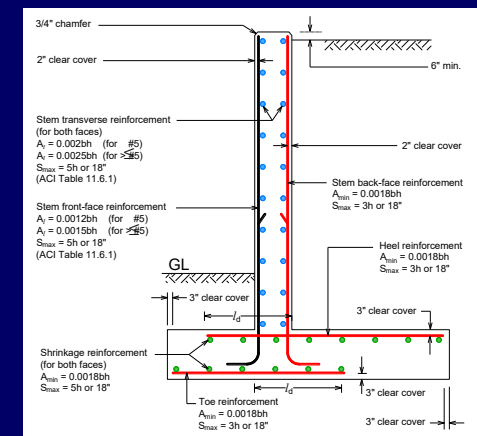
Concrete Clear Cover

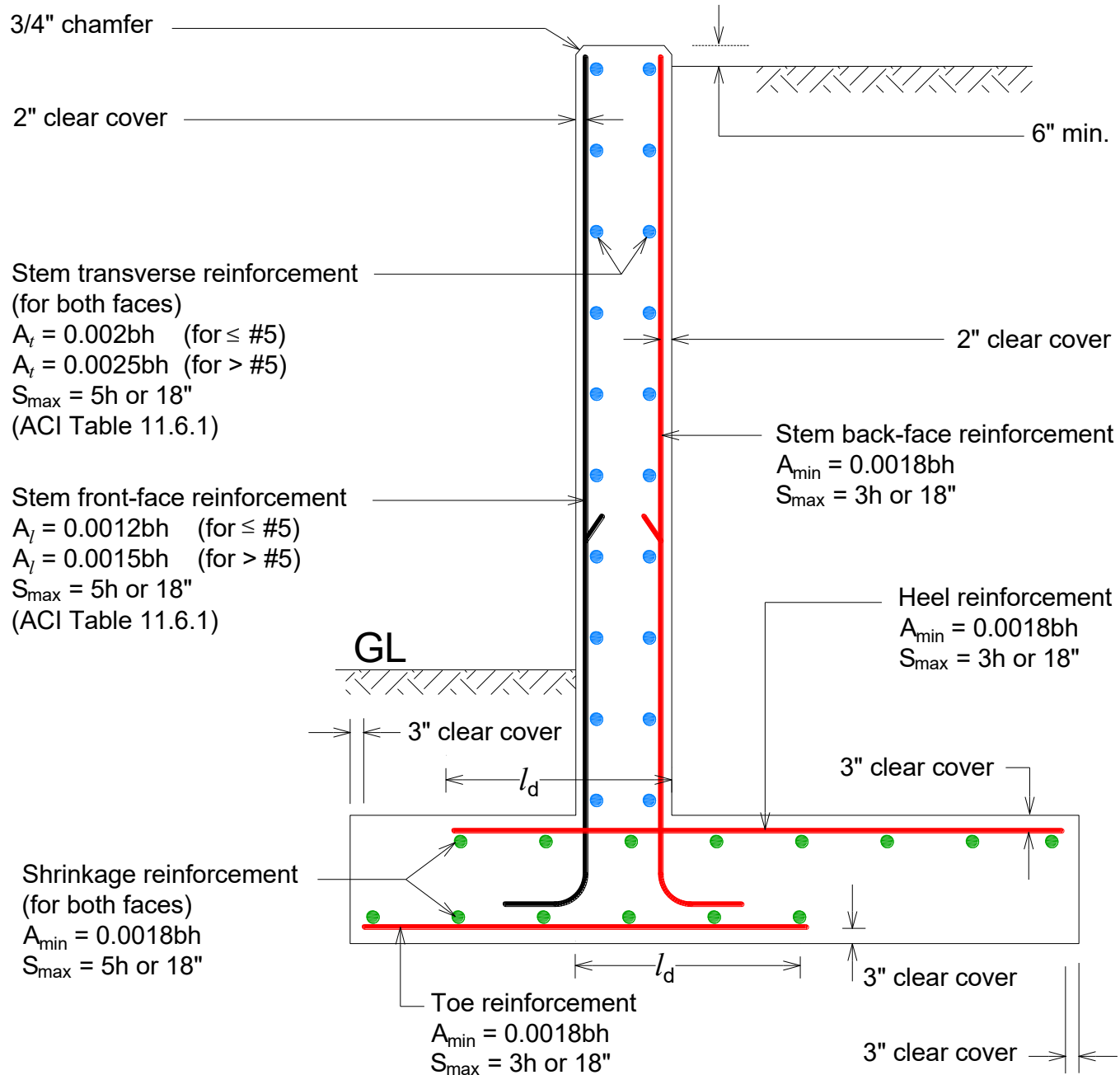
- The minimum specified clear cover is 3 inches for horizontal concrete members cast permanently in contact with the ground, and 2 inches for vertical members (ACI Table 20.5.1.3.1).



Reinforcement Limits

- Overall reinforcement requirement of retaining wall is shown in figure.



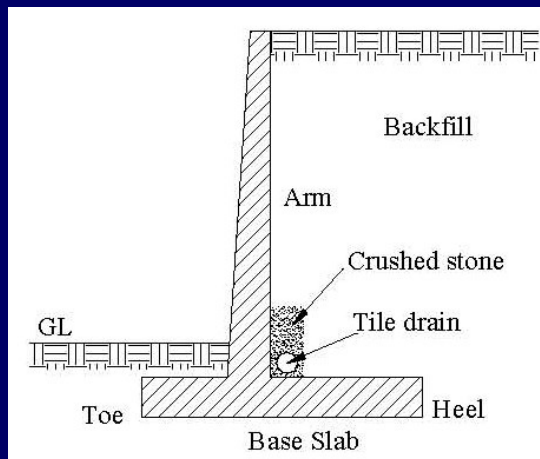




Design of RC Cantilever Retaining Wall

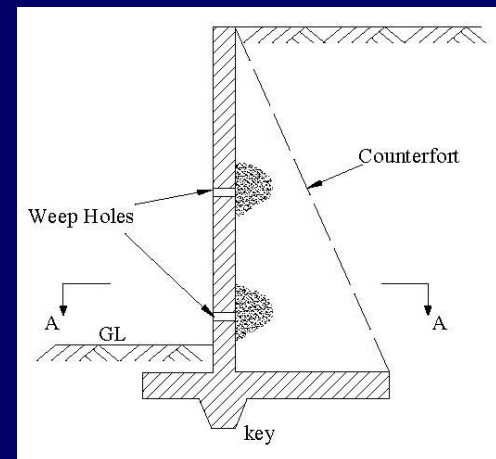
□ Drainage and Other Details

- A proper drainage system must be provided to retaining wall for collecting and redirecting rainwater away from the wall, otherwise the retaining wall may fail. Drainage can be provided in various ways, but two common ways are:



Longitudinal Drains

To prevent outflow to seep into the soil underneath the wall



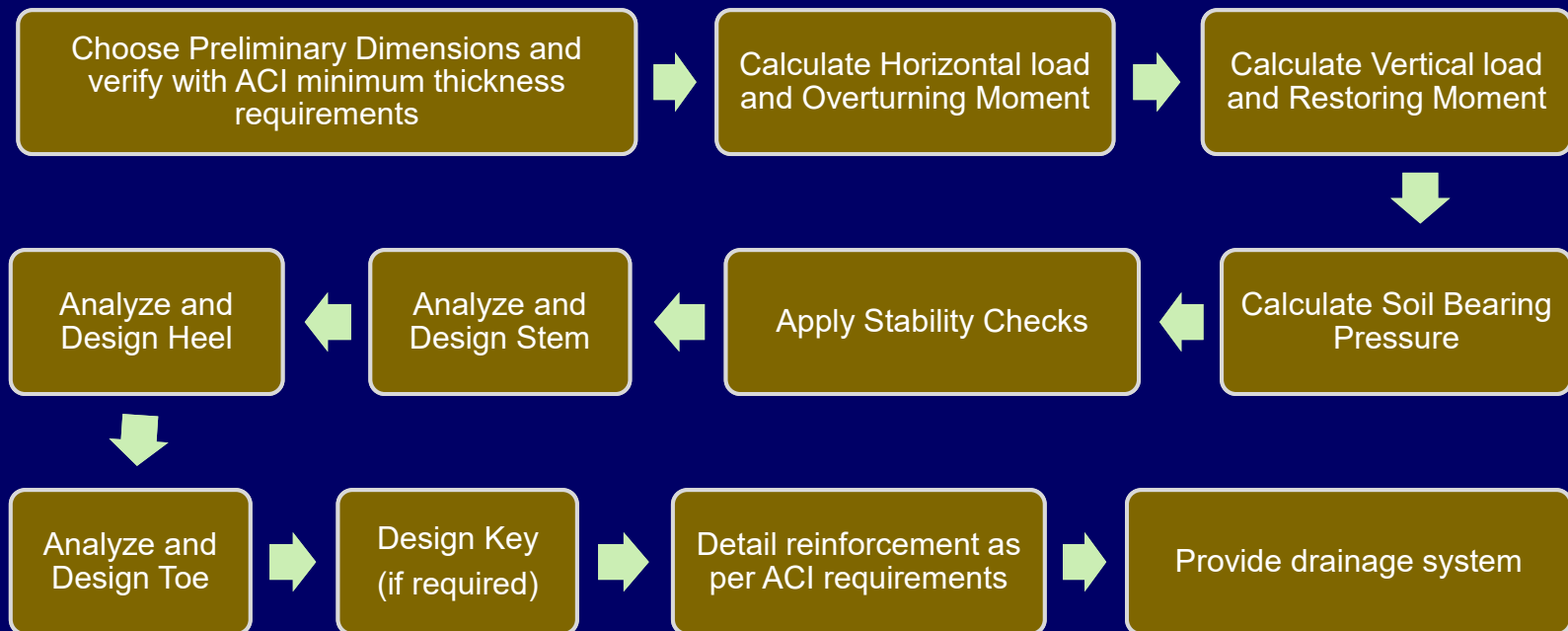
Weep Holes

usually spaced horizontally at an interval of 5 to 10 ft.



Design of RC Cantilever Retaining Wall

□ Summary of Design Steps



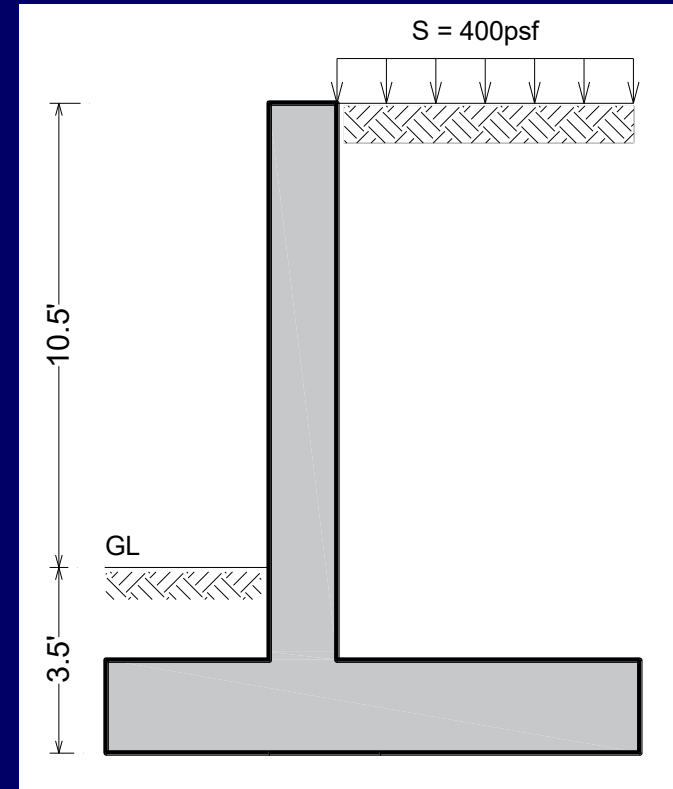


Example 6.1

□ Problem Statement

- **Design** the cantilever retaining wall for the following data:

- Height of stem from GL, $h_1 = 10.5'$
- Depth of base from GL, $h_2 = 3.5'$
- Surcharge, $S = 400$ psf
- Allowable bearing capacity, $q_a = 8$ ksf
- Base friction coefficient, $\mu = 0.6$
- Internal friction angle, $\phi = 30^\circ$
- Unit weight of Soil, $\gamma_s = 120$ pcf
- $f'_c = 4.5$ ksi and $f_y = 60$ ksi





Example 6.1

□ Solution

➤ Step 1: Preliminary dimensions

Equivalent depth of surcharge is calculated as

$$h_s = \frac{S}{\gamma} = \frac{400}{120} = 3.33'$$

$$h_{eq} = h + h_s = 14 + 3.33 = 17.33'$$

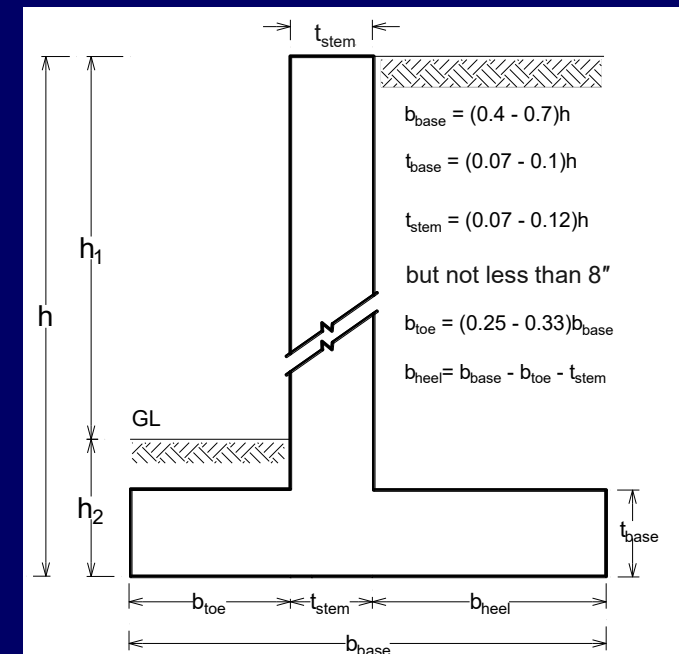
$$\text{Let } b = 0.58h_{eq} = 0.58(17.33) = 10'$$

$$b_{toe} = 0.3b = 0.3(10) = 3'$$

$$b_{heel} = 10 - 3 - 1.25 = 5.75'$$

$$t_{base} = 0.1(17.33) \times 12 = 21''$$

$$t_{stem}(top \& bot) = 0.07(17.33) \times 12 = 15''$$





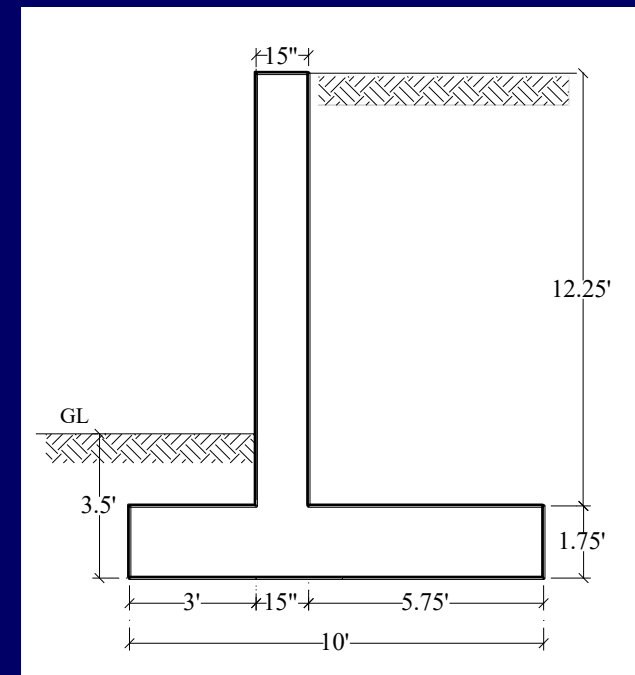
Example 6.1

□ Solution

➤ Step 1: Preliminary dimensions

Check the dimensions against the minimum thickness requirements provided in Table 7.3.1.1.

Minimum thickness requirements			
Component	Assumed depth	Minimum $h = l / 10$	Remarks
Stem/Arm	15"	$\frac{12.25 \times 12}{10} = 14.7"$	OK
Heel	21"	$\frac{5.75 \times 12}{10} = 6.9"$	OK
Toe	21"	$\frac{3 \times 12}{10} = 3.6"$	OK





Example 6.1

□ Solution

➤ Step 2: Calculation of Horizontal load and Overturning Moment

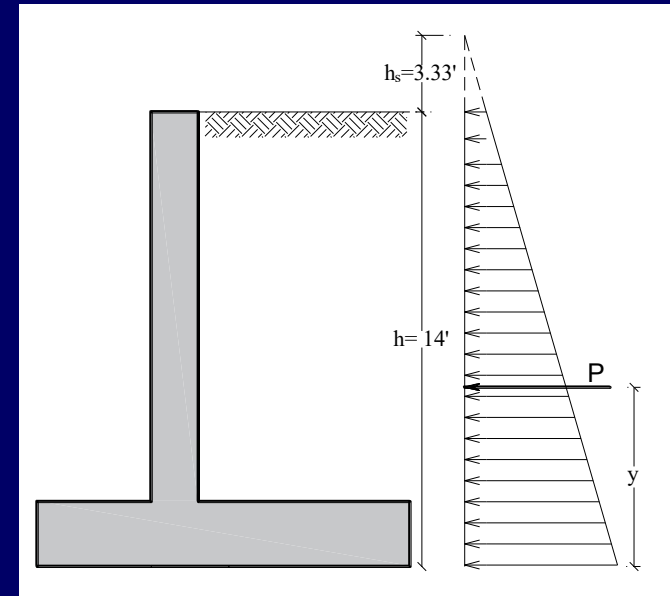
$$K_a = \frac{1 - \sin(\phi)}{1 + \sin(\phi)} = \frac{1 - \sin(30)}{1 + \sin(30)} = 0.33$$

$$P = \frac{1}{2} [K_a \gamma_s h (h + 2h_s)]$$

$$P = \frac{1}{2} [0.33 \times 0.120 \times 14 (14 + 2 \times 3.33)]$$

on solving, we get

$$P = 5.73 \text{ kips}$$





Example 6.1

□ Solution

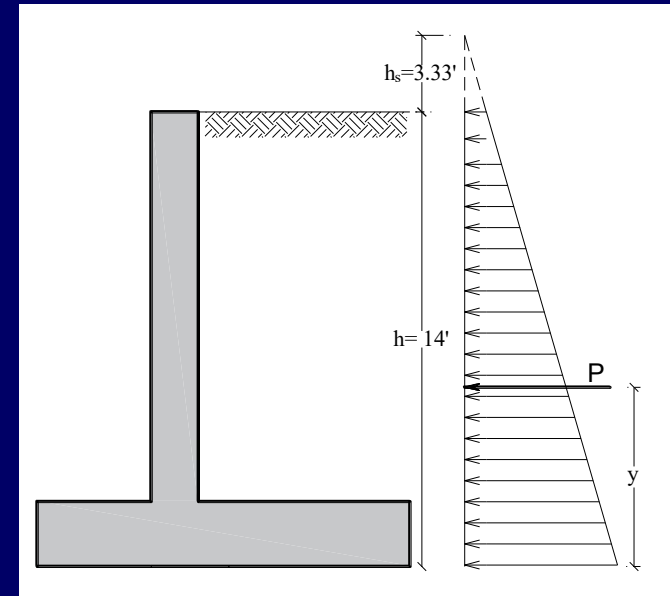
➤ Step 2: Calculation of Horizontal load and Overturning Moment

$$y = \frac{h^2 + 3hh_s}{3(h + 2h_s)}$$

$$y = \frac{14^2 + 3 \times 14 \times 3.33}{3(14 + 2 \times 3.33)} = 5.42'$$

So, overturning moment about the tip of toe can be determined as,

$$M_{OT} = P \times y = 5.73 \times 5.42 = 31.06 \text{ ft. kip}$$





Example 6.1

□ Solution

➤ Step 3: Calculation of Vertical load and Restoring Moment

- Vertical load and restoring or stabilizing moment due to the weight of retaining wall and the soil is calculated as follows:

- **Self weight of Retaining wall**

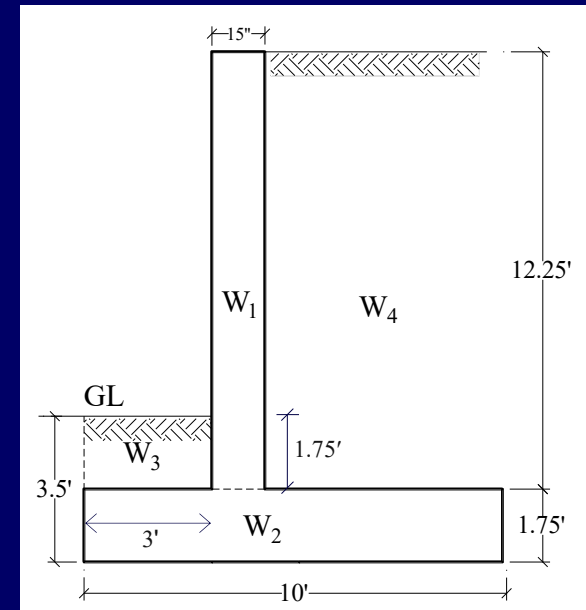
$$W_1 = A_1 \gamma_c = (1.25 \times 12.25) 0.150 = 2.30 \text{ k/ft}$$

$$W_2 = A_2 \gamma_c = (1.75 \times 10) 0.150 = 2.63 \text{ k/ft}$$

- **Self weight of Soil**

$$W_3 = A_3 \gamma_s = (1.75 \times 3) 0.120 = 0.63 \text{ k/ft}$$

$$W_4 = A_4 \gamma_s = (5.75 \times 12.25) 0.120 = 8.45 \text{ k/ft}$$





Example 6.1

□ Solution

➤ Step 3: Calculation of Vertical load and Restoring Moment

- Vertical load and restoring or stabilizing moment due to the weight of retaining wall and the soil is calculated and shown below

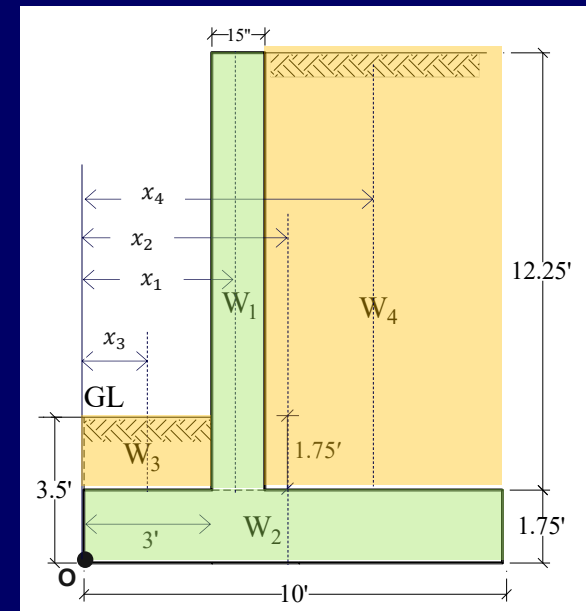
▪ Moment arms about Point O

$$x_1 = 3 + \frac{1.25}{2} = 3.63 \text{ ft}$$

$$x_2 = \frac{10}{2} = 5.0 \text{ ft}$$

$$x_3 = \frac{3}{2} = 1.5 \text{ ft}$$

$$x_4 = 10 - \frac{5.75}{2} = 7.13 \text{ ft}$$





Example 6.1

□ Solution

➤ Step 3: Calculation of Vertical load and Restoring Moment

S. No.	W (kip/ft)	x (ft)	Wx (kip.ft/ft)
1	2.30	2.63	8.35
2	2.63	5.0	13.15
3	0.63	1.5	0.95
4	8.45	7.13	60.25
Sum	14.01		82.69

Hence, we get;

Vertical Load, $= \sum W = 14.01 \text{ kips/ft}$

Restoring Moment, $M_R = \sum Wx = 82.69 \text{ kip. ft/ft}$



Example 6.1

□ Solution

➤ Step 4: Calculation of Bearing pressure

To find the point of action “a”, we have

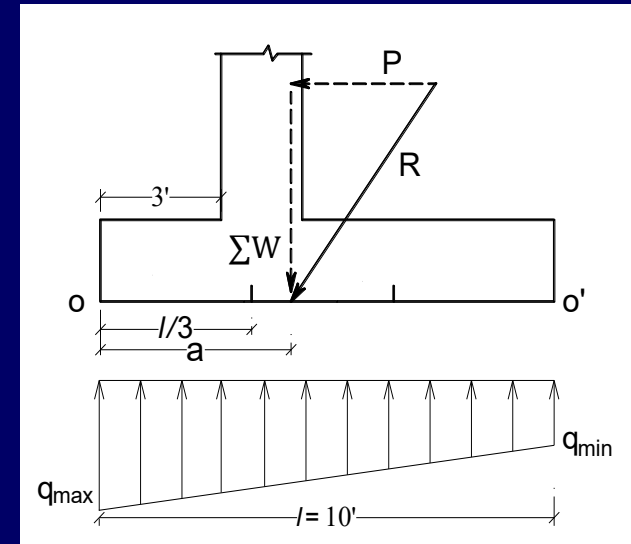
$$a = \frac{M_R - M_{OT}}{\Sigma W}$$

$$a = \frac{82.69 - 31.06}{14.01} = 3.68'$$

Now,

$$\frac{l}{3} = \frac{10}{3} = 3.33' < 3.68'$$

Hence, the resultant lies within the middle third of the base.





Example 6.1

□ Solution

➤ Step 4: Calculation of Bearing pressure

$$q_{max} = (4l - 6a) \frac{\Sigma W}{l^2}$$

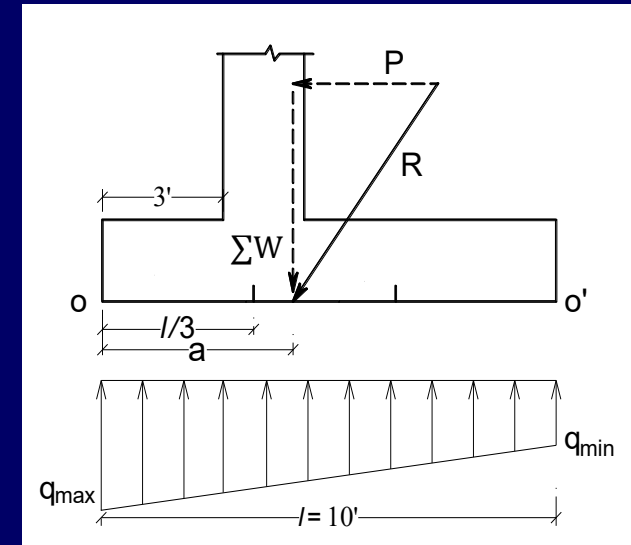
By substituting values;

$$q_{max} = (4 \times 10 - 6 \times 3.68) \frac{14.01}{10^2} = 2.51 \text{ksf}$$

Similarly,

$$q_{min} = (6a - 2l) \frac{\Sigma W}{l^2}$$

$$q_{min} = (6 \times 3.68 - 2 \times 10) \frac{14.01}{10^2} = 0.29 \text{ksf}$$





Example 6.1

□ Solution

➤ Step 5: Applying stability checks

1. F.O.S against Overturning

$$FS_{OT} = \frac{M_R}{M_{OT}} = \frac{82.69}{31.06} = 2.67 > 2 \Rightarrow OK!$$

2. F.O.S against Sliding

$$FS_{SL} = \frac{\mu \Sigma W}{P} = \frac{0.6(14.01)}{5.73} = 1.47 \approx 1.5 \Rightarrow OK!$$

3. FOS against Bearing

$$FS_{BP} = \frac{q_a}{q_{max}} = \frac{8}{2.51} = 3.19 > 3 \Rightarrow OK!$$



Example 6.1

□ Solution

➤ Step 5: Applying stability checks

□ Concluding remarks

- The retaining wall preliminary dimensions are adequate to resist overturning, sliding and prevent bearing failure of soil.
- In the subsequent steps, each component of retaining wall is designed for strength.
- If any of the determined dimensions are not satisfactory, then all the previous steps must be revised.

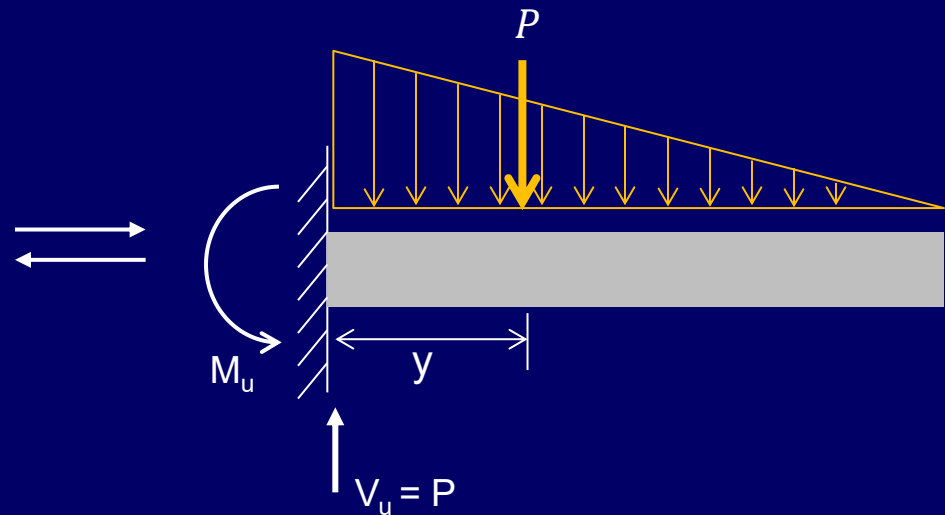
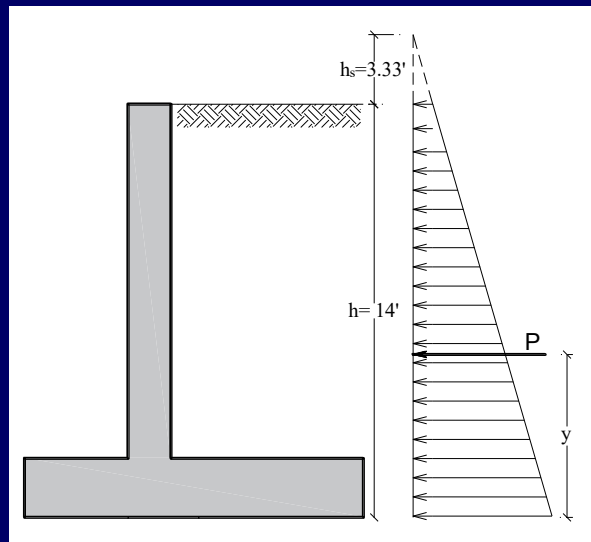


Example 6.1

□ Solution

➤ Step 6: Analysis and Design of Stem

- Analysis of stem of the cantilever wall is just like the analysis of a cantilever beam subjected to uniformly varying (triangular) load as shown below.



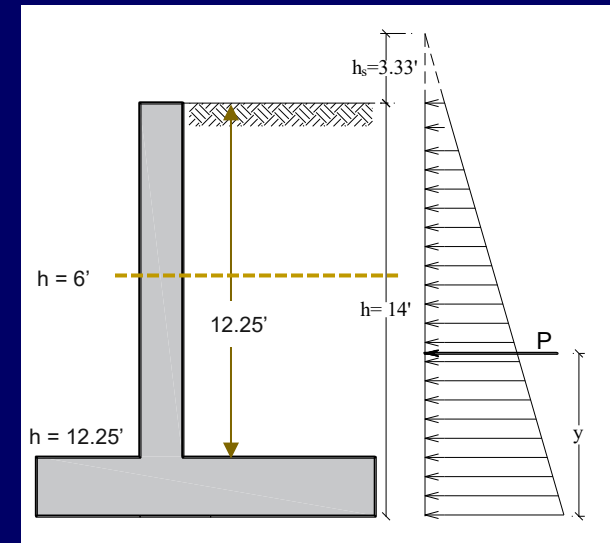


Example 6.1

□ Solution

➤ Step 6: Analysis and Design of Stem

- A better design approach is to divide the stem height into appropriate intervals and then to calculate shear force and bending moment values at those intervals.
- In the current case, we will divide the stem height into two intervals.





Example 6.1

□ Solution

➤ Step 6: Analysis and Design of Stem

For shear demand

$$P = \frac{1}{2} [K_a \gamma_s h (h + 2h_s)]$$

Setting $h = 12.25'$ and 6 ft , we have

$$P_{@12.25} = \frac{1}{2} [0.33 \times 0.120 \times 12.25 (12.25 + 2 \times 3.33)]$$

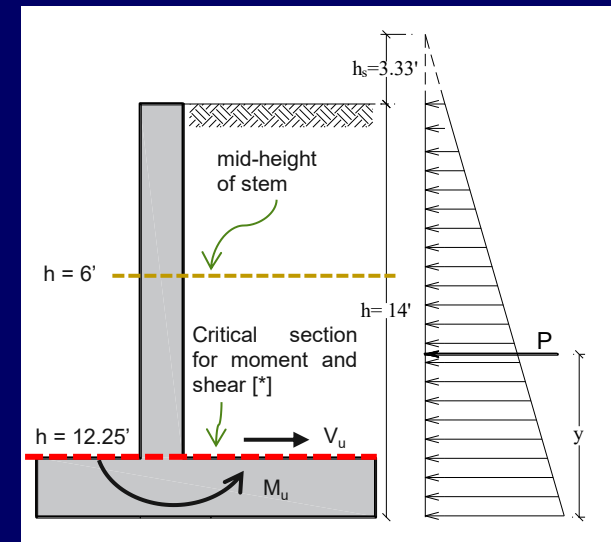
$$P_{@12.25} = 1/2(9.17) = 4.59 \text{ kip}$$

$$P_{@6} = 1.51 \text{ kip}$$

Now,

$$V_{u@12.25} = \text{load factor} \times P = 1.6 \times 4.59 = 7.34 \text{ kip}$$

$$V_{u@6} = 1.6 \times 1.51 = 2.42 \text{ kip}$$



[*] ACI 318-19, section 13.3.6.3



Example 6.1

□ Solution

➤ Step 6: Analysis and Design of Stem

For bending moment

$$M_u = V_u \times y = V_u \times \frac{h^2 + 3hh_s}{3(h + 2h_s)}$$

At $h = 12.25'$

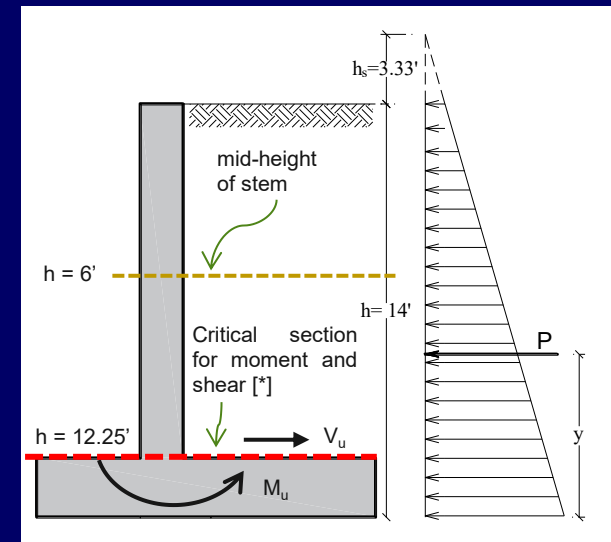
$$M_{u@12.25} = V_{u@12.25} \times \frac{12.25^2 + 3(12.25) \times 3.33}{3(12.25 + 2 \times 3.33)}$$

$$M_{u@12.25} = 7.34 \times 4.8 = 35.2 \text{ ft.kip}$$

Similarly, for $h = 6\text{ft}$, we have

$$M_{u@6} = V_{u@6} \times \frac{6^2 + 3(6) \times 3.33}{3(6 + 2 \times 3.33)}$$

$$M_{u@6} = 2.42 \times 2.53 = 6.1 \text{ ft.kip}$$



[*] ACI 318-19, section 13.3.6.3

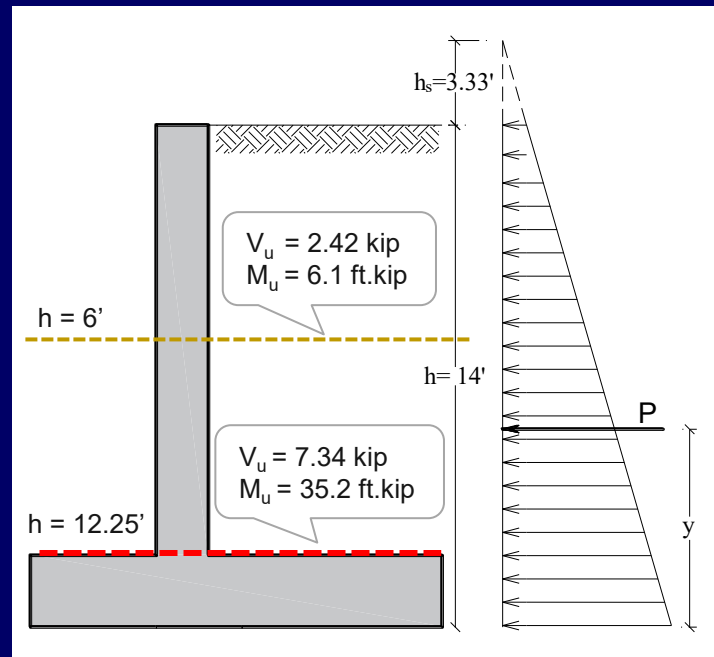


Example 6.1

□ Solution

➤ Step 6: Analysis and Design of Stem

□ Analysis Summary





Example 6.1

□ Solution

➤ Step 6: Analysis and Design of Stem

□ Design for Shear

$$V_u = 7.34 \text{ kips}$$

$$d = t_{stem} - Cc - \frac{d_b}{2}$$

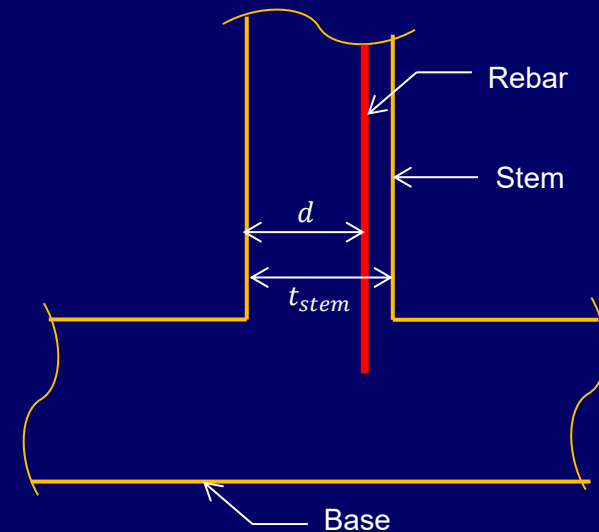
Using 2" clear cover, and assuming #6 bar,

$$d = 15 - 2 - 6/16 = 12.63''$$

$$\phi V_c = \phi 2\sqrt{f'_c}bd$$

$$= (0.75)2\sqrt{4500} \times 12 \times 12.63 = 15.24 \text{ kips}$$

$$\phi V_c > V_u \Rightarrow OK!$$





Example 6.1

□ Solution

➤ Step 6: Analysis and Design of Stem

□ Design for Flexure (Vertical reinforcement at the back face)

For $h = 12.25'$; $M_u = 35.2 \text{ ft-kip/ft} = 422.4 \text{ in-kip/ft}$

$$a = d - \sqrt{d^2 - \frac{2.614M_u}{f'_c b}} = 12.63 - \sqrt{12.63^2 - \frac{2.614 \times 422.4}{4.5 \times 12}} = 0.85''$$

$$A_s = \frac{M_u}{0.9f_y \left(d - \frac{a}{2}\right)} = \frac{422.4}{0.9 \times 60 \left(12.63 - \frac{0.85}{2}\right)} = 0.65 \text{ in}^2$$

$$A_{s,min} = 0.0018bh = 0.0018(12 \times 15) = 0.324 \text{ in}^2 < A_s \rightarrow OK$$

$$\text{Using \#6 bar, spacing} = 12A_b/A_s = 12(0.44)/0.65 = 8.1'' \text{ c/c}$$

$$S_{max} = \min(3h = 3 \times 15 = 45'', 18'') \Rightarrow OK! , \text{ Hence, finally provide \#6 @ 8'' c/c}$$



Example 6.1

□ Solution

➤ Step 6: Analysis and Design of Stem

□ Design for Flexure (Vertical reinforcement at the back face)

For $h = 6'$; $M_u = 6.1 \text{ ft-kip/ft} = 73.2 \text{ in-kip/ft}$

$$a = d - \sqrt{d^2 - \frac{2.614M_u}{f'_c b}} = 12.63 - \sqrt{12.63^2 - \frac{2.614 \times 73.2}{4.5 \times 12}} = 0.14"$$

$$A_s = \frac{M_u}{0.9f_y \left(d - \frac{a}{2}\right)} = \frac{73.2}{0.9 \times 60 \left(12.63 - \frac{0.14}{2}\right)} = 0.11 \text{ in}^2$$

$$A_{s,min} = 0.324 \text{ in}^2 > A_s \rightarrow A_{s,min} \text{ governs}$$

Using #6 bar, spacing = $12A_b/A_s = 12(0.44)/0.324 = 16.3" \text{ c/c}$

$S_{max} = \min(3h = 3 \times 15 = 45", 18") \Rightarrow \text{OK!}$, Hence, finally provide #6 @ 16" c/c

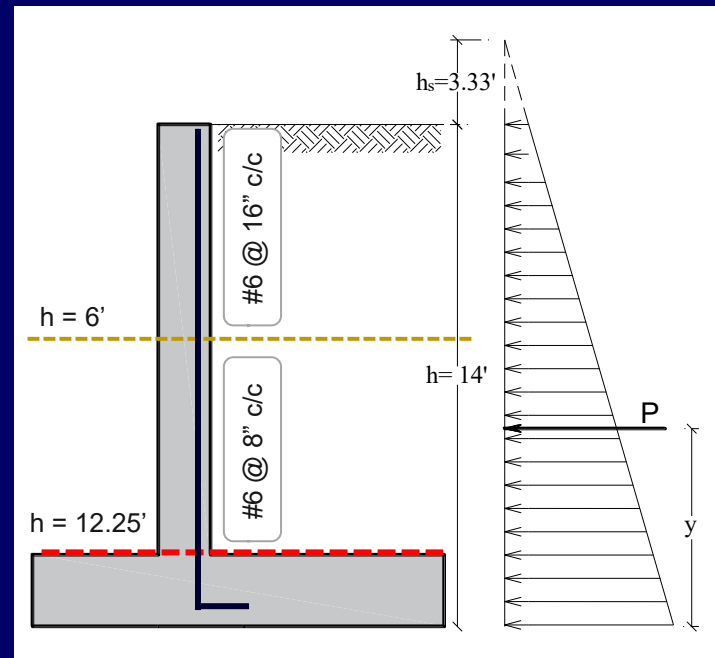


Example 6.1

□ Solution

➤ Step 6: Analysis and Design of Stem

□ Design for Flexure (Vertical reinforcement at the back face)





Example 6.1

□ Solution

➤ Step 6: Analysis and Design of Stem

□ Vertical reinforcement at the front face

$$A_s = 0.0012 \times 12 \times 15 = 0.216 \text{ in}^2 \quad (\text{From slide No. 31})$$

Using #4 bar with area $A_b = 0.20 \text{ in}^2$

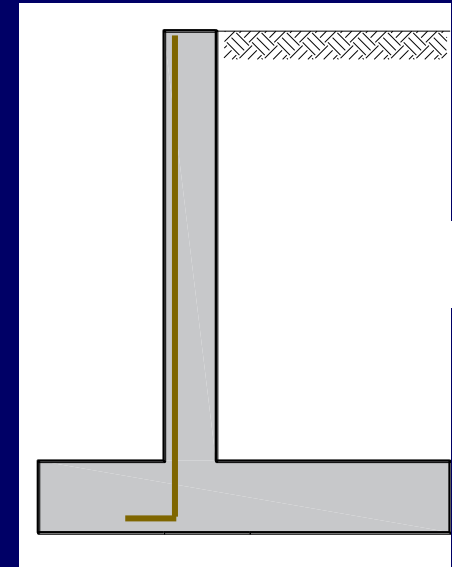
Spacing = Area of one bar A_b / A_{st}

$$= (0.20 / 0.216) \times 12 = 11.1''$$

Maximum spacing should not exceed;

- $5h = 5 \times 15 = 75''$ or $18''$

Provided spacing is OK!. Finally provide #4 @ 10"





Example 6.1

□ Solution

➤ Step 6: Analysis and Design of Stem

□ Horizontal /Transverse reinforcement

$$A_{st} = 0.0020bh \quad (\text{From slide No. 31})$$

$$A_{st} = 0.0020 \times 12 \times 15 = 0.36 \text{ in}^2/\text{ft}$$

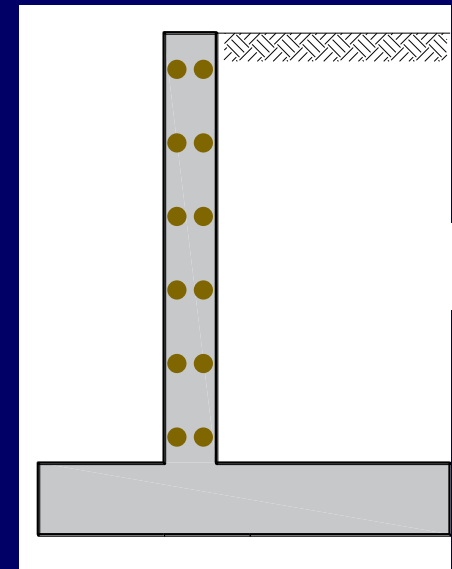
This is the total required area of horizontal reinforcement, i-e for both faces.

$$\text{So, for each face, } A_{st} = 0.36/2 = 0.18 \text{ in}^2/\text{ft}$$

Using #4 bar with $A_b = 0.20 \text{ in}^2$

$$\text{Spacing} = (0.20/0.18) \times 12 = 13.3'' < 18'' \Rightarrow \text{OK!}$$

Finally, Use #4 @ 10" c/c





Example 6.1

□ Solution

➤ Step 6: Analysis and Design of Stem

□ Calculation of development length

As per ACI 318, section 25.4.3.1, development length l_{dh} for deformed bar, is largest of :

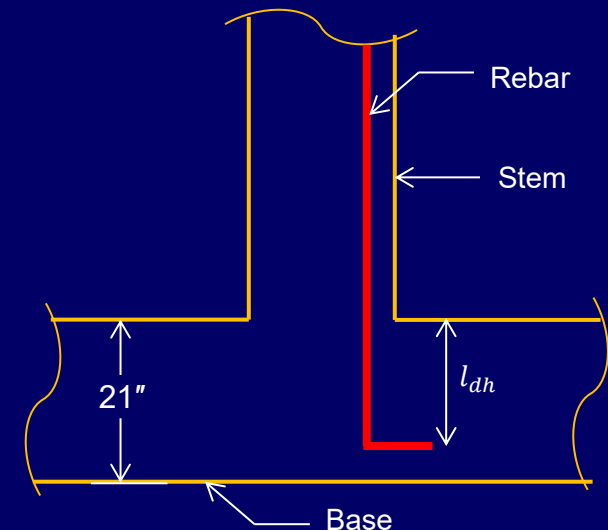
$$1. \frac{9f_y}{550\sqrt{f'_c}} d_b = \frac{9(60000)}{550\sqrt{4500}} (0.75) = 11"$$

$$2. 8d_b = 8(0.75) = 6"$$

$$3. 6"$$

Therefore, $l_{dh} = 11"$

Since, $l_{dh} = 11" < \text{depth of base} \Rightarrow \text{OK!}$



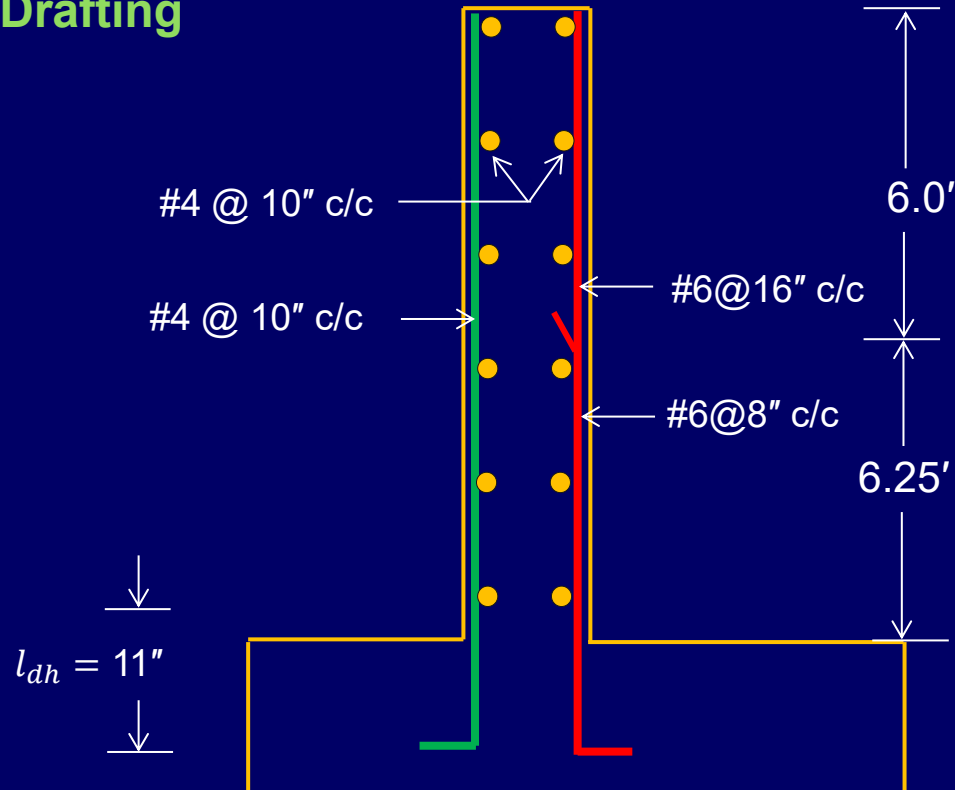


Example 6.1

□ Solution

➤ Step 6: Analysis and Design of Stem

□ Drafting





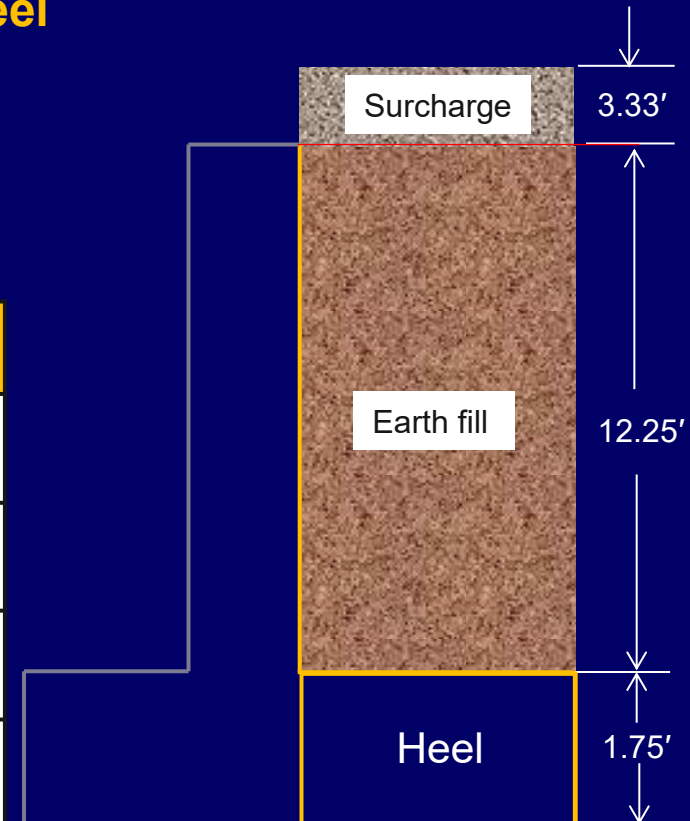
Example 6.1

□ Solution

➤ Step 7: Analysis and Design of Heel

□ Loads

Component	Factored Load (k/ft)
Self weight of heel	$1.2\gamma h_b = 1.2(0.15)(1.75 \times 1) = 0.32$
Earth fill load	$1.6\gamma h b = 1.6(0.12)(12.25 \times 1) = 2.35$
Surcharge Load	$1.6\gamma h_s b = 1.6(0.12)(3.33 \times 1) = 0.64$
Total:	$W_u = 0.32 + 2.35 + 0.64 = 3.31 \text{ k/ft}$





Example 6.1

□ Solution

➤ Step 7: Analysis and Design of Heel

□ Analysis for Shear

Taking 3" clear cover, and assuming #6 bar,

$$d = 21 - 3 - 6/16 = 17.63''$$

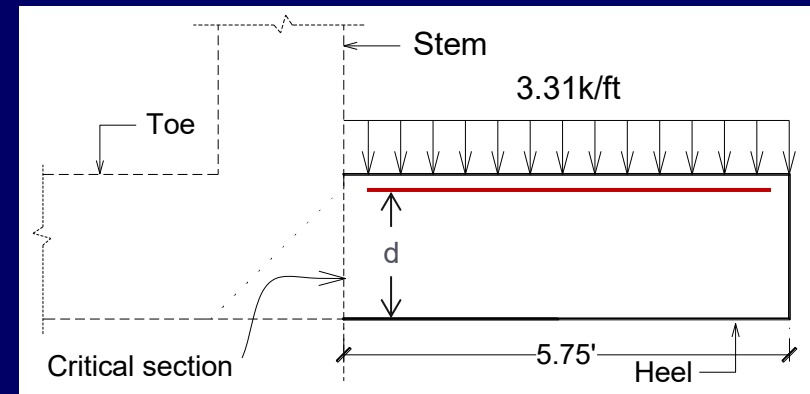
Shear force at critical location is,

$$V_u = 3.31 \times (5.75) = 19.0 \text{ kips}$$

□ Analysis for Flexure

$$M_u = 3.31 \times 5.75 \times \left(\frac{5.75}{2} \right)$$

$$M_u = 54.71 \text{ ft. kip or } 656.62 \text{ in. kip/ft}$$





Example 6.1

□ Solution

➤ Step 7: Analysis and Design of Heel

□ Design for Shear

$$V_u = 19.0 \text{ kips}$$

Design shear capacity of concrete is given as:

$$\phi V_c = \phi 2\sqrt{f'_c}bd$$

$$\phi V_c = (0.75)2\sqrt{4500} \times 12 \times 17.63 = 21.29 \text{ kips}$$

$$\phi V_c > V_u \Rightarrow OK!$$



Example 6.1

□ Solution

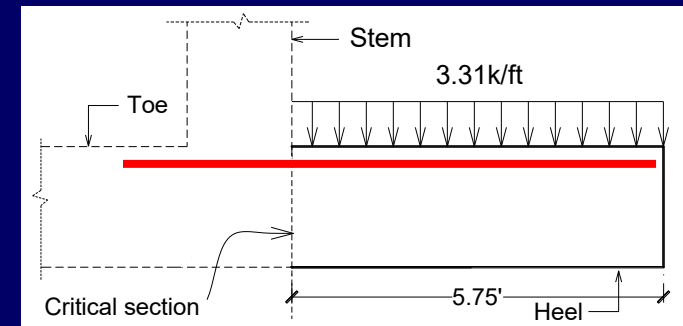
➤ Step 7: Analysis and Design of Heel

□ Design for Flexure (Longitudinal reinforcement)

$$M_u = 656.62 \text{ in-kip/ft}$$

$$a = 17.63 - \sqrt{17.63^2 - \frac{2.614 \times 656.62}{4.5 \times 12}} = 0.93''$$

$$A_s = \frac{656.62}{0.9 \times 60 \left(17.63 - \frac{0.93}{2}\right)} = 0.71 \text{ in}^2$$





Example 6.1

□ Solution

➤ Step 7: Analysis and Design of Heel

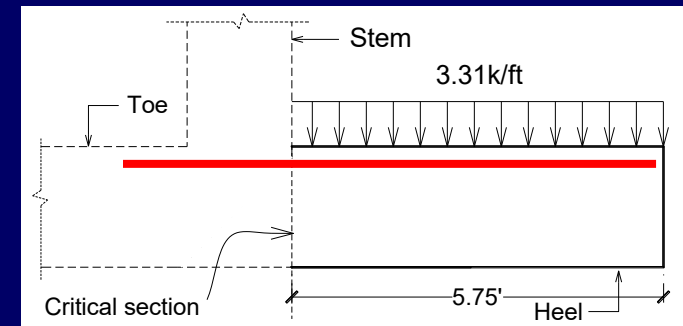
□ Design for Flexure (Longitudinal reinforcement)

$$A_{min} = 0.0018bh = 0.0018(12 \times 21) = 0.454 \text{ in}^2$$

$$\text{Using \#6 bar, spacing} = 12A_b/A_s = 12(0.44)/0.71 = 7.4 \text{ c/c}$$

$$S_{max} = \min(3h = 3 \times 21 = 63", 18") \Rightarrow \text{OK!}$$

Hence, finally provide #6 @ 7" c/c





Example 6.1

□ Solution

➤ Step 7: Analysis and Design of Heel

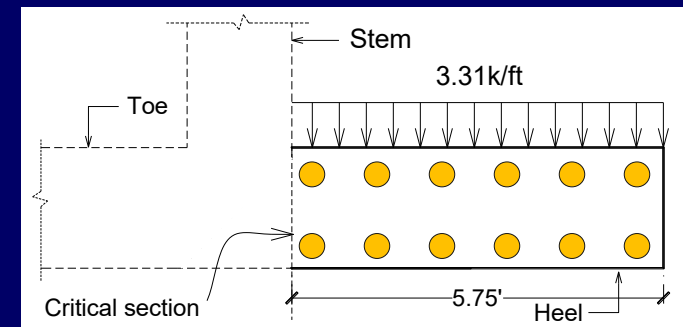
□ Transverse reinforcement

Minimum shrinkage reinforcement is given by

$$A_{shrinkage} = 0.0018bh = 0.0018(12 \times 21) = 0.454 \text{ in}^2/\text{ft}$$

Divide this equally between two faces,

$$A_{shrinkage} = \frac{0.454}{2} = 0.227 \text{ in}^2/\text{ft}$$





Example 6.1

□ Solution

➤ Step 7: Analysis and Design of Heel

□ Transverse reinforcement

Using #4 bar with $A_b = 0.20 \text{ in}^2$

Spacing = $(0.20/0.227) \times 12 = 10.57'' \text{ c/c}$

Maximum spacing for shrinkage should be least of

- $5h = 5 \times 21 = 105''$ or $18''$

Provided spacing is OK!.

Finally provide #4@10'' c/c on top and bottom face of heel



Example 6.1

□ Solution

➤ Step 7: Analysis and Design of Heel

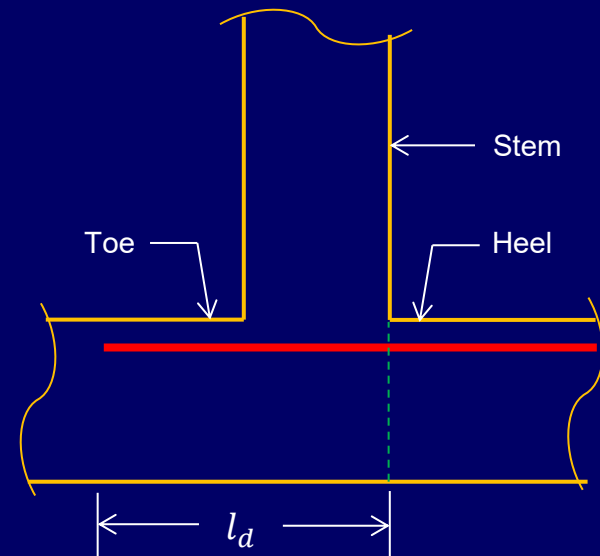
□ Calculation of development length

Development length of Heel reinforcement into Toe as per ACI 318-19, section 25.4.2.4 is given by

$$l_d = \frac{f_y}{25\sqrt{f'_c}} d_b$$

$$l_d = \frac{60000}{25\sqrt{4500}} (0.75) = 26.8" \approx 27"$$

Therefore, $l_d = 2.25'$



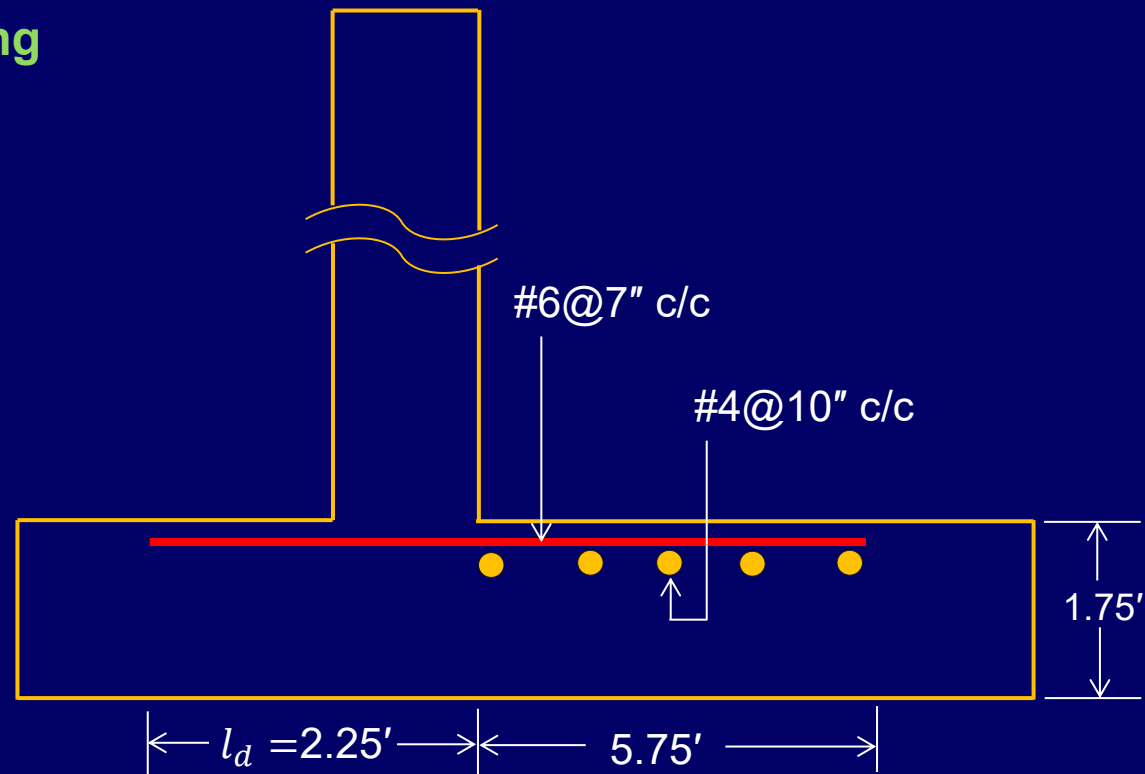


Example 6.1

□ Solution

➤ Step 7: Analysis and Design of Heel

□ Drafting





Example 6.1

□ Solution

➤ Step 8: Analysis and Design of Toe

□ Loads

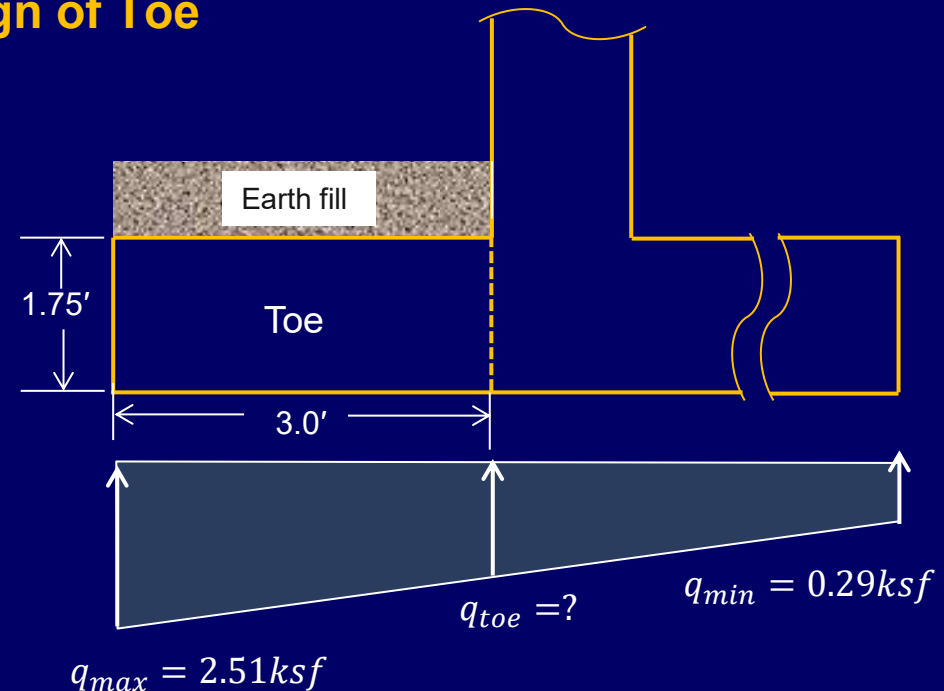
Weight of Earth fill = Ignored

Self weight of Toe = Ignored

Factored $q_{max} = 1.6 \times 2.51 = 4.01$

Factored $q_{min} = 1.6 \times 0.29 = 0.46$

Now, the factored soil pressure at interior end of toe slab can be determined using similarity of triangles as shown on next slide.



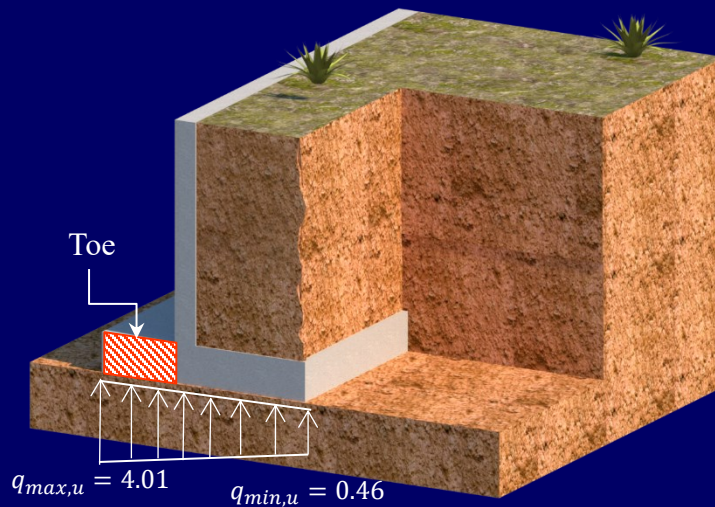


Example 6.1

□ Solution

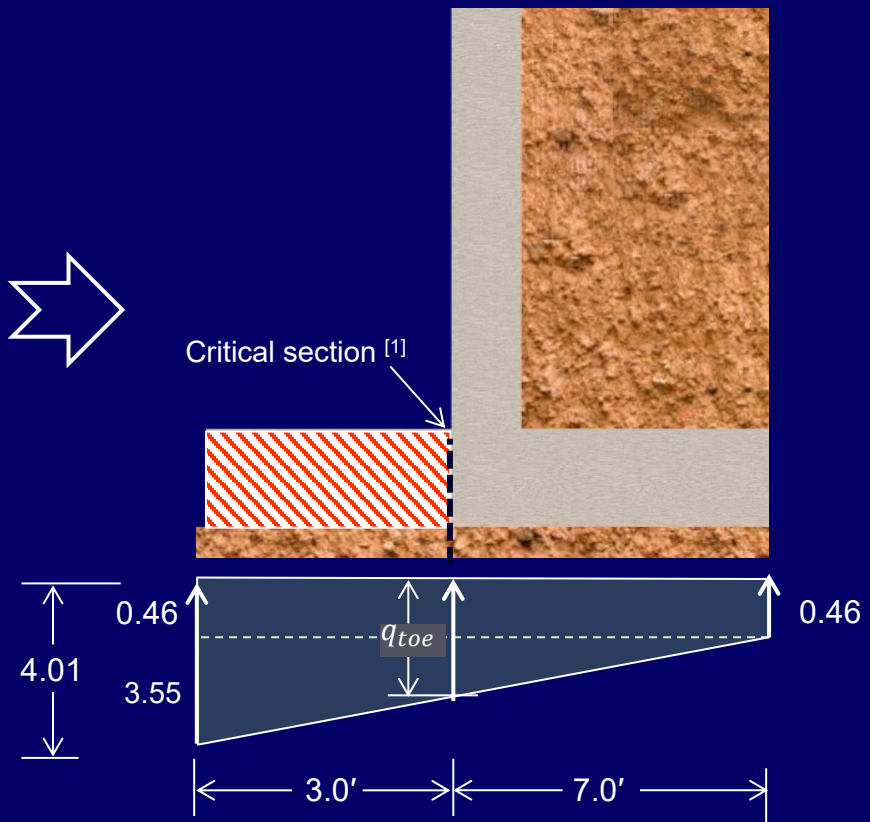
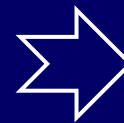
➤ Step 8: Analysis and Design of Toe

□ Loads



Note:

[1] Although the ACI Code recommends taking the critical section of the toe at a distance of d from the face of support, here we chose to take the critical section at the interface to make the case simple.





Example 6.1

□ Solution

➤ Step 8: Analysis and Design of Toe

□ Loads

From $\triangle ACD \Leftrightarrow \triangle BDC$

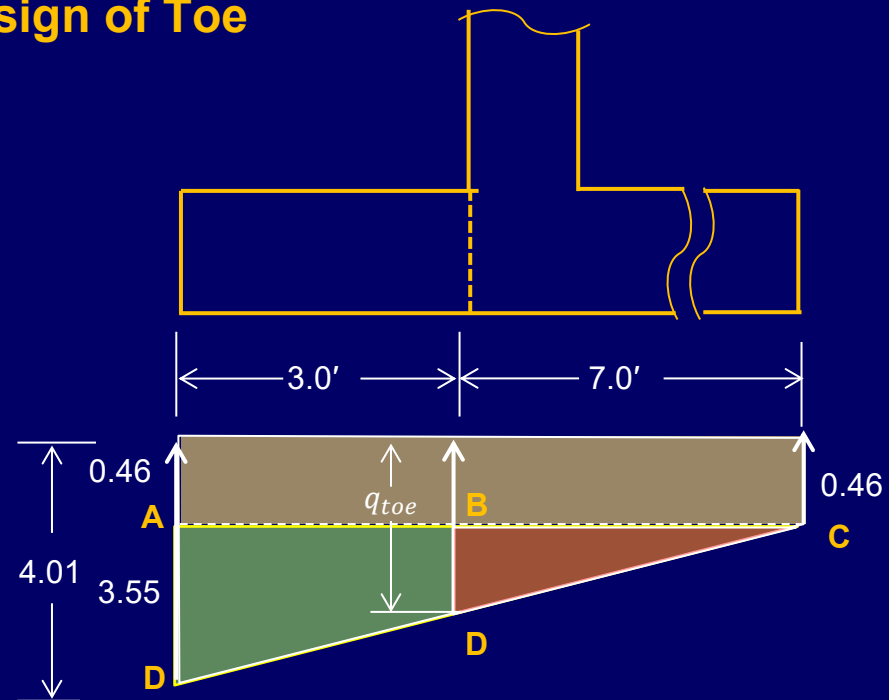
$$\frac{BD}{AD} = \frac{BC}{AC}$$

$$\frac{BD}{3.55} = \frac{7}{10}$$

$$BD = 2.48 \text{ ksf}$$

Now,

$$q_{toe} = 0.46 + 2.48 = 2.94 \text{ ksf}$$



General formula:

$$q_{toe} = q_{min} + \frac{l_{base} - l_{toe}}{l_{base}} (q_{max} - q_{min})$$



Example 6.1

□ Solution

➤ Step 8: Analysis and Design of Toe

□ Analysis for Shear

$$V_u = W_u = \text{Area of shaded region}$$

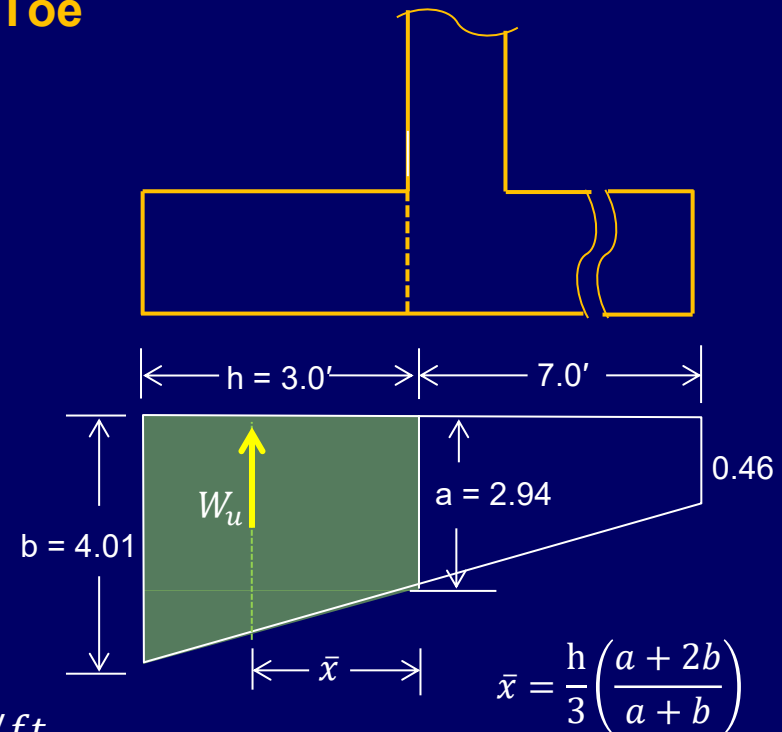
$$V_u = \frac{4.01 + 2.94}{2} \times 3 = 10.42 \text{ kips}$$

□ Analysis for flexure

$$M_u = W_u \times \bar{x} = 10.42 \bar{x}$$

From figure, $\bar{x} = 1.58'$

$$\text{So, } M_u = 10.42 \times 1.58 = 16.46 \text{ ft.kip/ft}$$





Example 6.1

□ Solution

➤ Step 8: Analysis and Design of Toe

□ Design for Shear

$$V_u = 10.42 \text{ kips}$$

Design shear capacity of concrete is given as:

$$\phi V_c = \phi 2\sqrt{f'_c}bd = 0.75 \times 2\sqrt{4500} \times 12 \times 17.63 = 21.29 \text{ kips}$$

$$\phi V_c > V_u \Rightarrow OK!$$



Example 6.1

□ Solution

➤ Step 8: Analysis and Design of Toe

□ Design for Flexure

$$M_u = 16.46 \times 12 = 197.52 \text{ in-k/ft}$$

$$a = d - \sqrt{d^2 - \frac{2.614M_u}{f'_c b}} = 12.63 - \sqrt{12.63^2 - \frac{2.614 \times 197.52}{4.5 \times 12}} = 0.27"$$

$$A_s = \frac{M_u}{0.9f_y \left(d - \frac{a}{2}\right)} = \frac{197.52}{0.9 \times 60 \left(17.63 - \frac{0.27}{2}\right)} = 0.21 \text{ in}^2$$

$$A_{s,min} = 0.454 \text{ in}^2 > A_s \rightarrow A_{s,min} \text{ governs}$$

$$\text{Using \#6 bar, spacing} = 12A_b/A_s = 12(0.44)/0.454 = 11.63" \text{ c/c}$$

$$S_{max} = \min(3h = 3 \times 21 = 63", 18") \Rightarrow \text{OK! , Hence, finally provide \#6 @ 10" c/c}$$



Example 6.1

□ Solution

➤ Step 8: Analysis and Design of Toe

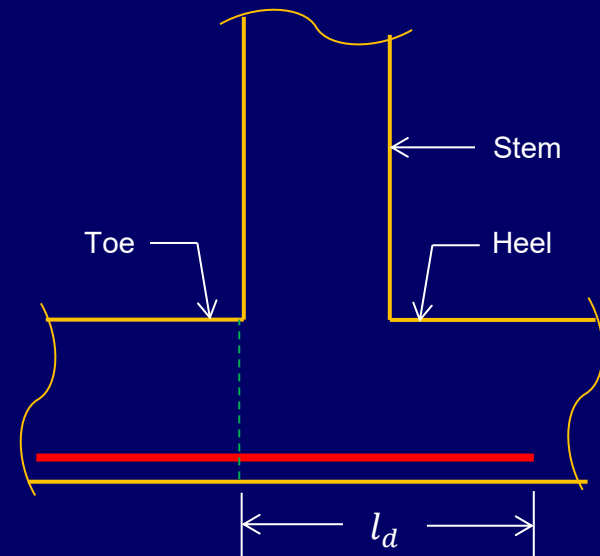
□ Calculation of development length

Development length of Toe reinforcement into heel as per ACI 318-19, section 25.4.2.4 is given by

$$l_d = \frac{f_y}{25\sqrt{f'_c}} d_b$$

$$l_d = \frac{60000}{25\sqrt{4500}} (0.75) = 26.8" \approx 27"$$

Therefore, $l_d = 2.25'$



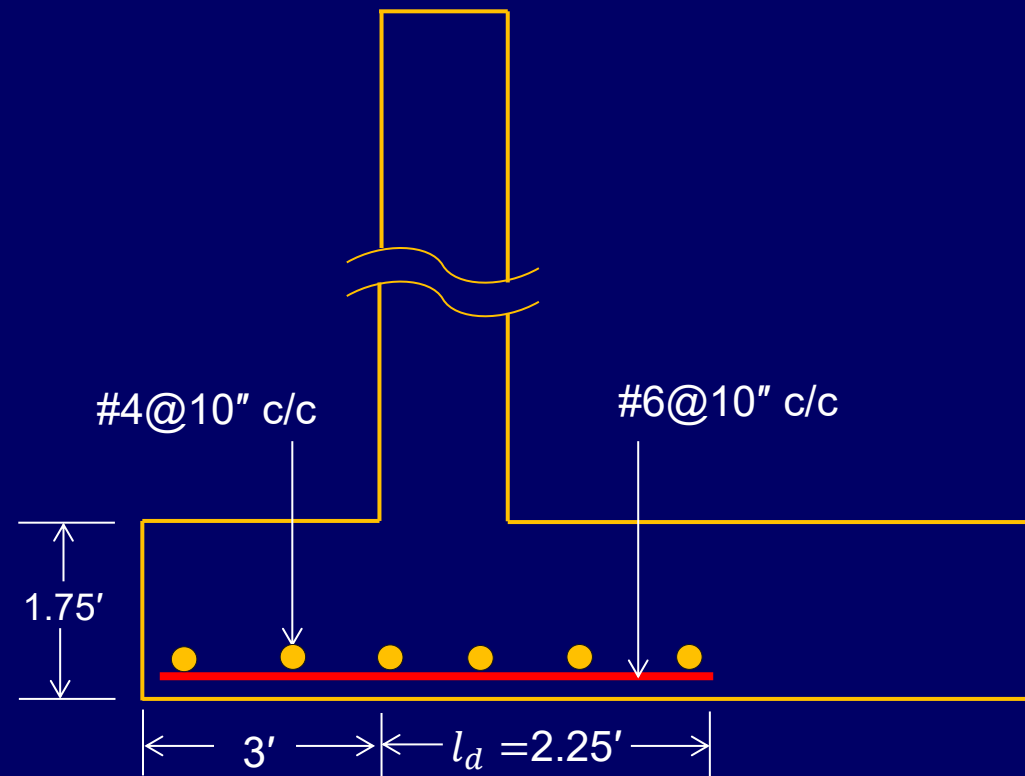


Example 6.1

□ Solution

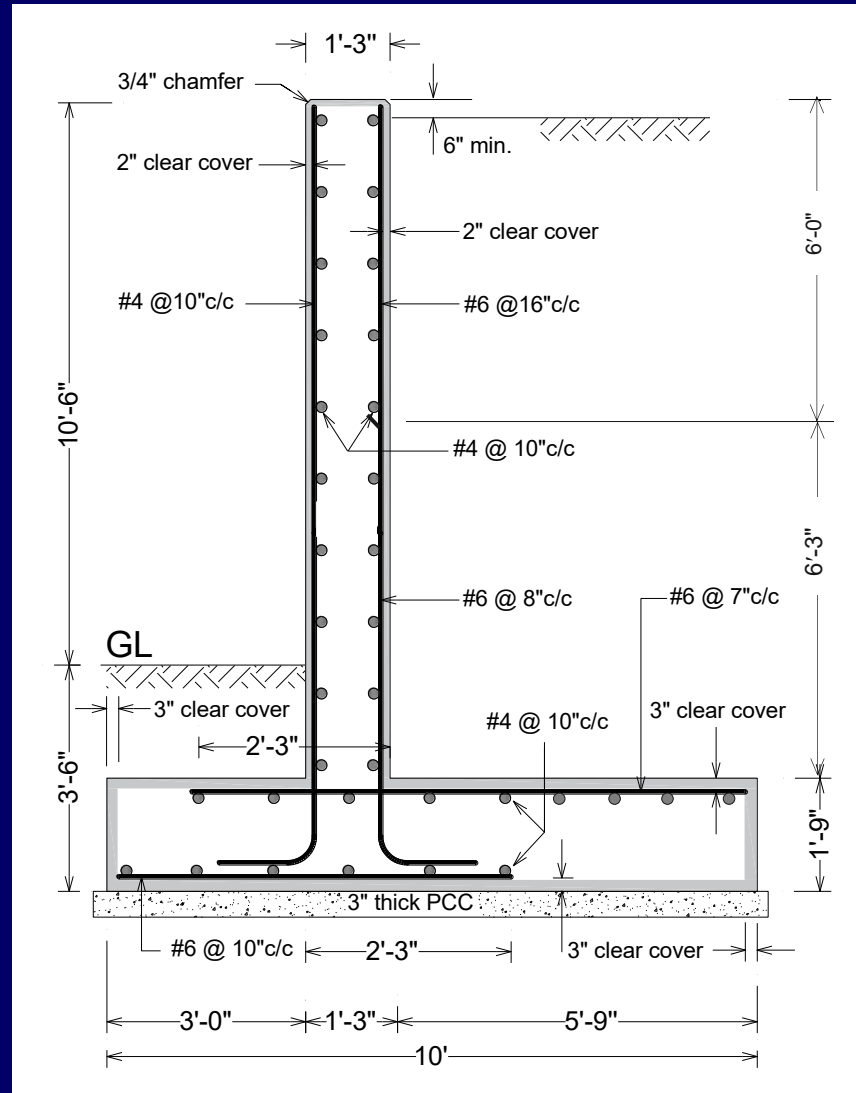
➤ Step 8: Analysis and Design of Toe

□ Drafting





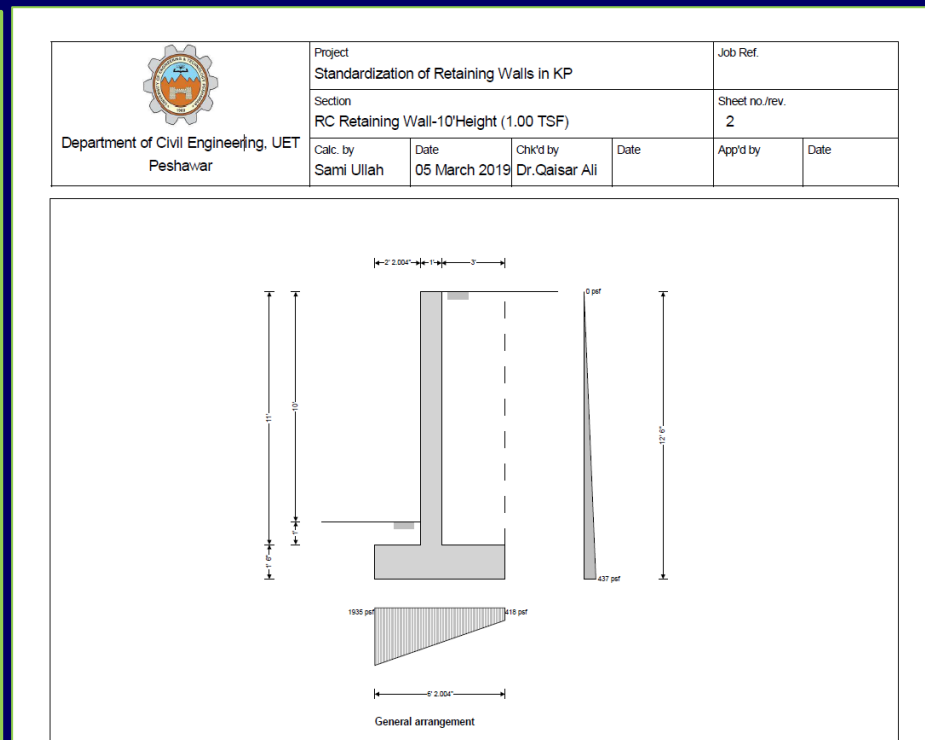
Example 6.1





Practical Examples

□ Tekla Tedds for Analysis and Design of RC Cantilever Retaining Wall



Various failures of Retaining wall at Nathia Gali



Practical Examples

❑ Retaining Wall Construction





Practical Examples

❑ Retaining Wall Construction





Practical Examples

❑ Retaining Wall Construction





Practical Examples

❑ Retaining Wall Construction





Practical Examples

❑ Retaining Wall Construction





Practical Examples

❑ Retaining Wall Construction





Practical Examples

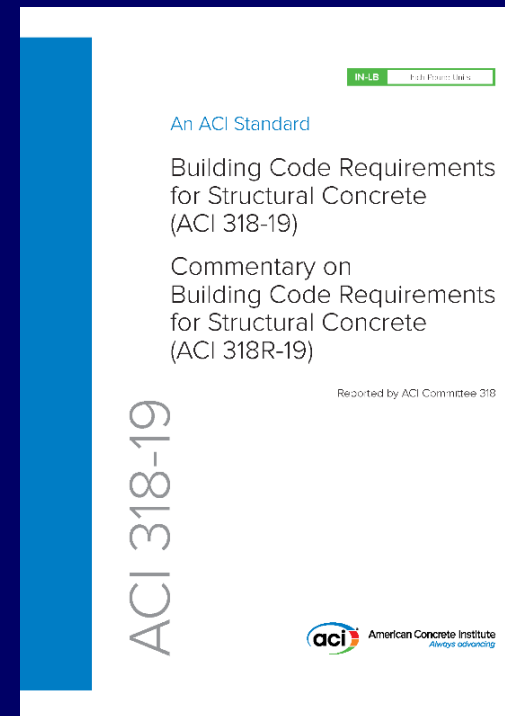
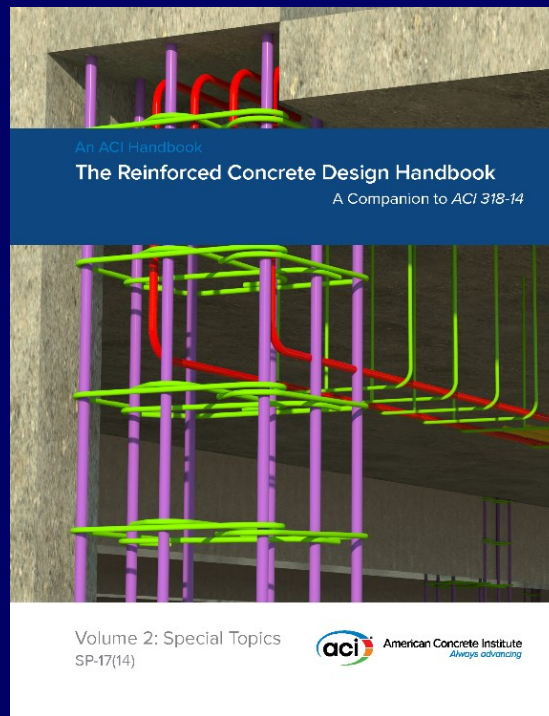
❑ Retaining Wall Construction





References

- ACI Reinforced Concrete Design Handbook (Volume: 2)_Special Topics, MNL - 17(21)
- Building Code Requirements for Structural Concrete (ACI 318-19)





Appendix

□ Derivation of Equations for Case 1

Considering the figure shown, let a be the distance from the front edge O to the intersection of the resultant R with the base. From axial plus bending formula, we have:

$$q_{\max}^{\min} = \frac{N}{A} \pm \frac{Mc}{I}$$

Where;

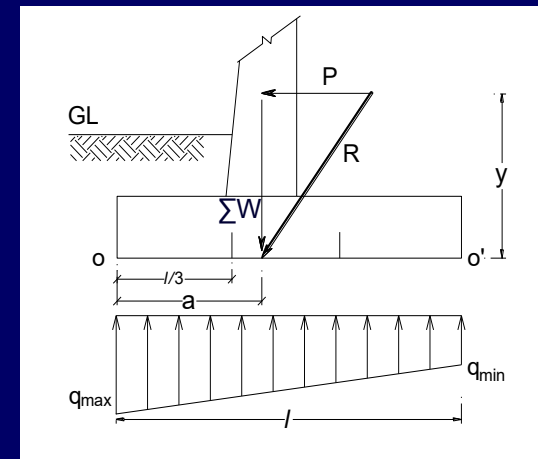
N = normal/axial force acting on the base

M = bending moment about centroid

c = distance of farthest edge from centroid

I = moment of inertia of base

Case 1: $a > l/3$





Appendix

□ Derivation of Equations for Case 1

For the current case,

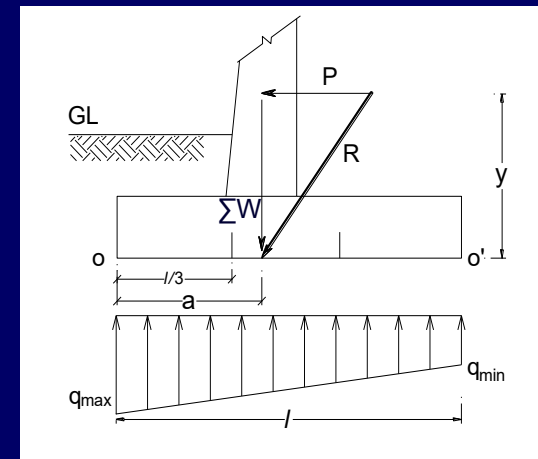
$$N = \Sigma W, \quad M = \Sigma W \left(\frac{l}{2} - a \right), \quad A = b \times l \quad c = \frac{l}{2} \quad \text{and} \quad I = \frac{bl^3}{12}$$

Considering unit width strip ($b = 1\text{ft}$) and
substituting values in the equation

$$q_{\max}^{\min} = \frac{N}{A} \pm \frac{Mc}{I} = \frac{\Sigma W}{1 \times l} \pm \frac{\Sigma W \left(\frac{l}{2} - a \right) \times l/2}{1 \times l^3/12}$$

$$q_{\max}^{\min} = \frac{\Sigma W}{l} \pm \frac{6\Sigma W \left(\frac{l}{2} - a \right)}{l^2}$$

Case 1: $a > l/3$





Appendix

□ Derivation of Equations for Case 1

$$q_{\max}^{\min} = \frac{\Sigma W}{l} \pm \frac{6\Sigma W \left(\frac{l}{2} - a \right)}{l^2}$$

$$q_{\max}^{\min} = \frac{\Sigma W}{l^2} [l \pm (3l - 6a)]$$

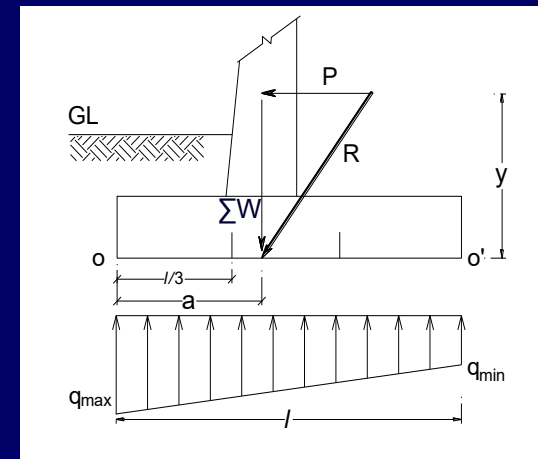
Now

$$q_{\max} = \frac{\Sigma W}{l^2} (l + 3l - 6a) = \frac{\Sigma W}{l^2} (4l - 6a)$$

and

$$q_{\min} = \frac{\Sigma W}{l^2} (l - 3l + 6a) = \frac{\Sigma W}{l^2} (6a - 2l)$$

Case 1: $a > l/3$



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